On coincident-peak and anytime-peak transmission charges

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Abstract

We develop a generalized Nash equilibrium model with two players to compare the effects of using coincident-peak transmission charges with anytime-peak transmission charges. Players are assumed to be able to shift load between periods with a cost that grows quadratically with the amount shifted. When the shifting costs are large compared with peak charges, the model has a unique equilibrium. Coincident-peak charging and anytime-peak charging give different outcomes when the peak load for one purchaser does not coincide with the coincident peak. Coincident-peak charges favour purchasers whose peaks do not coincide with the system peak. They are more effective than anytime-peak charges at decreasing peak loads and therefore lowering peak charges.

1 Introduction

In this paper we use a simple game-theory model to compare two different peak pricing regimes in an electricity transmission network. In this model the transmission network owner wishes to recover the costs of the network by charging consumers for their peak consumption of electricity. We assume a horizon of two years in each of which there is two trading periods. The transmission network owner estimates the costs to be recovered at the start of each year, and then these are divided amongst the purchasers. We assume that these costs are proportional to the network owner's estimate of peak load in this year (which may differ from that actually realized).

We assume throughout the paper that purchasers respond only to peak load transmission charges (i.e. they do not respond to high energy charges). Of course this is a simplification, and they will attempt to respond to both high energy prices and peak-load transmission prices. A simple model that examines this from the perspective of the grid owner and the energy supplier is analysed in [3].

There are a number of approaches that the network owner might adopt to determine how much to charge each purchaser to recover the estimated total cost for each year. In this paper we compare two models:

1. the *coincident* peak tariff model charges each customer in proportion to his load at the *system* peak time;
2. the *anytime* peak tariff model charges each customer in proportion to his individual peak load;

It is easy to see that the charging regimes give the same network charges when the peak load for each purchaser coincides with the system peak. When the peak load for some purchaser is at a different time from the system peak (we call these *off-peak* purchasers), then the two systems might give different allocations of cost. It is easy to see that an off-peak purchaser would prefer a coincident peak tariff model, while others whose charges coincide with the system peak prefer an anytime-peak model.

It is not obvious which tariff model is preferred when purchasers can shift load between periods. For example, a purchaser with a high load in a system-peak period might prefer a coincident-peak tariff model because they can shift load inexpensively out of the system peak. A different (less flexible) purchaser with a high load in a system-peak period might prefer an anytime-peak model which discourages the first customer from avoiding their (coincident) peak charges by load shifting. Our contribution in this paper is to illuminate situations such as these.

There is a long literature (see [1] for a recent survey) on peak-load pricing. This literature is primarily devoted to the welfare effects of different coincident-peak-load pricing schemes. Our focus in this paper is on comparing coincident-peak and anytime-peak schemes, and investigating the incentives that they give to agents to shift consumption. We do not explicitly compare welfare effects, although some conclusions about these can be made by observing the outcomes of some simple examples.

The paper is laid out as follows. In the next section we describe an equilibrium model for a coincident-peak tariff system in which each purchaser can shift load between trading periods (at some cost). In section 3 we repeat this analysis for any-time peak tariffs. In sections 4, 5, and 6 we work through three examples that compare outcomes under each tariff system. Section 7 shows that our model can produce many Nash equilibria when the cost of shifting is low compared with peak charges. The results of the paper are then summarized in a final section.

2 A coincident peak demand model

Consider a situation in which there are two purchasers X and Y. They both consume electricity and have loads defined by Table 1. Here TP1 and TP2

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	u_1	u_1'	u_2	u_2'
Y	v_1	v_1'	v_2	v_2'

Table 1: Loads before load shifting

are trading periods, and we assume that

$$u_i + v_i > u_i' + v_i', \quad i = 1, 2,$$

so the system peaks occur in TP1 in both years. In year 1, the transmission owner charges a fixed amount R_1 to be split between both players according to their maximum coincident peak demand in year 1. The amount R_1 to be charged does not depend on the observed peak demand in that year but on an estimate of this made in advance by the network owner. On the other hand, the allocation of R_1 to purchasers depends on their actual demand levels. Given the loads this amounts to a charge of $\frac{u_1}{u_1+v_1}R_1$ to X and $\frac{v_1}{u_1+v_1}R_1$ to Y.

Now, if we consider year 1 on its own, then one might suppose that each purchaser might seek to reduce their peak load by shifting it into TP2 (at some cost to each). Observe however that this reduction might not achieve the desired result since if u_1 and v_1 are decreased proportionally then the peak charges remain the same. The purchasers are faced with a form of prisoner's dilemma: if both shift load then they both incur costs but might gain no transmission-charge benefit, yet if one were to decrease load unilaterally then he would be better off.

The incentives to shift load become clearer when a two-period model is introduced. Here each agent might not reduce their share of R_1 by load reductions, but by reducing the system peak in year 1 then they will lower the cost forecast of the network owner, and so provide lower charges in year 2 for both players. This provides sufficient incentive for each of them to incur the cost in year 1. A multi-period version of this model is also straightforward to derive but since this leads to little further insight, we will restrict attention to a two-period model. Observe also that we are assuming no uncertainty in this model - both purchasers can simultaneously choose their optimum demand levels for both years at the beginning of year 1.

