Appendix for the paper titled: Reserve Constraints in Co-Optimised Electricity Markets - A theoretical and empirical study of the New Zealand market

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1 Single Node - Reserve Constraining Generation

1.1 Diagram

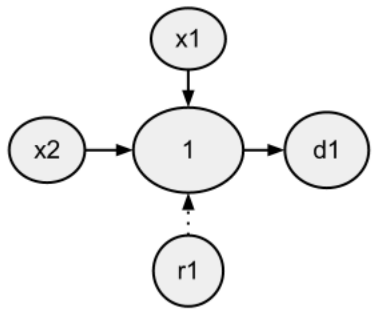

Figure 1: Single Node Diagram

1.2 Primal Formulation

The linear program for this system is thus formulated

$$min\ \ 0.01x_1 + 100x_2 + 30r_1 \tag{1}$$

subject to

$$\begin{aligned}
x_1 + x_2 = 350 &\quad \perp \pi_1 \\
x_1 \leq 400 &\quad \perp \gamma_1 \\
x_2 \leq 400 &\quad \perp \gamma_2 \\
r_1 \leq 400 &\quad \perp \rho_1 \\
x_1 - r_1 \leq 0 &\quad \perp \lambda_1 \\
x_2 - r_1 \leq 0 &\quad \perp \lambda_1 \\
x_1, x_2, r_1 \geq 0 &
\end{aligned}$$

From this we can construct a primal matrix as follows

0.01	100	30	
x_1	x_2	r_1	
1	1		π_1
1			γ_1
	1		γ_2
		1	ρ_1
1		-1	λ_1
	1	-1	λ_1

1.3 Dual Formulation

We may develop a dual formulation as follows

$$max \;\; 350\pi_1 + 400\gamma_1 + 400\gamma_2 + 400\rho_1 \tag{2}$$

subject to

$$\begin{array}{cc}
\pi_1 + \gamma_1 + \lambda_1 \leq 0.01 & \perp x_1 \\
\pi_2 + \gamma_2 + \lambda_1 \leq 1000 & \perp x_2 \\
\rho_1 - \lambda_1 \leq 30 & \perp r_1 \\
\lambda_1, \gamma_1, \gamma_2 \leq 0 & \\
\pi_1 free &
\end{array}$$

1.4 Shadow Pricing

Shadow price calculations were completed by using the solutions to the primal problem. It is noted that this is not the only method of determining these. However, for the purposes of the modelling approach it was sufficient for the requirements.

We note here that given the parameters the following is true, $\gamma_1, \gamma_2, \rho_1 = 0$ as these constraints are all non-binding. Total system energy demand is less than 400 MW (the capacity of the generation units) and sufficient reserve to cover up to the maximum capacity was present

Shadow Price Calculation:

$$\pi_1 + \gamma_1 + \lambda_1 = 0.01 \tag{3}$$
$$\gamma_1 = 0 \tag{4}$$
$$\pi_1 = 0.01 - \lambda_1 \tag{5}$$
$$\rho_1 - \lambda_1 = 30 \tag{6}$$
$$-\rho_1 = 0 \tag{7}$$
$$-\lambda_1 = 30 \tag{8}$$
$$\pi_1 = 30.01 \tag{9}$$

