An Electricity Market Game between Consumers, Retailers and Network Operators

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December 17, 2002

Abstract

We consider a simple game-theoretical model in which an electricity retailer and a network owner offer incentives to consumers to shift load from a peak period to an off-peak period. Using a simple example we compare the market outcomes from collusion with those from the equilibrium of a non-cooperative game, and examine the behaviour in this game when it is repeated in a situation in which agents have imperfect information.

1 Introduction

Over the past couple of decades, most OECD countries have deregulated, or have started the process of deregulating, their electricity markets. Different countries have approached this deregulation in different ways (see [6]). Production of electricity is subject to competition in all deregulated markets, but there are differences as to how the wholesale markets and the end-user markets are organized.

The deregulation process in Norway started in 1990 and was initiated due to a desire to improve the efficiency and profitability of the electricity sector (see ([4]) and ([15])). The formal legislation was effective as of January 1st 1991, and from this date on, the wholesale market was fully liberalized. Also, end users were allowed to have their electricity delivered from any retailer, but the first years they were charged a fee for this, making it economically meaningful only to large end users. The end user market was not fully liberalized until 1997, when all consumers were allowed to change retailer at no cost. We refer to [12] for a discussion of the development and effects of the rules and regulations that liberalized the Norwegian end user market.

During the second half of the 1990's the markets in Sweden, Finland and Denmark were also deregulated and an integrated Nordic market emerged. These four countries now have a common wholesale market for electricity where any producer in any of the countries may deliver electricity to the entire region. The market is built up around the electricity exchange Nord Pool, which provides a common spot market and a transparent exchange place for electricity derivatives. The Nordic market has a large share of hydro plants[1], giving a mix of production technologies that provides great flexibility.

In the Nordic market, production, transmission and retail of electricity have been split into three independent business areas. Briefly explained, the roles of these in the market are as follows:

- The *producers* produce electricity to be sold into the wholesale market.
- The *retailers* purchase electricity in the wholesale market, either through bilateral agreements with the producers or through Nord Pool. The purchased electricity is then sold to the end users. Hence, the retailers function as intermediaries between the wholesale market and the end users. Because of the large number of retailers, and partly also because of the invariable focus on the electricity prices from authorities and the news media, the competition among the retailers is fierce and most retailers experience very small profit margins, if any at all.

[1]Norway has 99% hydro power, Sweden 48% and Finland 22%. Denmark has no hydro power.

- The *network operators* are responsible for transmitting the electricity from the production plants to the end users. The system operators are responsible for maintaining the main national grid that transmits electricity between regions. The lines that transport the electricity to the end users are the responsibility of local network operators. The local network operators have monopoly on the transmission of electricity in their designated area and they are obliged to maintain a network that is capable of carrying the power needed at any time to all customers in their area at the same per kWh price for all customers[2]. The network operators are financed through transmission fees paid by the end users, and to prevent them from enjoying monopoly profits, their profit margins are regulated by the Norwegian Water Resources and Energy Administration (NVE).

In the wholesale market, the prices of electricity vary from hour to hour. Usually the variations are more extreme in winter than in summer because in winter the water level in the hydro reservoirs may be low, while the cold weather puts transmission and production capacity under pressure in the peak periods. The intra-day price fluctuations suggest that production capacity and/or transmission capacity are scarce resources that are priced in the market.

The vast majority of end-users, however, do not have their consumption metered by the hour[3]. Instead, they have their consumption metered four times a year, and they are billed based on their accumulated consumption over the last three months. This means that the incentive structure in the end-user market is directed only towards the consumer's total energy consumption. Hence, to attract customers from their competitors, the retailers have focused on offering as low energy prices as possible. Since short term price fluctuations are not observed by the consumer (at least with current metering methods), the only way for her to reduce her electricity bill, besides changing to a cheaper retailer, is to reduce her total consumption of energy. If the consumers had metering instruments installed that made them exposed to the short-term fluctuations in the wholesale electricity prices, they would also have incentives to alter their daily consumption habits in order to save costs.

Since the retailers must purchase electricity at hourly varying prices in the wholesale market and sell it to the consumers at flat prices, this system of metering and billing end users introduces considerable risk to the retailers. This is because the consumers tend to use more electricity while the wholesale prices are high, but do not pay the market price for it. Therefore, customers with some consumption profiles would be valuable to a retailer, and if a retailer were able to meter his customers' load by the hour, then he could provide incentives for them to shift load from peak to off-peak periods and thereby make a profit from the ability to source cheaper power to his customers.

