Reserve Constraints in Co-Optimised Electricity Markets - A theoretical and empirical study of the New Zealand market

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Abstract

In this paper we will discuss the implications of reserve constraints acting upon energy prices in a co-optimised electricity market. We will identify five situations and illustrate them using simplified linear programs based upon the Scheduling, Pricing and Dispatch model used in New Zealand. We will then use the insights gleaned from our analysis to screen the empirical data from the 2008-2010 calendar years. We will identify over 9000 constrained periods in total and theoretically demonstrate, in a simplified setting, how a generator may utilise these reserve constraints to profit. This demonstration will illustrate the primary cash flows between participants and identify the possibility for a integrated participant to exert market power to extract rentals.

Paper Outline

Section one will cover an introduction to the issue along with an overview of the relevant literature from both academic and industrial publications. Section two is a background section which will provide a brief geographical understanding of the nature of the New Zealand grid. Section three will familiarise the reader with the crucial aspects of the Scheduling, Pricing and Dispatch model (SPD) which is used to dispatch the grid. In Section four simplified linear programming models will be developed to showcase the identified mechanisms. Section five will highlight the real world effects, highlighted through a statistical filter based upon our models. Section six will detail the incentives a generator may have to withhold reserve whilst section seven will highlight the aggregate effects of these constraints. Finally, in section eight we will draw several conclusions from this paper and develop avenues for further research.

1 Introduction

The New Zealand electricity grid is a co-optimised reserve and energy constrained system. Every thirty minutes a model known as the Scheduling, Pricing and Dispatch model utilizes offers and bids from generators, consumers and reserve providers to construct an optimal dispatch. This optimal dispatch is for a nodal based system [Schweppe et al., 1988] and is a large network flow linear programming model operated by the System Operator, a unit of Transpower. This system is robust and since the deregulation of the grid in 1996 has been successfully dispatching optimal solutions, given the market determined input data, to the grid dispatch problem [Goodwin, Douglas, 2006].

This system, although robust, is not perfect and subject to manipulation due to the unique nature of the grid itself. The grid may be classified as "long and skinny" and was designed to transmit energy from the South Island hydro schemes to the North Island load centres. This design has led to transmission and reserve becoming key constraints. Spring washer situations, due to the transmission constraints, heavily influence the optimal dispatched solution [Transpower, 2010b] leading to complex nodal pricing. Furthermore, the total size of the grid is small with a peak load of approximately 6.6 GW in 2011 [EA, 2012]. This creates difficulty in securing the grid against instantaneous frequency drops due to generator disconnection. Frequency keeping (Frequency Response) as a rule of thumb is approximately 1% of peak load [NERC, 2011] or approximately 60 MW, far below the hundreds of MW required to secure against a large Combined Cycle Gas Turbine (CCGT)unit disconnecting. Thus there is insufficient "frequency response" to halt a fall in the operating frequency in the New Zealand grid. To overcome this limitation the New Zealand System Operator is forced to dispatch separate fast and sustained reserves to secure the grid and maintain N-1 security.

This reserve is procured from two major sources, Interruptible Load (IL) and Generation, with hydro generation providing either Partially Loaded Spinning Reserve (PLSR) or Tail Water Depressed Reserve (TWDSR). It is this provision of reserves which leads to the co-optimisation of both energy and reserve dispatches to create the optimal dispatch in the SPD model [Transpower, 2007a]. However, an issue of this reserve dispatch is the potential for new avenues of strategic market power which can be induced [Chakrabarti, 2007a]. This paper will discuss five scenarios where reserve constrains the energy dispatch using simplified linear programming models to shed light upon these insights. These scenarios include reserve constraining generation below the rated capacity of a unit. Reserve constraining a transmission line between two nodes and the special case of the transmission line constraint incorporating the set of reverse bath tub constraints of mixed energy and reserve dispatch from a hydro unit. These situations are different to the common transmission congestion constraint identified by earlier authors [Chakrabarti, 2006] and are a direct consequence of the energy-reserve co-optimization. Following the identification of these mechanisms empirical data is presented to showcase the effect of these constraints on the New Zealand Grid.

Several parties have identified the occurrence of a binding reserve constraint [Commission, 2010, Energy, Genesis, 2010, Smith, 2010] but no prior literature exists surrounding the mechanism through which it binds. Furthermore, as a consequence of this identification by the regulatory bodies a series of rule changes have been implemented [EA, 2010]. These rule changes were a direct result of model alterations made by the system operator to solve the constrained linear programming model including variable reserve adjustment factors and the treatment of net free reserves. However these modifications lead to a less secure system as insufficient reserve will be dispatched increasing the likelihood of an AUFLS (Automatic Under Frequency Load Shedding) type event. Furthermore, they reduce the information value of prices through the suppression of final prices [Smith, 2010]. This paper will identify that these periods are due to the inherent design of the market, not minor rule changes. The SPD model functions appropriately during the majority of situation and in the situations identified the constrained pricing was due to a shortfall in the participant offers. Thus, mechanisms to rectify these shortfalls will be examined for future consideration.

2 Background

New Zealand is a country of two islands located in the south pacific. Peak New Zealand demand is approximately 6.6 GW with the majority of the residential demand located in the far North around the major population hub of Auckland. However, New Zealand under the previously centrally operated system invested heavily in South Island hydro schemes which play a major role in meeting the countries generation needs. These hydro schemes have low water storage, 6 weeks at peak demand, and as such the New Zealand grid is sensitive to adverse weather conditions. Dry year situations are common with a number of provisions including conservation campaigns in place for such an occurrence.

The two islands are linked through an HVDC line running from Benmore in the South Island to Haywards in the North Island. This linkage currently consists of two poles and is rated to 700 MW northward flow and 500 MW southward flow [Transpower, 2010a]. This link is currently undergoing a major upgrade with the commissioning of a new Pole 3 and subsequent decommissioning of Pole 1 to occur in stages culminating in 2017. This upgrade will improve the rated capacity of the linkage to 1400 MW northward and 1000 MW southward. Additionally, Transpower is currently undertaking other major investments around the Whakamaru line (a constrained transmission line) in the North Island and around Auckland city [Transpower, 2012]. A graphical representation is found in Figure 7.

3 SPD Model

The SPD model is a joint energy and reserve co-optimised linear programming network flow model which maximises total system welfare. The SPD model matches an inelastic demand curve with a supply curve fixed at least two hours in advance from generator offers. No day ahead market is used in the NZ market. At this point all reserve offers are also input to the model and the reserve requirements and dispatch are co-optimised with energy as part of the total system dispatch. Here, in the interests of brevity, it will be assumed that the reader has a basic knowledge of linear programming and the development of energy dispatch models. For further information regarding the SPD model itself please see the model documentation provided by the System Operator [Transpower, 2011b].

The objective function of the SPD model is as shown in Equation 1. For completeness it is noted that constraint violation penalties are not included in this objective function. This expression shows that the provision of reserves is a net cost to the system and that a least cost dispatch will be procured. This section will use the nomenclature as set out in Table 1

$$max \sum p_P P - \sum p_g G - \sum p_R R \qquad (1)$$

Thus, conceptually we understand that reserves are a net cost to the system to be optimised. It is now worthwhile to identify the purpose of reserve from both the conceptual framework and its mathematical implementation. From this conceptual framework our simplified linear programming models will be constructed.