In year 2, the transmission owner charges a fixed amount R_2 to be split between both players according to their maximum coincident peak demand in year 2. In the second year, we assume that in normal circumstances the first year's charge would be increased by the ratio of coincident peak demands, so the charge to be recovered in year 2 is

$$R_2 = \left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1.$$

Suppose now that agents X and Y shift some of their load from period 1 to period 2 in each year. We adopt the notation for the new loads as shown in Table 2.

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	x_1	x'_1	x_2	x'_2
Y	y_1	y'_1	y_2	y'_2

Table 2: Loads after load shifting

If agents in the first year decrease their load in the peak period to x_1+y_1, then we assume that the transmission operator reduces the peak charge R_2 in year 2 proportionally by $\left(\frac{x_1+y_1}{u_1+v_1}\right)$. For example if total peak demand in year 1 is reduced to 80% of $u_1 + v_1$ through load shifting, then the load forecast for Year 2 (i.e. $u_2 + v_2$) is assumed to be reduced by 20%. In general, in year 2 the transmission owner will charge

$$R_2 = \left(\frac{x_1 + y_1}{u_1 + v_1}\right) \left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1.$$

Now suppose that it costs X an amount $c_1 w^2$ to shift an amount of load w from period 1 to period 2, and similarly it costs Y $c_2 z^2$ to shift an amount of load z from period 1 to period 2. We shall assume that these costs do not vary from year to year. Then X seeks to choose x_1, x'_1, x_2, and x'_2 to minimize his load-shifting costs and peak charges in both periods, assuming that y_1, y'_1, y_2, and y'_2 are all fixed. We assume that no load can be totally shed, so in year 1 and year 2 the total load for each player remains the same. This gives

$$x_1 + x_1' = u_1 + u_1', \tag{1}$$

$$y_1 + y_1' = v_1 + v_1', \tag{2}$$

$$x_2 + x_2' = u_2 + u_2', \tag{3}$$

$$y_2 + y_2' = v_2 + v_2'. \tag{4}$$

These equations uniquely determine x_1' and x_2', once x_1 and x_2 are chosen (similarly y_1' and y_2', once y_1 and y_2 are chosen).

We now optimize the peak charges for X and Y, by optimizing over x_1 and x_2 and y_1 and y_2 respectively. For X, the cost summed over both years is

$$X(x_1, x_2) = c_1(x_1 - u_1)^2 + \frac{x_1}{x_1 + y_1} R_1 + c_1(x_2 - u_2)^2 + \frac{x_2}{x_2 + y_2} R_2$$

$$= c_1(x_1 - u_1)^2 + \frac{x_1}{x_1 + y_1} R_1 +$$
$$c_1(x_2 - u_2)^2 + \frac{x_2}{x_2 + y_2} \left(\frac{x_1 + y_1}{u_1 + v_1}\right) \left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1$$

and for Y the cost is

$$Y(y_1, y_2) = c_2(y_1 - v_1)^2 + \frac{y_1}{x_1 + y_1} R_1$$
$$+ c_2(y_2 - v_2)^2 + \frac{y_2}{x_2 + y_2} \left(\frac{x_1 + y_1}{u_1 + v_1}\right) \left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1.$$

Taking derivatives we obtain

$$\frac{\partial X}{\partial x_1} = R_1 \frac{y_1}{(x_1 + y_1)^2} + 2c_1(x_1 - u_1)$$
$$+ \frac{x_2}{x_2 + y_2} \left(\frac{1}{u_1 + v_1}\right) \left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1,$$

$$\frac{\partial X}{\partial x_2} = \left(\frac{x_1 + y_1}{u_1 + v_1}\right) \left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1 \frac{y_2}{(x_2 + y_2)^2} + 2c_1(x_2 - u_2),$$

$$\frac{\partial Y}{\partial y_1} = R_1 \frac{x_1}{(x_1+y_1)^2} + 2c_2(y_1 - v_1)$$
$$+ \frac{y_2}{x_2+y_2}\left(\frac{1}{u_1+v_1}\right)\left(\frac{u_2+v_2}{u_1+v_1}\right) R_1,$$

$$\frac{\partial Y}{\partial y_2} = \left(\frac{x_1+y_1}{u_1+v_1}\right)\left(\frac{u_2+v_2}{u_1+v_1}\right) R_1 \frac{x_2}{(x_2+y_2)^2} + 2c_2(y_2 - v_2).$$

We model the system as a one-shot game with Cournot conjectural variations, under which X chooses x_1 and x_2, assuming that y_1 and y_2 are fixed, and Y chooses y_1 and y_2, assuming that x_1 and x_2 are fixed.