We have mentioned that the network owner's profit margins are regulated by the authorities. The regulatory regime was revised in 2001, and from 2002 the revenue caps of the local network operators are calculated as follows[4]

$$RC = (OM + D + NL + RIC)(1 - EI) + NI \tag{1}$$

where
OM is operating and maintenance costs. The calculation of this parameter is based on the utilities' financial statements for the years 1996 through 1999.
D is depreciation.
NL denotes the network losses. The value of this parameter depends on the market price of power at the time the losses occur.
RIC is return on invested capital, calculated as $RIC = IC_{99} r_{NVE}$, where IC_{99} is the invested capital at the end of 1999 and r_{NVE} is the interest rate set by NVE. $r_{NVE} = r_f + RP$, where r_f is the risk free rate of return and RP is a risk premium.
EI is a rate denoting the efficiency improvement requirement. EI is set on an individual basis for each network operator and depends i.a. on how the network operator's efficiency increases compared to other network operators. This term provides an element of competition among the network operators.
NI is an adjustment parameter for new investments added to cover the need for new investments in the network. The calculation of this parameter is based on the nationwide increase in energy consumption

[2]In fact the network operators are allowed to charge different prices to different customer segments such as households, vacation homes, small businesses and large businesses, but the segments are defined by the authorities.

[3]Some large consumers, but only very few small and medium sized consumers, do have their consumption metered by the hour and therefore have incentives to adjust their consumption according to the short-term price fluctuations in the market.

[4]Our representation of the calculation of the revenue cap is a little simplified, but sufficient for our purpose.

and the number of new customers in the network operator's region.
The revenue cap is constrained by a paragraph stating that the arithmetic average of the return over a five year period must be between 2% and 20%.

For a network operator, a new investment will generate income due to the book value of the investment and the appurtenant depreciations forming the basis for future revenue caps. However this income will come too late to fully compensate for the capital cost of the investment, and therefore the profitability of a new investment will always be negative if the adjustment parameter NI in Equation (1) were not introduced. The parameter NI is to provide the network operators with funds to undertake necessary and profitable grid expansions. It is, however, important to note that they will recieve those funds regardless of whether investments are undertaken or not. Hence, the network operators will have strong incentives to invest as little money as possible in grid expansions. While the parameter is determined based on nationwide and local increases in energy consumption, the needed grid capacity is determined by the maximum instantaneous load carried through the grid. The network operators are obliged by law to carry any realistic load needed to the consumers in their region. Having the consumers curtail their peak loads would therefore benefit the network operators. If they, for example, need to connect a new housing estate to their network, lower consumption peaks may enable them to connect the new housing estate to an existing network node instead of building a new and expensive one. Therefore, the network operators may want to incentivise consumers to move consumption from peak load periods to low-peak periods. A detailed review of the network owners' revenue cap may be found in ([17])[5] and ([18])[4] . A profound analysis of the adjustment parameter for new investments is given in ([7])[4] . ([3]) investigates possible peculiarities in the network operators' investment behavior under the regulatory regime that was valid from 1997 to 2001.

From this discussion we see that both the retailers and the network operators could have an interest in making consumers shift load from peak to off-peak periods. One way of doing this may be to install equipment that allows retailers and/or network operators to physically cut load when there is danger of an extreme peak. The consumers should then get some sort of compensation for allowing them to do this. Another approach is to provide the consumers with economic incentives to cut load in peak periods. This could for example be done by offering the customers hourly rates instead of flat rates, which would require that hourly metering instruments be installed. Customers would then react to the hourly differentiated rates by shifting load from peak to off-peak periods without the electricity utilities needing to physically cut off the supply. Especially for small and medium sized consumers we believe that providing economic incentives is a more practical and feasible solution than cutting supply. Hourly metering equipment is regarded as rather expensive compared to the potential economic savings such instruments could provide[6]. A cheaper approach is to install instruments that distinguish peak and off-peak consumption. Then, the consumers could be offered two-part tariffs, normally a higher price in the peak period than in the off-peak period. See e.g. ([1]), ([2]), ([5]), ([8]), ([10]), ([11]), ([13]), ([14]), ([16]) and ([19]) for studies on consumer responsiveness to two-part tariffs.

We have seen that two independent parties could be willing to offer the consumers some sort of incentive to make them shift load. The consumers, however, are only concerned about the total incentive payment that they receive, and they would not care about who provides the incentive, only how much they can save in total. Therefore, if the retailer offers an incentive to a consumer, then there is an opportunity for the network operator to free ride on this. This will affect the incentive to be offered by the network operator. The question is then how large an incentive each of them should offer to maximize their own profit.