Reserve, at the conceptual level, is dispatched to protect the integrity of the grid from unforeseen events. These events, commonly known as contingent events or under frequency events, are relatively common with varying severity [Transpower, 2007b].Of these events two are considered to be of the most risk to the system itself:

1. Loss of a major generating unit (typically thermal CCGT plants)

2. Partial or complete loss of the interconnecting HVDC line between the islands.

The System Operator uses a custom, proprietary, tool known as the Reserve Management Tool (RMT) to determine any mitigating factors present with a particular grid dispatch [Transpower, 2011a]. This mitigation factor, or risk offset, is subtracted from the raw numerical risk to determine an island risk to be secured against.

Thus, conceptually reserve is an imposed constraint upon the maximum generation capacity of a thermal unit, or the rated capacity of a transmission line. From this conceptual basis, equations 2, 3 and 4 are

Table 1: Notation Used

P	Sets of Purchase bids and estimates of load quantities used by the System Operator
G	Sets of Generation offers from all units within the system
R	Sets of Reserve offers from all units within the system
p_i	Price of the respective bids and offers from the market participants
$F_{transfer}$	HVDC transfer between islands
$Offset$	Offset factor determined using the RMT
RAF	Binary value set by the System Operator. The value is set to 1 except following a grid emergency when the SO will set it to zero to allow additional freedom in reconfiguring the grid. It is noted here that mechanisms exist for incrementally decreasing the RAF in the event of an infeasible solution. This indicates that insufficient reserve to arrest grid frequency falls may be dispatched.
$Risk$	Island wide risk which must be secured using reserve
G_{risk}	The risk quantity associated with the largest thermal generation unit operating
R_{risk}	Any reserve dispatched from the large thermal generation unit associated with the marginal risk must also be secured
$R_{dispatch}$	Dispatched Reserve from the reserve providers
$R_{spinning}$	The dispatched quantity of reserve from a spinning provider
$G_{spinning}$	The dispatched quantity of energy from a spinning provider
ρ	A fraction specified by the generation unit in its offers to the SO
$R_{maximum}$	The maximum quantity of reserve which may be dispatched from a particular spinning unit
$G_{maximum}$	The total maximum quantity of reserve and generation which may be dispatched from a unit (i.e. unit rated capacity).

developed. These equations govern the security risk of the current system dispatch as follows.

$$Risk \leq \sum R_{dispatch} \tag{2}$$

$$Risk \geq RAF(F_{receiving} - Offset) \tag{3}$$

$$Risk \geq RAF(G_{risk} - Offset) + \sum R_{risk} \tag{4}$$

We now proceed to the special inverse bathtub constraint which governs the dispatch of spinning reserve within the system. The relevant equations within the SPD model have been rewritten as equations 5, 6 and 7. These equations may also be depicted graphically as shown in Figure 1 [Chakrabarti, 2007b], the so called "reverse bath tub" constraint.

$$R_{spinning} \leq \kappa G_{spinning} \tag{5}$$

$$R_{dispatch} \leq R_{maximum} \tag{6}$$

$$G_{spinning} + R_{spinning} \leq G_{maximum} \tag{7}$$

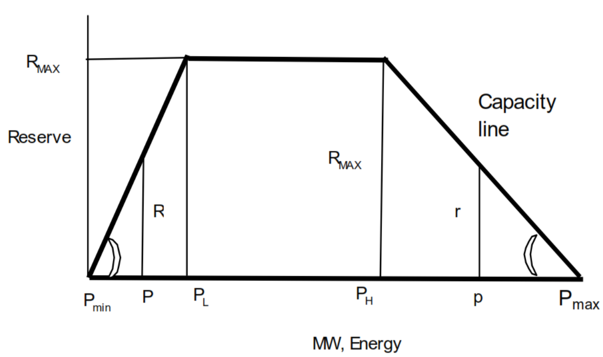

Figure 1: Graphical Depiction of the Reverse Bathtub Constraints [Chakrabarti, 2007b]

The full set of reverse bath tub constraints is relevant for Spinning Reserve generation units only. In conceptual terms it states the following:

1. That reserve may not be dispatched in excess of a proportion of the energy dispatch [1]
2. That an upper limit with respect to reserve dispatch exists
3. That the total quantity of reserve and generation dispatched must be less than the total unit rated capacity.

[1] This is due to the physical ramping capabilities of each unit as well as the market requirement for $\leq$ 6s dispatch speed

In the subsequent section we will develop simple linear programming models that will identify constrained situations caused by the co-optimisation of energy and reserve. Using this we will analyse the impact of such situations on price in a simplified setting.

4 Simplified LPs showcasing Constrained Situations

This section will develop the situations through which a constraint in the reserve dispatch impacts the subsequent energy dispatch. We will use simplified linear programming models, based upon insights gleaned from the SPD model to highlight these effects. The following nomenclature will be used. Full descriptions of the linear programs are located in the Appendix at
http://www.epoc.org.nz/publications.html.

Table 2: Notation used in developing Linear Programs

x_g	Energy dispatch from unit g
r_r	Reserve dispatch from unit r
$f_{i,j}$	Flow between nodes i and j
d_i	Demand at node i
π_i	Energy price at node i
λ_i	Reserve price at node i
κ_i	Proportion constant for unit i
σ_i	Combined Generation/Reserve Constraint
ρ_i	Reserve Maximal Offer Constraint

4.1 Marginal Generation constrained by Reserve

This small model pertains to a single node with two large generation units. Reserve must be procured to secure against the risk these units provide to the system. Figure 2 describes the system, we note here that for brevity we have included the parameters directly in the primal LP as described below, for a full tabular reference please see the Appendix.

The Linear program may be formulated as follows:

Equations 8a) represents the nodal balance and associated nodal price while equations 8b) and 8c) represent the reserve balances and nodal reserve price. From this primal formulation we may determine the

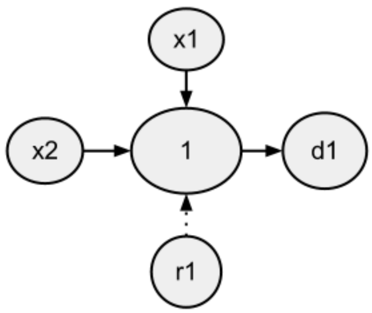

Figure 2: Single Node System

$$min \;\; 0.01x_1 + 100x_2 + 30r_1 \tag{8}$$

subject to:

$$\begin{aligned}
8a) \quad & x_1 + x_2 = d_1 \quad \perp \pi_1 \\
& x_1 \leq 400 \\
& x_2 \leq 400 \\
& r_1 \leq 400 \\
8b) \quad & r_1 - x_1 \geq 0 \quad \perp \lambda_1 \\
8c) \quad & r_1 - x_2 \geq 0 \quad \perp \lambda_1 \\
& x_1, x_2, r_1 \geq 0
\end{aligned}$$

Table 3: Optimal Solution to Model One

Variable	Value
x_1	350
x_2	0
r_1	350
π_1	30.01
λ_1	30

associated dual program and back calculate the shadow prices. For this particular primal the following holds true.

$$\pi_1 = 0.01 - \lambda_1 \tag{9}$$

Where $\pi_1, -\lambda_1$ s the energy price at node one and reserve for the system respectively. Here we recognise that the energy price consists of the offer price for unit one plus the reserve clearing price. This coupling is a direct consequence of the co-optimisation of energy and reserve. The full results for this primal are contained in Table 3, with the results as expected from the shadow price calculation. This situation is equivalent to a large CCGT unit operating as risk setter and marginal generator.