2 Two Node - Reserve Constraining Transmission

2.1 Diagram

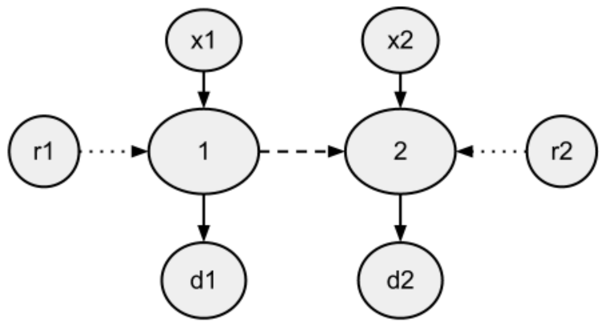

Figure 2: Two Node Independent Reserve

2.2 Primal Formulation

The primal linear program is formulated as follows

$$min \;\; 0.01x_1 + 100x_2 + 30r_1 + 30r_2 \tag{10}$$

subject to

$$\begin{aligned}
x_1 - f_{12} = 50 \quad & \perp \pi_1 \\
x_2 + f_{12} = 300 \quad & \perp \pi_2 \\
-f_{12} - r_1 \leq 0 \quad & \perp \lambda_1 \\
f_{12} - r_2 \leq 0 \quad & \perp \lambda_2 \\
x_1 \leq 400 \quad & \perp \gamma_1 \\
x_2 \leq 400 \quad & \perp \gamma_2 \\
r_1 \leq 400 \quad & \perp \rho_1 \\
r_2 \leq 400 \quad & \perp \rho_2 \\
f_{12} \leq 1000 \quad & \perp \mu_1 \\
-f_{12} \leq 1000 \quad & \perp \mu_2 \\
x_1, x_2, r_1, r_2 \geq 0 \quad & \\
f_{12} free \quad &
\end{aligned}$$

With a primal matrix

0.01	100	30	30		
x_1	x_2	r_1	r_2	f_{12}	
1				-1	π_1
	1			1	π_2
			-1	-1	λ_1
			-1	1	λ_2
1					γ_1
	1				γ_2
		1			ρ_1
			1		ρ_2
				1	μ_1
				-1	μ_2

2.3 Dual Formulation

The corresponding dual problem is thus formulated as follows:

$$max 50\pi_1 + 300\pi_2 + 400\gamma_1 + 400\gamma_2 + 400\rho_1 + 400\rho_2 + 1000(\mu_1 + \mu_2) \tag{11}$$

subject to

$$\begin{array}{cr} -\pi_1 + \pi_2 + \lambda_2 - \lambda_1 + \mu_1 - \mu_2 = 0 & \perp f_{12} \\ -\lambda_2 + \rho_2 \leq 30 & \perp r_2 \\ -\lambda_1 + \rho_1 \leq 30 & \perp r_1 \\ \pi_1 + \gamma_1 \leq 0.01 & \perp x_1 \\ \pi_2 + \gamma_2 \leq 1000 & \perp x_2 \\ \lambda_1, \lambda_2, \gamma_1, \gamma_2, \rho_1, \rho_2, \mu_1, \mu_2 \leq 0 & \\ \pi_1, \pi_2 free & \end{array}$$

2.4 Shadow Pricing

Shadow price calculations were completed by using the solutions to the primal problem to obtain the optimal values for each variable. Further notes as follows.

- Dual variables which have been set to zero are those that relate to non-binding constraints within the system
- There exists a nodal price separation between the nodes due to the treatment of reserve and energy prices
- The nodal price at node 1 is unconstrained as no reserve constraint is in effect at this node, this explains the reduced price.

Calculations:

$$
\begin{aligned}
-\pi_1 + \pi_2 + \lambda_2 - \lambda_1 + \mu_1 - \mu_2 &= 0 &(12)\\
\lambda_1, \mu_1, \mu_2 &= 0 &(13)\\
\pi_2 &= \pi_1 - \lambda_2 &(14)\\
-\lambda_2 + \rho_2 &= 30 &(15)\\
\rho_2 &= 0 &(16)\\
-\lambda_2 &= 30 &(17)\\
\pi_1 + \gamma_1 &= 0.01 &(18)\\
\gamma_1 &= 0 &(19)\\
\pi_1 &= 0.01 &(20)\\
\pi_2 &= 30.01 &(21)
\end{aligned}
$$