It is easy to see that $\frac{\partial X}{\partial x_1} \geq 0$ for any choice of x_2, y_1 and y_2, assuming that $x_1 \geq u_1$. Thus X has no incentive to increase their load above u_1 and we may impose the condition

$$x_1 \leq u_1 \tag{5}$$

without loss of generality. A similar argument yields

$$y_1 \leq v_1, \tag{6}$$

$$x_2 \leq u_2, \tag{7}$$

$$y_2 \leq v_2. \tag{8}$$

Given y_1 and y_2, X must choose x_1 and x_2 to minimize his costs. Suppose x_2 is fixed. As x_1 decreases from u_1, $X(x_1, x_2)$ gets smaller until either there is some point at which $\frac{\partial X}{\partial x_1} = 0$ or

$$\begin{aligned} x_1 &= x_1' + y_1' - y_1 \\ &= u_1 + u_1' + v_1 + v_1' - x_1 - 2y_1 \end{aligned}$$

i.e. when

$$x_1 = \frac{u_1 + u_1' + v_1 + v_1'}{2} - y_1$$

At this point the peak in year 1 changes from TP1 to TP2, so the cost $X(x_1, x_2)$ switches to

$$\begin{aligned}\hat{X}(x_1,x_2) &= c_1(x_1-u_1)^2 + \frac{x_1'}{x_1'+y_1'}R_1 + c_1(x_2-u_2)^2 + \\ &\quad \frac{x_2}{x_2+y_2}\left(\frac{x_1'+y_1'}{u_1+v_1}\right)\left(\frac{u_2+v_2}{u_1+v_1}\right)R_1 \\ &= c_1(x_1-u_1)^2 + \frac{u_1+u_1'-x_1}{u_1+u_1'-x_1+y_1'}R_1 + c_1(x_2-u_2)^2 + \\ &\quad \frac{x_2}{x_2+y_2}\left(\frac{u_1+u_1'-x_1+y_1'}{u_1+v_1}\right)\left(\frac{u_2+v_2}{u_1+v_1}\right)R_1\end{aligned}$$

Now

$$\begin{aligned}\frac{\partial}{\partial x_1}\hat{X}(x_1,x_2) &= 2c_1(x_1-u_1) - R_1\frac{y_1'}{(u_1+u_1'-x_1+y_1')^2} \\ &\quad -\frac{x_2}{x_2+y_2}\left(\frac{1}{u_1+v_1}\right)\left(\frac{u_2+v_2}{u_1+v_1}\right)R_1\end{aligned}$$

which is negative. Thus $\hat{X}(x_1, x_2)$ will start increasing if we decrease x_1 below $\frac{u_1+u_1'+v_1+v_1'}{2} - y_1$. A similar argument shows that Y has no incentive to decrease y_1 below $\frac{u_1+u_1'+v_1+v_1'}{2} - x_1$. This shows that if the system peak is in TP1 without load shifting then it remains in TP1 in equilibrium.

In the model we enforce a system peak in TP1 in both years by imposing the inequalities

$$x_1 + y_1 \geq x_1' + y_1',$$

$$x_2 + y_2 \geq x_2' + y_2',$$

which amounts to imposing the equivalent conditions

$$2(x_1 + y_1) \geq u_1 + u_1' + v_1 + v_1',$$

$$2(x_2 + y_2) \geq u_2 + u_2' + v_2 + v_2',$$

thus providing lower bounds on x_1 and x_2 and y_1 and y_2 (assuming that the other purchaser's decisions are fixed).

Given the other purchaser's actions, each purchaser minimizes their costs subject to simple bounds on their purchase amount. Thus the choice of x_1 and x_2 satisfies optimality conditions:

$$\frac{\partial X}{\partial x_1} < 0 \Rightarrow x_1 = u_1,$$

$$\frac{\partial X}{\partial x_1} > 0 \Rightarrow 2(x_1 + y_1) = u_1 + u_1' + v_1 + v_1',$$

$$x_1 < u_1 \Rightarrow \frac{\partial X}{\partial x_1} = 0,$$

$$2(x_1 + y_1) > u_1 + u_1' + v_1 + v_1' \Rightarrow \frac{\partial X}{\partial x_1} = 0.$$

This can be written as the complementarity conditions

$$\frac{\partial X}{\partial x_1} = s_1 - t_1,$$

$$0 \leq 2(x_1 + y_1) - (u_1 + u_1' + v_1 + v_1') \perp s_1 \geq 0,$$

$$0 \leq (u_1 - x_1) \perp t_1 \geq 0.$$

We can repeat this for x_2, y_1 and y_2 to give the following mixed complementarity system:

$$\begin{array}{ccccccc}
 & & \frac{\partial X}{\partial x_1} & = & s_1 - t_1 & & \\
0 & \leq & 2(x_1 + y_1) - (u_1 + u_1' + v_1 + v_1') & \perp & s_1 & \geq & 0 \\
0 & \leq & u_1 - x_1 & \perp & t_1 & \geq & 0 \\
 & & \frac{\partial X}{\partial x_2} & = & s_2 - t_2 & & \\
0 & \leq & 2(x_2 + y_2) - (u_2 + u_2' + v_2 + v_2') & \perp & s_2 & \geq & 0 \\
0 & \leq & u_2 - x_2 & \perp & t_2 & \geq & 0 \\
 & & \frac{\partial X}{\partial y_1} & = & p_1 - q_1 & & \\
0 & \leq & 2(x_1 + y_1) - (u_1 + u_1' + v_1 + v_1') & \perp & p_1 & \geq & 0 \\
0 & \leq & v_1 - y_1 & \perp & q_1 & \geq & 0 \\
 & & \frac{\partial X}{\partial y_2} & = & p_2 - q_2 & & \\
0 & \leq & 2(x_2 + y_2) - (u_2 + u_2' + v_2 + v_2') & \perp & p_2 & \geq & 0 \\
0 & \leq & v_2 - y_2 & \perp & q_2 & \geq & 0
\end{array}$$

This system can be solved using GAMS/PATH [2] to yield a Nash equilibrium for the one-shot game.