4.2 Nodal Transmission constrained by non-generator reserve

This small model pertains to a two node system with a single transmission line, now reserve is offered to secure against the loss of a transmission line between Node 1 and Node 2 as shown in Figure 3. Once again for brevity we have directly included the system parameters in the primal with a full tabular description in the Appendix.

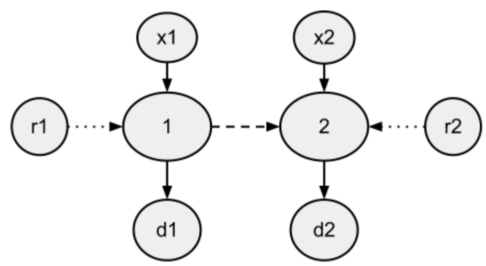

Figure 3: Two Node System with Transmission

The LP for this situation is as follows

$$min \;\; 0.01x_1 + 100x_2 + 30r_1 + 30r_2 \tag{10}$$

subject to

$$\begin{aligned}
10a) \quad & x_1 - f_{12} = 50 \quad \perp \pi_1 \\
10b) \quad & x_2 + f_{12} = 300 \quad \perp \pi_2 \\
10c) \quad & -f_{12} - r_1 \leq 0 \quad \perp \lambda_1 \\
10d) \quad & f_{12} - r_2 \leq 0 \quad \perp \lambda_2 \\
& x_1 \leq 400 \\
& x_2 \leq 400 \\
& r_1 \leq 400 \\
& r_2 \leq 400 \\
& f_{12} \leq 1000 \\
& -f_{12} \leq 1000 \\
& x_1, x_2, r_1, r_2 \geq 0 \\
& f_{12} free
\end{aligned}$$

Now, Equations 10c) and 10d) indicate that reserve must be procured to secure the nodal transfer, not the generation units. From the associated dual solution we determine that the following equation will hold true.

$$\pi_2 = \pi_1 - \lambda_2 \tag{11}$$

That is, the price at node 2 is equal to the marginal energy cost at node 1, plus the reserve cost associated with transferring it to node 2. This model predicts that nodal price separation will occur during these situations with the difference equal to the reserve cost as shown in Table 4. Physically, this situation is based upon HVDC inter-island transfer in the New Zealand grid.

Table 4: Optimal Solution to the Transmission Constrained LP model

Variable	Value
x_1	350
x_2	0
r_1	0
r_2	300
f_{12}	300
π_1	0.01
π_2	30.01
λ_2	30

4.3 Nodal Transmission Constrained by Bathtub Constrained Reserve

This scenario has three possible outcomes depending upon the type of constraint binding. Three separate models will be developed to highlight this using a two node system. Reserve is dispatched in conjunction with energy, that is generation units serve a dual role. Reserve is dispatched to secure against the nodal transfer.

4.3.1 Proportionality Constraint

This constraint is the first initial slope. It states that reserve may not be dispatched independently of generation as specified in equation (5). The diagram is depicted in Figure 4. A proportionality constraint of 0.5 will be used in the primal and is denoted by κ. Here κ is a system variable as specified by the generators in their offers. Typical values can range from 0% to 250% with limits subject to trader decisions, unit capability and desired generation unit's configuration. The system parameters are once again specified directly within the LP.

The primal is formulated as follows:

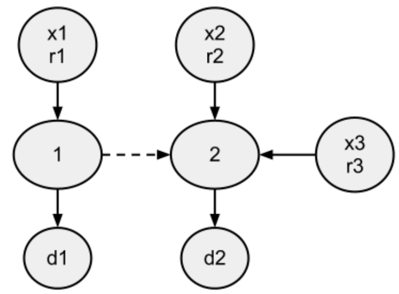

Figure 4: Transmission Constrained Model System with Bathtub Constraints

$$min \;\; 0.01x_1 + 1000x_2 + 10x_3 + 0r_1 + 10r_2 + 0.01r_3 \tag{12}$$

subject to:

$$\begin{array}{lcr}
\text{12a)} & x_1 - f_{12} = 50 & \perp \pi_1 \\
\text{12b))} & x_2 + x_3 + f_{12} = 305 & \perp \pi_2 \\
\text{12c)} & f_{12} - r_2 - r_3 \leq 0 & \perp \lambda_2 \\
\text{12d)} & -f_{12} - r_1 \leq 0 & \perp \lambda_1 \\
 & x_1 \leq 300 & \\
 & x_2 \leq 50 & \\
 & x_3 \leq 300 & \\
 & r_1 \leq 300 & \\
 & r_2 \leq 50 & \\
 & r_3 \leq 300 & \\
 & f_{12} \leq 1000 & \\
 & -f_{12} \leq 1000 & \\
 & x_1 + r_1 \leq 300 & \\
 & x_2 + r_2 \leq 50 & \\
 & x_3 + r_3 \leq 300 & \\
\text{12e)} & r_1 - \kappa_1 x_1 & \perp \omega_1 \\
\text{12f)} & r_2 - \kappa_2 x_2 & \perp \omega_2 \\
\text{12g)} & r_3 - \kappa_3 x_3 & \perp \omega_3 \\
 & x_1, x_2, x_3, r_1, r_2, r_3 \geq 0 & \\
 & f_{12} free & \\
 & \kappa_1, \kappa_2, \kappa_3 = 0.5 &
\end{array}$$

Equations 12e), 12f) and 12g) are new here and represent the proportionality constraint. From this primal we may determine the dual program and given a solution back calculate the mechanism of the shadow price calculation. To determine the shadow prices we set the unconstrained shadow price at node one, π_1 equal to the offer price \$0.01. From there we may use the following equations to determine the node two energy price(π_2), reserve price (λ_2) and the applied constraint charge (ω_2). Where ω_2 is the constraint charge related to equation (5). After several manipulations the nodal energy prices may finally be calculated as follows:

$$\pi_1 = 0.01$$
$$\pi_2 = 1000 - \frac{989.9\kappa_2}{1+\kappa_2}$$
$$-\lambda_2 = \frac{10\kappa_2 + 999.9}{1+\kappa_2}$$

Here, we note that these calculations quickly become more complex due to the simultaneous equations which must be solved to determine the shadow values. Given that due to equation (5) reserve may not be dispatched independent of reserve. Thus, to meet the marginal MW pricing requirement a combination of expensive energy and low cost reserve (to cover the low cost energy from Node 1) is dispatched. In this situation the model must dispatch the expensive peaking plant type generator due to its inability to obtain reserve via other means. The full solution to this model is described in Table 5.