3 Two Node - Proportionality Constraint

3.1 Diagram

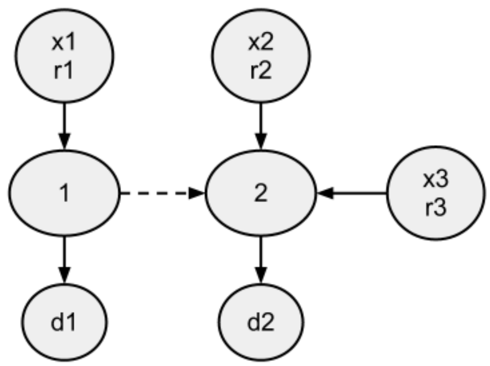

Figure 3: Two Node Non-Independent Reserve

3.2 Primal Formulation

$$min 0.01x_1 + 1000x_2 + 10x_3 + 0r_1 + 10r_2 + 0.01r_3 \tag{22}$$

subject to

$$
\begin{aligned}
x_1 - f_{12} = 50 & \quad \perp \pi_1 \\
x_2 + x_3 + f_{12} = 305 & \quad \perp \pi_2 \\
f_{12} - r_2 - r_3 \leq 0 & \quad \perp \lambda_2 \\
-f_{12} - r_1 \leq 0 & \quad \perp \lambda_1 \\
x_1 \leq 300 & \quad \perp \gamma_1 \\
x_2 \leq 50 & \quad \perp \gamma_2 \\
x_3 \leq 300 & \quad \perp \gamma_3 \\
r_1 \leq 300 & \quad \perp \rho_1 \\
r_2 \leq 50 & \quad \perp \rho_2 \\
r_3 \leq 300 & \quad \perp \rho_3 \\
f_{12} \leq 1000 & \quad \perp \mu_1 \\
-f_{12} \leq 1000 & \quad \perp \mu_2 \\
x_1 + r_1 \leq 300 & \quad \perp \sigma_1 \\
x_2 + r_2 \leq 50 & \quad \perp \sigma_2 \\
x_3 + r_3 \leq 300 & \quad \perp \sigma_3 \\
r_1 - \kappa_1 x_1 \leq 0 & \quad \perp \omega_1 \\
r_2 - \kappa_2 x_2 \leq 0 & \quad \perp \omega_2 \\
r_3 - \kappa_3 x_3 \leq 0 & \quad \perp \omega_3 \\
x_1, x_2, x_3, r_1, r_2, r_3 \geq 0 & \\
f_{12} free & \\
\kappa_1, \kappa_2, \kappa_3 = Constant = 0.5 &
\end{aligned}
$$

Primal matrix

0.01	1000	10	0	10	0.01		
x_1	x_2	x_3	r_1	r_2	r_3	f_{12}	
1						-1	π_1
	1	1				1	π_2
				-1	-1	1	λ_2
			-1			-1	λ_1
1							γ_1
	1						γ_2
		1					γ_3
			1				ρ_1
				1			ρ_2
					1		ρ_3
						1	μ_1
						-1	μ_2
1			1				σ_1
	1			1			σ_2
		1			1		σ_3
$-\kappa_1$			1				ω_1
	$-\kappa_2$			1			ω_2
		$-\kappa_3$			1		ω_3

3.3 Dual Formulation

The dual linear program may be developed as follows

$$max \;\; 50\pi_1+305\pi_2+300\gamma_1+50\gamma_2+300\gamma_3+300\rho_1+50\rho_2+300\rho_3+1000(\mu_1+\mu_2)+300\sigma_1+50\sigma_2+300\sigma_3 \quad (23)$$

$$\begin{array}{cr}
\pi_1 + \gamma_1 + \sigma_1 - \kappa_1\omega_1 \leq 0.01 & \perp x_1 \\
\pi_2 + \gamma_2 + \sigma_2 - \kappa_2\omega_2 \leq 1000 & \perp x_2 \\
\pi_2 + \gamma_3 + \sigma_3 - \kappa_3\omega_3 \leq 10 & \perp x_3 \\
-\lambda_1 + \rho_1 + \sigma_1 + \omega_1 \leq 0 & \perp r_1 \\
-\lambda_2 + \rho_2 + \sigma_2 + \omega_2 \leq 10 & \perp r_2 \\
-\lambda_2 + \rho_3 + \sigma_3 + \omega_3 \leq 0.01 & \perp r_3 \\
-\pi_1 + \pi_2 + \lambda_2 - \lambda_1 + \mu_1 - -mu_2 = 0 & \perp f_{12} \\
\lambda_1, \lambda_2, \gamma_1, \gamma_2, \gamma_3, \rho_1, \rho_2, \rho_3, \sigma_1, \sigma_2, \sigma_3, \omega_1, \omega_2, \omega_3, \mu_1, \mu_2 \leq 0 & \\
\pi_1, \pi_2 free & \\
\kappa_1, \kappa_2, \kappa_3 = Constant &
\end{array}$$

