Supply Function Equilibria in Markets with Reserve Constrained Transmission Lines

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***Abstract*—We develop a Supply Function Equilibria model to assess the impact of uncompetitive Contingency Reserve markets on participant behaviour in a market where Transmission is a risky asset. Companies may submit energy and reserve offers with the requirement for reserve specified by the co-optimised dispatch of energy assets subject to the reserve offers. The results of this model suggest that withholding reserve may be an optimal strategy. A generator at the sending end of a reserve constrained transmission line should self withhold in order to minimise price differences between the nodes. A preliminary empirical assessment suggests that one provider systematically offered less combined energy and reserve capacity to the market during periods of reserve constraints.**

***Index Terms*—Ancillary services, reserve markets, supply function equilibria, strategic behaviour.**

NOMENCLATURE

Parameters

$\beta_{n,i}$	Constant portion of marginal energy cost for unit at node n, firm i
$\gamma_{n,i}$	Linear portion of marginal energy cost for unit at node n, firm i
$\alpha_{n,i}$	Constant marginal cost for reserve dispatch for unit at node n, firm i
d_n	Demand for energy at node n
$SP_{n,i}$	Strike price for a CFD a company i has at node n
$\delta_{n,i}$	Quantity component of a CFD a company i has at node n
ϕ	User defined convergence parameter

Optimisation Variables

$\beta^*_{n,i}$	Generator submitted offer for constant energy cost at node n, firm i
$\gamma^*_{n,i}$	Generator submitted offer for the linear portion of the energy offer at node n, firm i
$\alpha^*_{n,i}$	Generator submitted offer for marginal cost of reserve from the unit at node n, firm i

Market Clearing Variables

$x_{n,i}$	Dispatched energy quantity for generator at node n, firm i
$r_{n,i}$	Dispatched reserve quantity for the unit at node n, firm i
f	Transmission flow between nodes
χ_n	Value of the largest risk setter at node n

Dual Variables

λ_n	Nodal energy price at node n
μ_n	Nodal reserve price for security at node n
$\nu_{n,i}$	Shadow price for security constraint upon generation dispatch at node n, firm i
τ_n	Shadow price for security constraint upon transmission at node n

I. INTRODUCTION

In electricity markets the unforeseeen failure of assets must be accounted for during the dispatch of generation units for energy. Modern grids are interconnected and the failure of one asset may lead to the subsequent failures of other assets. This cascade collapse of the generation units may lead to blackouts or localised disconnections with subsequent economic or social impacts. To protect against this Contingency Reserve (CR) is procured from both the demand and supply side [1], [2]. Contingency Reserve is dispatched in two forms, to arrest the decline in frequency associated with unit disconnection (supply shortfall), known as primary reserve and to restore the system to the set operating point using secondary reserve. Regulating Reserve (load following) may also be procured, however this is associated with load fluctuations and falls outside the scope of this paper.

One innovation has been the co-optimisation of energy and reserve through the simultaneous dispatch of Energy and Ancillary Service (AS) markets. Such markets include New Zealand (NZ) [3] and Singapore. New Zealand and Singapore differ from other co-optimised markets throughout the world via the following key features. Single clearing auction (no day ahead market), Low gate closure period (two hours), co-optimised contingency reserves, non co-optimised regulating reserves[1], five minute re-dispatch period of generation assets (thirty minute offers) and integration of interruptible load as a provider of contingency reserves. This differs from other markets, such as Spain, in which secondary reserve is procured for regulation purposes, primarily due to the one hour dispatch window.

In Europe, we note that terminology differences arise and that regulating reserve (Secondary Reserve) is partially optimised in some markets [4]. There exists substantial reference material in the literature regarding both market design and pricing in these co-optimised markets [5]–[10]. A key aim of these models is to improve economic efficiency by integrating the dispatch of reserve within a single linear program as

[1] these are procured through a separate bidding process and their integration into the wider dispatch model is loose

opposed to sequentially dispatching different products [11]. These models establish separate market designs for the provision of co-optimised energy and reserve as theoretical constructs. This differs from our work as we base our equilibrium model upon an implemented market design (the NZEM).

Generation units are able to provide both up and down ramping regulation services as well as CR. The Demand Side is integrated as well, although the focus is typically on reducing consumption, that is providing up regulating reserve or CR. The Integration of the demand side is seen as necessary for economically efficient markets [12]. We note that whilst [12] considered a slightly different perspective Ancillary Service markets provide a stop gap mechanism through which the demand side may be integrated. Papers considering the integration of demand side response for reserve purposes include [13]–[15].

