Electricity Distribution Network Expansion Planning

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Abstract

In this paper we focus on planning the expansion of electricity distribution networks. The objective of the distribution network expansion planning problem is to determine an investment schedule to ensure an economic and reliable energy supply. This is done by constructing a minimum cost radial distribution network under the constraints of network line load capacities, voltage drops, reliability, and load demands. The complexity comes from the combinatorial nature imposed by the radial network constraint and: various options for transformer and substation location; several alternatives for cable or line sizes and routes; multistage investment decisions; complex objectives; and uncertainty about demand variation and location, equipment availability, and faults.

The aim is to solve a multistage stochastic programming problem, however, solving even a deterministic mixed integer programming problem is very computationally expensive. To overcome this, we use Lagrangian relaxation to decompose the original problem into smaller, easy-to-solve dynamic programming subproblems for each arc. This approach looks promising for efficiently solving multistage stochastic programming problems, for which a stochastic subproblem would be solved using stochastic dynamic programming.

1 Introduction

Electrical power is the most useful form of energy. Every residential, industry and commercial customer in our country is served by the electric power system. This system renders power to provide our everyday lighting, heating, and electrical equipment needs. Our everyday electrical necessities are met so well that we take this high quality service for granted.

The role of an electric power system can simply be described as the generation and supply of electrical energy in an efficient, economic and reliable manner to meet customer demands. The electric power system is made up of several systems such as the generation, the transmission, and the distribution system. Power is generated at a location where it gives the most overall economical selling cost. The transmission system is used to transport large amounts of energy from the point of generation to the distribution system. The distribution system then distributes the energy to all the customers in the area. In general, individual organizations own and operate only one of the components of the electric power system. These three abovementioned systems together consist of millions of components and cost billions of dollars to construct, operate and maintain. A diagram of an electric power system is shown in figure 1.

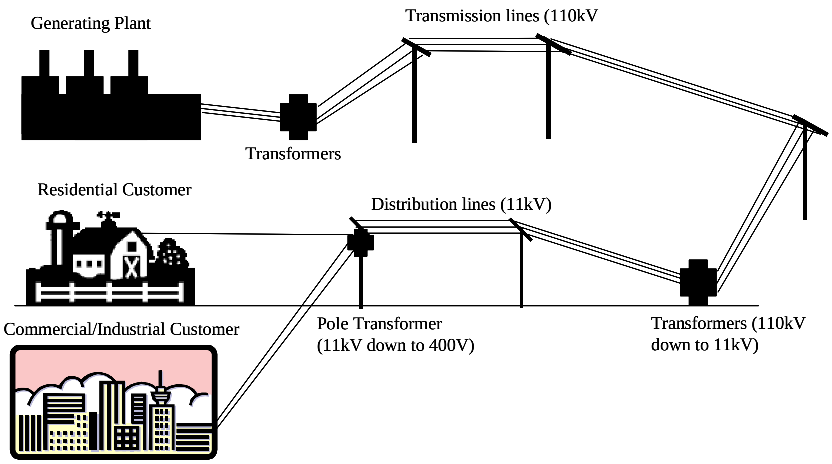

Figure 1: The electric power system.

1.1 The Electricity Distribution Network

Commonly, a distribution network is operated in a radial configuration (Lakervi and Holmes 1995). Radial networks have a treelike structure. This means that each load point is connected to the point of supply (substation) through a single path. The radial network is the simplest relative to a wide range of network operating configurations. Furthermore, radial networks are the least expensive to maintain and operate. Figure 2 shows a schematic diagram of a radial distribution network.

The electricity distribution network is an important part of the highly complex electric power system. The capital investment in the distribution system constitutes a significant portion of the total amount spent in the entire power system. Thus effective design and efficient operation of distribution networks have become very important.