In this paper we analyze model in which a network owner and a retailer together offer a consumer an incentive to shift load out of a peak period and into an off-peak period. For simplicity, we analyze a simple case in which the day is divided into two periods. In Section 2 we give a formal description of the market participants by presenting our assumptions on the players' profit functions and the consumer's cost function. In Section 3 we compare the solutions to be found by the network owner and the retailer when they collude to offer incentives. In Section 4 we describe a Nash equilibrium based on the assumptions in Section 2, and we discuss the dynamic behaviour of this game if it is played repeatedly with incomplete information. In Section 5 we discuss strategic behaviour from the consumer's viewpoint.

[5]The document is only available in Norwegian.

[6]Such equipment is, however, getting increasingly common, and the price of this may decrease as more instruments are produced.

2 Description of the market participants

2.1 Consumer

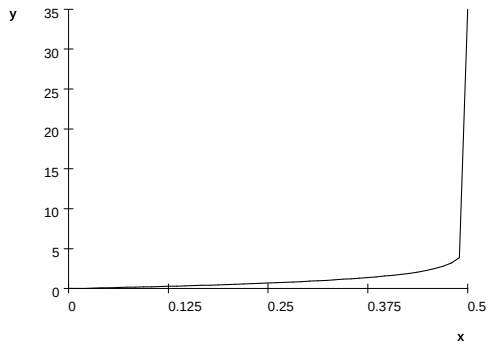
The first type of participant is the consumer, who is paid to shift load out of the peak period into the off peak period. Suppose the consumer is offered an amount px to shift x units of demand out of the peak period. In the simplest model, the consumer has a cost function $f(x)$, for shifting load. We assume that f is a twice differentiable strictly convex function with $f(0) = 0$.

Example 1:

As an example, consider the following cost function

$$f(x) = \begin{cases} -\beta \log \frac{\alpha - x}{\alpha} & , \quad x \geq 0 \\ 0 & , \quad \text{otherwise} \end{cases} \tag{2}$$

plotted below for $\alpha = 0.5$ and $\beta = 1$.



This cost function has some nice properties. First, $f'(x) > 0$ for positive x which makes sense since the consumer is likely to be more unhappy the more load that is shifted. Second, $f''(x) > 0$, indicating an increasing marginal cost. Third, the function has a vertical asymptote at $x = \alpha$. This asymptote makes sense because there will always be a limit to how much load that it is physically possible to shift.

Consider the problem faced by the consumer. To minimize the cost of shifting load, the consumer seeks to

$$\max_x \{px - f(x)\}.$$

Let the optimal solution to this problem be denoted $\bar{x}(p)$. Since $f''(x) > 0$, $\bar{x}(p)$ is the unique solution to

$$f'(x) = p. \tag{3}$$

This shows that the incentive $\bar{p}(x)$ required to induce an optimal shift in load of x is $f'(x)$. Since $\bar{p}'(x) = f''(x) > 0$, $\bar{p}(x)$ is strictly increasing in x.

At this point we shall make an additional assumption on the behaviour of the consumer. Although the price $\bar{p}(x)$ is increasing in x, there is a limit to how much load the consumer can shift, and it is clear that she will demand increasing incentives at the margin as this limit is approached. We assume therefore that $\bar{p}(x)$ is strictly convex. This amounts to the assumption that $f'''(x) > 0$.

Differentiating (3) at the optimal solution gives

$$\bar{x}'(p) f''(\bar{x}(p)) = 1, \tag{4}$$

which shows that $\bar{x}(p)$ (the inverse of $p(x)$) is also a strictly increasing function of p.

Example 1 (continued)

For the cost function

$$f(x) = -\beta \log \frac{\alpha - x}{\alpha}$$

we obtain

$$\begin{aligned} f'(x) &= \frac{\beta}{\alpha - x}, \\ f''(x) &= \frac{\beta}{(\alpha - x)^2}, \\ f'''(x) &= \frac{2\beta}{(\alpha - x)^3}, \end{aligned}$$

and

$$\bar{x}(p) = \frac{-\beta + p\alpha}{p}.$$

Throughout this paper (apart from Section 5) we assume that the consumer reacts to the incentive offered by the other parties. In this sense she is a follower in a Stackelberg-type game.

