On stochastic auctions in risk-averse electricity markets with uncertain supply

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Abstract

This paper studies risk in the context of a stochastic auction designed to facilitate the integration of renewable generation in electricity markets. We model market participants who are risk averse, when their risk aversion is reflected through coherent risk measures. We uncover a closed form characterization of the optimal pre-commitment behaviour for a given real-time policy, with arbitrary risk aversion: when participants cannot trade risk, generators provide less pre-commitment than when participants are risk-neutral, alternatively, when participants trade a rich set of financial instruments, generators provide more pre-commitment than when they are risk-neutral.

Keywords: OR in Energy, Stochastic programming, Risk-aversion, Risky equilibria.

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1. Introduction

Renewable power generation is an increasingly attractive investment option for participants in electricity pool markets, as it does not emit carbon and has a marginal cost of zero. Furthermore, investment in intermittent renewable generation is attractive from a regulatory standpoint, as wind and solar generators reduce the expected dispatch cost and do not emit carbon. However, renewable investment increases supply-side uncertainty. This creates difficulties for independent system operators (ISOs) when clearing electricity pool markets, as inflexible coal and nuclear generators may require several hours or more to implement a dispatch and intermittent power output is unknown this far in advance.

When intermittent renewable generators supply a small proportion of electricity, the ISO can efficiently manage deviations from forecasts by procuring suitable amounts of frequency-keeping and reserve generation. However, when intermittent generators supply a larger proportion of electricity, procuring suitable amounts of reserve generation becomes expensive and more efficient grid management strategies are required. Consequently, some pool markets employ a two-market structure, which employs the following strategy:

1. Clear a pre-commitment market by assuming that renewable generation takes its forecast value.
2. Let nature select a realization of uncertainty.

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3. Clear a real-time market, to balance deviations between renewables' forecast and realised generation output.

This two-market structure allows inflexible generators to implement a dispatch, by providing them with a pre-commitment setpoint. However, the pre-commitment and real-time nodal prices might fail to converge in expectation, as the expected adjustment cost is not priced when solving for the pre-commitment setpoint. Indeed, [34] claims this price distortion is a market design flaw which may lead to systematic arbitrage opportunities.

More sophisticated uncertainty management strategies comprise modelling intermittent renewable generation as a random variable and providing a pre-commitment setpoint by solving a two-stage stochastic program. The stochastic program minimizes the expected cost of generation plus adjustment, and the real-time market minimizes the cost of generating electricity plus adjusting to manage fluctuations from forecast renewable generation. This market-clearing strategy is known as stochastic dispatch, and has been studied by authors including [8], [31], [26], and [33]. Stochastic dispatch almost-surely induces efficiency savings in the long-run (see, e.g, [6]), because it explicitly prices the expected cost of deviating from a pre-commitment setpoint in the first stage, while deterministic dispatch mechanisms do not.

In spite of the almost-sure existence of cumulative system savings, we cannot guarantee that all market participants benefit from a pool-market implementing stochastic dispatch. Indeed, implementing stochastic dispatch could leave generators out of pocket, and cost recovery is only guaranteed in expectation. This raises the question: *What happens if the participants are risk averse?*

This question is also of interest in a different context. One main criticism of stochastic dispatch, is that the system as a whole must have a unified view of the future distribution of outcomes, e.g. all agents must agree with the distribution of wind in the next hour. Allowing for risk aversion offers some flexibility here. Perceiving a different distribution of future outcomes by an agent, can often be equivalently modelled as that agent being risk averse, and equipped with a coherent risk measure. This perspective is often taken in finance, where a martingale measure emerges through a complete market and agents risks are traded [5]. We are interested in investigating the outcomes of such markets and we will return to this point in Section 3.

1.1. Contributions and structure

The main contributions of this paper are (1) a characterization of the impact of changes in pre-commitment on real-time nodal prices, and (2) a characterization of the impact of risk-aversion on changes in pre-commitment. The structure of the paper is as follows:

- In Section 2, we revisit and elaborate upon the main results obtained by Zakeri et al. in [33].
- In Section 3, we study SDM in a risk-averse context. When generators are endowed with law-invariant coherent risk measures and cannot trade risk, we establish that the resultant risk-averse competitive equilibrium admits a solution, whenever nodal prices are capped. We obtain a closed-form relationship between each generator's pre-commitment and their real-time dispatch, and demonstrate that generator risk aversion without financial instruments results in pre-commitment supply shortfalls. Alternatively, when generators trade Arrow-Debreu securities, a second risked equilibrium exists, which provides excess pre-commitment.
- In Section 4, we illustrate the impact of risk-aversion on pre-commitment, by studying the six-node toy network of ([26], [11]) in a risk-averse context.