Table 5: Optimal solution to model transmission constrained LP with proportionality constraints

Variable	Value
x_1	151.667
x_2	3.333
x_3	200
r_1	0
r_2	1.667
r_3	100
f_{12}	100
π_1	0.01
π_2	670.003
λ_2	669.993
ω_2	-659.933

4.3.2 Total Reserve Constraints

This constraint binds when the total upper bound on reserve dispatch is insufficient for an optimal solution. Here, designed reserve is offered to secure against the loss of nodal flow and is interruptible load independent of generation as described in Figure 5. Once

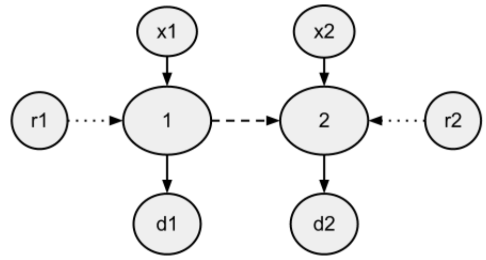

Figure 5: Total Reserve Constrained System

The primal is formulated as follows

$$min \;\; 0x_1 + 1000x_2 + 0r_1 + 0r_2 \qquad (13)$$

subject to

$$\begin{array}{lcl}
13a) & x_1 - f_{12} = 150 & \perp \pi_1 \\
13b) & x_2 + f_{12} = 150 & \perp \pi_2 \\
13c) & -f_{12} - r_1 \leq 0 & \perp \lambda_1 \\
13d) & f_{12} - r_2 \leq 0 & \perp \lambda_2 \\
 & x_1 \leq 500 & \\
 & x_2 \leq 200 & \\
13e) & r_1 \leq 50 & \perp \rho_1 \\
13f) & r_2 \leq 50 & \perp \rho_2 \\
 & f_{12} \leq 200 & \\
 & -f_{12} \leq 200 & \\
 & x_1, x_2, r_1, r_2 & \\
 & f_{12} free &
\end{array}$$

again we have directly incorporated the system parameters within the LP.

Equations 13e) and 13f) state that the total upper bound on reserve dispatch is very low. Given this situation and the associated dual value the solution to the primal may be formulated with the shadow price mechanism back calculated to give the following equations:

$$\pi_2 = \pi_1 - \lambda_2$$
$$-\lambda_2 = 0 - \rho_2$$
$$\pi_2 = \pi_1 - \rho_2$$

Here, the constraint charges, ρ_2 are incorporated into the reserve cost, and through this reserve cost into the nodal energy price at node two. The reserve price is equal to the constraint charge as a relaxation in the constraint would reduce \$1000 peaking plant generation by 1 MW in favour of the \$0 priced generation at node 1. Full results of this LP are depicted in Table 6

Table 6: Optimal Solution to Model Total Reserve Constrained System

Variable	Value
x_1	200
x_2	100
r_1	0
r_2	50
f_{12}	50
π_1	0
π_2	1000
$-\lambda_1$	0
$-\lambda_2$	1000
ρ_2	1000

4.3.3 Total Reserve and Generation Dispatch Constraint

This is the final bathtub constraint which provides an upper bound to the total reserve and energy dispatched from a unit to be less than that units physical capacity. The physical situation is described in Figure 6.

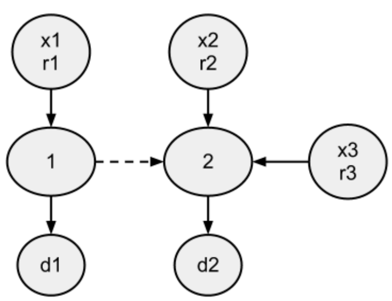

Figure 6: Total Reserve and Generation Dispatch Model

This situation requires the re-visitation of an earlier assumption made in the previous models. Earlier it was assumed that a 1:1 requirement for reserve to secure against a risk was required. However, experience and the SPD model itself has shown this assumption to not be accurate. Thus, we will incorporate a value, δ to represent the proportion of the security risk which must be secured. Thus, a MW of reserve may be used to secure more than 1 MW of security risk. With this revision in mind we formulate the following LP:

$$min\ \ 0x_1 + 1000x_2 + 50x_3 + 0r_1 + 100r_2 + 70r_3 \quad (14)$$

subject to:

$$\begin{array}{lcl}
 & \sigma = 0.75 & \\
14a) & x_1 - f_{12} = 150 & \perp \pi_1 \\
14b) & x_2 + x_3 + f_{12} = 180 & \perp \pi_2 \\
14c) & -\sigma f_{12} - r_1 \leq 0 & \perp \lambda_1 \\
14d) & \sigma f_{12} - r_2 - r_3 \leq 0 & \perp \lambda_2 \\
 & x_1 \leq 500 & \\
 & x_2 \leq 50 & \\
 & x_3 \leq 150 & \\
 & r_1 \leq 500 & \\
 & r_2 \leq 50 & \\
 & r_3 \leq 150 & \\
14e) & x_1 + r_1 \leq 500 & \perp \sigma_1 \\
14f) & x_2 + r_2 \leq 125 & \perp \sigma_2 \\
14g) & x_3 + r_3 \leq 150 & \perp \sigma_3 \\
 & f_{12} \leq 500 & \\
 & -f_{12} \leq 500 &
\end{array}$$

Here, equations 14e), 14f) and 14g) are the combined dispatch constraints whilst equations 14c) and 14d) have been rewritten to reflect the reduction in reserve requirements to secure flow. Based upon this primal we may determine the associated dual and solution to the primal as shown in Table 8. The shadow prices are determined by simultaneously solving the equation below with $\pi_1 = 0$.

$$\begin{array}{c}
-\pi_1 + \pi_2 + \sigma\lambda_2 = 0 \\
\pi_2 + \sigma_3 = 50 \\
-\lambda_2 + \sigma_3 = 70
\end{array}$$

Table 8 showcases that unit 3 is delivering a combined energy and dispatch to meet the demand in Node 2. Given that the reserve multiplier exists a trade off becomes apparent. Reserve, is more expensive than energy from unit 3. However, there is insufficient energy to meet demand without also dispatching the expensive peaking unit. Thus, a combination of low cost energy, and higher cost reserve is dispatched to meet the demand at node 2. The constraint charge σ_3 reflects the trade off which exists and is dependent upon the respective prices of energy, reserve and the proportionality constant σ. In general terms an equation for σ may be formulated as follows. It is noted here that the σ constraint will bind only when negative.

$$\sigma = \frac{P_E - \sigma P_R}{1-\sigma} \tag{15}$$

For the two potential generation units, x_2, x_3 the following constraint charges may be determined.

Table 7: Unit Constraint Charges

Unit	P_E	P_R	σ
x_2	1000	100	3700
x_3	50	70	-10

Thus, given that σ_3 is negative the constraint is in effect and a trade off between energy and reserve for that unit exists.

Table 8: Solution to Combined Dispatch Model

Variable	Value
x_1	270
x_2	0
x_3	60
r_1	0
r_2	0
r_3	90
f_{12}	120
π_1	0
π_2	60
$-\lambda_1$	0
$-\lambda_2$	80
σ_3	-10

5 Empirical Evidence

In this section we will utilise the insights gain from our small models to develop data filters which we have applied to the available price data. We have used the results of our simplified LP models, specifically the dual equations formulated for reserve and energy shadow prices. By manipulating these equations we were able to plot simplistic linear fitted diagrams show casing the effects of reserve constraints on energy prices. For completeness, the two primary dual price calculations are included here. Equation (16) refers to the single node case, whilst Equation (17) refers to all of the two node scenarios.

$$\pi - p = \lambda_F + \lambda_S \tag{16}$$

$$\pi_N - \pi_S = \lambda_F + \lambda_S \tag{17}$$

where

π	Nodal energy price
p	Energy offer price for the marginal generator
λ	Reserve Price
N/S	North/South Island
F/S	FIR or SIR

5.1 Generation Constrained by Reserve

Generation units are rarely constrained by reserves in the New Zealand grid and a complex filtering mechanism is needed to identify its occurrence. Initially the dataset was limited to the Otahuhu CCGT plant which is conveniently at the Otahuhu node, a major reference node for price in the North Island. Furthermore, the data set was limited to the 2008, 2009 and 2010 calendar years. The filtering mechanism was applied to the finalized generation offers and the finalized Otahuhu energy price and North Island reserve price through the application of the following steps. These steps were designed to identify the trading periods where the Otahuhu thermal unit was both the risk setter and marginal generation unit and was based upon the insights gleaned from Section 4.1.

