Optimizing Demand-Side Bids in Day-Ahead Electricity Markets

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Abstract— **We present a model of a purchaser of electricity in Norway, bidding into a wholesale electricity pool market that operates a day ahead of dispatch. The purchaser must arrange purchase for an uncertain demand that occurs the following day. Deviations from the day-ahead purchase are bought in a secondary market at a price that differs from the day-ahead price by virtue of regulating offers submitted by generators. Under an assumption that arbitrageurs are absent in these markets, we study conditions under which the purchaser should bid their expected demand, and examine the two-period game played between a single generator and purchaser in the presence of a competitive fringe. In all our models it is found that purchasers have an incentive to underbid their expected demand, and so the day-ahead prices will be below expected real-time prices. We also derive conditions on the optimal demand curve that purchasers should bid if the behavior of the other participants is unknown, but can be modeled by a market distribution function.**

Keywords— **Games, optimization methods, power system economics.**

I. Introduction

THERE has been much attention paid in recent years to optimizing the policies of generators who sell electricity in wholesale electricity pool markets. Much of this attention has focussed on equilibrium analyses that endeavour to quantify the extent of the market power generators might have (see e.g. [7], [11]). Although the effects of demand elasticity on the exercise of market power is well documented in Cournot models of electricity markets (see e.g. [7]), comparatively little attention has been paid to the effect of strategic demand bidding on market outcomes. Exceptions are the recent paper by Rassenti *et al* [18] who report on the results of demand behavior in a set of market simulation experiments, and Anderson and Hu [5], who carry out a Nash-equilibrium analysis of electricity pool markets in which generators and retailers hold contracts for differences.

In this paper we use some simple optimization models to study the optimizing behaviour of a large purchaser of electricity in a particular form of wholesale market, namely one in which the purchaser makes a day-ahead purchase bid, which is cleared against supply offers. The dispatch of power is then balanced in real time in an auxiliary market on the day of dispatch. The supply offers in the auxiliary market we study are constrained by the dispatch and price occurring in the day-ahead market.

An example of such a market structure is found in the Nordic market encompassing Norway, Denmark, Sweden and Finland. By noon each day, all generators and purchasers in this market submit respective supply and demand curves to the common electricity exchange, Nord Pool, giving production and purchase of electricity for the next day. Supply curves are required to be nondecreasing and demand curves must be nonincreasing. Based on these bids, the spot prices for each hour of the next day are derived. The real-time auxiliary market is somewhat different for each of the countries in the region.

In this paper we focus on the particular form of the Norwegian real-time market, called the *regulating* market, in which generators submit offers to increase or decrease the production level compared with their day-ahead market dispatch. If the market demand at the time of physical dispatch turns out to be higher than the quantity purchased in the day-ahead market, then the market is said to be *up-regulated*. In this circumstance each purchaser who is short of power must buy their shortfall at the regulating price. This single price is determined from where the total market demand meets the aggregate regulating-market offer curve. In the event of up-regulation, the regulating price is, by market design, always no less than the price in the day-ahead market.

On the other hand, if the market demand at the time of physical dispatch turns out to be lower than the quantity purchased in the day-ahead market, then the market is said to be *down-regulated*. In this case, each purchaser who has bought too much power must sell their excess at the regulating price. Equivalently generators who are called on in the regulating market must buy back electricity to maintain the physical balance. When the market is down-regulated, the regulating price is always no greater than the day-ahead price. Further details of the Norwegian regulating market can be found in Skytte [19].

In this paper we investigate the opportunities for speculation in these two markets, in the sense that purchasers might place a bid that differs from their expected demand in the day-ahead market in order to manipulate the regulating price. In all our models we do not explicitly include the possibility of arbitrage between the markets (by agents who buy in one market and sell in another), and the only players are generators and purchasers. In practice Statnett, the Norwegian system operator, prohibits demand-side speculation in the regulating market even by these players, and requires that the purchasers bid for their expected demand in the day-ahead market, and only generators are permitted to bid in the regulating market.

One circumstance in which a purchaser might deviate from bidding its expected demand occurs when the market as a whole is down-regulated, and the purchaser is up-

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regulated. This situation would occur if a purchaser bid for less power than it actually needed in the day-ahead market, while the market as a whole bid for too much. Then, since the regulating price is lower than the day-ahead price, this purchaser would get to buy its excess demand at a lower price than if it had ordered the correct amount in the day-ahead market. It is tempting to conclude that a rational purchaser should systematically bid low in the day-ahead market, but it is not clear that this gives an equilibrium strategy. In other words, if all purchasers bid low then the market will be up-regulated, and the advantages of buying excess power at a lower price might not be realized.

An interesting feature of the Norwegian market from the perspective of market participants is the requirement that the regulating price is determined with respect to the day-ahead price and quantity. At first sight this might appear to be the same as regulating pool markets in most other countries, in which generators bid in a real-time market after being dispatched a quantity and price in a day-ahead market, or bid into a real-time pool market while holding contracts for differences. However in most market designs the real-time offers of generators are not constrained to pass through their day-ahead dispatch point, and are free to be chosen to be any non-decreasing function consistent with the market design rules (e.g. in New Zealand being piecewise constant with at most five steps). In contrast, the requirement that the generators in Norway bid regulating offer curves that pass through their day-ahead dispatch point gives rise to different behaviour of both generators and purchasers than is seen in other models.

When both generators and purchasers are acting strategically, the market can be modeled as game played in two stages by these agents. One way of simplifying this game is to assume that the day-ahead price will equal the expected regulating price (as done in the seminal work of Allaz and Vila [1] under a perfect foresight assumption, and in the electricity context by [10], [13], and [9]). This is a form of "no-arbitrage" assumption, which appears to be reasonable if trading is allowed between the two markets. If a price difference is observed then speculators will buy electricity in one market and sell it in the other, which will tend to reduce the difference. In situations in where there is asymmetric uncertainty, risk-averse participants, or generators with the opportunity to exercise market power it is not clear that day-ahead prices should match expected regulating prices. In [8] an argument is advanced under an assumption of risk neutrality that the day-ahead price will equal the expected real-time price even in the presence of market power. On the other hand, Anderson and Hu [5] present an argument based on a supply-function duopoly that contract prices will trade at a premium, because generators who are not contracted have incentives to exercise their market power. In such an environment an arbitrageur who seeks to offer a lower-priced contract to a purchaser will be exposed to the risk that a generator who loses the contract to the arbitrageur will offer so as to raise the spot price in the real-time market above what would be expected if he was contracted.