A key consideration in many markets is to provision the optimal level of reserve. For example, some markets may specify reserve as a fraction of demand, as an N-1 deterministic requirement [3] or under probabilistic considerations [16]–[19]. A key consideration is the *optimal* quantity of reserve dispatched, where optimal may have both engineering and economic objectives. Over scheduling of reserve is economically inefficient, yet under scheduling reduces system security.

The shift towards markets for ancillary services requires an improved understanding of their competitive interactions with the wider energy market. One approach to understanding the dynamics of electricity markets is through the application of economic models of competition. These models attempt to understand the optimal behaviour of participants in equilibrium using competitive approaches such as Bertand [20], Cournot [21] or Supply Function Equilibria [22], [23]. Such models often use simplifications to gain insight to more complex market situations. That is, by examining a simple case and through extrapolation applying this to a wider situation, noting that the real system may be too complex to be modelled fully within the limitations of the competitive framework. Authors with differing backgrounds typically focus upon approaches more suitable for their field. A key consideration is the depiction of the underlying physical network which the competitive agents interact within. Efforts to improve the depiction of the network are discussed in [24], [25].

Previous attempts to include reserve in equilibrium models have been undertaken by [26]–[32]. Some authors have also considered the bidding problem faced by generators and have sought to create optimal offer strategies [33]. For a fuller overview of competition models applied to Genco strategies we refer the reader to [11], [34].

For brevity we limit our discussion of the previous models to their differences with our undertaken approach. Our model is based upon a situation which occurs in the New Zealand Electricity Market (NZEM). The NZEM has an N-1 deterministic security requirement. That is, the requirement for reserves will change depending upon the energy offers submitted. Likewise, the energy dispatch may be constrained by the reserve offers submitted. Previous attempts, such as [27] do not consider this subtlety.

The introduction of this deterministically procured reserve introduces an interesting competitive dynamic. In a market where reserve is scarce a dominant reserve provider may be able to "block" competing energy offers from being dispatched due to security considerations. We present an equilibrium model which examines this case within a market with security constrained transmission lines. Based upon the insights gathered from this model we undertake a novel statistical analysis of the NZEM focussing upon the total unit utilisation of generation assets. By comparing this utilisation under both reserve constrained and unconstrained periods during periods of high prices we find a significant bias toward under utilisation of the total capacity available. We see the results of this model as having a novel application towards Transmission Grid Investment Tests (GIT) in markets with security co-optimisation. The effect of a transmission upgrade, in particular that of the asset utilisation - risk profile should be considered in the wider context of the reserve availability.

II. Background

We seek to design a model which will permit a greater understanding of the NZEM, in particular regarding incentives for offering reserve. We devise a simple case study which is intended to mimic a specific market situation which arises. NZ has two AC grids joined via a reserve constrained HVDC connection. The market structure is dominated by five vertically integrated gentailers (generator-retailers).

One outcome of the deregulation process has been the creation of generation companies with strong geographical and technological tendencies. For example, excluding a small quantity of wind capacity the entirety of Meridian Energy's generation portfolio is made up of lower South Island hydro. Whereas the majority of North Island hydro is owned by Mighty River Power in the form of the Waikato chain.

Partially Loaded Spinning Reserve (PLSR) procured from hydro generators is a major source of CR in both the primary and secondary markets. This places the hydro generators in an interesting position, if they schedule additional reserve they may enable greater HVDC transfers from their competitors in the opposing island. This creates the novel situation in which reserve withholding may *block* the transfer of lower cost hydro energy from an island. The model we develop is not intended to be quantitative. Instead we seek to develop qualitative models to understand the incentive structure seen by the various participants. This problem has been touched upon in [35] but not covered in depth.

The remainder of this paper is divided into the following sections. Section III outlines a SFE model we use to generate qualitative insights into the model. Section IV applies this model to a specific case. In Section V we extend the qualitative results of Section IV in a quantitative fashion using market data from the NZEM.

III. Equilibrium Model Formulation

There are a number of techniques suitable for equilibrium models in liberalised electricity markets. At the two extremes of competition lie Bertrand and Cournot competition. Colloquially, games in prices and quantities respectively. In

Bertrand competition a market participant assumes they can monopolise supply by offering a fractionally lower price. Whilst in Cournot competitors may set quantities without a price component within their bid.

A middling source of competition is Supply Function Equilibria (SFE). This produces a level of competition between Bertrand and Cournot. SFE models of equilibrium are particularly suitable to electricity markets as their structure mimics the price-quantity bids which GenCos must submit to an SO Although the requirement for a continuous, smooth, supply function is dissimilar to the limited flat tranches present in bid-quantity pairs.