The New Zealand electricity market was deregulated in 1996. The deregulation aims to lower electricity prices and shift the focus of distribution companies to consumers. Thus this provides a strong incentive for distribution companies to lower their total cost since their profit is the difference between the price limit set by the network authority and the actual costs incurred in distributing electricity to consumers. This can be achieved by improving efficiency, maximizing asset

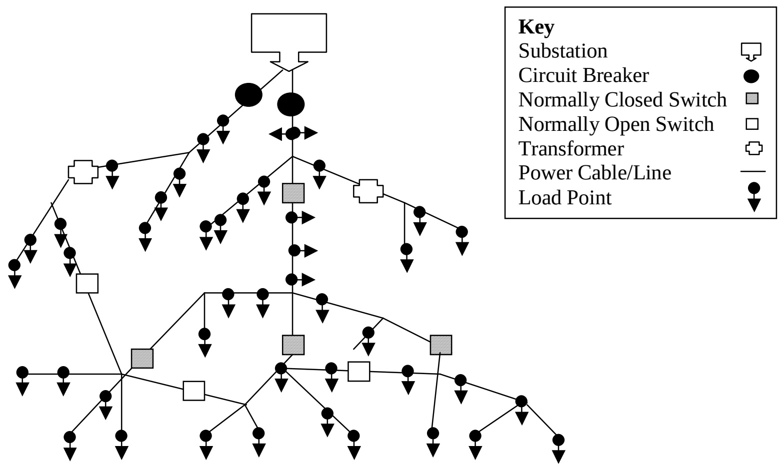

Figure 2: A radial distribution network.

utilization and operational economics. In addition to these tasks the distribution company must provide for increasing demand in electric energy. Typically this is done in the form of network expansion projects (this may include reinforcing current assets). These projects usually require capital investments in the order of millions of dollars; consequently it is of utmost importance to the distribution company to get the best return on investment.

1.2 Demand Growth

Normally, even small distribution networks may have several hundred nodes, of which most correspond to load points and the others correspond to switching points (Ferreira et al 1999). Furthermore there may be hundreds of arcs corresponding to cables/lines and switching busbars resulting in complex systems. Maintaining and operating such a large distribution network within an accepted service level is a difficult task. Moreover, with a growing population, the demand for electricity is also increasing. The increasing demand and the future demand levels will not only influence the size of each network equipment, but also the current network configuration. Therefore an economic design of the distribution network should be such that no network reinforcement or expansion is needed for a certain number of years.

In cases where the load in a particular feeder or line is nearing capacity, a distribution company first tries to reconfigure their network to accommodate such demand increase. To clarify this, consider the example in figure 3 where a simple radial network is given with 2 switches and 3 lines. Figure 3a shows the original configuration of the radial network with switch 1 closed, switch 2 open, 2 load points on line 1, 1 load point on line 2 and no load on line 3 i.e. a line currently not in use, represented by the dashed line. Suppose that each line has a capacity of 3 units and each load point occupies a single unit. Thus observe that line 1 supplies its own 2 loads and also the load on line 2. Consequently it is running at its capacity limit of 3 units. In figure 3b there has been a unit increase in load on line 2. In the current configuration this increases the load requirement on line

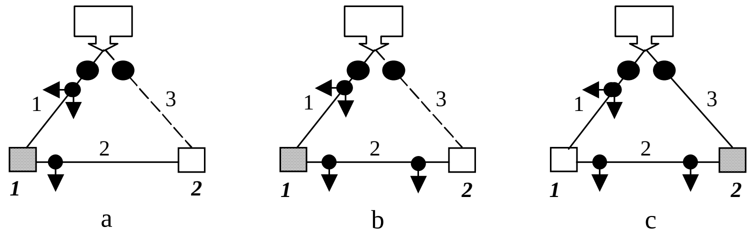

Figure 3: a) The original configuration. b) Unit increase in demand on line 2. c) Open switch 1, close switch 2 to reconfigure network to accommodate increased demand on line 2.

2 to 4 units. This is not feasible and therefore distribution company reconfigures the network by opening switch 1 and closing switch 2, as shown in figure 3c. This brings line 3 into service and frees up 1 unit of capacity in line 1 and 1 unit capacity in lines 2 and 3 combined.

The above example illustrates the idea that for given radial network configuration, all the arcs (line or cable) that are not part of the tree can be used to change the topology of the network to improve its performance or in the above case make it feasible again. Thus in the reliability context a reliable cable can be brought into service while taking an unreliable cable out of service somewhere else in the network. This may result in slightly altered but a more reliable radial network configuration. However, if in case of load growth there are no possible network configurations that satisfy the network operational constraints, then a distribution company must consider investing capital into reinforcement of existing equipment or installing of new equipment.