3.4 Primal Solution

Variable	Value
x_1	150
x_2	1.667
x_3	200
r_1	0
r_2	3.333
r_3	100
f_{12}	103.333
π_1	0.01
π_2	670.0033
$-\lambda_2$	669.9933
ω_2	-659.9333

3.5 Shadow Pricing

solutions multiple variables and constraints could be safely eliminated to determine the dual value solutions. Specific insights from this process include:

- Equation 7 is the combination of Equations 3 and 6. Here, there are two binding constraints which must be simultaneously satisfied to determine an optimal solution for the price at Node 2.
- Equations 10, 11 and 12 also reflect the inclusion of the proportionality constraint into the system. From this we incorporate a value 2 which relates to the constraint pricing present in the system.
- Equation 13 shows a clear reliance upon the value of 2 for the constraint pricing. Due to the system configuration any non-negative value of κ_2 is allowable. In some non-simple systems the value of κ may affect the system result.
- The shadow price calculations are presented in a simplified format, the full derivation is not presented.

Shadow Price Calculation

$$
\begin{aligned}
-\pi_1 + \pi_2 + \lambda_2 - \lambda_1 + \mu_1 - \mu_2 &= 0 && (24)\\
\lambda_1, \mu_1, \mu_2 &= 0 && (25)\\
\pi_2 &= \pi_1 - \lambda_2 && (26)\\
\omega_1, \omega_3, \gamma_3, \gamma_2, \gamma_3, \rho_1, \rho_2, \rho_3, \sigma_2 &= 0 && (27)\\
\pi_2 + \gamma_2 + \sigma_2 - \kappa_2\omega_2 &= 1000 && (28)\\
\pi_2 &= 1000 + \kappa_2\omega_2 && (29)\\
\pi_1 - \lambda_2 &= 1000 + \kappa_2\omega_2 && (30)\\
\pi_1 &= 0.01 && (31)\\
\lambda_2 + \rho_2 + \sigma_2 + \omega_2 &= 10 && (32)\\
-\lambda_2 + \omega_2 &= 10 && (33)\\
-\lambda_2 - \kappa_2\omega_2 &= 999.9 && (34)\\
-\omega_2(1 + \kappa_2) &= 989.9 && (35)\\
\omega_2 &= \frac{-989.9}{1 + \kappa_2} && (36)\\
\kappa_2 &= 0.5 && (37)\\
\omega_2 &= -659.9333 && (38)\\
-\lambda_2 &= 10 - \omega_2 && (39)\\
-\lambda_2 &= 669.9333 && (40)\\
\pi_2 &= \pi_1 - \lambda_2 && (41)\\
\pi_2 &= 0.01 + 669.9333 && (42)\\
\pi_2 &= 700.0033 && (43)
\end{aligned}
$$