1. All offer tranches which are priced beneath the final nodal energy price were eliminated.
2. All zero offer tranches were eliminated.
3. All offer tranches priced below \$1 were eliminated.
4. The following equation must be satisfied.

$$-0.01 \leq \pi_{Ota} - p_{Ota} - \lambda_{FIR} - \lambda_{SIR} \leq 0.01 \tag{18}$$

Where π_{Ota} is the Otahuhu nodal energy price, p_{Ota} is the offer tranche price, $\lambda_{FIR}, \lambda_{SIR}$ are the North Island FIR and SIR prices respectively.

In total 168 out of 52,608 trading periods or 0.3% were identified as meeting these criteria and are displayed in Figure 8. This minuscule amount was partially self-induced, as for simplicity we only considered the Otahuhu unit, and partly structural as the probability of a thermal unit acting as both marginal generator and risk setter is low. It is expected that the Huntly E3P and Stratford CCGT units would also display similar behaviour. However, as these units are not located at major reference nodes the possibility of line losses or transmission constraints influencing the data set is large. A linear trend line, $y = x$ has been fitted to the data. $R^2 = 0.9999$.

5.2 Transmission Constrained by Reserve

In New Zealand with the critical importance of the HVDC link to national grid operations the presence of a reserve constraint binding upon the transmission line is relatively simple to identify. However, as we have previously identified multiple mechanisms for the final prices a secondary filtering step must be utilized to classify the trading periods by scenario. The first stage of the filter was to identify when the constraint exists used the following two constraints. Here \$10/MWh was used as the cut-off to allow for minor deviations due to transmission losses. The results are not sensitivity to this cut-off value and we note that this could be reduced substantially without obfuscating the relevant data.

$$\pi_{Hay} - \pi_{Ben} \geq \$10/MWh \tag{19}$$

$$\pi_{Hay} - \pi_{Ben} - \lambda_{FIR,N} - \lambda_{SIR,N} \leq \$10/MWh \tag{20}$$

$$\pi_{Ben} - \pi_{Hay} - \lambda_{FIR,S} - \lambda_{SIR,S} \leq \$10/MWh \tag{21}$$

Where the satisfaction of (19) and (20) indicates the constraint exists on transfer from the South to North Island, When (19) and (21) are satisfied the constraint exists on North to South transfer. Note, this is an either or designation, equations (20) and (21) cannot be in effect simultaneously.

However, this blanket filter will capture the occurrence of all occurrences of the identified transmission constraints. A second stage filter mechanism was added to categorise the data set as belonging to either a simple reserve constraint or the combined bathtub constraints. Significant difficulty exists in classifying each individual bathtub constraint due to the differing roles of constraint charges in each circumstance. To categorise the data set as belonging to the simple reserve constraint both FIR and SIR final prices were checked against the final reserve offer stack. If both of the prices could be associated with a reserve offer they were classified as a simple reserve constraint, if not, the bathtub constraint. However, this filter is insufficient to distinguish between individual bathtub constraints. Simply, this filter is a brute force search method which checks each reserve price against the offer prices for the trading period in question.

The identification of offered reserve acting as a constraint is relatively rare with just 182 periods in the North Island and 361 in the South Island identified. However, the occurrence of scenario three where the reserve prices cannot be found on the offer curves is extremely frequent with 6886 and 1946 occurrences identified in the North and South islands respectively. To explain this disparity an understanding of the reserve offer curve is required. Reserve offers exhibit a hockey stick shape with significant zero priced offers. As we are performing a retroactive analysis the trading periods where flow is constrained by zero priced reserve is essentially invisible as no price deviations will occur. Figures 9, 10, 11, 12 showcase the empirical data identified using the two stage filter. Here Figure 9, offered reserve constraining north to south flow, exhibits the noisiest data set. During these periods the tolerance used of \$10 was insufficient to filter out some of the less conclusive data points. This in turn led to the greater deviation. However, given that losses still exist on the HVDC cable it is expected that some deviation will occur. This deviation in all cases will be biased towards an increased nodal price separation which cannot be fully explained by reserve. This is consistent with pricing under transmission losses scenarios. No modification was made to the filter to exclude these data as the trend is still readily apparent.

6 System Implications of these Constraints

Given the two node system used throughout the model formulation we can begin to form plausible possibilities for the implications of these effects on generators. Here we will identify three broad hypothetical scenarios:

1. Generator situated at sending node
2. Generator situated at receiving node
3. Generator situated at both nodes

In these scenarios we will assume that there are many other generators who are not otherwise specified. Furthermore a generator will be classified as either a net buyer or a net seller at a specific node subject to their exposure to the spot market and their contractual obligations.

6.1 Generator at sending node

This generators profits and losses depend upon its exposure to the spot market. If the generator is a net buyer at node 2, which it is attempting to meet with low cost generation at node 1 then it will suffer losses proportional to any constraint charges incurred. However, if this generator has no contractual obligations to supply in the opposite node, it simply has an excess of supply (for example, overflowing hydro lakes). This generator will receive additional profits proportional to the quantity of excess energy supplied to the receiving node.

6.2 Generator at receiving node

If this generator is a net buyer at the receiving node it is in the generators interest to provide a maximal quantity of reserve at low cost to reduce the constraint upon the line and maximize the availability of low cost generation. This behaviour is consistent with a retailer attempting to minimize the cost of its contractual obligation to supply. However, if the generator is a net seller at the node it is in the generators interest to heavily constrain the transmission line. By constraining the line the generator will restrict the availability of low cost generation to the node which must be replaced by the generators own higher priced generation. In this situation the generator will obtain additional profit equal to its net generation position multiplied by the nodal price difference.

6.3 Generator at both nodes

This is the most interesting of the three scenarios. To simulate potential motives simple contractual requirement for two different generation companies were set up at each Node. A four generator system with each generator containing a single tranche at each node was introduced and profit and loss calculations for each generator under unconstrained and constrained reserves were developed. We have deliberately set up this scenario in a manner such that constraint charges will eventuate to highlight this situation. A summary of the results of the two situations evaluated is depicted in Table 9. For a full summary of this dispatch problem please see the accompanying Appendix.

Table 9: Profit and Loss for Two Generators

	Unconstrained Reserves	Constrained Reserves
π_1	\$100/MWh	\$200/MWh
π_2	\$100/MWh	\$100/MWh
G_1 Profit	\$0	-\$15,000
G_2 Profit	\$0	\$10,000

Here, Generator 2 has clearly profited in the reserve constrained situation at the expense of Generator 1. Thus, we conclude that in certain situations it is in the generators best interests, based upon their net buy and sell positions, to constrain reserve production.