2.2 Retailer

The second type of participant in the market is the retailer. He offers an incentive to the consumer to shift load. Shifts in load produce benefits for the retailer that can be modelled as a function $A(x)$ of the amount of load shifted. In our model we shall assume that A is (or may be approximated by) a concave increasing function with $A(0) = 0$. Suppose the incentive payment per unit of shifted load is s. Then the retailer seeks to

$$\max_s \{A(\bar{x}(s)) - s\bar{x}(s)\}.$$

This gives a first-order optimality condition of

$$\bar{x}'(s)A'(\bar{x}(s)) - s\bar{x}'(s) - \bar{x}(s) = 0.$$

Substituting for $\bar{x}'(s)$ using (4) we obtain

$$f''(\bar{x}(s))\bar{x}(s) = A'(\bar{x}(s)) - s. \tag{5}$$

As s varies the optimal response $\bar{x}(s)$ varies. Since $\bar{x}(s)$ is strictly increasing in s one may consider the optimal choice of s satisfying (5) as an equivalent choice of x satisfying

$$f''(x)x = A'(x) - s(x). \tag{6}$$

Since $s(x)$ is increasing and $A(x)$ is concave, the right-hand side of this equation is a decreasing function. Since the derivative of the left-hand side is

$$(f''(x)x)' = xf'''(x) + f''(x) > 0,$$

it follows that $f''(x)x$ is strictly increasing from 0 and so any solution to (6) will give a unique s, and therefore a unique $\bar{x}(s)$.

For simplicity we shall assume from now on that $A(x) = Ax$, yielding

$$f''(\bar{x}(s))\bar{x}(s) = A - s. \tag{7}$$

Observe that since $(A - s)\bar{x}(s) = 0$ at $s = 0$ and $(A - s)\bar{x}(s) \leq 0$ for $s \geq A$, we need only consider $s \in [0, A]$ in seeking an optimal value of s.

2.3 Network owner

Now consider a network owner offering an incentive to the consumer to shift load. Shifts in load produce benefits for the network owner that are assumed to be a concave function $B(x)$ of the amount of load shifted. (Observe that in reality B is likely to be a discontinuous function with jumps at points at which

the network capacity needs to be expanded to meet load.) Suppose the incentive payment per unit of shifted load is t. Then the supplier seeks to

$$\max_t \{B(\bar{x}(t)) - t\bar{x}(t)\}.$$

The network owner has the same equations as the supplier, but the incentive offered by the network owner is t. This gives a first-order optimality condition of

$$\bar{x}'(t)B'(\bar{x}(t)) - t\bar{x}'(t) - \bar{x}(t) = 0.$$

Substituting for $\bar{x}'(t)$ we obtain

$$f''(\bar{x}(t))\bar{x}(t) = B'(\bar{x}(t)) - t.$$

For simplicity we shall assume from now on that $B(x) = Bx$, yielding

$$f''(\bar{x}(t))\bar{x}(t) = B - t. \tag{8}$$

Using the same argument of the previous section we may show that (8) has a unique solution t (with unique $\bar{x}(t)$).

3 Independence and Collusion

In this section we introduce the situation in which the retailer and network owner operate in isolation and compare this with the benefits to be obtained by colluding. We assume throughout that given the incentives offered by either the retailer or network owner, the consumer acts as to minimize cost.

The retailer seeks to induce the consumer to shift load from the peak period. The optimal amount to offer is s satisfying

$$f''(\bar{x}(s))\bar{x}(s) = A - s.$$

Example 1: (continued)
As before let the cost function of the consumer be

$$f(x) = -\beta \log \frac{\alpha - x}{\alpha}.$$

Then at the optimal solution $\bar{x}(s)$, the marginal cost of the consumer equals the payment so

$$f'(x) = -\frac{\beta}{-\alpha + x} = s$$

giving

$$x = \frac{-\beta + s\alpha}{s}.$$

However if β is large and α and s are small then x will become negative. The optimal choice is

$$\bar{x}(s) = \max\{\frac{-\beta + s\alpha}{s}, 0\}. \tag{9}$$

Now we solve

$$f''(\bar{x}(s))\bar{x}(s) = A - s,$$

using

$$f''(x) = \frac{\beta}{(-\alpha + x)^2}.$$

Therefore

$$\frac{\beta}{(-\alpha+\bar{x}(s))^2}\bar{x}(s) = A - s.$$

Substituting (9) gives (assuming $s \geq \frac{\beta}{\alpha}$)

$$\frac{\beta}{\left(-\alpha+(\frac{-\beta+s\alpha}{s})\right)^2}\frac{-\beta+s\alpha}{s} = -s + \frac{1}{\beta}s^2\alpha,$$