4 Two Node - Total Reserve Constraint

4.1 Diagram

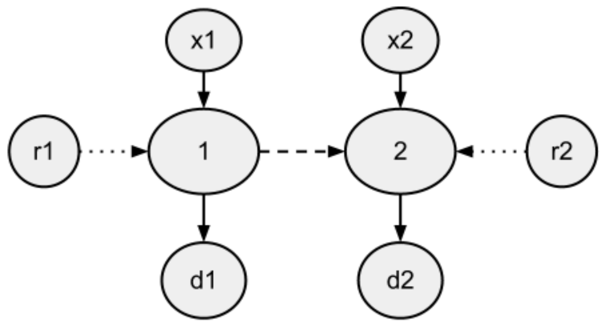

Figure 4: Two Node Non-Independent Reserve

4.2 Primal Formulation

$$min\ \ 0x_1 + 1000x_2 + 0r_1 + 0r_2 \tag{44}$$

$$\begin{array}{cr}
x_1 - f_{12} = 150 & \perp \pi_1 \\
x_2 + f_{12} = 150 & \perp \pi_2 \\
-f_{12} - r_1 \leq 0 & \perp \lambda_1 \\
f_{12} - r_2 \leq 0 & \perp \lambda_2 \\
x_1 \leq 500 & \perp \gamma_1 \\
x_2 \leq 200 & \perp \gamma_2 \\
r_1 \leq 50 & \perp \rho_1 \\
r_2 \leq 50 & \perp \rho_2 \\
f_{12} \leq 200 & \perp \mu_2 \\
-f_{12} \leq 200 & \perp \mu_2 \\
x_1, x_2, r_1, r_2 \geq 0 & \\
f_{12} free &
\end{array}$$

0	1000	0	0		
x_1	x_2	r_1	r_2	f_{12}	
1				-1	π_1
	1			1	π_2
		-1		-1	λ_1
			-1	1	λ_2
1					γ_1
	1				γ_2
		1			ρ_1
			1		ρ_2
				1	μ_1
				-1	μ_2

4.3 Dual Formulation

$$max \;\; 150\pi_1 + 150\pi_2 + 500\gamma_1 + 200\gamma_2 + 50\rho_1 + 50\rho_2 + 200(\mu_1 + \mu_2) \tag{45}$$

subject to

$$\begin{array}{cc}
\pi_1 + \gamma_1 \leq 0 & \perp x_1 \\
\pi_2 + \gamma_2 \leq 1000 & \perp x_2 \\
-\lambda_1 + \rho_1 \leq 0 & \perp r_1 \\
-\lambda_2 + \rho_2 \leq 0 & \perp r_2 \\
-\pi_1 + \pi_2 - \lambda_1 + \lambda_2 + \mu_1 - \mu_2 = 0 & \perp f_{12} \\
\gamma_1, \gamma_2, \lambda_1, \lambda_2, \rho_1, \rho_2, \mu_1, \mu_2 \leq 0 & \\
\pi_1, \pi_2 free &
\end{array}$$

4.4 Primal Solution

Variable	Value
x_1	200
x_2	100
r_1	0
r_2	50
f_{12}	50
π_1	0
π_2	1000
$-\lambda_1$	0
$-\lambda_2$	1000

4.5 Shadow Pricing

Shadow price calculations are completed using the primal solution. In this instance a total constraint on the amount of reserve present within the system is enforced. Transfer from node 1 to node 2 is limited by reserve availability and thus the ρ constraint binds in this instance.

$$\pi_1, \lambda_1, \gamma_1, \gamma_2, \rho_1, \mu_1, \mu_2 = 0 \tag{46}$$
$$\pi_2 = \pi_1 - \lambda_2 \tag{47}$$
$$-\lambda_2 = 0 - \rho_2 \tag{48}$$
$$\pi_2 = \pi_1 - \rho_2 \tag{49}$$
$$\pi_1 = 0 \tag{50}$$
$$\pi_2 = 1000 \tag{51}$$
$$-\lambda_2 = 1000 \tag{52}$$
$$\rho_2 = -1000 \tag{53}$$