The empirical evidence on differences between day-ahead prices and observed real-time prices seems to support the view that these are reducing as markets mature, and trading becomes more common. Price differences between forward and real-time electricity markets have been observed in a number of real electricity markets (see e.g. [6] for data comparing PJM prices from April 1997 to July 2000), but there is recent evidence (see [12], [15]) from electricity markets in New York and New England that day-ahead and real-time prices are converging over time. This is attributed to the introduction of *virtual bidding* that allows speculators to exploit any price deviations between the day-ahead and real-time markets.

In this paper we investigate the relationship between the day-ahead and regulating markets in Norway using a collection of simple models of increasing complexity. Speculating in these markets is not permitted under the Norwegian market rules, so market outcomes in which day-ahead and regulating prices deviate in expectation are not susceptible to the argument that speculating traders will drive these together. Nevertheless, even in the absence of speculators, one might expect a discrepancy in prices to be absent in equilibrium, since any difference would result in a change in behaviour of generators and purchasers over time that would tend to decrease this difference.

The equilibrium models that we describe do not support this conclusion. We find in nearly all our models that purchasers should bid for less than their expected demand. This is because the regulating price is centered around the clearing price of the day-ahead market, and by making this value small (by underbidding) the purchasers can effectively pay a smaller marginal price on each of two segments of their load. The result of this underbidding is to decrease the clearing price in the day-ahead market relative to the real-time market. It is tempting to suppose that this situation is not sustainable as the generators are encouraged to increase the prices of their day-ahead offers. We investigate this with a simple model of a single generator and a single purchaser both bidding strategically. The result of the model indicates that in equilibrium the efforts of a generator to increase price in the day-ahead market are countered by the purchaser bidding for very little demand in this market.

The paper is laid out as follows. In the next section we look at a model for a single purchaser. We first prove a result that gives conditions under which the purchaser should bid their expected load when there is a fixed price in the day-ahead market We then consider the case where the purchaser can influence the day-ahead price by bidding for supply from a known industry supply function. In this case it is optimal to buy less than the expected demand in the day-ahead market. In section 3 we extend the model of section 2 to n purchasers and construct a Nash equilibrium *a la* Cournot. In section 4, we study a single generator and a single purchaser both bidding strategically in a two-stage model, and show that bidding low in the day-ahead market remains an optimal strategy for the purchaser. In section 5 we move from equilibrium to an arguably more realistic probabilistic model. Here the offers and bids of the other agents are assumed to be unknown, but can be represented by a market distribution function of a similar form to that introduced in [2]. This allows the optimality conditions for generators to be applied to a purchaser, to yield an optimal bid curve for the day-ahead market. We illustrate the procedure with a simple example.

II. A single purchaser model

We first consider the case where all generators offer at the same price $\overline{p}$ in the day-ahead market, and the market has a single purchaser who is to choose an amount x to order in the day-ahead market at this price. Following this, a random demand H is observed, and the purchaser must purchase the extra energy (or sell it back to the market) at the regulating price.

The regulating market price is determined by offers of generators into the regulating market. These take the form of non-decreasing supply functions passing through the point $(\overline{p}, x)$. The clearing price is determined by the inverse $\tau(\cdot)$ of the aggregate regulating supply function. Note that τ depends on x and $\overline{p}$ but for simplicity we choose to suppress this in the notation. To make the dependence explicit, observe that $\tau(x) = \overline{p}$, so we may represent τ by

$$\tau(H) = \overline{p} + \delta\left(H - x\right) \tag{1}$$

where $\delta(\cdot)$ denotes the difference between the regulating price and the day-ahead price (see Figure 1). Since H is a random variable, the regulating price will also be a random variable.

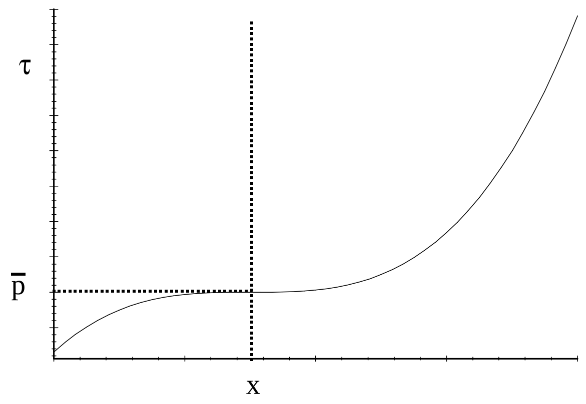

Fig. 1. Plot of $\tau(\cdot)$

The purchaser now faces the problem of minimizing the cost of meeting this random demand. His optimization problem is then

$$\text{P:}\quad \min_x \left\{\overline{p}x + E\left[\left(\overline{p} + \delta\left(H - x\right)\right)\left(H - x\right)\right]\right\}.$$

Observe that this is equivalent to

$$\text{P:}\quad \min_x \left\{\overline{p}E[H] + E\left[\delta\left(H - x\right)\left(H - x\right)\right]\right\},$$

so the purchaser should seek x^* to solve

$$\bar{\text{P}}\text{:}\quad \min_x \left\{E\left[\left(H - x\right)\delta\left(H - x\right)\right]\right\}.$$

It is easy to see that when $\delta(y) = ay$ for some $a \geq 0$ we have

$$E\left[\left(H - x\right)\delta\left(H - x\right)\right] = aE\left[\left(H - x\right)^2\right] \tag{2}$$

and so the optimal choice of x is $E[H]$. To obtain this optimal policy for more general forms of the regulating market, we must place some conditions on this as well as the probability distribution of demand. Recall that function f on the real line is called *even* if for every x, $f(-x) = f(x)$, and *odd* if for every x, $f(-x) = -f(x)$. Then we have the following proposition.

Proposition 1: Suppose $\delta(y)$ is an odd nondecreasing function with $y\delta(y)$ convex, and H has a symmetric probability distribution around $E[H]$. Then the optimal solution to P is $x^* = E[H]$.

Proof: See Appendix 1. ■

It is easy to construct examples in which $y\delta(y)$ is convex but $\delta(y)$ is not an odd function (e.g. $\delta(y) = \max\{0, y\}$) and for which a bid of $E[H]$ is not optimal. Moreover the convexity of $y\delta(y)$ is needed in Proposition 1 as shown by the following example.