so

$$\frac{1}{\beta}s^2\alpha = A,$$

giving

$$\bar{s} = \sqrt{\frac{A\beta}{\alpha}}.$$

Observe that $A \geq \frac{\beta}{\alpha}$ if and only if $\bar{s} \geq \frac{\beta}{\alpha}$. If $A \leq \frac{\beta}{\alpha}$then any incentive $s \in [0, A]$ will satisfy $s \leq \frac{\beta}{\alpha}$, and so $\bar{x}(s) = 0$. Therefore we can assume without loss of generality that $A \geq \frac{\beta}{\alpha}$. Under this assumption $\bar{s} \in [0, A]$ and

$$\bar{x}(s) = \frac{-\beta+\bar{s}\alpha}{\bar{s}}. \tag{10}$$

To illustrate these formulae, consider an example where $A = 6$, $B = 7$, $\alpha = 0.5$ and $\beta = 1$. Acting independently, the retailer would offer $s = 3.464$, and the network owner would offer $t = 3.742$. This gives respectively

$$\bar{x}(s) = 0.2113$$

$$\bar{x}(t) = 0.2327$$

and respective profits for the retailer and network owner of

$$(6)(0.2113) - (3.464)(0.2113) = 0.5359$$

$$(7)(0.2327) - 3.741(0.2327) = 0.7583.$$

Consider now the situation where the supplier and the network owner collude. They seek a joint incentive payment p that solves

$$\max_p\{A\bar{x}(p) + B\bar{x}(p) - p\bar{x}(p)\}$$

Thus

$$p = \sqrt{\frac{(A+B)\beta}{\alpha}} = 5.0990,$$

using the choice $A = 6$, $B = 7$, $\alpha = 0.5$ and $\beta = 1$. This gives

$$\bar{x}(p) = 0.3039$$

and total profit equal to

$$A\bar{x}(p) + B\bar{x}(p) - p\bar{x}(p) = 2.401.$$

A choice of

$$s = 2.0495$$

$$t = 3.0495$$

can be shown to give equal profits of 1.200 for the retailer and the network owner. The customer incurs a cost of

$$-\beta \log \frac{\alpha - 0.3039}{\alpha} = 0.936.$$

Observe that these values of s and t are both lower than what the retailer and network owner would offer in isolation, and yield higher profits. However this situation does not represent an equilibrium, because the network operator could reduce their incentive payment to $t = 2.2048$ and make more than they are currently making. Assuming that s remains at 2.0495, we get

$$\bar{x}(s+t) = \frac{-\beta + (2.2048 + 2.0495)\alpha}{(2.2048 + 2.0495)} = 0.2649.$$

The network operator now makes a profit of

$$B(0.2649) - (2.2048)(0.2649) = 1.2704.$$

4 Nash equilibrium

The analysis of the previous section leads us to a Nash equilibrium in the one-shot game in which the retailer and the network operator offer independently to induce a response from the consumer. The retailer offers the consumer an amount sx to shift x units of demand out of this period, and the network offers the consumer an amount tx to shift x units of demand out of this period. (The consumer will make $(s+t)x$ from this transaction.) The supplier then seeks to

$$\max_s \{A(\bar{x}(s+t)) - s\bar{x}(s+t)\},$$

and the network seeks to

$$\max_t \{B(\bar{x}(s+t)) - t\bar{x}(s+t)\}.$$

The first-order optimality conditions for the supplier (given a fixed offer of t from the network) are:

$$A\bar{x}'(s+t) - s\bar{x}'(s+t) - \bar{x}(s+t) = 0,$$

yielding

$$f''(\bar{x}(s+t))\bar{x}(s+t) = A - s.$$

Similarly, the first-order optimality conditions for the network (given a fixed offer of s from the supplier) are:

$$B\bar{x}'(s+t) - t\bar{x}'(s+t) - \bar{x}(s+t) = 0,$$

yielding

$$f''(\bar{x}(s+t))\bar{x}(s+t) = B - t.$$

Example 1 (continued):

We continue to use the consumer's cost function (2). This gives

$$-(s+t) + \frac{1}{\beta}(s+t)^2\alpha = A - s,$$

$$\frac{1}{\beta}(s+t)^2\alpha = A+t,$$

$$s = \sqrt{\frac{(A+t)\beta}{\alpha}} - t,$$

Similarly

$$t = \sqrt{\frac{(B+s)\beta}{\alpha}} - s$$

Observe that since $s \in [0, A]$ and $t \in [0, B]$, the optimal solutions will in fact solve