3 Anytime peak demand

We now study the same players in a setting in which the tariff is based on anytime-peak demand. Recall that two players X and Y both consume electricity and have loads defined by Table 3.

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	u_1	u'_1	u_2	u'_2
Y	v_1	v'_1	v_2	v'_2

Table 3: Loads before load shifting

In year 1, the transmission owner charges a fixed amount R_1 to be split between both players according to their maximum anytime peak demand in year 1. In year $i = 1, 2$ we continue to assume that

$$u_i + v_i > u'_i + v'_i$$

but now we make the further assumption that

$$u_i > u'_i, \text{ and } v'_i > v_i.$$

This means that Y's peak period is not the same as X's, which we assume is the coincident peak. Thus in response to a peak charge X will decrease u_i and Y will increase v_i.

The transmission charge R_i in each year will be split between the purchasers according to their anytime peak loads. Thus in year $i = 1, 2$, X pays $\frac{u_i}{u_i+v'_i}R_i$ and Y pays $\frac{v'_i}{u_i+v'_i}R_i$. In the second year, in normal circumstances the charge R_2 would be increased by the ratio of coincident peak demands, so

$$\begin{aligned} R_2 &= \frac{\max\{u_2 + v_2, u'_2 + v'_2\}}{u_1 + v_1} R_1 \\ &= \frac{u_2 + v_2}{u_1 + v_1} R_1. \end{aligned}$$

Suppose now that agents X and Y shift some of their load between periods in each year to give the loads shown in Table 4. Recall it costs X an amount $c_1 w^2$ to shift an amount of load w from period 1 to period 2, and similarly it costs Y $c_2 z^2$ to shift an amount of load z from period 2 to period 1. We shall assume that these costs do not vary from year to year. Then X seeks to choose x_1, x'_1, x_2, and x'_2 to minimize his peak charges in both periods, assuming that y_1, y'_1, y_2, and y'_2 are all fixed. Again we assume that no load

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	x_1	x'_1	x_2	x'_2
Y	y_1	y'_1	y_2	y'_2

Table 4: Loads after load shifting

can be totally shed, so in each year the total load for each player remains the same as it was before shifting. This gives

$$x_1 + x'_1 = u_1 + u'_1, \tag{9}$$

$$y_1 + y'_1 = v_1 + v'_1, \tag{10}$$

$$x_2 + x'_2 = u_2 + u'_2, \tag{11}$$

$$y_2 + y'_2 = v_2 + v'_2, \tag{12}$$

This uniquely determines x'_1 and x'_2, once x_1 and x_2 are chosen. (Similarly y'_1 and y'_2 are determined once y_1 and y_2 are chosen.)

We now optimize the peak charges for X and Y, by optimizing over x_1 and x_2 and y_1 and y_2 respectively. For X, the cost summed over both years is

$$X(x_1, x_2) = c_1(x_1 - u_1)^2 + \frac{x_1}{x_1 + y'_1} R_1 + c_1(x_2 - u_2)^2 + \frac{x_2}{x_2 + y'_2} R_2$$

$$= c_1(x_1 - u_1)^2 + \frac{x_1}{x_1 + y'_1} R_1 +$$
$$c_1(x_2 - u_2)^2 + \frac{x_2}{x_2 + y'_2} \left(\frac{x_1 + y_1}{u_1 + v_1} \right) \left(\frac{u_2 + v_2}{u_1 + v_1} \right) R_1$$

Taking derivatives we obtain

$$\frac{\partial X}{\partial x_1} = R_1 \frac{y'_1}{(x_1 + y'_1)^2} + 2c_1(x_1 - u_1)$$
$$+ \frac{x_2}{x_2 + y'_2} \left(\frac{1}{u_1 + v_1} \right) \left(\frac{u_2 + v_2}{u_1 + v_1} \right) R_1$$

$$\frac{\partial X}{\partial x_2} = \left(\frac{x_1 + y_1}{u_1 + v_1}\right)\left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1 \frac{y_2'}{(x_2 + y_2')^2} + 2c_1(x_2 - u_2)$$

For Y the cost is

$$\begin{aligned} Y(y_1, y_2) &= c_2(y_1 - v_1)^2 + \frac{y_1'}{x_1 + y_1'} R_1 \\ &+ c_2(y_2 - v_2)^2 + \frac{y_2'}{x_2 + y_2'}\left(\frac{x_1 + y_1}{u_1 + v_1}\right)\left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1. \end{aligned}$$

$$\begin{aligned} &= c_2(y_1 - v_1)^2 + \frac{v_1 + v_1' - y_1}{x_1 + v_1 + v_1' - y_1} R_1 \\ &+ c_2(y_2 - v_2)^2 + \frac{v_2 + v_2' - y_2}{x_2 + v_2 + v_2' - y_2}\left(\frac{x_1 + y_1}{u_1 + v_1}\right)\left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1. \end{aligned}$$

giving

$$\begin{aligned} \frac{\partial Y}{\partial y_1} &= -R_1 \frac{x_1}{(x_1 + v_1 + v_1' - y_1)^2} + 2c_2(y_1 - v_1) \\ &+ \frac{v_2 + v_2' - y_2}{x_2 + v_2 + v_2' - y_2}\left(\frac{1}{u_1 + v_1}\right)\left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1, \end{aligned}$$

$$\frac{\partial Y}{\partial y_2} = -\left(\frac{x_1 + y_1}{u_1 + v_1}\right)\left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1 \frac{x_2}{(x_2 + v_2 + v_2' - y_2)^2} + 2c_2(y_2 - v_2).$$