Example 1

Suppose the price in the regulating market is determined by

$$\delta(y) = \begin{cases} -1 + e^{3y} & y \leq 0 \\ 1 - e^{-3y} & y > 0 \end{cases} \tag{3}$$

so

$$y\delta(y) = \begin{cases} -y + ye^{3y} & y \leq 0 \\ y - ye^{-3y} & y > 0 \end{cases}. \tag{4}$$

It is easy to verify that $y\delta(y)$ is not convex. Suppose H has a discrete probability distribution with $\Pr(\frac{1}{2}) = \frac{1}{2}$ and $\Pr(\frac{7}{2}) = \frac{1}{2}$. Then

$$E\left[\left(H - x\right)\delta\left(H - x\right)\right] = \frac{1}{2}g(\frac{1}{2} - x) + \frac{1}{2}g(\frac{7}{2} - x) \tag{5}$$

for which $x = E[H] = 2$ is a local maximum (not a minimum) as shown in Figure 2.

The symmetry of the probability distribution of H is also needed as shown by the following example.

Example 2

Suppose H has a discrete probability distribution with $\Pr(0) = 0.1$ and $\Pr(1) = 0.9$. Now let the price in the regulating market be determined by

$$\delta(y) = y^3. \tag{6}$$

We thus seek to minimize

$$E[(H - x)^4] = 0.1x^4 + 0.9(1 - x)^4 \tag{7}$$

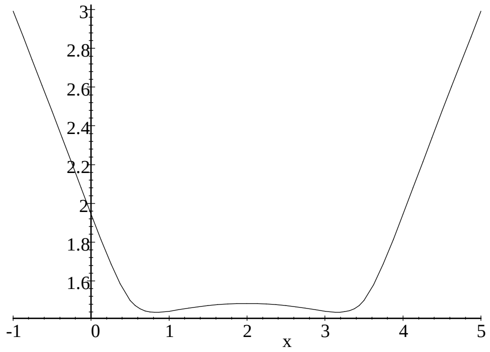

Fig. 2. Plot of $E[(H-x)\delta(H-x)]$ for different bids x.

which has a minimum at $x = 0.67533$ rather than $x = E[H] = 0.9$.

We conclude this section by considering the case where, instead of all generators offering at $\bar{p}$, the market is supplied by n generators who offer supply functions, where we denote by $S_i(p)$ the quantity offered at price p in the day-ahead market by generator i. The purchaser will face a random market demand H the following day. If the amount bought on the day-ahead market is different from H, the difference must be offset on the regulating market. We assume in this section that the aggregate supply function (although perhaps not each S_i) is known to the purchaser, and is strictly increasing.

We observe that there is no advantage in the purchaser offering a demand curve. With perfect knowledge of $S(\cdot) = \sum_i S_i(\cdot)$ he can decide on a price p and choose any decreasing curve that passes through $(p, S(p))$. Equivalently, he can determine an optimum quantity x to buy, for which he will pay $p = S^{-1}(x)$ per unit. (In this model we assume that the generators do not game their offers in response to the purchaser's bids.)

For convenience let $T(\cdot) = S^{-1}(\cdot)$. If the purchaser selects x to buy in the day-ahead market then he will pay $xT(x)$. In addition he will face a cost in the regulating market of $(H-x)\tau(H)$, where $\tau(\cdot)$ is the regulating market price at demand H and day-ahead dispatch volume x.

The purchaser should choose x to solve

$$\text{P:} \quad \min_x \{xT(x) + E[(H-x)\tau(H)]\},$$

so setting

$$\tau(H) = T(x) + \delta(H-x) \tag{8}$$

gives

$$\text{P:} \quad \min_x \{\bar{h}T(x) + E[(H-x)\delta(H-x)]\}$$

where $\bar{h} = E[H]$.

It is interesting to observe that in contrast to the situation in which the day-ahead price is a known constant, the optimal offer is not necessarily $\bar{h}$ even if $\tau(\cdot)$ is linear. To see this suppose $\delta(y) = ay$, for $a > 0$. Then

$$\begin{aligned} E[(H-x)\delta(H-x)] &= aE\left[(H-x)^2\right] && (9)\\ &= aE\left[H^2\right] - 2a\bar{h}x + ax^2 && (10) \end{aligned}$$

Now, the purchaser solves

$$\min_x \left\{\bar{h}T(x) - 2a\bar{h}x + ax^2\right\}$$

Differentiating gives $\bar{h}T'(x) - 2a\bar{h} + 2ax = 0$, whereby

$$x^* + \frac{\bar{h}}{2a}T'(x^*) = \bar{h} \tag{11}$$

The purchaser therefore should purchase less than $\bar{h}$ in the day-ahead market. (The solution $\bar{h}$ for a constant price is recovered by setting $T'(x^*) = 0$.)

III. Many purchasers

In the previous section we looked at the behaviour of a single purchaser. In this section we use a similar framework to calculate the market outcomes in a market with n purchasers having random demands H_i, $i = 1, \ldots, n$. We assume Cournot conjectural variations, namely that each purchaser i bids x_i in the day-ahead market assuming that x_j, $j \neq i$ is fixed. We will assume that $T(x) = bx$ and the price in the regulating market to be

$$bx + a(H-x), \tag{12}$$

where $x = \sum_{i=1}^n x_i$ and $H = \sum_{i=1}^n H_i$.