$$\begin{aligned} s &= u(t) \\ t &= v(s) \end{aligned}$$

where

$$u(t) = \begin{cases} 0 & \text{if} \quad \sqrt{\frac{(A+t)\beta}{\alpha}} < t \\ \sqrt{\frac{(A+t)\beta}{\alpha}} - t & \text{if} \quad t \leq \sqrt{\frac{(A+t)\beta}{\alpha}} \leq A+t \\ A & \text{if} \quad \sqrt{\frac{(A+t)\beta}{\alpha}} > A+t \end{cases} \tag{11}$$

$$v(s) = \begin{cases} 0 & \text{if} \quad \sqrt{\frac{(B+s)\beta}{\alpha}} < s \\ \sqrt{\frac{(B+s)\beta}{\alpha}} - s & \text{if} \quad s \leq \sqrt{\frac{(B+s)\beta}{\alpha}} \leq B+s \\ B & \text{if} \quad \sqrt{\frac{(B+s)\beta}{\alpha}} > B+s \end{cases} \tag{12}$$

Let $X = [0, A] \times [0, B]$. If we consider the mapping $F : X \to X$ defined by

$$F((s,t)) = (u(t), v(s))$$

then it is clear that F is continuous and X is convex and compact. Therefore there always exists an equilibrium by Brouwer's fixed point theorem.

Suppose $A = 6$, $B = 7$, $\alpha = 0.5$ and $\beta = 1$. The players' response functions are shown in Figure 1.

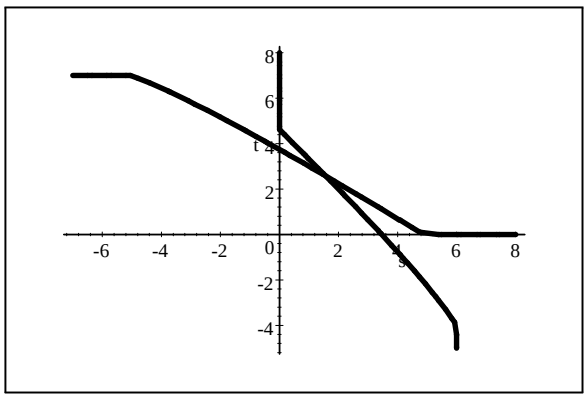


Figure 1: The players' response functions for $A = 6$, $B = 7$

An equilibrium is given by

$$s_{eq} = 1.57, \quad t_{eq} = 2.57$$

Now $s_{eq} + t_{eq} = 4.14$, so

$$\bar{x}(s_{eq} + t_{eq}) = \frac{-\beta + 4.14\alpha}{4.14} = 0.2585$$

The supplier makes a profit of

$$A\bar{x}(s_{eq} + t_{eq}) - s_{eq}\bar{x}(s_{eq} + t_{eq}t) = (6)(0.2585) - (1.57)0.2585 = 1.1449.$$

The network makes a profit of

$$B\bar{x}(s_{eq} + t_{eq}) - t_{eq}\bar{x}(s_{eq} + t_{eq}) = (7)(0.2585) - (2.57)0.2585 = 1.1449.$$

The customer incurs a cost of

$$-\beta \log \frac{\alpha - 0.2585}{\alpha} = 0.7277,$$

which is lower than they incur under collusion. The profits for the retailer and network owner are less than the profits (i.e. 1.2) to be made in collusion. A colluding retailer and network owner therefore find themselves in a Prisoner's Dilemma situation, in which it is profitable to renege from the collusion as long as the other player does not. The equilibrium strategy (when both players renege) has a poorer payout for both players.

Figure 1 illustrates some appealing features of this model that are true for all choices of parameters. First observe that $u(t)$ and $v(s)$ defined by (11) and (12) are both nonincreasing functions. This makes sense, as the optimal incentive to offer should not increase as the other agent offers more incentive. To show this, first observe that $u(t)$ is continuous, and is not constant only where $t \leq \sqrt{\frac{(A+t)\beta}{\alpha}} \leq A + t$. In this range

$$\sqrt{\frac{(A+t)\beta}{\alpha}} \leq A + t \Rightarrow \sqrt{\frac{\beta}{\alpha(A+t)}} \leq 1$$

so

$$u'(t) = \sqrt{\frac{\beta}{4\alpha\,(A+t)}} - 1 < 0. \tag{13}$$

A similar argument shows that

$$v'(s) = \sqrt{\frac{\beta}{4\alpha\,(B+s)}} - 1 < 0, \tag{14}$$

if $s \leq \sqrt{\frac{(B+s)\beta}{\alpha}} \leq B + s$.