As before X would not wish to increase the load in their peak period, and so we impose the constraints

$$x_1 \leq u_1, \tag{13}$$

$$x_2 \leq u_2. \tag{14}$$

In the anytime-peak tariff case it is possible for y_1 to decrease below v_1 in equilibrium. This will happen if the year 2 savings from a lower system peak in year 1 overides the extra costs incurred by Y having a larger anytime peak. To deal with this case we relax the conditions on y to give

$$y_1 \geq 0, \tag{15}$$

$$y_2 \geq 0. \tag{16}$$

As in the previous section, we model the game with Cournot conjectural variations, under which X chooses x_1 and x_2, assuming that y_1 and y_2 are fixed, and Y chooses y_1 and y_2, assuming that x_1 and x_2 are fixed. It is easy to see by repeating the argument of the previous section that x_1 can decrease at most until

$$x_1 = \frac{u_1 + u_1' + v_1 + v_1'}{2} - y_1,$$

when $X(x_1, x_2)$ switches to a different form that will increase with decreasing x_1. So we impose the following lower bounds

$$x_1 \geq \frac{u_1 + u_1' + v_1 + v_1'}{2} - y_1,$$

$$x_2 \geq \frac{u_2 + u_2' + v_2 + v_2'}{2} - y_2.$$

What happens to $Y(y_1, y_2)$ if y_1 becomes too large? If

$$y_1 \geq y_1',$$

then Y's peak load in year 1 becomes TP1 and so $Y(y_1, y_2)$ switches to

$$\begin{aligned}\hat{Y}(y_1, y_2) &= c_2(y_1 - v_1)^2 + \frac{y_1}{x_1 + y_1} R_1 \\ &+ c_2(y_2 - v_2)^2 + \frac{y_2'}{x_2 + y_2'} \left(\frac{x_1 + y_1}{u_1 + v_1}\right) \left(\frac{u_2 + v_2}{u_1 + v_1}\right) R_1.\end{aligned}$$

which is increasing in y_1. So we impose the constraint

$$y_1 \leq y_1'.$$

A similar restriction applies to y_2. Together this gives rise to the constraint system:

$$0 \leq y_1 \leq \frac{v_1 + v_1'}{2},$$

$$0 \leq y_2 \leq \frac{v_2 + v_2'}{2}.$$

Given the other purchaser's actions, each purchaser minimizes their costs subject to simple bounds on their purchase amount. Thus the choice of x_1 satisfies optimality conditions:

$$\frac{\partial X}{\partial x_1} < 0 \Rightarrow x_1 = u_1,$$

$$\frac{\partial X}{\partial x_1} > 0 \Rightarrow x_1 = \frac{u_1 + u_1' + v_1 + v_1'}{2} - y_1.$$

This can be written as the complementarity conditions

$$\frac{\partial X}{\partial x_1} = s_1 - t_1,$$

$$0 \leq 2(x_1 + y_1) - (u_1 + u_1' + v_1 + v_1') \perp s_1 \geq 0,$$

$$0 \leq (u_1 - x_1) \perp t_1 \geq 0.$$

We can repeat this for x_2. For y_1 we get

$$\frac{\partial Y}{\partial y_1} < 0 \Rightarrow y_1 = \frac{v_1 + v_1'}{2},$$

$$\frac{\partial Y}{\partial y_1} > 0 \Rightarrow y_1 = 0,$$

whence

$$\frac{\partial Y}{\partial y_1} = p_1 - q_1,$$

$$0 \leq y_1 \perp p_1 \geq 0,$$

$$0 \leq (v_1 + v_1') - 2y_1 \perp q_1 \geq 0.$$

Repeating this for y_2, and collecting all expressions we obtain the following mixed complementarity system:

$$\begin{array}{ccccccc}
 & & \frac{\partial X}{\partial x_1} & = & s_1 - t_1 & & \\
0 & \leq & 2(x_1 + y_1) - (u_1 + u_1' + v_1 + v_1') & \perp & s_1 & \geq & 0 \\
0 & \leq & u_1 - x_1 & \perp & t_1 & \geq & 0 \\
 & & \frac{\partial X}{\partial x_2} & = & s_2 - t_2 & & \\
0 & \leq & 2(x_1 + y_1) - (u_1 + u_1' + v_1 + v_1') & \perp & s_2 & \geq & 0 \\
0 & \leq & u_2 - x_2 & \perp & t_2 & \geq & 0 \\
 & & \frac{\partial X}{\partial y_1} & = & p_1 - q_1 & & \\
0 & \leq & (v_1 + v_1') - 2y_1 & \perp & q_1 & \geq & 0 \\
0 & \leq & y_1 & \perp & p_1 & \geq & 0 \\
 & & \frac{\partial X}{\partial y_2} & = & p_2 - q_2 & & \\
0 & \leq & (v_2 + v_2') - 2y_1 & \perp & q_2 & \geq & 0 \\
0 & \leq & y_2 & \perp & p_2 & \geq & 0
\end{array}$$

As before this system can be solved using GAMS/PATH [2] to yield a Nash equilibrium for the one-shot game.