7 System Cost

The previous sections have detailed the mechanisms, empirical evidence and incentives surrounding reserve constraints in a co-optimised settings based upon the New Zealand grid. We have identified an allocation inefficiency that may distort long term signals within the market. Here we must segue into a brief understanding of the cash flows within the market. Reserve is dispatched by the SO to maintain N-1 security. To pay for this, it levies the cost of reserve onto the generators who are providing risky generation. This cost is thus borne fully by the generators and will be ultimately passed through to the consumer either directly or through higher levies. However, as our analysis has shown the consumer is also liable to pay the cost of reserve, through spot transactions, during those periods when a constraint is in effect. During these periods the consumer will pay the security price directly to the generators in the form of higher energy prices. To further evaluate these scenarios we have identified the two primary cash flows, for a single period, as follows in Equations 22 and 23.

$$P_{C,G,Reserve} = (\lambda_{F,I} + \lambda_{S,I})_\theta \times L_{I,\theta} \tag{22}$$

$$P_{G,R} = \lambda_{F,I} \times D_{F,I} + \lambda_{S,I} \times D_{S,I} \tag{23}$$

where

$P_{i,j}$	Payments from i to j
$\lambda_{c,I}$	Reserve price for type c, island I
$L_{I,\theta}$	Energy Load for island I, constrained period θ
$D_{i,I}$	Reserve dispatch for type i, island I
C, G, R	Consumers, Generators and Reserve providers respectively
F, S	Fast and Sustained Reserves respectively
θ	A period where a reserve constraint charge exists

To further our analysis of the system we aggregate these cash flows over time to determine total payments for a calendar year. We have studied three years in question, 2008 to 2010 and have identified that these cash flows were greatest in 2008 (South Island) and 2009 (North Island). This empirical first order estimation is set out in Table 10 and clearly shows that overpayment for reserve has occurred. We have not attempted to assess individual generator net positions, nor individual cash flows between participants as some difficulty exists in identifying contract positions. However, we reflect that increased volatility in wholesale prices will lead to higher hedge and retail costs for all consumers. Thus, any increase in the wholesale cost of electricity due to these constraint charge will be eventually passed on to consumers. A further assessment of these cash flows has been made in Figures 13 and 14. This assessment highlights the impact of the national hydrological position and its effect upon binding constraint charges within the system.

Table 10: Aggregated System Costs

	2009 NI (millions NZD)	2008 SI (millions NZD)
$P_{C,G,Total}$	\$1,242	\$1,939
$P_{G,R}$	\$64	\$39
$P_{C,G,Reserve}$	\$319	\$63
$P_{C,G,Reserve} - P_{G,R}$	\$255	\$24

These first order calculations clearly indicate that an overpayment, or rental, exists in the current system. We will mathematically simplify the above equations to identify the form of these overpayments which can occur. During the constrained periods the rental payment simplifies to the difference between island wide load and reserve dispatch multiplied by the reserve prices. For completeness we note that these rental cash payments will only eventuate when a constraint exists. During all other periods no method of direct cash flow due to reserve, from consumers to generators, exists within the system.

$$P_{C,G,Rentals} = P_{C,G,Reserve} - P_{G,R} \tag{24}$$

$$P_{C,G,Rentals} = \lambda_{F,I}(L_{I,\theta} - D_{F,I,\theta}) + \lambda_{S,I}(L_{I,\theta} - D_{S,I,\theta}) \tag{25}$$

Given that $L_I \gg D_I$ by an order of magnitude in some cases a generator will clearly profit due to these distribution effects. These situations when a generator may profit are relatively infrequent, yet the nature of the distortion is still clear and situations can arise where constraints may pose a significant cost to the system as shown in Figure 14. We note that there exists the potential for a participant in both markets to express market power to obtain these rental payments. This may produce misaligned incentives reducing the long term efficiency of market operation resulting in higher prices for all consumers.

Solving this issue of misaligned cash flows is not a simple matter however within the current system. The issue itself revolves around the appropriate alignment of incentives between the different stakeholders. Removing the constraint costs from the energy price and directly charging consumers, not generators for the reserve costs would resolve the cash flow issue. Yet, generators would then have no incentive to increase reliability as they are fire walled from the costs of their actions. The full analysis of a method to realign these incentives properly is left for further work.

8 Conclusions

In this paper we have discussed both the mechanism and potential systemic effects of reserve constraints on energy prices. Through an analysis of the governing Scheduling Pricing and Dispatch mode we have created simplified linear programming models to examine constrained reserve pricing. A common aspect of these mechanisms was the effect of a marginal source of generation being constrained by the procurement of reserve. Furthermore, the procurement of reserve as subject to its own set of constraints may incur significant constraint charges which may impact the energy price. In total, five separate mechanisms for elevated energy prices caused by a lack of reserve were detailed.

In section five we used a data set for the 2008 to 2010 calendar years and create a series of filters to determine the periods when reserve constraints were binding. This analysis indicated that reserve binds far more frequently than first estimated at a substantial cost to the system as a whole. Furthermore, this binding occurs over a range of prices from less than $20 up to $4000 (NZ Dollars) in one case. Based upon the linear programs developed and the filtered data set we were able to produce an estimate of the additional security cost incurred through co-optimisation. Our analysis showed that this cost is heavily situational dependent and ranged up to 29% of the total cost paid by load.

We have briefly examined the market incentive implications of the co-optimisation and have concluded that potential exists within the current market structure to obtain rental payments. These rental payments may occur as appropriate marginal price signals interact between co-optimised markets to elevate prices in both markets. These rental payments could result in an incentive for a participant in both market to express market power in one of the markets to constrain the situation. Our analysis has been conducted in the context of the New Zealand grid and is specific to New Zealand to our knowledge. However, as other markets may be considering the inclusion of reserve through a co-optimisation we consider the insights gleaned from our analysis to be worthwhile to a broader audience.