A second observation is that the players' response functions can intersect in at most one point. To see this consider the two curves plotted in Figure 1. The inverse of $u(t)$ (as plotted in Figure 1 as a function of s) has two vertical sections and a downward sloping section. The slopes of these are all strictly more negative than the slope of $v(s)$ at the same s. This is because (11) and (12) imply

$$-1 < u'(t) < 0, \quad \text{and} \quad -1 < v'(s) < 0,$$

so

$$\left[\frac{du^{-1}}{ds}\right]_{s=u(t)} = \frac{1}{u'(t)} < v'(s).$$

It follows that the curves may intersect at most once.

The arguments above have shown the following proposition. For all strictly positive choices of A, B, α, and β there exists a unique Nash equilibrium in the noncooperative game played between the retailer and the network owner.

As a second example of an equilibrium for the model we discuss, suppose $A = 30$, $B = 6$, $\alpha = 8$ and $\beta = 1$. The players' response functions are shown in Figure 2, where the equilibrium is

$$s_{eq} = 2, \quad t_{eq} = 0.$$

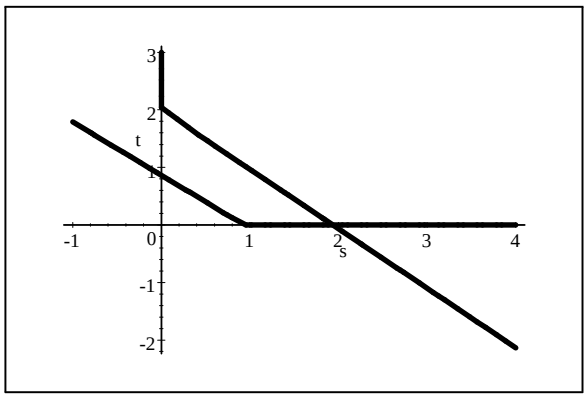


Figure 2: The players' response functions for $A = 6$, $B = 30$, $\alpha = 8$ and $\beta = 1$.

4.1 Repeated game

In this section we examine the case when the retailer and the network operator do not know anything about each other, but have perfect information about their own profit functions.

We have analyzed a situation where the players draw non-simultaneously and at each draw the drawing player chooses the incentive that maximizes his own profit with respect to the incentive already offered by the opponent. We have assumed that both players will draw and they will continue drawing as long as it is possible to make a decision that increases profit. By assuming this we implicitly make two important assumptions. The first is that as the game starts none of the players know anything about the opponent's response function. The player that starts the game just notices that the consumer has not been offered any incentive by someone else and will therefore simply offer the incentive that maximizes his profit. The second is that as the game progresses, the players do not learn anything about the opponent's response function. At each draw, they consider p, that is the total incentive offered to the consumer, and figure out how they can change this in order to maximize profit. They do not really understand that the opponent will respond by changing his incentive again.

In the table below we have presented some results from this process. We have assumed that the network owner draws first and that he also gains a slightly higher turnover per unit of load shifted. In this example $A = 6$, $B = 7$, $\alpha = 0.5$ and $\beta = 1$.

Period	*s*	*t*	*Retailer's profit*	*Network's profit*
1	0,000	3,742	1,396	0,758
2	0,672	3,742	1,457	0,891
3	0,672	3,245	1,304	0,919
4	1,055	3,245	1,322	1,004
5	1,055	2,959	1,240	1,014
6	1,274	2,959	1,246	1,066
7	1,274	2,794	1,201	1,069
8	1,400	2,794	1,203	1,100
9	1,400	2,699	1,178	1,101
10	1,472	2,699	1,178	1,119
,				
29	1,569	2,571	1,145	1,145
30	1,570	2,571	1,145	1,145

In period 1 the network owner draws and makes his incentive decision, t_1, based on $s_1 = 0$. This gives the network owner a profit of 0.758, but observe that the retailer earns much more. This is so, because the retailer gets the advantage of the load shifting, but does not pay anything for it. Now the game has started and, in period 2 the retailer draws. The reason why the retailer draws is that he realizes that he can make an even better profit by making a contribution to the incentive payment to the consumer. Therefore the retailer makes an incentive decision based on $t_2 = t_1 = 3.742$, giving $s_2 = 0.672$. Now, both players are actually better off than in period 1, but the network owner realizes that, given s_2, he would be even better off by offering $t_3 = 3.245$. This, however, makes the retailer even worse off than

he has been in any earlier stage of the game. Therefore he makes a draw, s_4, to catch up some of the lost profit. The game goes on and on like this (theoretically forever) until none of the players can make a profit by changing his incentive offer.