4 Example 1

Suppose $c_1 = 0.5$, $c_2 = 0.5$, $R_1 = 10$. Table 5 shows the loads before shifting occurs.

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	8	3	9	4
Y	5	6	6	7

Table 5: Loads before load shifting

Before computing the results of our equilibrium models, it is instructive to examine the tariff share of each consumer in the absence of load shifting. Under a coincident peak tariff the first year's payment (10) is divided in the ratio 8:5, so X's share = 6.154, and Y's share = 3.846. The second year's payment is

$$R_2 = \frac{u_2 + v_2}{u_1 + v_1} R_1 = 11.538,$$

which is divided in the ratio 9:6, so X's share = 6.923, and Y's share = 4.615. The total cost incurred by X is 13.077. The cost incurred by Y is 8.462.

Under an anytime peak tariff the first year's payment (10) is divided in the ratio 8:6, so X's share = 5.714, and Y's share = 4.286. The second year's payment corresponds to the coincident peak, so it is the same as before, i.e. $R_2 = 11.538$, and this is divided in the ratio 9:7, so X's share = 6.490, and Y's share = 5.048. The total cost incurred by X is 12.205. The cost incurred by Y is 9.333.

Without load shifting it is clear that X prefers anytime peak charging and Y prefers coincident peak pricing. We now examine the outcomes of an equilibrium with load shiftting.

4.1 Coincident peak tariff with load shifting

If we solve the complementarity problem in section 2 then we obtain the results shown in Table 6. This shows the new loads after each purchaser has shifted its load from the peak period (TP1 in each year). It is easily verified (by carrying out the optimization for each purchaser separately) that this is a Nash equilibrium. Observe that Y shifts load into its peak period TP2 (though this does not become a coincident peak). We will see that Y has the opposite incentive for an anytime-peak tariff. The share of peak charges incurred by X and Y are shown in Table 7.

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	7.13375	3.86625	8.7286	4.2714
Y	4.0875	6.91249	5.5751	7.4249

Table 6: Loads after load shifting - coincident peak tariff

	Year 1	Year 2	Total Cost(including shifting)
X	6. 357	6. 078	12. 847
Y	3. 643	3.882	8.031
Total	10.000	9.960	

Table 7: Shares of peak charges - coincident peak tariff

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	7.20507	3.79493	8.70282	4.29718
Y	5.03257	5.96743	6.40386	6.596132

Table 8: Loads after load shifting - anytime peak tariff

	Year 1	Year 2	Total Cost(including shifting)
X	5.470	6.179	12.009
Y	4.530	4. 683	9.295
Total	10	10.862	

Table 9: Shares of peak charges - anytime peak tariff

4.2 Anytime peak tariff with load shifting

If we solve the complementarity problem in section 3 then we obtain the results shown in Table 8 and Table 9. Observe that Y's peak charges have increased and X's have decreased. So X still prefers the anytime-peak system and Y still prefers coincident-peak tariffs. Observe that to lessen its anytime-peak charge Y now shifts load out of its peak period into the coincident peak period. This increases the total payment sought in year 2 to 10.862. Under the equilibrium with load shifting, both X and Y are better off than they were with no load shifting.

5 Example 2

As in example 1 we suppose $c_1 = 0.5$, $c_2 = 0.5$, $R_1 = 10$, but Y now has a larger proportion of the load. Table 10 shows the loads before shifting occurs.

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	8	3	9	4
Y	20	21	21	22

Table 10: Loads before load shifting

Before computing the results of our equilibrium models, we examine the tariff share of each consumer in the absence of load shifting. Under a coincident peak tariff the first year's payment (10) is divided in the ratio 8:20, so X's share = 2.857, and Y's share = 7.143. The second year's payment is

$$R_2 = \frac{u_2 + v_2}{u_1 + v_1} R_1 = 10.714,$$

which is divided in the ratio 9:21, so X's share = 3.214, and Y's share = 7.500. The total cost incurred by X is 6,071.The cost incurred by Y is 14.643.

Under an anytime peak tariff the first year's payment (10) is divided in the ratio 8:21, so X's share = 2.759, and Y's share = 7.241. The second year's payment corresponds to the coincident peak, so it is the same as before, i.e. $R_2 = 10.714$, and this is divided in the ratio 9:22, so X's share = 3.111, and Y's share = 7.603. The total cost incurred by X is 5.870 .The cost incurred by Y is 14.844.

Without load shifting it is clear that X prefers anytime peak charging and Y prefers coincident peak pricing. We now examine the outcomes of an equilibrium with load shiftting.