In choosing a form of competition all knowledge about the system must be taken into account. As we are seeking to gain insight, not replicate final pricing situations a form of equilibrium which is capable of replicating prices is not required. We must carefully consider what is exogenously defined as the competitive structure, demand and cost functions will also influence the equilibrium obtained.

In the N-1 deterministic security case we have no exogenous (fixed) demand for reserve. Instead, the demand for reserve is endogenous to a unique combination of each generators supply functions for energy. Cournot equilibria which require the careful formulation of the demand elasticity curve are not applicable in this situation. Furthermore, it is likely given the interplay between energy and reserve that withholding will occur via price, not quantity. Bertrand competition at the opposite end of the spectrum is too extreme for meaningful comparisons especially in a multiple (co-optimised) product setting.

This leaves SFE as it is analytically tractable and may be formulated for the N-1 case. We extend the SFE approaches used by [23], [27], [36], [37] to assess the market. For completeness we note that conjectured supply functions are a final option, however these are difficult to formulate for the co-optimised case.

Co-optimised markets have been studied before in [25]–[27] yet these approaches did not give sufficient weighting to the potential to use the reserve markets to influence energy dispatch. The aforementioned papers were concerned with AC power formulations primarily. We seek to understand the role of reserve in a setting similar to that studied in [38]. In [38] the competitive effects of transmission were studied and the role of transmission capacity in competitive behaviour was studied. In this section we will develop a model to extend this analysis to reserve constrained transmission lines. This incorporates the competitiveness of the reserve markets as a critical factor, not just the underlying transmission capacity. We note that we use a simplified representation as many engineering considerations such as loop flows, networks and other AC considerations specific to stable operation are beyond the scope of this paper.

A. Generator Problem

We develop our equilibrium model as a game between two profit maximising entities. These entities are competing with one another by offering two products, energy and reserve to the market. In this market reserve refers to CR and is assumed to be entirely separate from the energy offers. We do not consider the co-optimisation of energy and reserve at the unit level and note that generation and reserve plants are distinct from one another. Given that we are considering CR we formulate reserve as a constant marginal cost "availability" charge in a similar formulation to [27].

The market is nodal [39] for both energy and reserve with separate energy and reserve clearing prices. The link between the energy and reserve market is through the deterministic security requirement which forms the demand for reserve. Companies are able to offer energy with linearly non decreasing marginal costs as well as fixed reserve costs (which may be zero). There are no capacity limits upon either energy or reserve. Each generator takes turns to maximise profit under an assumed state of its opponent via diagonalisation. When no participant can unilaterally increase their outcomes Nash Equilibrium is reached.

Total costs for generation and reserve are:

$$C(x_{i,n}) = (\beta_{i,n} + \frac{1}{2}\gamma_{i,n}x_{i,n})x_{i,n} \tag{1}$$

$$C(r_{i,n}) = \alpha_{i,n}r_{i,n} \tag{2}$$

We assume that each company may have only one unit at any specific node n. These nodes may have CFDs present with a strike price SP and quantity δ which we retain for generality. The nodal revenue obtained for each generation and reserve unit is thus:

$$R(x_{i,n}) = \lambda_n x_{i,n} + (SP_{i,n} - \lambda_n)\delta_{i,n} \tag{3}$$

$$R(r_{i,n}) = \mu_n r_{i,n} \tag{4}$$

The total profit earned by each generation company is thus the sum of the revenue it earns from generation and reserve less the costs of providing this and taking into account any contractual obligations. For the ith company profit Π_i is thus the sum of all revenue and costs over each of the nodes.

$$\begin{aligned} \max \quad & \sum_n \{\lambda_n x_{i,n} + (SP_n - \lambda_n)\delta_n\} + \sum_n \{\mu_n r_{i,n}\} \\ & - \sum_n \{(\beta_{i,n} + \gamma_{i,n}x_{i,n})x_{i,n}\} - \sum_n \{\alpha_{i,n}r_{i,n}\} \end{aligned} \tag{5}$$

In this maximisation problem the key variables are the generation and reserve outputs $x_{i,n}, r_{i,n}$ as well as the clearing prices λ_n, μ_n. These variables are determined by solving the related ISO least cost dispatch problem at equilibrium by submitting the bid parameters $\beta^*_{i,n}, \gamma^*_{i,n}, \alpha^*_{i,n}$.

B. ISO Clearing Problem

We develop the ISO clearing problem using a simplified flow model across two nodes and dynamic security requirements. The ISO takes submitted cost functions from each of the generators which it uses to meet demand given the security requirements. The generators are not required to submit true costs to the ISO, hence β^* may be (but does not have to be) different from β. We formulate demand as inelastic, but known, no stochastic demand realisation is present.