To cope with growing demand, reconfiguring the distribution network is an extremely low-cost solution compared with investing in reinforcement or network expansions. However, as mentioned, this is not always an option. Clearly, it is very uneconomical for a distribution company to carry out reinforcements or expansions on a yearly basis as the demand growth is realized. From this follows the question, what is the least cost expansion plan which will ensure that all distribution network operational requirements are satisfied and every demand is met at all times for a given planning horizon? This forms the basis for the distribution network expansion planning problem.

1.3 The Distribution Network Expansion Planning Problem

The objective of the distribution network expansion planning problem is to determine an investment schedule to ensure an economic, efficient and reliable energy supply. This is done by constructing a minimum cost distribution network under: load balance constraints that ensure that supply meets all demand; load flow constraints to ensure that capacity is not exceeded; voltage restrictions to ensure quality of supply; constraints for the acceptable level of network reliability related to frequency and duration of outages; and the network radial configuration requirement. The set of radial configurations is an unknown discrete set as there are definite states for switches that control the configuration. The switches are either on or off, there is no in between. As mentioned above, even small distribution networks can have hundreds of arcs and thus millions of unknown possible radial configurations. Hence the radial configuration requirement imposes a very difficult constraint on the optimization process.

This optimization problem may have several objectives (multi-criteria), commonly that of minimizing: 1) operational costs such as the day-to-day network monitoring and maintenance; 2) expansion costs to meet growing demand; 3) heat energy losses that occur due to conductor impedance; and 4) maximizing network reliability to reduce supply outages hence increase customer satisfaction. The multi-criteria approach adds to the problem dimensionality as there may be several variables and constraints associated with each piece of equipment.

2 Mixed Integer Programming Formulation

First we formulate a deterministic mixed integer linear program for the distribution network expansion planning problem based on the aforementioned constraints. In a deterministic model the future demand is known with certainty. For simplicity we do not consider any reliability or voltage aspects. Nonetheless we intend to evaluate reliability and voltage quality of the optimal solution after the optimization process using a specialized power systems software, in which a solution will be considered feasible if it satisfies the reliability service and voltage levels. Recall that an arc represents a cable or line, and a node represents either a demand load point or supply point. It is assumed that each demand point has a switch to disconnect it from any arc incident upon it.

Parameters

$A_{i,k}$	$=$	Arc incidence matrix.
B_l	$=$	Size of block expansion in capacity gained from using technology l (A).
$C_{k,l,t}$	$=$	Cost per block of expansion in arc k using technology l in year t.
D_{it}	$=$	Demand at node i in year t (A).
I_k	$=$	Initial capacity of arc k, i.e., capacity at time, $t = 0$.
K	$=$	Total number of arcs in the electricity network.
L	$=$	Total number of expansion technologies available.
M_k	$=$	Maximum capacity (A) of arc k (including expansions over years $1, ..., t$).
N	$=$	Total number of nodes in the electricity network.
T	$=$	Final time period of the planning horizon.

Decision variables

$x_{k,t}$	$=$	Current flow in arc k in year t (A).
$y_{k,l,t}$	$=$	1 if technology l is chosen for expansion on arc k at time t, 0 otherwise.
$z_{k,t}$	$=$	1 if arc k is in radial network at time t, 0 otherwise.

Objective function

$$\min \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{t=1}^{T} C_{klt} y_{klt}$$

subject to:

Total capacity of arc k at time t must be equal to initial capacity plus the sum of possible block expansions over the years $1, ..., t$:

$$\begin{array}{ll} x_{kt} \leq I_k + \sum_{s=1}^{t} \sum_{l=1}^{L} B_l y_{kls}, & k = 1, ..., K, t = 1, ..., T. \\ x_{kt} \geq -I_k - \sum_{s=1}^{t} \sum_{l=1}^{L} B_l y_{kls}, & k = 1, ..., K, t = 1, ..., T. \end{array} \tag{1}$$

There is flow in arc k of the network at time t if and only if it is part of the network:

$$\begin{array}{ll} x_{kt} \leq z_{kt} M_k, & k = 1, ..., K, t = 1, ..., T. \\ x_{kt} \geq -z_{kt} M_k, & k = 1, ..., K, t = 1, ..., T. \end{array} \tag{2}$$