2. Background

We start this section by reviewing a stochastic market clearing mechanism for risk-neutral electricity markets, and its properties. Stochastic Dispatch Mechanism (SDM) is a mechanism which explicitly models uncertainty in wind supply using a probability distribution when determining the pre-commitment setpoint and takes a recourse action after renewable generation output is revealed. SDM is a relaxation of the dispatch mechanism introduced in [26], where non-physical constraints on the first stage are relaxed (see [33] for a justification of relaxing non-physical constraints). We follow [26] in assuming that all generator offers are submitted before the pre-commitment setpoint is determined, remain intact in real time, and the dispatch model is coupled with a single-settlement payment scheme, wherein charges to participants under different realizations of renewable generation are known before and incurred after uncertainty is realised.

Notation

We follow [6] and [29] in letting uncertainty be represented by the scenario $\omega \in \Omega$, which occurs with probability $\mathbb{P}(\omega) > 0$, and prescribes all uncertainty caused by intermittency in our stochastic dispatch models. We denote a random variable by $Z : \Omega \mapsto \mathbb{R}$ and its realised value in scenario ω by $Z(\omega)$. We assume the sample space Ω is finite, which is an approximation obtained by sampling from the true distribution of uncertain outcomes. Consequently, the dispatch models considered here are Sample Average Approximations for which one can derive asymptotic convergence results as the sample size increases (see [29]).

The sets and indices used throughout this paper are defined as follows:

- $\mathcal{F}$ is the set of flows which obey thermal limits, line capacities and the DC load-flow constraints imposed by Kirchhoff's laws. We assume that $\mathcal{F}$ is closed, convex and non-empty.
- i is the index of a generation unit. We assume perfect competition, meaning the ownership of each generation unit is irrelevant and each generation tranche can be thought of as operated by a separate generator.
- $j(i)$ is the index of the node j where generator i is located.
- $\mathcal{N}$ is the set of all nodes in the network.
- $\mathcal{T}(n)$ is the set of all generators located at node n.

The problem data used throughout this paper are defined as follows:

- c_i is generator i's marginal generation cost.
- $D_n(\omega)$ is the inelastic demand at node n in scenario ω.
- $G_i(\omega)$ is generator i's production capacity in scenario ω.
- $r_{u,i}$ is generator i's marginal cost of upward deviation.
- $r_{v,i}$ is generator i's marginal cost of downward deviation.

To avoid out-of-merit-order dispatches we follow [33] in requiring that $r_{u,i}$ and $r_{v,i}$ are equal for all tranches offered by a generation unit (see Section 2.1 of [33] for a justification of this requirement). We also require that $r_{u,i}, r_{v,i} > 0$ for some generator i, as otherwise all generation units are infinitely flexible and we can implement a wait-and-see market clearing mechanism wherein we dispatch all generation units after uncertainty is realised.

The variables used throughout this paper are defined as follows:

- x_i is generator i's pre-commitment setpoint, the amount generator i prepares to produce before uncertainty is realised.
- $X_i(\omega)$ is generator i's real-time dispatch in scenario ω.
- $U_i(\omega)$ is generator i's upward deviation from its setpoint in scenario ω. It is equal to $\max(X_i(\omega) - x_i, 0)$.
- $V_i(\omega)$ is generator i's downward deviation from its setpoint in scenario ω. It is equal to $\max(x_i - X_i(\omega), 0)$.
- $F(\omega)$ is the vector of branch flows through the network in scenario ω.
- $\tau_n(F(\omega))$ is the net amount of energy injected from the grid into node n in scenario ω.