The Primal Optimal Power Flow Problem (POPF) is to minimise the cost of meeting demand subject to the constraint that demand is satisfied at each node and security constraints are met. We formulate the security requirement through the inclusion of a risk variable χ_n which specifies the largest risk setter (generation or transmission) in each reserve zone. Note that this is a variable and the quantity of reserve required will thus depend upon the dispatch of different generation units and transmission lines. For convenience we specify that each risk must be secured via reserve from the node it is located at. No transmission of reserve is possible as transmission is seen as a risk setter.

The primal problem may be initially written as a quadratic cost minimisation problem with objective function (6) and constraints (7)-(10).

$$
\begin{aligned}
\min \quad & \sum_{i,n}\{\beta^*_{i,n}x_{i,n} + \frac{1}{2}\gamma^*_{i,n}x^2_{i,n}\} \\
& + \sum_{i,n}\alpha^*_{i,n}r_{i,n} && (6)\\
\text{s/t} \quad & \sum_{i\in n(i)} x_{i,n} + \sigma_n f = d_n \quad \forall n && [\lambda] \quad (7)\\
& \chi_n \geq x_{i,n} \quad \forall i,n && [\nu] \quad (8)\\
& \chi_n \geq \sigma_n f \quad \forall n && [\tau] \quad (9)\\
& \sum_{i(n)} r_{i,n} \geq \chi_n \quad \forall n && [\mu] \quad (10)\\
& x,r,\chi \geq 0 \\
& f \quad \text{free}
\end{aligned}
$$

This is an optimisation problem with a convex quadratic objective function. However, it is not currently in a suitable form for embedding into the generator problem in order to determine equilibrium. To achieve this we use the methods outlined in [40], [41].[2]

As such we require the form:

$$
\begin{aligned}
\min \quad & \Theta(x) \\
\text{s/t} \quad & x \in X^0 \\
& g(x) \leq 0
\end{aligned}
$$

We rewrite (6)-(10) as follows:

$$
\begin{aligned}
\min \quad & \sum_{i,n}\{\beta^*_{i,n}x_{i,n} + \frac{1}{2}\gamma^*_{i,n}x^2_{i,n}\} \\
& + \sum_{i,n}\alpha^*_{i,n}r_{i,n} && (11)\\
\text{s/t} \quad & -\sum_{i\in n(i)} x_{i,n} - \sigma_n f + d_n \leq 0 && [\lambda] \quad (12)\\
& x_{i,n} - \chi_n \leq 0 && [\nu] \quad (13)\\
& \sigma_n f - \chi_n \leq 0 && [\tau] \quad (14)\\
& \chi_n - \sum_{i\in n(i)} r_{i,n} \leq 0 && [\mu] \quad (15)\\
& x,r,\chi \geq 0 \\
& f \quad \text{free}
\end{aligned}
$$

along with the following complementarity conditions:

$$
\begin{aligned}
\lambda_n\big(\sum_{i\in n(i)} x_{i,n} - \sigma_n f + d_n\big) = 0 \quad \forall n && (16)\\
\nu_{i,n}\big(x_{i,n} - \chi_n\big) = 0 \quad \forall i,n && (17)\\
\tau_n\big(\sigma_n f - \chi_n\big) = 0 \quad \forall n && (18)\\
\mu_n\big(\chi_n - \sum_{i\in n(i)} \big) = 0 \quad \forall n && (19)\\
\lambda,\nu,\tau,\mu \geq 0
\end{aligned}
$$

These are embedded into each generators maximisation problem and solved for equilibrium. As such the full maximisation problem is the objective function (5) and the primal constraints (7)- (10) along with the complementarity conditions given by (16)-(19)

C. Solving for Equilibrium

This is a non linear (quadratic) program which has been implemented using LINGO. The solution is found by iteratively maximising each generators profits using an assumption of the opposing generators position. This position is given by an intermediate position between their two most recent offers in the same fashion as [46] with the weighting given to each offer given by the variable ϕ in (20). Iterations are halted once Nash equilibrium is reached which is identified by a sequential inability to improve profits for each of the optimising generators.

$$\beta^*_{i+1} = \phi\beta^*_i + (1-\phi)\beta^*_{i-1} \quad (20)$$

Multiple start positions chosen randomly in order to determine that an equilibrium is unique. That is, they all converge to a similar profit and unit configuration even if parameter values are not exactly equal. Cyclic behaviour can occur, each generator is optimising individually and thus assumes that they will monopolise quantity in certain configurations, we note these when they occur but seek to avoid this issue through the careful selection of unit parameters.