In a Nash equilibrium each purchaser i chooses a quantity x_i to solve

$$\text{P(n)}: \quad \min_{x_i \geq 0} x_i bx + E[(H_i - x_i)(bx + a(H-x))]. \tag{13}$$

The objective function of P(n) can be written

$$\begin{aligned} f_i(x) &= x_i bx + E[H_i bx + H_i aH \\ &\quad - H_i ax - x_i bx - x_i aH + x_i ax] && (14)\\ &= h_i bx + aE[H_i H] - h_i ax - x_i a\bar{h} + x_i ax, && (15) \end{aligned}$$

where we let $E[H_i] = h_i$, $E[H] = \bar{h}$. The optimal bid x_i for each purchaser will satisfy

$$\frac{\partial f_i}{\partial x_i} = h_i b - ah_i - a\bar{h} + ax_i + ax = \begin{cases} = & 0, \quad x_i > 0, \\ \geq & 0, \quad x_i = 0. \end{cases} \tag{16}$$

We first study the case where each $x_i > 0$. This is guaranteed to be a minimum because

$$\frac{\partial^2 f_i}{\partial x_i^2} = 2a > 0. \tag{17}$$

Now $\sum_{i=1}^n h_i = \bar{h}$, so summing over i gives

$$\bar{h}b - a\bar{h} - an\bar{h} + ax + nax = 0. \tag{18}$$

Thus

$$x = \bar{h}(1 - \frac{b}{a\,(1+n)}) \tag{19}$$

and so the optimal bid for purchaser i is

$$x_i = h_i + \frac{b}{(n+1)\,a}\left(\bar{h} - (n+1)h_i\right) \tag{20}$$

$$= (1 - \frac{b}{a})h_i + \frac{b}{a}\frac{\bar{h}}{(n+1)}. \tag{21}$$

Observe that we require $(1 - \frac{b}{a})h_i + \frac{b}{a}\frac{\bar{h}}{(n+1)} \geq 0$ for this set of bids to be optimal.

Recall that for a single purchaser the optimal bid x^* in the day-ahead market satisfies

$$x^* = \overline{h} - \frac{\overline{h}}{2a}T'\left(x^*\right) = \bar{h}\left(1 - \frac{b}{2a}\right). \tag{22}$$

For n purchasers, the total amount bid in equilibrium is

$$\sum_{i=1}^{n} x_i = \bar{h} + \frac{b}{(n+1)\,a}\left(n\bar{h} - (n+1)\bar{h}\right) \tag{23}$$

$$= \bar{h}(1 - \frac{b}{(n+1)\,a}). \tag{24}$$

So in aggregate, more demand will be bid for in the day-ahead market as the number of purchasers increases, and as $n \to \infty$, the limiting optimal bid for each purchaser is to bid their expected demand.

Observe that in this model the amount bid into the day-ahead market by each player depends only on the expected demand of each player. There is no dependence on the correlation between H_i and H. As a consequence of the linearity of the regulating market, this correlation appears only in the constant term $aE[H_iH]$ in the objective function. A negative correlation means that purchaser i is likely to be down-regulated when the market as a whole is up-regulated, and up-regulated when the market is down-regulated. However, although he makes a windfall profit from such a market, his equilibrium bidding strategy in the day-ahead market is independent of this correlation.

The degree to which purchasers reduce their bids depends upon the relative magnitudes of a and b. In practice we would expect $a \geq b$, since generating plant in the regulating market will be less flexible with a shorter time horizon, but since these numbers are determined by generators it might be possible for b to be larger than a.

First observe that $a \geq b$ guarantees that the smooth optimality conditions apply (i.e. the optimal $x_i > 0$.) When $a = b$, all purchasers bid for the same amount in equilibrium even if their expected demands are very different. When $a > b$, then the purchasers increase their bids in the day ahead market, and these bids may differ amongst players in equilibrium. For example with two purchasers with expected demands of $h_1 << h_2$, it is not hard to see that purchaser 1 bids for more than h_1, and purchaser 2 bids for less than h_2.

We now look at the case where $b > a$. For simplicity we will restrict attention to the case where purchasers have identical expected demand so

$$x_i = \frac{\bar{h}}{n}(1 - \frac{b}{(n+1)\,a}). \tag{25}$$

Now $x_i > 0$, as long as $b < (n+1)\,a$. Recall

$$\frac{\partial f_i}{\partial x_i} = h_i b - a h_i - a\bar{h} + a x_i + a x \tag{26}$$

$$= h_i b - a h_i - a n h_i + a x_i + a n x_i \tag{27}$$

$$= h_i(b - (n+1)\,a) \tag{28}$$

when $x_i = 0$. So if $b \geq (n+1)\,a$, then $\frac{\partial f_i}{\partial x_i} \geq 0$, and $x_i = 0$ is optimal for each purchaser.

IV. Strategic purchasers and generators

In the model of the previous section, we assumed that the purchasers have perfect knowledge of the aggregate generator supply function in the day-ahead market, and constructed a Nash equilibrium in the single-shot game played against other purchasers. In practice the supply function offers of the generators will be chosen at the same time as each purchaser's demand-function bids, and so one might expect over time for suppliers to increase the price of their offers in the day-ahead market until some equilibrium is reached.

In this section we consider a simple model in which generators bid strategically in the day-ahead market. The structure of our model is similar in style to that of Anderson and Hu [5]. They seek a sub-game perfect Nash equilibrium in a game in which the first stage is played by a retailer offering contracts to two generators with quadratic generation costs $C_i(q)$. The generators then decide whether to accept the contract on offer. In the final stage, given their respective contracts, the generators offer linear supply functions to a real-time market with a linear demand function with a random demand shock. Anderson and Hu give a formula for the supply function equilibrium in the last stage of this game in which generator i has contract quantity x_i. The linear supply function equilibrium is unique, and each supply curve passes through the point $(x_i, C'(x_i))$.

Unfortunately this result does not apply to the Norwegian regulating market,for which each supply curve must pass through $(x_i, \bar{p})$ where $\bar{p}$ will be determined by the outcome in the day-ahead market, and in general will be different from $C'(x_i)$. Indeed, when $\bar{p} \neq C'(x_i)$ it is possible to show that an optimal offer in the regulating market will have a horizontal and vertical section meeting at $(x_i, \bar{p})$, and it is difficult to see under this condition how an equilibrium might be constructed in the absence of a "sharing rule" that determines how ties are broken in the real-time dispatch to offers at $\bar{p}$.

To overcome this problem we consider a simpler model with a single generator A offering with costs

$$C(q) = 0.5q^2 \tag{29}$$

and a single purchaser and a competitive fringe represented by a generator B who offers at marginal cost q in both the regulating market and the day-ahead market. Suppose in the day-ahead market that generator A is dispatched x, at day-ahead price $\bar{p} > x$, so the competitive fringe generator is dispatched $\bar{p}$. This means that the consumer buys $(x + \bar{p})$ at price $\bar{p}$ in the day-ahead market. We suppose that generator A chooses k, defining a linear supply function $x = \frac{\bar{p}}{k}$, at the same time as the purchaser chooses a quantity x to buy in the day-ahead market. Then given x and $\bar{p}$, generator A chooses an optimal supply function to offer through $(\bar{p}, x)$ in the regulating market. (Note that B's offer in the regulating market will pass through $(\bar{p}, \bar{p})$ by construction.) We seek a Nash equilibrium in k and x.