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	7.6227	3.37727	8.752118	4.24788
Y	19.628	21.3723	20.89618	22.1038

Table 11: Loads after load shifting - coincident peak tariff

	Year 1	Year 2	Total Cost(including shifting)
X	2.797	3.0781	5. 9773
Y	7.203	7.3493	14.6267
Total	10.000	10.4274	

Table 12: Shares of peak charges - coincident peak tariff

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	7.65225	3.3477	8.764169	4.23583
Y	19.8188	21.181	21.09796	21.9020

Table 13: Loads after load shifting - anytime peak tariff

	Year 1	Year 2	Total Cost(including shifting)
X	2.6539	3.0042	
Y	7.3461	7.5077	
Total	10	10.5119	

Table 14: Shares of peak charges - anytime peak tariff

5.1 Coincident peak tariff with load shifting

If we solve the complementarity problem in section 2 then we obtain the results shown in Table 11. This shows the new loads after each purchaser has shifted its load from the peak period (TP1 in each year). It is easily verified (by carrying out the optimization for each purchaser separately) that this is a Nash equilibrium. The share of peak charges incurred by X and Y are shown in Table 12.

5.2 Anytime peak tariff with load shifting

If we solve the complementarity problem in section 3 then we obtain the results shown in Table 13 and Table 14. X's payment has decreased in both years under an anytime-peak tariff so it is still the case that X prefers anytime-peak charging. Y's payment has increased in both years under an anytime-peak tariff so it is still the case that Y prefers coincident-peak charging.

6 Example 3

In the final example we suppose $c_1 = 0.05$, $c_2 = 0.5$, $R_1 = 10$, so X can shift load much more readily than Y, and will do so to avoid peak charges. At the same time Y will respond to this shifting (albeit in a more modest fashion). Table 15 shows the loads before shifting occurs.

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	10	5	15	10
Y	20	10	25	15

Table 15: Loads before load shifting

Before computing the results of our equilibrium models, we examine the tariff share of each consumer in the absence of load shifting. Under both a coincident and anytime peak tariff the first year's payment (10) is divided in the ratio 10:20, so X's share = 3.3333, and Y's share = 6.6667. The second year's payment is

$$R_2 = \frac{u_2 + v_2}{u_1 + v_1} R_1 = 13.3333,$$

which is divided in the ratio 15:25, so X's share = 5, and Y's share = 8.3333. The total cost incurred by X is 8.3333.The cost incurred by Y is 15.

We now examine the outcomes of an equilibrium under coincident-peak pricing with load shiftting.

6.1 Coincident-peak tariff with load shifting

If we solve the complementarity problem in section 2 then we obtain the results shown in Table 16. This shows the new loads after each purchaser has shifted its load from the peak period (TP1 in each year). It is easily verified (by carrying out the optimization for each purchaser separately) that this is a Nash equilibrium. The share of peak charges incurred by X and Y are shown in Table 17. Here it is easy to see that X's share of the peak payment has dropped significantly after load shifting.

6.2 Anytime-peak tariff with load shifting

We now compare the equilibrium with that obtained under an anytime-peak tariff. This model must be constructed carefully as the cost function $X(x_1, x_2)$ is not smooth at $x_1 = \frac{u_1+u_1'}{2}$. At this point the anytime-peak load for X shifts into TP2. It may still be advantageous for X to continue increasing this peak load as long as the savings from year 2 (from a lower coincident peak in year 1) compensate for the extra expense of the anytime peak.

With the data used in this example X does not gain any further benefit from decreasing x_1 below $\frac{u_1+u_1'}{2}$, so $x_1 = x_1' = 7.5$ in equilibrium. The remaining values are shown in Table 18. The share of peak charges incurred by X and Y are shown in Table 19. One can observe that the peak charges are now shared more equally between the purchasers, albeit with an increase in total peak charge in year 2, because X has not been given sufficient incentive to move load to TP2 in year 1.

7 Multiple Equilibria

When the cost of shifting is large compared with the benefits of saving on peak charges, the amount of load shifting in equilibrium will be too small to make either of the constraints

$$2(x_1 + y_1) \geq u_1 + u_1' + v_1 + v_1',$$

$$2(x_2 + y_2) \geq u_2 + u_2' + v_2 + v_2',$$

binding at equilibrium. In other words TP1 remains the unique peak period in each year. If either of these constraints is binding then both periods

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	5.31295	9.6870	13.08766	11.9123
Y	19.6232	10.37676	24.89948	15.1005

Table 16: Loads after load shifting - coincident peak tariff

	Year 1	Year 2	Total Cost(including shifting)
X	2.131	3.818	7.2302
Y	7.869	7.264	15.2098
Total	10.000	11.082	

Table 17: Shares of peak charges - coincident peak tariff

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	7.5	7.5	12.9005	12.0995
Y	19.6052	10.3948	24.8912	15.1088

Table 18: Loads after load shifting - coincident peak tariff

	Year 1	Year 2	Total Cost(including shifting)
X	2.767	4.112	7.4121
Y	7.233	7.935	15. 2514
Total	10.000	12.047	

Table 19: Shares of peak charges - coincident peak tariff

in that year are peaks. If it is inexpensive to shift then we might expect that the above constraints are binding at equlibrium. Unfortunately in these circumstances there can be an infinite number of Nash equilibria.