D. Scenarios Considered

The scenarios we are consider are drawn from specific cases in the NZEM. In particular the N-1 risk setting behaviour for GenCos when a participant has market dominance in the reserve market, even if this is not true in the full energy market. We consider four situations, a zone based risk for either of generation or transmission (or a combination). An a system wide risk for generation. Figure 1 describes the different configurations considered.

Companies one and two are the optimising generators who compete by submitting changing supply functions to the ISO. Companies three and four provide a cap upon the market but do not actively optimise their bids to the ISO. Company One is an Energy only bidder. In particular we seek to understand the effect of Company Two's reserve actions upon the profits obtained by Company One. For real world context this is similar to the interplay between Mighty River Power and Meridian Energy. Cost parameters for each unit are shown in table I.

[2] We have implemented the model using a slightly different formulation based upon methods taken from [42], [43] and using the concept of the zero duality gap at optimality [44], [45].

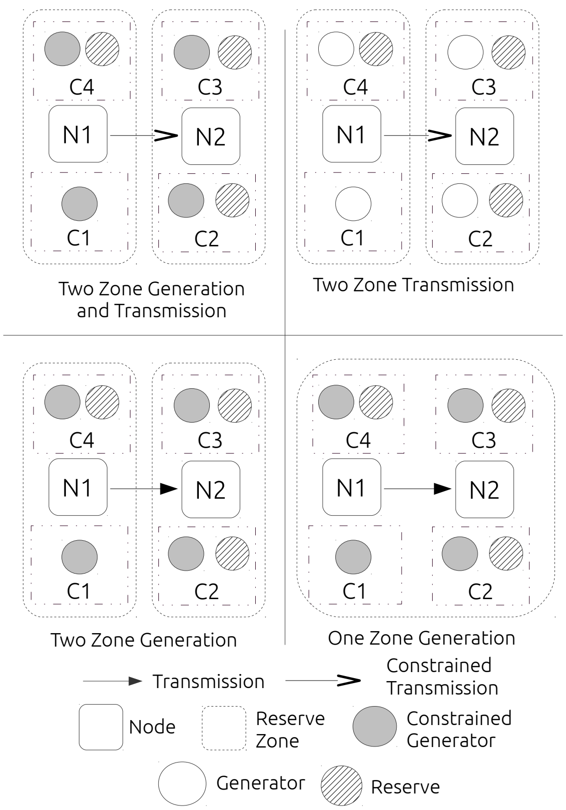

Fig. 1. Example structures considered, the captions indicate which assets are considered "risky" within each reserve zone.

TABLE I
SIMULATION COST PARAMETERS FOR ALL EXAMPLES INCLUDING IDENTIFICATION OF COMPANIES AS COST BIDDERS, OR OPTIMISERS

Company	β	γ	α	Leader
1	5	1	-	Y
2	5	1	5	Y
3	150	0.05	40	N
4	150	0.05	5	N

IV. COMPETITIVE EFFECTS OF RESERVE CONSTRAINED TRANSMISSION

In markets with LMP and constrained transmission networks the spatial positions of participants can have a strong effect on market outcomes. In particular, transmission constraints may limit the ability of GenCos to compete in markets outside of their local sphere of influence. A natural method of resolving this problem is to invest in transmission capacity. Increased capacity in effect can open an area to competition which may lead to improved consumer outcomes.

For example, consider the case outlined in [38], a market with limited transmission capability between two regions. In the markets modelled (under Cournot competition) the presence of a constrained transmission line led to uncompetitive behaviour by the major participants. Additional capacity (if sufficient to resolve the constraint) would lead to substantially improved outcomes even in the case that the line isn't used. That is, if a participant cannot constrain the line it is in their best interests to compete which could lead to the line being underutilised.

An extension to this result is to consider the case of a reserve constrained transmission line. Consider a two node marketplace under three separate transmission regimes. 1) No transmission capacity, 2) Unlimited transmission capacity, 3) Unlimited transmission capacity but reserve constrained. We are not considering the case of optimal transmission capacity, but instead seeking to develop insight into behaviour in a novel situation. We note that 1) and 2) are broadly a replication of the result of [38] in a SFE setting. Considering the cases in Table II we see that the introduction of a transmission line reduces prices for consumers as expected even if the line is not utilised.