Power supply must equal demand, i.e., power balance constraints:

$$\sum_{k=1}^{K} A_{ik} x_{kt} = D_{it}, \qquad i = 1, ..., N, t = 1, ..., T. \tag{3}$$

Distribution network must be in a radial configuration:

$$\sum_{k=1}^{K} z_{kt} = N - 1, \qquad t = 1, ..., T. \tag{4}$$

$$x_{kt} \in \Re, \quad y_{klt}, \; z_{kt} \in \{0, 1\}.$$

Note that the parameter M_k in constraint (2) tends to introduce fractionality in the problem. This effect can be decreased by making M_k as small as possible without affecting the actual optimal solutions. A possibility is to assign it a value that is some multiple of the initial capacity I_k. Furthermore, one can introduce a parameter α to scale the values of M_k in proportion to the demand for a particular year t. For example, if demand is low than the α could be 1.5 or 2, and if demand is high then to allow possibly larger flows, one could increase α to 3 or 4.

This integer program has been implemented and gives optimal solutions to relatively small problems. For example, consider a problem with 6 nodes, 9 arcs, and a 4 year planning horizon for which the future demand is known. There are two technologies available for expansion of each arc k, i.e., $L = 2$. One allows for a block expansion of 5 units and the other of 10. Table 1 shows the costs of expansion based on the block expansion type B_l used and year t of expansion.

Table 1:

		t			
		1	2	3	4
B_l	5	100	105	110	118
	10	150	158	165	174

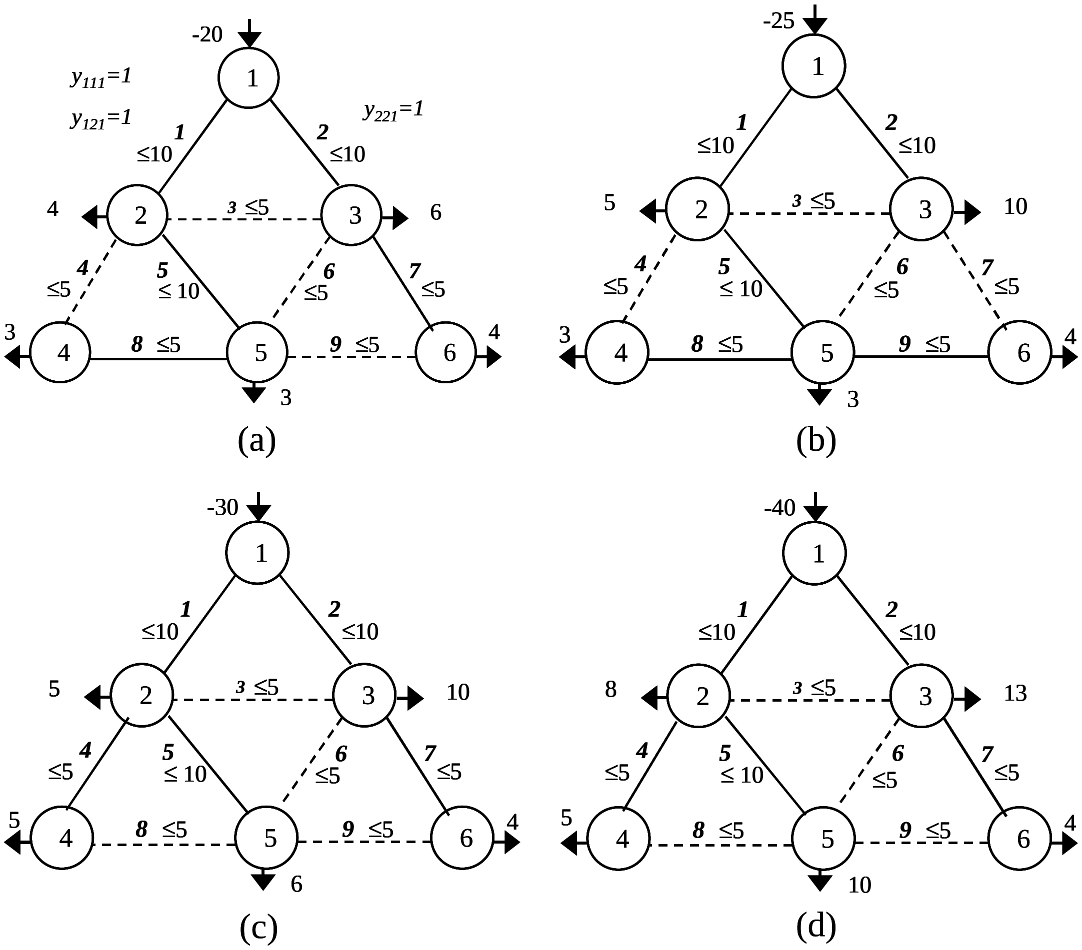

Figure 4: Illustrates the solution of the example problem with 6 nodes, 9 arcs, and a 4 year planning horizon.

The minimum cost expansion solution is illustrated in figure 4 part a, b, c and d, representing year 1, 2, 3 and 4, respectively. Here, a solid line represents a power line in service and a dashed line represents a redundant power line. The number next to the '$\leq$' sign indicates the initial capacity of the corresponding line. The load points and the load amounts are represented by the arrows and the adjacent numbers, respectively. Note that the negative load indicates a point of supply. The solution to this problem ($y_{111} = 1, y_{121} = 1, y_{221} = 1$) indicates that all the expansions should be done in year 1, i.e., arc 1 is expanded using both block expansion technologies and arc 2 is expanded using a 10 unit block. For this example, the reason for expanding in year 1 is apparent. That is, the deterministic information about the future demand is exploited by expanding in the year of least cost, that given by expansions in year 1. Observe that the radial network configurations change in year 2 and year 3, relative to year 1 and 2, respectively. By close examination, the change in configuration in year 2 is not necessary as having arc 7 in service and arc 9 out of service will not violate the network capacity constraints. However this is advantageous in the operational sense as it indicates the flexibility in network configurations. Thus a distribution company could reconfigure their network during maintenance or repairs without having to compromise service to customers. The change in configuration in year 3 however is necessary since the flow in arc 10 in the configuration of year 2 is at maximum capacity. Hence if it were to stay in the same configuration as year 2, it would not be able to supply the increased demand in nodes 4 and 5.

As mentioned above, problems can be solved to optimality. However the computational effort is large relative to the size of the problem. For instance, the small problem for the above example took approximately 5 seconds of CPU time to solve using CPLEX6.6. A problem with 10 nodes and 18 arcs for a 4 year planning horizon takes longer than 30 minutes to solve. The time to solve increases exponentially relative to the size of the problem, hence the complex nature (NP-hard) of the electricity distribution network expansion planning problems.

We intend to solve multistage stochastic formulations of such problems, where there is uncertainty in demand. That is, there may be many probable outcomes for each demand point for each stage of the multistage planning horizon. Thus the multistage stochastic formulation will contain many scenarios, where each scenario will represent a sequence of realizations of an outcome of a demand at each stage of the multistage planning horizon. However with real life problems easily exceeding hundreds of arcs and nodes, using the abovementioned approach for multistage stochastic programming would be very computationally impractical. To circumvent this obstacle, next we look at solving a Lagrangian relaxation formulation of this problem.

3 Lagrangian Relaxation Formulation

In the past, the Lagrangian relaxation technique has proven successful with discrete optimization problems of similar scale and complexity. This approach has led to improved algorithms for a number of important and large integer programming problems in the areas of routing, capacitated facility location, scheduling, assignment and set covering (Fisher 1981). Furthermore, stochastic unit commitment problems for power generation have been solved using Lagrangian relaxation in which the complex problem is decomposed into simpler stochastic single unit subproblems (Dentcheva and Romisch 1998).

The Lagrangian relaxation technique is based on the observation that many difficult integer programming problems can be modelled as relatively easy problems complicated by a set of side constraints (Fisher 1985). Thus we create a Lagrangian problem in which the complicating constraints are replaced with a penalty term in the objective function. This penalty term involves the amount of constraint violation and their corresponding dual variables or Lagrange multipliers (cost of violation). Replacing these constraints produces a Lagrangian relaxation problem that is relatively easy to solve. For example, consider the following integer program:

$$z = \min\{cx : Ax \leq b, Dx \leq d, x \in Z_+^n\}. \qquad (IP)$$

Let us suppose that $Ax \leq b$ are the easy constraints and $Dx \leq d$ are the complicating constraints. If we relax the complicating constraints, then the Lagrangian relaxation of (IP) is:

$$z(x,\lambda) = \min\{cx + \lambda(d - Dx) : x \in X, \lambda \geq 0\}, \qquad (L(x,\lambda))$$

where $X = \{x : Ax \leq b, x \in Z_+^n\}$ and λ are the dual variables corresponding to the relaxed constraints. Since the Lagrangian problem $L(x,\lambda)$ is a relaxation of the integer program IP, for any $\lambda \geq 0$, the optimal value of $L(x,\lambda)$ provides a lower bound on the optimal value of the IP problem, i.e., $z(x,\lambda) \leq z$. Therefore with different values of duals λ, solving $L(x,\lambda)$ will provide different lower bounds. We want to obtain the maximum lower bound on the optimal value of IP. For this we need to solve the Lagrangian dual problem:

$$\phi(\lambda) = \max\{z(x,\lambda) : \lambda \geq 0\}. \qquad (LD(\lambda))$$

The Lagrangian dual problem $LD(\lambda)$ can be considered as the problem of maximizing a concave nonsmooth function $z(x,\lambda)$, piecewise linear in λ. An important property of the Lagrangian dual is that it gives a lower bound which is at least as good as the lower bound obtained from the linear programming relaxation. In many cases the bound is strictly better. The proof and more detailed description of other properties of Lagrangian Duality can be found in (Everett 1963) and (Wolsey 1998).

Note that the optimal solution for $LD(\lambda)$ which provides a lower bound, may not be feasible for the IP problem. In such cases one has to heuristically alter the infeasible solution to obtain a solution that is feasible. The objective function value $\overline{z}$ of the solution found by the heuristic will provide an upper bound on the optimal value of the IP problem z. From this we know that the optimal solution to the IP gives an objective value z that lies in the gap between the upper and lower bounds, i.e., $z(x,\lambda) \leq z \leq \overline{z}$. So the gap gives us a means of quantifying the relative accuracy of any IP feasible solution obtained.

In the distribution network expansion planning problem, the set of complicating constraints are the supply-demand (3) and radial configuration (4) constraints. These two constraints together force the feasible solution to give a treelike network structure. Therefore a possible Lagrangian problem is given by relaxing these constraints, which is shown in the following formulation.

$$\min \sum_{t=1}^{T} \left[\begin{array}{c} \sum_{k=1}^{K} \left(\sum_{l=1}^{L} C_{klt} y_{klt} + \mu_t z_{kt} + \sum_{i=1}^{N} \lambda_{it} A_{ik} x_{kt} \right) \\ - \left(\mu_t (N-1) + \sum_{i=1}^{N} \lambda_{it} D_{it} \right) \end{array} \right] \qquad (L(\mu,\lambda))$$

subject to:

$$\begin{array}{ll} x_{kt} \leq I_k + \sum_{s=1}^{t} \sum_{l=1}^{L} B_l y_{kls}, & k = 1,...,K, t = 1,...,T, \\ x_{kt} \geq -I_k - \sum_{s=1}^{t} \sum_{l=1}^{L} B_l y_{kls}, & k = 1,...,K, t = 1,...,T, \\ x_{kt} \leq z_{kt} M_k, & k = 1,...,K, t = 1,...,T. \\ x_{kt} \geq -z_{kt} M_k, & k = 1,...,K, t = 1,...,T, \\ x_{kt} \in \Re,\ y_{klt},\ z_{kt} \in \{0,1\}, & \end{array}$$

where:

λ_{it} = the Lagrange multiplier for the supply-demand constraint (3);
μ_t = the Lagrange multiplier for the radial configuration constraint (4).

Notice that Lagrangian problem $L(\mu, \lambda)$ can be decomposed into a Lagrangian subproblem for each arc k. Then the solution of each subproblem will specify whether or not: 1) the capacity on arc k will be expanded; and 2) if arc k will be a constituent of the radial network; for each year t of the multistage planning horizon. The Lagrangian subproblems would be easy to solve using dynamic programming. Therefore this approach looks promising for efficiently solving electricity distribution network expansion planning problems. More importantly, for solving multistage stochastic programming problems, we would solve a stochastic subproblem for each arc k (for each multistage scenario), using stochastic dynamic programming.

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