We can make some interesting observations from the table. The first one is that the network owner's profit is monotonically increasing towards equilibrium in each period, regardless off who makes the draw. If the network owner knew how the game would progress, he would have known that he would achieve a monotonic increasing profit all the way up to equilibrium. Therefore it could be optimal for the network owner to draw $t_1^* = t_{eq} = 2.570$, that is drawing his equilibrium incentive payment in the first period. However, this would mean that the network owner would be offering this amount alone for some period of time, until the retailer responds. There could be different reasons why it will take some time for the retailer to respond. First, it may take some time before he actually realizes that he benefits from the customer's load shifting. Second, there could be regulatory constraints preventing him from making a move. When offering t_1^*, the network owner will be worse off in period 1 than when offering t_1. Therefore, the time he will have to wait for the retailer to make his response would be crucial when deciding whether to offer t_1 or offering the equilibrium solution directly.

The second observation to make is that the retailer's (the follower's) profit is decreasing towards equilibrium. His profit is, however, not monotonic decreasing. Each time the retailer draws, he will earn more in the following period, but when the network owner replies, the retailer will be worse off than he ever was. This means that the retailer, if he had known how the game would progress, would not draw at all. In this case, the value of full information for the retailer is 0.251 i.e. the profit in period 1 minus the end-game profit (discounting excluded).

In the repeated game, we see that the retailer, if he knew how the game would progress, should not make his first move at all, but let the network owner provide the incentive alone. This way the retailer could just wait until the network owner realizes that he would be better off if they both offered an incentive and reduces his incentive down to the equilibrium level. If the network owner never realizes this, the game would be stuck in period one.

If the retailer, however, did not know how the game would progress, he would be tempted to make a move right away since he will earn more in the next period if he does so. Then, if none of them knew the other player's profit function, the game would progress as in the table, meaning that the players would learn the equilibrium.

5 Strategic consumer behaviour

In the discussion above we have assumed that the consumers' cost function $f(x)$ is known to the retailer and the network owner. Here we consider the situation in which this function is deduced by the retailer and network owner from observing behaviour of the consumer to the incentives offered by each. This raises the possibility of the consumer misrepresenting her true costs to these parties. For simplicity, we assume in this section that the retailer is the only agent offering incentives.

To model this suppose the consumer is represented by a single agent whose true cost function is

$$f(x) = -\beta \log \frac{\alpha - x}{\alpha},$$

where $\alpha = \beta = 0.5$. Suppose given an incentive s from the retailer, she were to shift load according to

$$g(x) = -\beta \log \frac{\alpha - x}{\alpha}.$$

where $\alpha = 0.5$ and $\beta = 1$. In other words she behaves as if her costs were twice as large.

Now we suppose that the retailer has experimented with different incentives to the extent that they have an accurate estimate of g, which they intend to use to compute an optimal incentive. As shown above, the optimal incentive given a response of g is

$$s = \sqrt{\frac{A\beta}{\alpha}} = 3.464,$$

assuming $A = 6$. The response of the consumer (still deceiving the retailer) is to shift a load of

$$x = \frac{-\beta + s\alpha}{s} = 0.2113.$$

Her true profit from this is

$$sx - f(x) = 0.457.$$

We compare this with the case where the consumer behaves according to her true cost function $f(x)$. Then $\alpha = \beta = 0.5$ gives

$$s = \sqrt{\frac{A\beta}{\alpha}} = 2.449$$

$$x = \frac{-\beta + s\alpha}{s} = 0.2959.$$

Her profit from this policy is

$$sx - f(x) = 0.277,$$

which is less than the profit she would make by misrepresenting her costs.

The conclusion here is that it is more profitable for the consumer to behave as though her costs were twice as high as they really were, since this induces the retailer to offer more inducement for her to shift load. So even if she does not optimize her profit (to maintain the deception) she is better off than if she were to optimize and reveal (over time) her true costs to the retailer.

As observed above, the retailer under this model will not offer the consumer any more than $s = A$, since this is the benefit he gets from a unit of load shifted. Thus there are bounds on how much profit the consumer can expect to extract from the retailer by inflating her true costs. For example suppose she were to choose to misrepresent her costs as

$$h(x) = -\beta \log \frac{\alpha - x}{\alpha}.$$

where $\alpha = 0.5$ and $\beta = 3$. Then the optimal offer from the retailer is

$$s = 6,$$

and the response of the consumer to this offer (while pretending to incur h) is to shift load of $x = 0$. The profit from this policy is clearly 0.

It is clear from the above discussion that for this model there is an optimal choice of β and α that will yield the best outcome for the consumer if they pretend to have cost function using that α and β. A more difficult question, that we leave unresolved, is the determination of an inflated cost function that produces the best response from the retailer.

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