TABLE II
EFFECT OF INTRODUCING NON-RESERVE-CONSTRAINED TRANSMISSION AND RESERVE-CONSTRAINED TRANSMISSION BETWEEN PRIOR MONOPOLY CONTROLLED NODES

	No Transmission	Free Transmission	Reserve Constrained Transmission
λ_1	150	55.1	150
λ_2	150	55.1	150
μ_1	-	-	5
μ_2	-	-	58.7
g_{11}	10	9.5	10
g_{22}	10	10.5	10
r_{22}	-	-	0
f	-	-0.5	0
π_1	1350	384	1350
π_2	1350	416	1350

However, the introduction of the reserve requirement in case 3) returns the situation to the monopoly state. That is the participant has incentives to withhold reserve in order to block the transmission of energy. This effectively returns the monopoly behaviour and the benefits of the transmission line are nullified.

In the presence of a binding reserve constraint the optimal behaviour for the displaced *sending* generator is to self withhold. Instead of offering competitively the GenCo realises that any reduction in their own bids (in an attempt to gain volume) will be blocked and result in lower prices at their own node. Thus, they restrict their bids in order to decrease any price difference arising between the nodes.

Interpreting this result, we see this as a generator self withholding to prevent the binding reserve constraint from occurring. When a binding reserve constraint binds energy prices between the nodes become linked via the marginal cost of reserves. That is, marginal energy prices at a receiving node are equal to the marginal energy price at the sending node plus the security cost of transmitting it as in (21).

$$\lambda_{receiving} = \lambda_{sending} + \mu_{receiving} \tag{21}$$

In the case that a participant has retail or contractual positions at the receiving node this price discrepancy represents a source of risk. An optimal response is to not attempt to compete with the reserve monopolist and instead monopolise their own node. The implication being they should also structure their contractual positions accordingly. It is worthy to note that introducing contractual positions at the opposite node for the reserve monopolist may lead to more competitive outcomes, but lower total profits.

Introducing a cap upon the reserve price in a market (through a competitive bidder) introduces a non-stable result with no equilibrium reached. This occurs as the best response for a participant at the cap price can be to reduce their energy offer and dominate all of the volume in the market. Naturally this causes a response and iterations will occur until the price reaches the cap and the cycle repeats itself. The occurrence of this behaviour in equilibrium is dependent upon the parameters specified. That is, there exists a threshold price at which this behaviour will occur. At any cap price beyond this the monopoly result will be returned. We note that the cycling behaviour may be a limitation of our implementation which introduces a constant marginal reserve cost.

A. *Grid Investment Tests*

In most liberalised markets transmission still remains a regulated monopoly. These monopolists have a guaranteed return upon their installed capital base and as such many of their investments must meet regulator imposed grid investment tests. One proposed approach is to consider the benefit a transmission line provides to the market and whether a positive result occurs. In a market with security constrained transmission the regulatory bodies must also consider the reserve markets.

For example, if the existing assets are risk constrained (not physically) then any subsequent upgrade of the assets which does not alter the initial risk profile is of little benefit. Consequentially the line could be utilised more effectively through the promotion of competition in the reserve market with additional reserve capacity increasing the utilisation of the existing assets.

In NZ the HVDC interconnection between the North and South Islands has recently been upgraded. This interconnection was security constrained and the upgrade increased capacity from 700 MW to 1240 MW through the introduction of a new parallel pole. We note that before this upgrade transfers very rarely reached the thermal limit and preliminary views from after the upgrade indicate that transfers do not reach substantially beyond the prior limit. However, the risk profile of the new asset has been completely changed severely limiting the reserve requirement.

As such in the context of our models and in light of the result of [38] we see the benefit of the line in its parallel nature. The decrease in the assets risk profile should reduce the ability of the participants to act competitively. As such the South Island generators, in particular Meridian Energy will not need to self withhold generation capacity in order to prevent the link from becoming security constrained. This should reduce price separation between the islands and enable greater competition, even if flows are not substantially increased. A Grid Investment Test which does not take into account these competitive results would understate the benefit of the HVDC interconnection. Although we note that the benefits may be localised to specific participants.

V. Empirical Market Assessment of the NZEM

A key result from the theoretical models is that an incentive exists to constraint the line via withholding reserve. This may occur via price or volume but price is more likely. Measuring this behaviour in the real market is a difficult task. In particular, energy and reserve are both complementary as well as substitutes for each other. To increase the dispatch of reserve often implies a reduction in the quantity of energy. Thus we may not consider offer based assessments in isolation. Instead, we consider general offers under unconstrained and constrained situations at different price points.

In (previous work) we described a simplistic test for assessing the presence of a reserve constraint. In the presence of a binding risk constraint upon the marginal energy unit the reserve price will be incorporated into the energy price as in (21).

We reformulate this as a test in order to establish a binary selection criteria, reserve constrained or unconstrained. In the NZEM we note that approximately 10% of trading periods were constrained by reserve on South to North flow from 2008 to early 2013. Upon establishment of an appropriate metric we may use this binary qualifier to assess differences in behaviour between the two situations.