2.1. The Stochastic Dispatch Mechanism (SDM)

In SDM, we model renewable generation output by a set of samples from a continuous distribution, which constitutes an ensemble forecast of future uncertainty. Consequently, SDM is a Sample Average Approximation which yields pre-commitment setpoints that asymptotically converge to the optimal setpoint as the number of scenarios considered increases (see [29]). We determine SDM's pre-commitment setpoint, x^*, by solving the following stochastic program (see [33]):

$$
\begin{aligned}
\text{SLP: } \min \quad & \mathbb{E}_\omega[c^\top X(\omega) + r_u^\top U(\omega) + r_v^\top V(\omega)] & \\
\text{s.t.} \quad & \sum_{i \in \mathcal{T}(n)} X_i(\omega) + \tau_n(F(\omega)) \geq D_n(\omega), & \forall \omega \in \Omega,\ \forall n \in \mathcal{N}, \\
& x + U(\omega) - V(\omega) = X(\omega), & \forall \omega \in \Omega, \\
& F(\omega) \in \mathcal{F}, & \forall \omega \in \Omega, \\
& 0 \leq X(\omega) \leq G(\omega), & \forall \omega \in \Omega, \\
& U(\omega),\ V(\omega),\ x \geq 0, & \forall \omega \in \Omega.
\end{aligned}
$$

After determining the pre-commitment setpoint x^*, nature selects the scenario $\hat{\omega}$, and the ISO solves the following recourse problem for the real-time dispatch $X(\hat{\omega})$:

$$
\begin{aligned}
\min \quad & c^\top X(\hat{\omega}) + r_u^\top U(\hat{\omega}) + r_v^\top V(\hat{\omega}) & \\
\text{s.t.} \quad & \sum_{i \in \mathcal{T}(n)} X_i(\hat{\omega}) + \tau_n(F(\hat{\omega})) \geq D_n(\hat{\omega}),\ \forall n \in \mathcal{N}, & [\lambda_n(\hat{\omega})], \\
& X(\hat{\omega}) - U(\hat{\omega}) + V(\hat{\omega}) = x^*, & [\rho(\hat{\omega})], \\
& F(\hat{\omega}) \in \mathcal{F}, & \\
& 0 \leq X(\hat{\omega}) \leq G(\hat{\omega}), & \\
& U(\hat{\omega}), V(\hat{\omega}) \geq 0, &
\end{aligned}
$$

where $\lambda_n(\hat{\omega})$ is the dual multiplier for the supply-demand balance constraint at node n.

After solving both stages, the ISO pays $\lambda_{j(i)}(\hat{\omega})X_i(\hat{\omega})$ to generator i and charges $\lambda_n(\hat{\omega})D_n(\hat{\omega})$ to consumer n. The ISO does not incur a penalty for deviating from f to $F(\hat{\omega})$, and therefore is never out of pocket ([33], Proposition 1). Moreover, generators recover their fuel and deviation costs in expectation (but not with probability 1), since both quantities are priced when clearing the first stage ([33], [11]).

2.2. Review of stochastic auction properties in presence of risk-neutrality

In this section, we revisit the main results derived by Zakeri et al. in [33], in order to motivate our subsequent analysis. They are laid out as follows:

Proposition 1. *Let all generation agents be risk-neutral price-takers who are dispatched under SDM; then for each generator i, x_i^* is a $\frac{r_{u,i}}{r_{u,i}+r_{v,i}}$ quantile of the probability distribution of $X_i^*(\omega)$, i.e., a $\frac{r_{u,i}}{r_{u,i}+r_{v,i}}$-*VaR *of $X_i^*(\omega)$.*

Proof. See [29] for the result, and [34], [33] for applications of the result to SDM. □

Proposition 2. *Let agent i be a risk-neutral price-taking generation agent who offers the deterministic quantity G_i in each scenario, incurs the marginal costs c_i, $r_{u,i}$, $r_{v,i} > 0$, makes a pre-commitment decision $x_i^* > 0$, and is dispatched under SDM; then $M_i(x_i^*, \omega)$, generator i's profit margin in scenario ω, is defined as the following function, which is non-decreasing in $X_i^*(\omega)$:*

$$M_i(x_i^*, \omega) = \begin{cases} -r_{v,i}x_i^*, & \text{if } 0 \leq X_i^*(\omega) < x_i^*, \\ (\lambda_{j(i)}(\omega) - c_i)x_i^*, & \text{if } X_i^*(\omega) = x_i^*, \\ r_{u,i}x_i^*, & \text{if } x_i^* < X_i^*(\omega) < G_i, \\ \big(\lambda_{j(i)}(\omega) - c_i - r_{u,i}\big)G_i + r_{u,i}x_i^* & \text{if } X_i^*(\omega) = G_i. \end{cases}$$