Suppose demand H is uniformly distributed on $[0, 1]$. Given that B offers $T_B(q) = q$, the optimal offer for generator A in the regulating market is

$$T_A(q) = \begin{cases} 2q - x, & 0 < q < x \\ \bar{p}, & x < q < \frac{x+\bar{p}}{2} \\ 2q - x, & \frac{x+\bar{p}}{2} < q \end{cases} . \tag{30}$$

Including generator B, the total regulating market offer is then

$$T(q) = \begin{cases} \frac{-x+2q}{3}, & 0 < q < 2x \\ -x + q, & 2x < q < x + \bar{p} \\ \bar{p} & x + \bar{p} < q < \frac{x+3\bar{p}}{2} \\ \frac{-x+2q}{3}, & \frac{x+3\bar{p}}{2} < q \end{cases} \tag{31}$$

The consumer buys H in the regulating market. Her total expected payment in the regulating market is then

$$E[(H - (x + \bar{p}))T(H)]. \tag{32}$$

So her total payoff (i.e. minus her payment) is

$$F(x, \bar{p}) = -(x + \bar{p})\bar{p} - E[(H - (x + \bar{p}))T(H)] \tag{33}$$

$$= -\frac{4}{3}x\bar{p} - \bar{p}^2 - \frac{1}{24}x^3 + \frac{1}{8}x^2\bar{p} + \frac{1}{24}\bar{p}^3 - \frac{1}{8}x\bar{p}^2 - \frac{2}{9} + \frac{1}{2}x + \frac{1}{3}\bar{p} - \frac{1}{3}x^2 \tag{34}$$

Suppose that generator A offers curve $\bar{p} = kx$ in the day-ahead market. Then $F(x, kx)$ is a cubic in x that has a maximum (for $x > 0$) where

$$x = \frac{1}{2\left(-9k^2 - 3 + 9k + 3k^3\right)}\Big(64k + 48k^2 + 16 - 4\sqrt{(361k^2 + 393k^3 + 107k + 138k^4 + 25)}\Big). \tag{35}$$

Now consider generator A. In the regulating market, generator B is dispatched $T(H)$, so generator A is dispatched to deliver $H - T(H)$, at a cost of $\frac{1}{2}(H - T(H))^2$. The expected profit of A is then

$$R(x, \bar{p}) = x\bar{p} + E[(H - T(H) - x)\,T(H) - \frac{1}{2}(H - T(H))^2] \tag{36}$$

$$= x\bar{p} + \frac{1}{12}x^3 - \frac{1}{4}x^2\bar{p} + \frac{1}{4}x\bar{p}^2 - \frac{1}{12}\bar{p}^3 + \frac{1}{18} - \frac{1}{3}x + \frac{1}{6}x^2. \tag{37}$$

If the purchaser bids x in the day-ahead market then the expected profit of A is

$$R(x, kx) = x^2k + \frac{1}{12}x^3 - \frac{1}{4}x^3k + \frac{1}{4}x^3k^2 - \frac{1}{12}k^3x^3 + \frac{1}{18} - \frac{1}{3}x + \frac{1}{6}x^2 \tag{38}$$

which has a unique maximum (where $k > 0$) at

$$k = \frac{1}{2x}\left(2x + 4\sqrt{x}\right). \tag{39}$$

Now a Nash equilibrium (assuming Cournot conjectural variations) is obtained by solving (35) and (39) simultaneously. This gives

$$k = 25.612$$
$$x = 6.6032 \times 10^{-3}$$
$$\bar{p} = 0.16912$$

It is easy to compute the expected regulating price which has a value of 0.33751, which is larger than $\bar{p}$.

In this framework, generator A increases the price of his offer in the day-ahead market, and at the same time the purchaser decreases her bid x. In equilibrium, the purchaser buys very little in the day-ahead market.

V. Optimal bidding with a market distribution

In this paper we have restricted attention to models that represent strategic behaviour in the one-shot game played between generators and purchasers. Electricity markets of the type we have been discussing are repeated games, and so a one-shot analysis might not give strategies that would be useful in practice. In any given trading period a participant in this market might elect not to play a one-shot equilibrium strategy because of possible punishment by competitors in later rounds.

Because of this feature participants might not look for equilibria at all, but seek optimal strategies to pursue as opportunities to exploit market power arise. For example a purchaser might seek a (nondecreasing) demand curve under the assumption that the generators' supply curves and the other purchasers' bid curves are not fixed at their one-shot optimal values but are drawn from some probability distribution. The purchaser (say purchaser 1) then might seek a bid curve to offer that will yield a (random) day-ahead purchase outcome that in expectation minimizes his costs of purchasing to meet his next day's demand.

To model this formally we follow the approach of [2] and define a *market distribution* function ϕ, where $\phi(r, p)$ denotes the probability of the demand of purchaser 1 being fully met if he requests a quantity r at price p from the day-ahead pool. The probability $\phi(\cdot)$ is decreasing in r and increasing in p. The market distribution function is a powerful construction to help estimate optimal bids in electricity markets. We shall assume that ϕ is known or may be estimated by a purchaser (possibly by experimenting with different bids, see [4] and [16]).

Suppose now that the purchaser is to submit a parameterised demand curve $\mathbf{s} = \{(r(t), p(t), 0 \le t \le T\}$ to the day-ahead market. We choose the parameter t so the curve is traversed from right to left, implying that $r(t)$, which traces the quantity component of the demand curve, is monotonic decreasing in t, and the component $p(t)$, which traces the quantity component of the demand curve, is monotonic increasing in t. This is illustrated by the solid curve in Figure 3. The dashed line to the right shows all points for which $\phi(r,p) = \phi_0 = 0$. The dashed line to the left shows all points for which $\phi(r,p) = \phi_1 = 1$. From this, we observe that between ϕ_1 and ϕ_0 there is a probability measure that lies on $\mathbf{s}$ that defines the probability of the purchaser being sold quantity r at price p.