In the NZEM hydro stations play a significant role as both energy and reserve providers. As reserve refers to CR which is called less than 10 times per year and only for short durations we note that there exists few reasons why unused capacity may not be offered as reserve barring technical issues. We consider Mighty River Power (MRP) in the North Island. MRP controls the Waikato river chain which consists of approximately 900 MW of installed capacity. Each station on the chain is offered via five energy tranches and three reserve tranches for each reserve product (Primary and Secondary CR). We have chosen MRP as other participants are in a less dominant position and face conflicting incentives. For example, Genesis Energy has the Huntly power station which is also a risky asset and as such bears some of the cost of the reserve market changing the incentives.

In any given trading period there a number of different environmental and market considerations at play. Given that we are conducting this analysis *ex post* with none of the internal knowledge as to why offers were structured in a particular case we cannot make an assessment of any particular trading period. However, we may develop an assessment of particular trends under the binary criteria described in (21).

In an electricity market with price quantity bids neither price nor quantity may be considered in isolation. A unit may be fully *offered* in quantity terms to the market but at such a high price that it is essentially withheld. This is further complicated by the inclusion of reserve. We present a short algorithm to establish a dynamic moving "desired utilisation" for each trading period under the assumption that contractual considerations between similar trading periods will be equivalent.

Using this desired utilisation we may establish a baseline through an averaging procedure and compare each period against the baseline of similar periods to assess if systemic trends are present. For example, does a particular company always underutilise their units during periods where constraints are present. This trend based approach helps correct for different environmental considerations for example demand and hydrology.

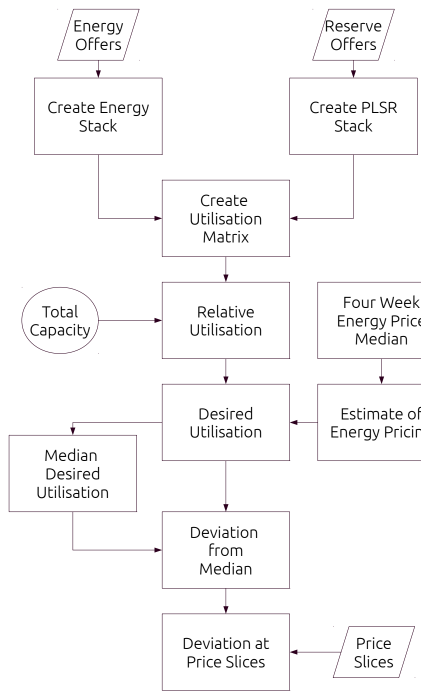

Fig. 2. Algorithm for evaluating instances of relative capacity utilisation through the development of a price and time sensitive baseline.

The algorithm as shown in Figure 2 is as follows: A price baseline is established by considering the previous four weeks of energy prices for a specific trading period and correcting for weekends. We take the median energy price from this situation which we multiply by two (although other values are possible). Offers above this price point are assumed to be outside the realm of the desired dispatch. That is, the GenCo does not wish for these tranches to be considered but retains them in the stack as either speculative offers or to ensure that a feasible dispatch occurs. Reserve prices are tail end distributed and average approximately $\$5/MWh$. We assume any reserve offer beyond $\$50/MWh$ to be undesirable from the point of view of the dispatch.

We construct a matrix of energy and reserve offers where each cell has an associated energy and reserve price and a value being the combined dispatch of the two. Using our price exclusion criteria we remove any rows or columns which exceed this point. The desired utilisation hence becomes the greatest combined quantity which satisfies the price criteria on either a unit or company level basis. The baseline used is the central rolling median for each trading period.

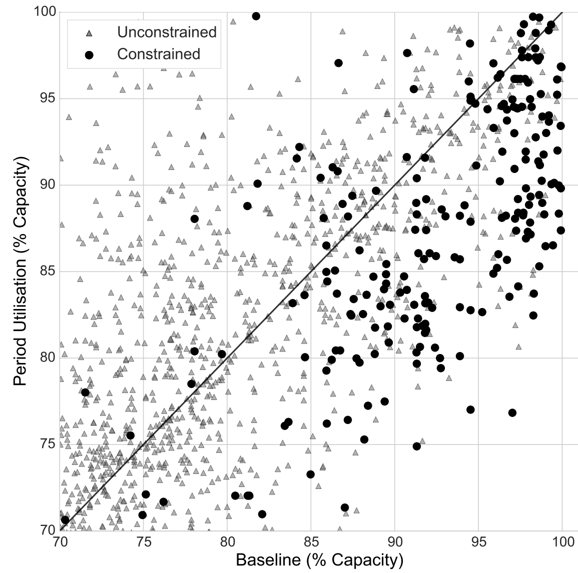

Fig. 3. Desired Utilisation as compared to the baseline for similar periods where price was above \$200/MWh, note the large number of periods at 100% desired utilisation, this is normal during peak periods when demand is high.