Acknowledgements

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Figures

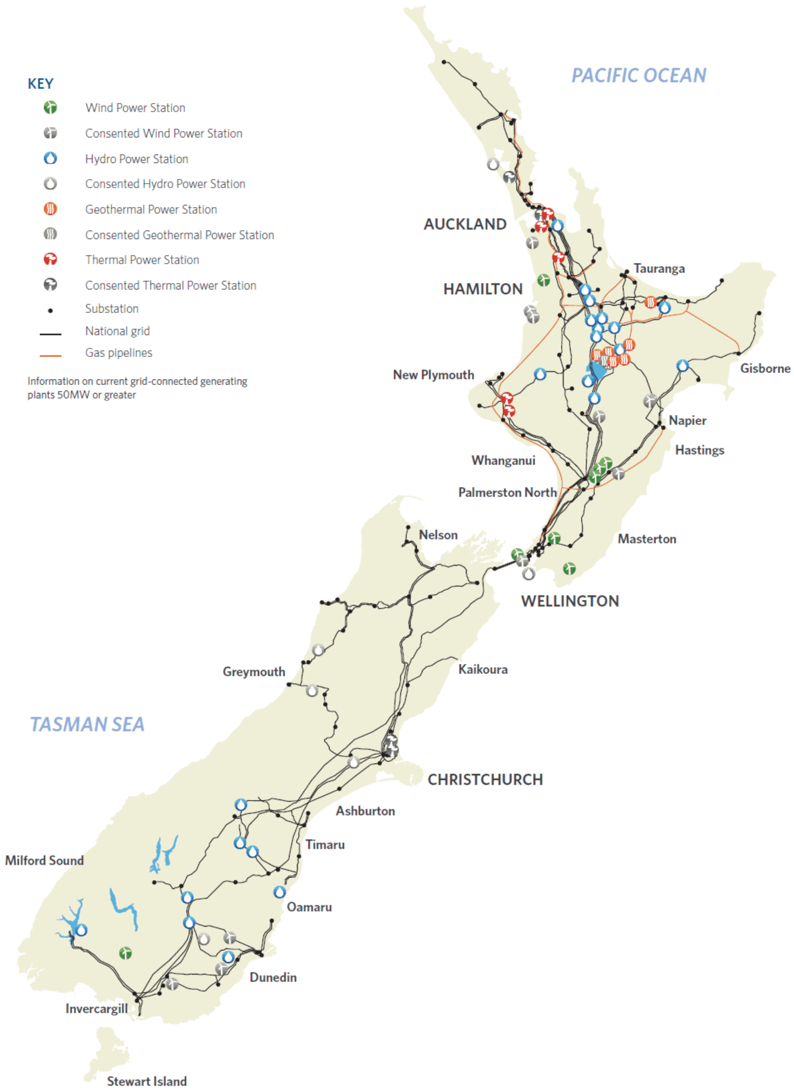

Figure 7: Representation of the New Zealand Grid

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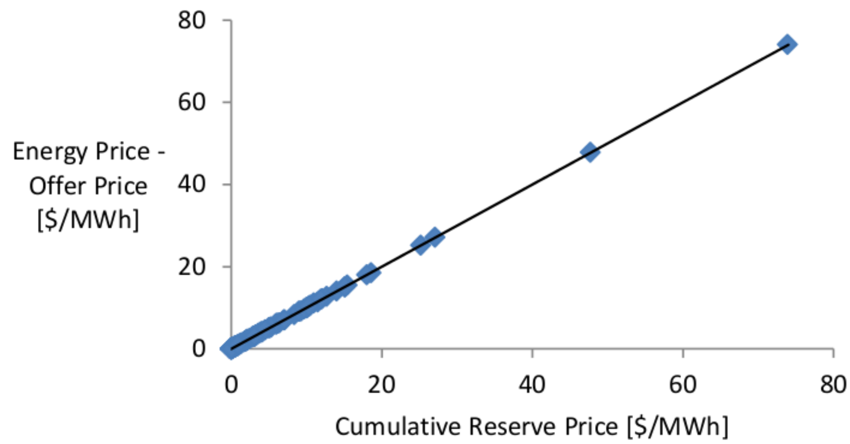