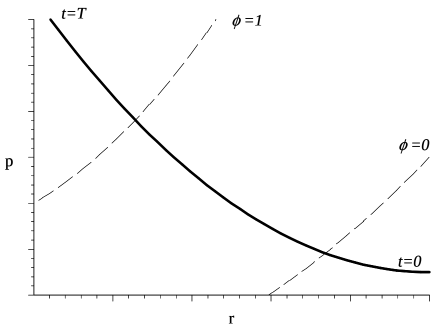

Fig. 3. The solid line is the demand curve. The dotted line to the right corresponds to $\phi(\cdot) = 0$, while the dotted line to the left corresponds to $\phi(\cdot) = 1$.

Now, purchaser 1 seeks to minimize his total cost of purchasing electricity. The total cost is the sum of the cost in the day-ahead market and the cost in the regulating market. The optimal bidding curve will be the solution to

$$\min_{s} \int_{s} [rp + C(r,p)]\, d\phi(r,p) \tag{40}$$

subject to

$$s = \{(r(t), p(t)), 0 \le t \le T\} \tag{41}$$

$$\frac{dr}{dt} \le 0 \ (r(\cdot) \text{ non-increasing}) \tag{42}$$

$$\frac{dp}{dt} \ge 0 \ (p(\cdot) \text{ non-decreasing}) \tag{43}$$

$$0 \le r(t) \le q_M \tag{44}$$

where $C(r,p)$ is the regulating cost and q_M is an upper bound on r.

The market distribution function ϕ is different from the standard market distribution function introduced in Anderson and Philpott [2], but we can make use of their framework. To do so, we may relate ϕ to a standard market distribution function by a simple transformation. To do this we let $q = q_M - r$. The parameter q_M is an upper bound on both r and q. Also, we define $\psi(q,p) = \phi(q_M - q, p)$. This corresponds to reversing the graphs in Figure 3 giving the graph in Figure 4.

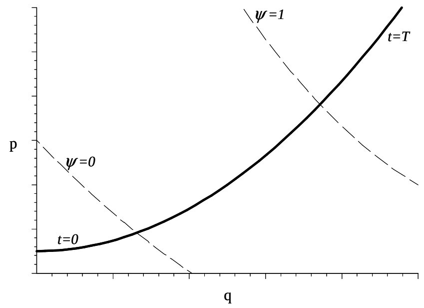

Fig. 4. The plots in Figure 3 reversed. The dashed line to the right corresponds to $\psi = 1$, and the dashed line to the left corresponds to $\psi = 0$.

Observe that on the curve s in Figure 3 the measure $d\phi(r,p)$ is the same as $d\psi(q_M - r, p)$ on the curve s' depicted by the solid line in Figure 4. The objective function (40) is then equivalent to

$$\min_{s'} \int_{s'} [(q_M - q)\, p + C(q_M - q, p)]\, d\psi(q,p). \tag{45}$$

Now we rewrite the expression (45) and get the following problem

$$\max_{s'} \int_{s'} \{-(q_M - q)\, p - C(q_M - q, p)\} d\psi(q,p) \tag{46}$$

subject to

$$s' = \{q(t), p(t), \quad 0 \le t \le T\} \tag{47}$$

$$\frac{dq}{dt}, \frac{dp}{dt} > 0 \ (q \text{ and } p \text{ non-decreasing}) \tag{48}$$

$$0 \le q(t) \le q_M. \tag{49}$$

We observe that this is equivalent to the problem of a generator maximizing the profit from an offer stack, where q is the quantity offered into the pool by the generator at price p as described in [2]. The market distribution function $\psi(q,p)$ denotes the probability of a generator not being fully dispatched at the price-quantity pair (p,q). We denote by $B(q,p)$ the integrand in the objective function (46). It is shown in [2] that the optimal solution to this problem must satisfy the first order condition

$$Z(q,p) = \frac{\partial B}{\partial q}\frac{\partial \psi}{\partial p} - \frac{\partial B}{\partial p}\frac{\partial \psi}{\partial q} = 0. \tag{50}$$

Computing the regulating cost in (40) is not entirely straightforward as the regulating price depends on the clearing price p in the day-ahead market and the amount of demand cleared in the day-ahead market. We shall denote by $C(r,p)$ the expected regulating cost for purchaser 1 conditional on its being dispatched r at clearing price p in the day-ahead market. To evaluate $C(r,p)$ we take expectations with respect to the conditional probability distribution of dispatch of the other purchasers.

To do this let $\delta(\cdot)$ be the difference between the regulating market price and the day-ahead price as a function of regulating market dispatch. Then the regulating price is $p+\delta(H-U(p)-r)$, where H is the total (random) demand of all purchasers, $U(p)$ is the total (random) day-ahead demand dispatched at price p to the other purchasers, and r is the amount of day-ahead demand that purchaser 1 is cleared at price p. The amount that purchaser 1 buys in the regulating market is H_1-r, where H_1 is the (random) demand of purchaser 1.

Using this notation we obtain

$$C(r,p) = E_{H,H_1,U}[(p+\delta(H-U(p)-r))(H_1-r)\mid(r,p)] \tag{51}$$

We then compute an offer curve s' that solves

$$\max_{s'}\int_{s'}\{-(q_M-q)\,p - C(q_M-q,p)\}d\psi(q,p).$$

The optimal bid curve will then be defined by $(r,p)=(q_M-q,p)$.

As an illustration of how one might compute an optimal demand curve to offer in the day-ahead market, suppose $\delta(y)=y$. Then

$$\begin{aligned} C(r,p) &= E_{H,H_1,U}[(p+(H-U(p)-r))(H_1-r)\mid(r,p)] \\ &= pE[H_1] - pr + E[HH_1|(r,p)] - E[H]\,r \\ &- E[U(p)H_1|(r,p)] + E[U(p)|(r,p)]r - rE[H_1] + r^2. \end{aligned}$$

To simplify this expression we now assume that both H and H_1 are statistically independent of p, r, and $U(p)$, but H and H_1 may be correlated. This independence is quite a restrictive assumption. Although the assumption allows the total amount $U(p)$ bid by other purchasers in the day-ahead market to depend on $E[H]$, a single other purchaser might well base their contribution to $U(p)$ on the current day's observed demand level which will typically be correlated with H.