5 Two Node - Combined Reserve and Generation Constraint

5.1 Diagram

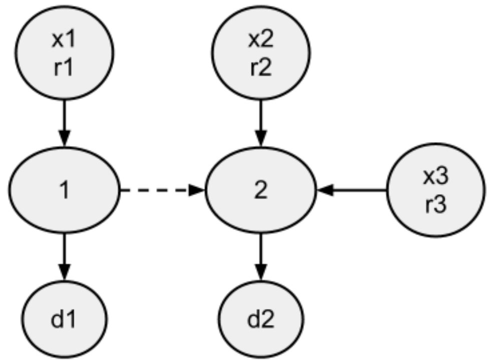

Figure 5: Two Node Non-Independent Reserve

5.2 Primal Formulation

$$min\ \ 0x_1 + 1000x_2 + 50x_3 + 0r_1 + 100r_2 + 70r_3 \tag{54}$$

subject to

$$\begin{aligned}
&\delta = 0.75 \\
&x_1 - f_{12} = 150 && \perp \pi_1 \\
&x_2 + x_3 + f_{12} = 180 && \perp \pi_2 \\
&-\delta f_{12} - r_1 \leq 0 && \perp \lambda_1 \\
&\delta f_{12} - r_2 - r_3 \leq 0 && \perp \lambda_2 \\
&x_1 \leq 500 && \perp \gamma_1 \\
&x_2 \leq 50 && \perp \gamma_2 \\
&x_3 \leq 150 && \perp \gamma_3 \\
&r_1 \leq 500 && \perp \rho_1 \\
&r_2 \leq 50 && \perp \rho_2 \\
&r_3 \leq 150 && \perp \rho_3 \\
&x_1 + r_1 \leq 500 && \perp \sigma_1 \\
&x_2 + r_2 \leq 50 && \perp \sigma_2 \\
&x_3 + r_3 \leq 150 && \perp \sigma_3 \\
&f_{12} \leq 500 && \perp \mu_1 \\
&-f_{12} \leq 500 && \perp \mu_2 \\
&x_1, x_2, x_3, r_1, r_2, r_3 \geq 0 \\
&f_{12} free
\end{aligned}$$

0	1000	50	0	100	70		
x_1	x_2	x_3	r_1	r_2	r_3	f_{12}	
1						-1	π_1
	1	1				1	π_2
			-1			$-\delta$	λ_1
				-1	-1	δ	λ_2
1							γ_1
	1						γ_2
		1					γ_3
			1				ρ_1
				1			ρ_2
					1		ρ_3
						1	μ_1
						-1	μ_2
1		1					σ_1
	1		1				σ_2
		1		1			σ_3

5.3 Dual Formulation

$$max \quad 150\pi_1+180\pi_2+500\gamma_1+50\gamma_2+150\gamma_3+500\rho_1+50\rho_2+150\rho_3+500\sigma_1+50\sigma_2+150\sigma_3+500(\mu_1+\mu_2) \quad (55)$$

subject to

$$\begin{array}{cc}
\pi_1+\gamma_1+\sigma_1 \leq 0 & \perp x_1 \\
\pi_2+\gamma_2+\sigma_2 \leq 1000 & \perp x_2 \\
\pi_2+\gamma_3+\sigma_3 \leq 50 & \perp x_3 \\
-\lambda_1+\rho_1+\sigma_1 \leq 0 & \perp r_1 \\
-\lambda_2+\rho_2+\sigma_2 \leq 100 & \perp r_2 \\
-\lambda_2+\rho_3+\sigma_3 \leq 70 & \perp r_3 \\
-\pi_1+\pi_2-\delta\lambda_1+\delta\lambda_2+\mu_1-\mu_2 = 0 & \perp f_{12} \\
\lambda_1,\lambda_2,\gamma_1,\gamma_2,\gamma_3,\rho_1,\rho_2,\rho_3,\sigma_1,\sigma_2,\sigma_3 \leq 0 & \\
\pi_1,\pi_2 free &
\end{array}$$

5.4 Results

Variable	Value
x_1	270
x_2	0
x_3	60
r_1	0
r_2	0
r_3	90
f_{12}	120
π_1	0
π_2	60
$-\lambda_1$	0
$-\lambda_2$	80
σ_3	-10