Figure 8: Reserve affecting Otahuhu Nodal Price

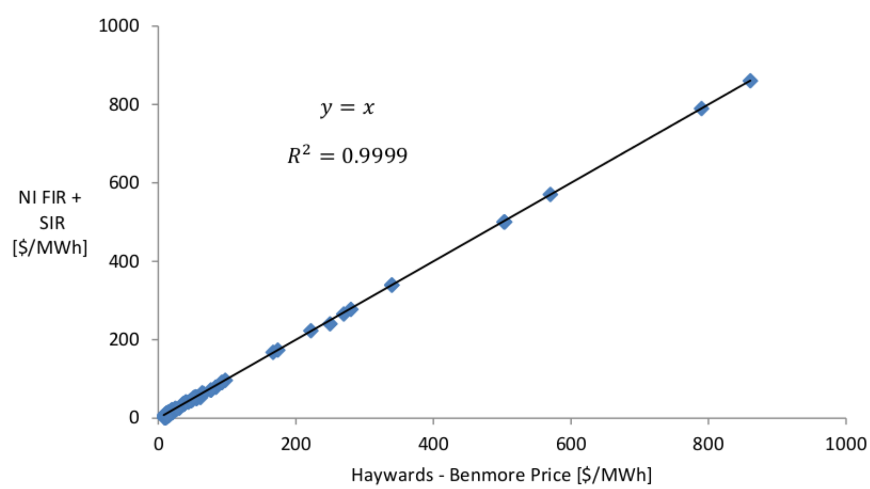

Figure 9: North Island Offer Constrained Scenario

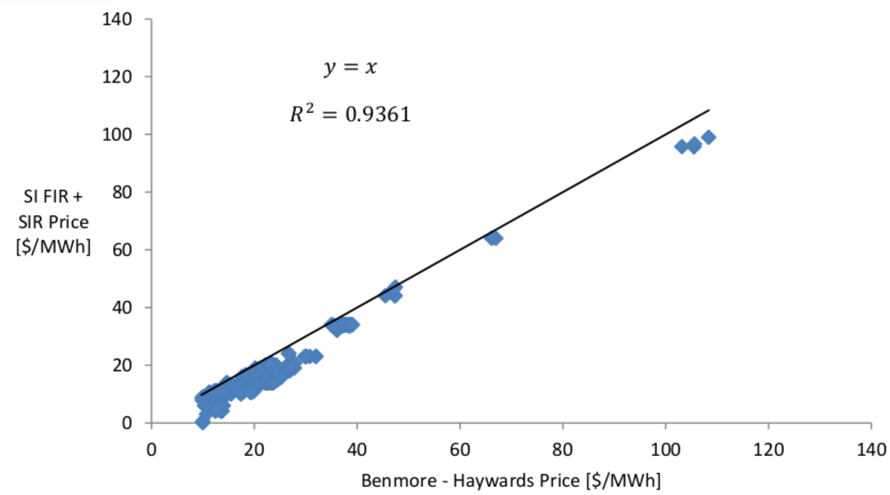

Figure 10: South Island Offer Constrained Scenario

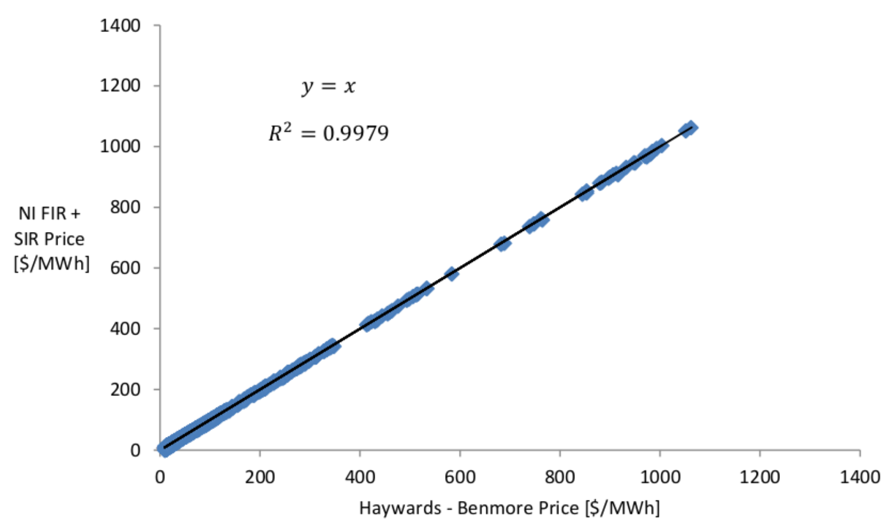

Figure 11: North Island Bath Tub Constrained Scenario

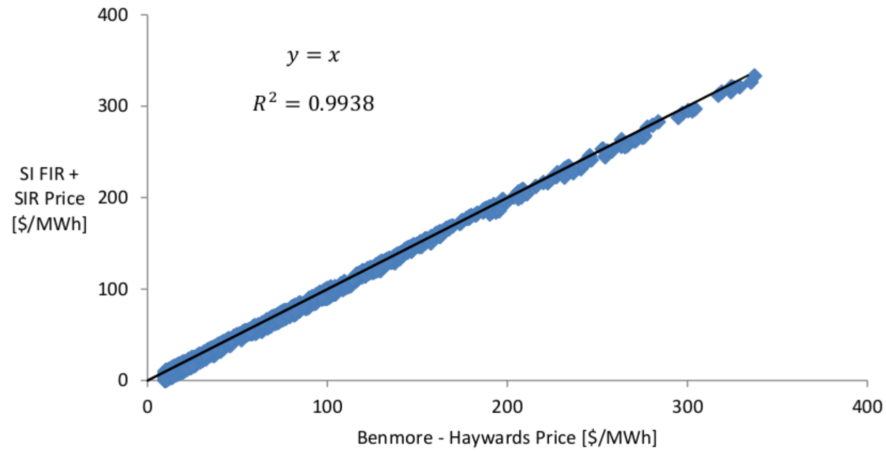

Figure 12: South Island Bath Tub Constrained Scenario

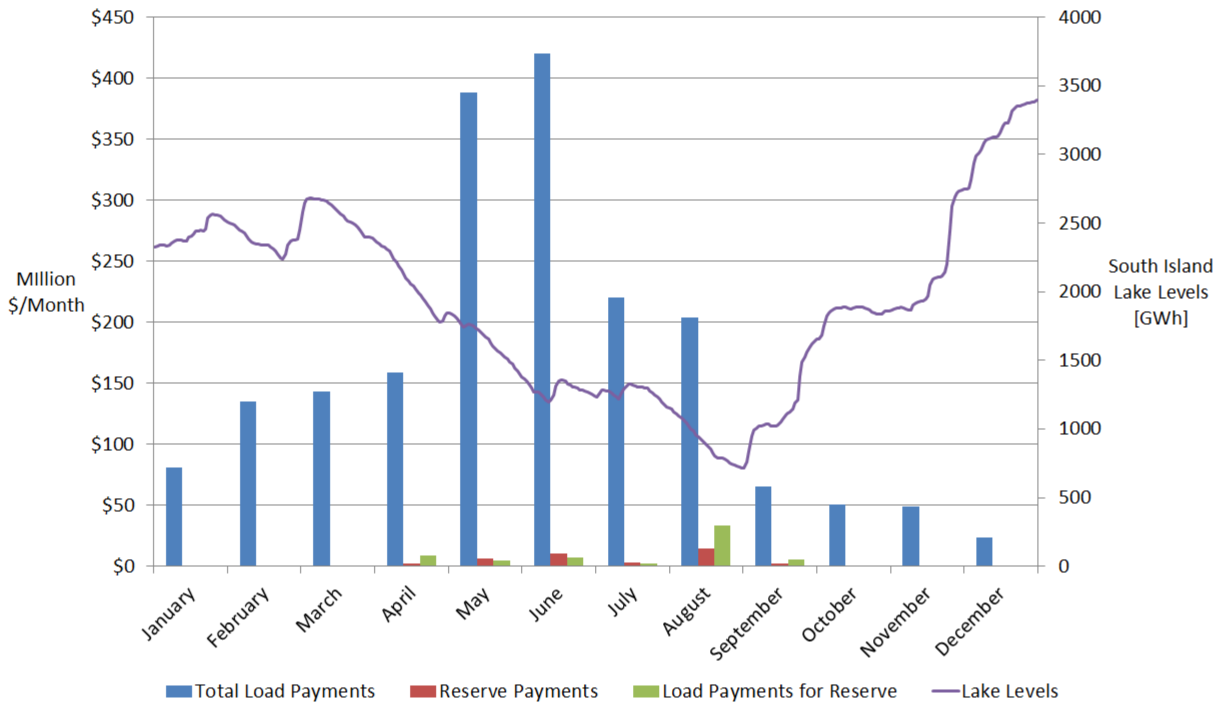

Figure 13: 2008 South Island Constrained Situations by month

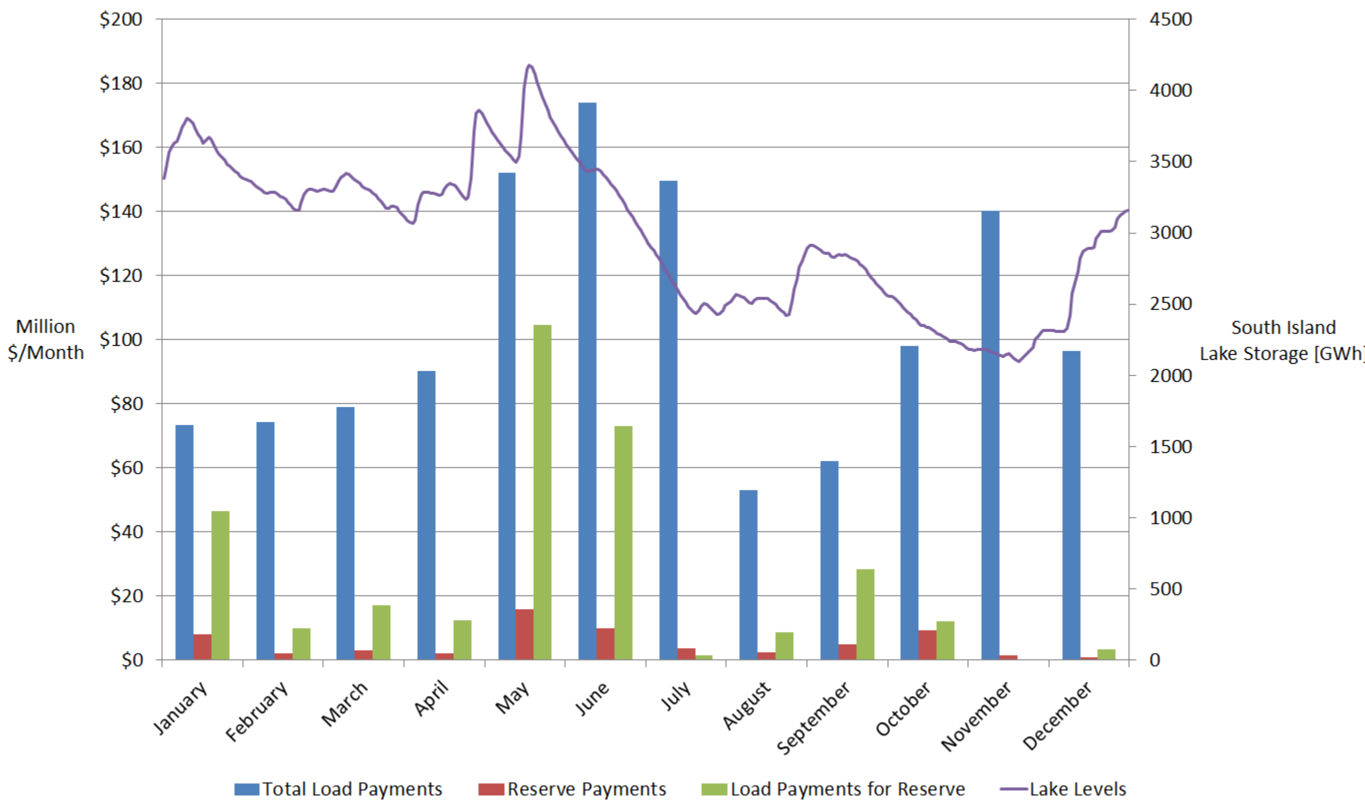

Figure 14: 2009 North Island Constrained Situations by month