Under the independence assumption the function $C(r,p)$ simplifies to

$$\begin{aligned} C(r,p) &= r^2 + \left(-p-\bar{h}+E[U(p)|(r,p)]-h_1\right)r \\ &\quad + ph_1 + E[HH_1] - h_1E[U(p)|(r,p)] \qquad (52) \\ &= r^2 + \left(-p-\bar{h}+u(p)-h_1\right)r + ph_1 \\ &\quad + E[HH_1] - h_1u(p), \qquad (53) \end{aligned}$$

where we denote $E[U(p)|(r,p)]$ by $u(p)$, $E[H]$ by $\bar{h}$, and $E[H_1]$ by h_1.

Example 3 (Single purchaser)

To illustrate how to derive an optimal bid in a particular case, we assume a single purchaser in a day-ahead market with $\psi(q,p)=\frac{qp}{4}$. For one purchaser, we have $H_1=H$, and $u(p)=0$. Now the integrand $-(q_M-q)\,p-C(q_M-q,p)$ becomes

$$\begin{aligned} & -(q_M-q)\,p - \{(q_M-q)^2 \\ & + \left(-p-\bar{h}+u(p)-h_1\right)(q_M-q) \\ & + ph_1 + E[HH_1] - h_1u(p)\} \\ & = -(q_M-q)^2 + 2\bar{h}(q_M-q) - p\bar{h} - E\left[H^2\right] \end{aligned} \tag{54}$$

so we seek to maximize

$$\int_{s'}\left(-(q_M-q)^2 + 2\bar{h}(q_M-q) - p\bar{h} - E\left[H^2\right]\right)d\left(\frac{qp}{4}\right)$$

over s'. As above

$$\begin{aligned} Z(q,p) &= B_q\psi_p - B_p\psi_q \\ &= \left(-2\bar{h}+2q_M-2q\right)\frac{1}{4}q + \frac{1}{4}\bar{h}p. \end{aligned} \tag{55}$$

Now suppose $E[H]=1$, $E[H^2]=2$, and $q_M=2$. Then

$$Z(q,p) = \frac{1}{4}(2-2q)\,q + \frac{1}{4}p. \tag{56}$$

The curve $Z(q,p)=0$ is plotted below in Figure 5, where $Z>0$ above the curve. This curve does not completely specify the optimal solution to the problem with objective (46) as the optimal curve must be non-decreasing.

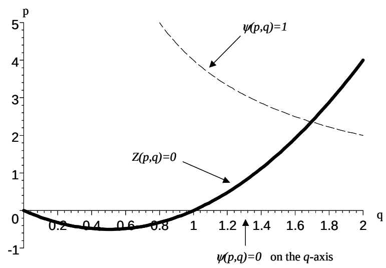

Fig. 5. The curve $Z(q,p)=0$.

However since $\psi(p,q)=0$ for $p\le 0$ the purchaser may choose any non-decreasing curve in this region joining $(0,0)$ to $(1,0)$. The same argument goes for the area where $\psi(p,q)=1$, which corresponds to $q\ge 1.6956$ and $p\ge 2.359$. Then, an optimal bidding curve would be

$$p(q) = \begin{cases} 0, & q\le 1, \\ 2q^2-2q, & 1<q\le 1.6956, \\ 2.359, & q>1.6956. \end{cases} \tag{57}$$

The optimal objective value of $p(q)$ is

$$\begin{aligned}
&\int_{s'} -\left(q_M - q\right)^2 + 2\bar{h}\left(q_M - q\right) \\
&\qquad\qquad - p\bar{h} - E\left[H^2\right] d\psi\left(q,p\right) \\
&= \int_{q=1}^{q=1.6956} \{-p(q) - 2q - (2-q)^2 + 2\} d\left(\frac{qp(q)}{4}\right) \\
&= \frac{1}{4}\int_{1}^{1.6956} (-18q^4 + 36q^3 - 28q^2 + 8q) dq \\
&= -2.5323.
\end{aligned}$$

This gives the following optimal bid curve for the purchaser:

$$b(r) = \begin{cases} 2.359, & r < 0.3043, \\ 4 - 6r + 2r^2, & 0.3043 \leq r < 1, \\ 0, & r \geq 1, \end{cases} \tag{58}$$

which is plotted in Figure 6. The cost of this policy is 2.5323 (minus the objective function value that we maximized).

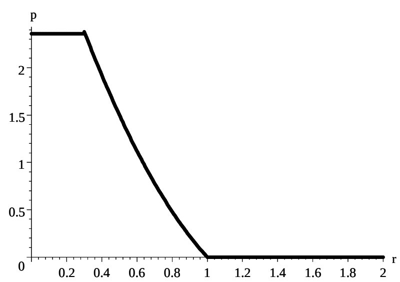

Fig. 6. Optimal demand bid curve.

We can compare this policy with several other candidates. A potential strategy for the purchaser may be to bid the expected demand, $r = \overline{h} = 1$, in the day-ahead market. This is defined by the offer curve $q = q_M - r = 1$. The objective value of this curve is

$$\begin{aligned}
&\int_{s'} \{-p\overline{h} - 2q\overline{h} - \left(q_M - q\right)^2 + K\} d\psi\left(q,p\right) \\
&= \int_{p=0}^{p=4} \{-p - 2 - (2-1)^2 + 2\} d\left(\frac{p}{4}\right) \\
&= \frac{1}{4}\int_{0}^{4} \left(-p-1\right) dp \\
&= -3.0,
\end{aligned}$$

giving an expected cost of 3.0 for the purchaser. Similarly if the purchaser chooses to buy the entire demand in the regulating market, then he would order $r = 0$ (or $q = 2$) in the day-ahead market. The objective value of this curve is also -3.0 giving a cost of 3.0. Bidding $r = 0.5$, in the day-ahead market, for example, gives an expected cost of 2.58.