5.5 Shadow Pricing

The shadow price calculations are slightly more complex in this case. The flow constraint is still binding, but a 1:1 relationship between risk and reserve is no longer present in the system. This is analogous to HVDC self cover, or the treatment of net free reserves by the system operator in dispatching the system. Here, reserve is more expensive than energy, however the next unit of energy is extremely expensive (peaking plant). This creates a unique trade off between the dispatch of reserve and energy which influences the prices accordingly.

$$\mu_1, \mu_2, \sigma_1, \sigma_2, \gamma_1, \gamma_2, \gamma_3, \rho_1, \rho_2, \rho_3, -\lambda_1 = 0 \quad (56)$$

$$-\pi_1 + \pi_2 + \delta\lambda_2 = 0 \quad (57)$$

$$\pi_1 = 0 \quad (58)$$

$$\pi_2 + \sigma_3 = 50 \quad (59)$$

$$-\lambda_2 + \sigma_3 = 70 \quad (60)$$

$$\pi_2 = \pi_1 - \delta\lambda_2 = 0 \quad (61)$$

$$\pi_2 = -\delta\lambda_2 \quad (62)$$

$$\sigma_3 = 50 + \delta\lambda_2 \quad (63)$$

$$-\lambda_2(1-\delta) = 20 \quad (64)$$

$$-\lambda_2 = 80 \quad (65)$$

$$\sigma_3 = 50 + \delta(-80) = -10 \quad (66)$$

$$\pi_2 = 50 - (-10) \quad (67)$$

$$\pi_2 = 60 \quad (68)$$

6 Profit & Loss Calculation - Nodally Located Generators

6.1 Description

A linear program was set up with contractually obligated to showcase the effects of constraining reserve on a security risk transmission line. This highlights the incentives an energy and reserve provider may have to constrain reserves. In this example, two generators each with generation offered at Node 1 and Node 2, but with different contractual obligations to supply will be used. A simple reserve constrained model without bathtub constraints will be used to showcase the effects.

Table 1: Parameters

Variable	Generator	Cost	Maximum
x_1	G_1	1	100
x_2	G_2	200	100
x_3	G_1	100	200
x_4	G_2	1	200
r_1		0	250/50
r_2		0	250/50
f_{12}			500
D_1			200
D_2			200

Table 2: Contractual Obligations

	G_1	G_2
D_1	200	0
D_2	0	200

6.2 Primal Formulation

$$min 1x_1 + 200x_2 + 100x_3 + 1x_4 + 0r_1 + 0r_2 \tag{69}$$

subject to

$$\begin{gathered}
x_1 + x_3 - f_{12} = D_1 \\
x_2 + x_4 + f_{12} = D_2 \\
f_{12} - r_2 \leq 0 \\
-f_{12} - r_1 \leq 0 \\
x_1 \leq 100 \\
x_2 \leq 100 \\
x_3 \leq 200 \\
x_4 \leq 200 \\
r_1 \leq 250/50 \\
r_2 \leq 250/50 \\
f_{12} \leq 500 \\
-f_{12} \leq 500 \\
x_1, x_2, x_3, x_4, r_1, r_2 \geq 0 \\
f_{12} free
\end{gathered}$$

6.3 Results

Using this primal we may determine two separate scenarios for an unconstrained reserve case with maximal reserve offers set to 250 MW. And a constrained reserve case with maximal offers set to 50 MW. The results of both scenarios are as follows.

6.3.1 Unconstrained Case

Table 3: Dispatch Solution

Variable	Value
x_1	100
x_2	0
x_3	100
x_4	200
r_1	100
r_2	0
π_1	100
π_2	100

Table 4: Profit and Loss

Participant	Profit
G_1	$0
G_2	$0

6.3.2 Constrained Case

Table 5: Dispatch Solution

Variable	Value
x_1	100
x_2	50
x_3	50
x_4	200
r_1	50
r_2	0
π_1	200
π_2	100

This shows a clear incentive for an integrated generator to exert market power.

Table 6: Profit and Loss

Participant	Profit
G_1	-$15,000
G_2	$10,000