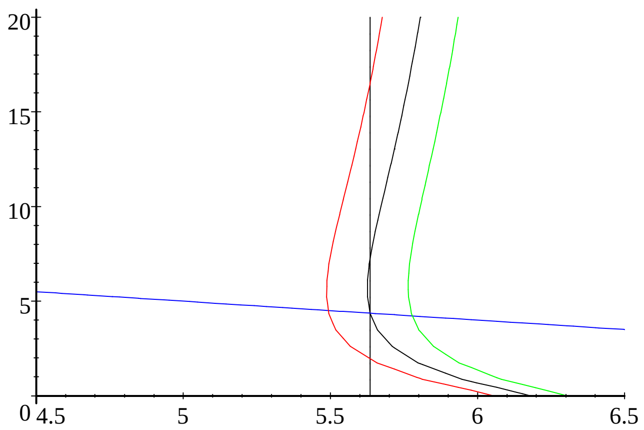
7.1 Example 4: Multiple equilibria

Suppose $c_1 = 0.4$, $c_2 = 0.02$, $R_1 = 10$. Here Y can shift a lot more cheaply than X. One equilibrium given by GAMS is shown in Table 20.

	Year 1		Year 2	
	TP1	TP2	TP1	TP2
X	5.63508681	4.36491319	9	4
Y	4.36491319	5.63508681	4	9

Table 20: Optimal loads after load shifting

There are many equilibria for this problem. It turns out that for all of them we have in year 2, $x_2 = 9$, $y_2 = 4$. We now consider the best responses in year 1.



The graph shows the best response x_1 to y_1 as the black curve. Here x_1 is the horizontal axis. The blue line is

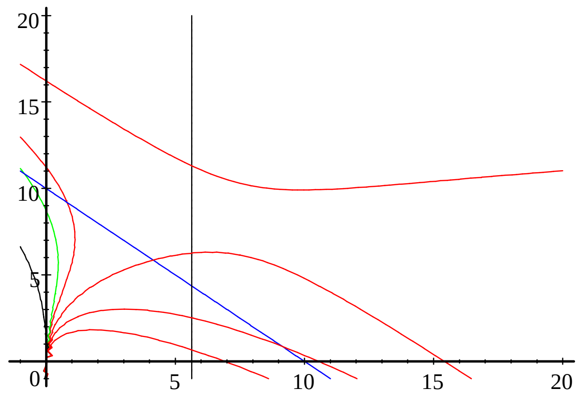
$$x_1 + y_1 = 10$$

and we must stay above it. The green contour shows where for any choice of y_1, the derivative of X with respect to x_1 is positive (the corresponding red contour is negative). Thus we wish to move x_1 to the left as much as possible, while staying to the right of the black curve and above the blue line. Thus for any choice of y_1 on the vertical axis, the optimal response is to choose $x_1 = 10 - y_1$, if $y_1 < 4.36491319$, and x_1 on the black curve otherwise.

The best response of y_1 to x_1 is less obvious. The red curves in the plot below are contours of the derivative of Y with respect to y_1, for fixed x_1. The blue line is

$$x_1 + y_1 = 10$$

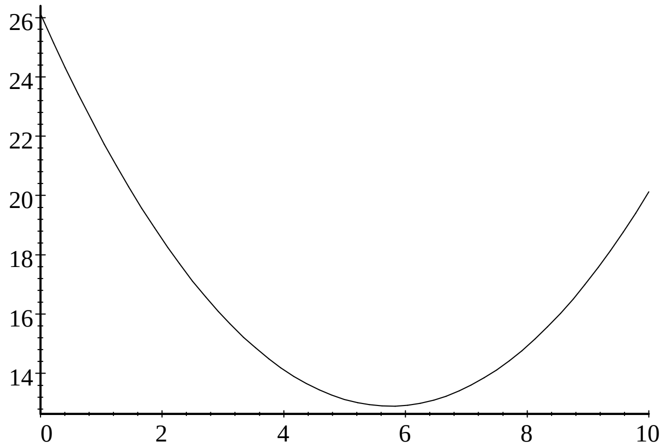
and we must stay above it. Thus we wish to move y_1 down as much as possible, while staying above the blue line.



The optimal response is thus the blue line. The set of Nash equilibria (given by the intersection of both optimal response curves) is then the segment of the blue straight line defined by

$$(x_1, 10 - x_1)\,, \quad x_1 \geq 5.635.$$

Is there a preferred equilibrium amongst these? It is easy to see that the best equilibrium for Y is when $10 - x_1$ is small, so Y prefers x_1 to be large. However plotting $X(x_1, 10 - x_1)$, gives a local minimum at $x_1 = 5.75$ as shown.



So the best equilibrium for X is $(5.75, 4.25)$. However Y would prefer an equilibrium in which x_1 is larger and so there is no equilibrium that is preferred by both players. This makes it difficult to predict likely outcomes from our models when it is inexpensive to shift load between periods compared with the level of peak payments.

8 Conclusion

In the absence of any ability to shift load, purchasers who peak when the system peaks will prefer an anytime-peak tariff, as this requires off-peak purchasers to contribute more than they would in an coincident-peak model. Off-peak purchasers prefer a coincident-peak tariff for the same reason.

When purchasers can shift load between periods, the situation becomes more complex, but essentially exhibits the same features. Given sufficient incentives we find that purchasers will shift load in equilibrium to avoid high peak charges in future years. Future peak charges depend on system peaks in year 1, so a coincident-peak tariff gives a greater reduction in system peak (when there are some off-peak purchasers) than an anytime-peak tariff. Since a key goal of peak tariffs is to reduce system peaks thus delaying network capacity expansion, this can be seen as a major advantage of coincident-peak tariff systems.

References

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