It is also interesting to compare expected clearing prices in the day-ahead market with those in the regulating market. In the day-ahead market the expected clearing price is given by

$$\begin{aligned}
&\int_{s'} p\left(q\right) d\psi\left(q,p\right) \\
&= \int_{1}^{1.6596} \left(2q^2 - 2q\right) \frac{d}{dq}\left(\frac{q\left(2q^2 - 2q\right)}{4}\right) dq \\
&= 1.102.
\end{aligned}$$

In this example the regulating price is $\tau = p + H - r$, so its expectation is

$$E[\tau] = E[p] + 1 - E[r]. \tag{59}$$

Here

$$\begin{aligned}
E[r] &= \int_{s} r d\phi\left(r,p\right) \\
&= \int_{s'} \left(q_M - q\right) d\psi\left(q,p\right) \\
&= \int_{1}^{1.6596} \left(2 - q\right) \frac{d}{dq}\left(\frac{q\left(2q^2 - 2q\right)}{4}\right) dq \\
&= 0.5373,
\end{aligned}$$

so

$$E[\tau] = 1.5647.$$

VI. Conclusions

In this paper we have seen that in nearly all of our models, a purchaser for electricity ought to bid for less than their expected demand in the day-ahead market. The central reason underlying this is that even with perfect knowledge of their demand, purchasers are likely to be better off buying their electricity in two marginally-priced tranches, rather than making a single bid. This behaviour is confirmed in both the equilibrium model and the model with uncertainty modeled using a market distribution function.

Despite the incentives to do so Norwegian purchasers do not underbid in the day-ahead Nord Pool as the system operator Statnett requires that the purchasers bid for their expected demand. If underbidding were allowed then according to the models described above one might expect purchasers to withdraw some of their demand from the day-ahead market. If this did happen then it is likely that generators who offer to this market will also reduce their supply offers. According to the models we have presented, we should still expect to see an equilibrium in which the day-ahead price is less than the expected spot price.

Note that we have ignored the risk attitude of purchasers in our models and adopted a risk-neutral stance. Risk-aversion (through variance) is one possible source of differences in forward and expected spot prices in electricity markets (see e.g. [6]). Because the regulating market is typically more volatile than the day-ahead market (being susceptible to constraints and outages) purchasers might be unwilling to rely too heavily on sourcing their electricity from this market, and so in these circumstances one might observe purchasers bids to be closer to their expected demand than we predict.

VII. Appendix 1: Proof of Proposition 1

Define $\bar{h} = E[H]$, $g(y) = y\delta(y)$, and suppose the probability distribution of H is defined by measure μ. Then the objective function of $\bar{\mathrm{P}}$ becomes

$$\phi(x) = E\left[(H-x)\,\delta\,(H-x)\right] \tag{60}$$
$$= \int_{-\infty}^{+\infty} g\,(h-x)\,d\mu(h). \tag{61}$$

Since $g(y)$ is convex (with left and right derivatives denoted by $g'_-(y)$ and $g'_+(y)$ respectively) by virtue of Proposition 4 in [17] we may compute the left and right derivatives of ϕ,

$$\phi'_+(x) = \int_{-\infty}^{+\infty} -g'_-\,(h-x)\,d\mu(h) \tag{62}$$
$$\phi'_-(x) = \int_{-\infty}^{+\infty} -g'_+\,(h-x)\,d\mu(h). \tag{63}$$

A change of variable yields

$$\begin{aligned}\phi'_+(x) &= \int_{-\infty}^{0} -g'_-\left(u+\bar{h}-x\right)d\mu(u+\bar{h})\\ &\quad+\int_{0}^{+\infty} -g'_-\left(u+\bar{h}-x\right)d\mu(u+\bar{h}) \qquad (64)\\ &= \int_{-\infty}^{+\infty} -g'_-\left(u+\bar{h}-x\right)d\mu(u+\bar{h}), \qquad (65)\end{aligned}$$

whence

$$\begin{aligned}\phi'_+(\bar{h}) &= \int_{-\infty}^{0} -g'_-\,(u)\,d\mu(u+\bar{h})\\ &\quad+\int_{0}^{+\infty} -g'_-\,(u)\,d\mu(u+\bar{h}). \qquad (66)\end{aligned}$$

Now since $\delta\,(\cdot)$ is an odd function, g is even, and since $\mu(u+\bar{h})$ is symmetric about $u=0$,

$$\int_{-\infty}^{0} -g'_-\,(u)\,d\mu(u+\bar{h}) = \int_{0}^{+\infty} g'_+\,(u)\,d\mu(u+\bar{h}) \tag{67}$$

giving

$$\begin{aligned}\phi'_+(\bar{h}) &= \int_{0}^{+\infty} g'_+\,(u)\,d\mu(u+\bar{h})\\ &\quad+\int_{0}^{+\infty} -g'_-\,(u)\,d\mu(u+\bar{h}) \qquad (68)\\ &= \int_{0}^{+\infty} \left(g'_+\,(u)-g'_-\,(u)\right)d\mu(u+\bar{h}) \qquad (69)\\ &\geq 0 \qquad (70)\end{aligned}$$

by the convexity of g. Similarly

$$\begin{aligned}\phi'_-(\bar{h}) &= \int_{-\infty}^{0} -g'_+\,(u)\,d\mu(u+\bar{h})\\ &\quad+\int_{0}^{+\infty} -g'_+\,(u)\,d\mu(u+\bar{h}) \qquad (71)\\ &= \int_{-\infty}^{0} -g'_+\,(u)\,d\mu(u+\bar{h})\\ &\quad-\int_{-\infty}^{0} -g'_-\,(u)\,d\mu(u+\bar{h}) \qquad (72)\\ &\leq 0, \qquad (73)\end{aligned}$$

demonstrating that $\bar{h}$ is a local minimizer of ϕ.

Furthermore, if $x > \bar{h}$, then by convexity $g'_-\left(u+\bar{h}-x\right) \leq g'_-\,(u)$, so

$$\begin{aligned}\phi'_+(x) &= -\int_{-\infty}^{+\infty} g'_-\left(u+\bar{h}-x\right)d\mu(u+\bar{h}) \qquad (74)\\ &\geq -\int_{-\infty}^{+\infty} g'_-\,(u)\,d\mu(u+\bar{h}) \qquad (75)\\ &= \phi'_+(\bar{h}), \qquad (76)\end{aligned}$$

and if $x < \bar{h}$, then $g'_+\left(u+\bar{h}-x\right) \geq g'_+\,(u)$, so

$$\begin{aligned}\phi'_-(x) &= -\int_{-\infty}^{+\infty} g'_+\left(u+\bar{h}-x\right)d\mu(u+\bar{h}) \qquad (77)\\ &\leq -\int_{-\infty}^{+\infty} g'_+\,(u)\,d\mu(u+\bar{h}) \qquad (78)\\ &= \phi'_-(\bar{h}), \qquad (79)\end{aligned}$$

which shows that $\bar{h}$ gives a global minimum.

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