There is a strong price component to withholding behaviour, the implicit reduction in volume is offset by the increased prices. As such, we may consider the behaviour under different price slices. For example, Figure 3 considers the desired utilisation against the baseline at fine prices above $\$200/MWh$ using the reserve constraint qualifier.

An alternative method is to assess the differences in the cumulative distributions of asset utilisation at the different price tranches. Figure 4 highlights three price slices $(0, 300, 500)$ and qualified across the binary reserve criteria. If no systematic trend was present we would assume the distributions for each slice to be centred around the 0% deviation point. Indeed, for unconstrained trading periods this is indeed the case at all of the assessed slices.

For constrained periods a clear bias is present. The assets are utilised less during reserve constrained periods at higher price points when reserve constraints are present. The NZEM is a uniform price auction with significant hockey shaped offer curves. Any reduction in the available supply may have a significant effect on price which may benefit companies that are net long in the market (dispatch exceeding contractual obligations).

The common statistical tests for significance are difficult to apply in this case primarily due to assumptions implicit in these tests regarding Gaussian distributions and sample sizes. One such test which is reasonably suitable (though with several caveats) is the one tailed Welch's t test. We applied this test at a confidence of $p = 0.05$ at different prices slices. For slices at $\$100, \$200, \$300$ a mean decrease of $1.52\%, 2.57\%$ and 2.3% exists respectively. Whilst not large we note this this is 10% of the available reserve capacity which is not offered.

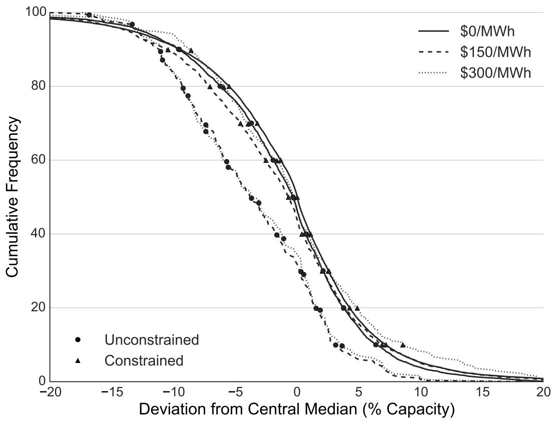

Fig. 4. Comparison of cumulative distribution showing asset utilisation between constrained and unconstrained periods at different price slices.

It is interesting that such a persistent trend is present at all. These periods are highly priced and as such any small increase in volume represents a potential windfall, except if such an increase would remove the constraint. We must also consider methodology assumptions as influencing these results, for example, the twice median figure for energy prices is relatively arbitrary. Hence, our presentation of the trends present between the two situations. During periods of reserve constraints a small, but observable, decrease in the utilisation of the assets are observed which is consistent with the incentive structures we observed from the equilibrium models.

VI. Conclusions

Markets with dynamic security requirements represent a challenge for modelling purposes. Dynamically determining the security requirement is hypothetically more efficient as under and over scheduling of reserves should be reduced. However, it increases the complexity of understanding the market and uncompetitive secondary markets may influence a competitive energy market. Regulators should assess whether the market designs as implemented permit a participant to influence the energy price via Ancillary Services. If they do then measures to increase secondary market competitiveness could be considered if appropriate.

We note that empirically market power in such co-optimised markets is difficult to assess. Given that energy and reserves are often substitutes for each other and are both priced a company may be able to "hide" withholding. A trend based approach, focussing upon behaviour relative to a common baseline under different qualitative considerations is appropriate. The question should not be "why did you offer in this way." But instead, "given that constraints are occurring why are you offering this way consistently."

For transmission operators the decision to invest in more capacity should be undertaken along parallel lines, not series. That is, additional smaller assets are preferential to larger riskier assets due to the reserve requirements which may be imposed. For GenCos in the presence of a constrained market it is optimal to self withhold in many situations in order to eliminate the price discontinuities introduced by the security requirements.

Further regions of study include assessing the impact of co-optimised units on equilibrium models with capacity constraints. At the empirical level new methods to assess the presence of reserve constraints on grid investment tests would be a worthwhile addition, as would measures to quantify the effect of uncompetitive reserve markets on the energy market empirically.

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