Proof. The result follows directly from ([33], Equation (3)). □

Corollary 3. *The relationship between agent i's optimal real-time dispatch in scenario ω, $X_i^*(\omega)$ and the marginal price at node $j(i)$ in scenario ω, $\lambda_{j(i)}(\omega)$, is given by the following expression:*

$$\lambda_{j(i)}(\omega) \begin{cases} \leq c_i - r_{v,i}, & \text{if } X_i^*(\omega) = 0, \\ = c_i - r_{v,i}, & \text{if } 0 < X_i^*(\omega) < x_i^*, \\ \in [c_i - r_{v,i}, c_i + r_{u,i}] & \text{if } X_i^*(\omega) = x_i^*, \\ = c_i + r_{u,i}, & \text{if } x_i^* < X_i^*(\omega) < G_i, \\ \geq c_i + r_{u,i}, & \text{if } X_i^*(\omega) = G_i. \end{cases}$$

Proof. The result follows directly from ([33], Proposition 2). □

2.3. The relationship between pre-commitment and real-time nodal prices

In this section, we provide some new results which supplement those provided in the previous section. They are laid out as follows:

Proposition 4. *Let $x, \hat{x} \geq 0$ be two feasible pre-commitment setpoints. Then, the corresponding optimal second-stage nonanticipavity dual multipliers, $\rho(\omega), \hat{\rho}(\omega)$ almost-surely obey the following relationship (c.f. Proposition* 7.14 *of [21]):*

$$\langle x - \hat{x}, \rho(\omega) - \hat{\rho}(\omega) \rangle \geq 0.$$

Proof. This follows directly from the observation that the subgradient of SLP with respect to x in scenario ω, $\rho(\omega)$, is a maximal monotone operator (see, e.g, [7]). □

Corollary 5. *Let generator i's dispatch under SDM in scenario ω be $0 < X_i(\omega) < G_i$. Then, for sufficiently small changes in x_i that the same inequality constraints remain binding in the real-time problem, we have that if $x_{i,new} = x_i + \delta > x_i$ then either $\Delta\lambda_{j(i),new}(\omega) = 0$ or $\Delta\lambda_{j(i),new}(\omega) = r_{u,i} + r_{v,i}$ and if $x_{i,new} = x_i + \delta < x_i$ then either $\Delta\lambda_{j(i),new}(\omega) = 0$ or $\Delta\lambda_{j(i),new}(\omega) = -r_{u,i} - r_{v,i}$.*

Proof. Observe that if $0 < X_i(\omega) < G_i$ then the real-time dispatch problem's KKT condition with respect to $X_i(\omega)$ is:

$$\lambda_{j(i)}(\omega) + \rho_i(\omega) = c_i,$$

meaning we have that $\Delta\lambda_{j(i)}(\omega) + \Delta\rho_i(\omega) = 0$.

Moreover, the KKT conditions with respect to $U_i(\omega)$, $V_i(\omega)$ imply that $-r_{u,i} \leq \rho_i(\omega) \leq r_{v,i}$. Therefore, there exists some optimal basis where $\rho_i(\omega) = r_{v,i}$ or $\rho_i(\omega) = -r_{u,i}$ for each generator i, without loss of generality. Invoking Proposition 4 then yields the result, where we use the continuity of the optimal primal solution in the problem data (see [21]) to ensure that the same primal constraints remain binding. □

Remark 6. *The real-time nodal price λ is a maximal monotone operator with respect to the realised demand D. Therefore, for two given real-time demand vectors $D, \hat{D}$ the real-time prices obey the relationship $\left\langle \hat{D} - D, \hat{\lambda} - \lambda \right\rangle \geq 0$, and, since generator supply margins are nondecreasing functions of the real-time price, deterministic generators prefer low wind periods. That is, if generators are risk-averse then their risk-aversion causes them to place additional emphasis on low-wind periods.*

3. Risk-averse theoretical results

Thus far, we have considered dispatching risk-neutral price-taking participants under SLP. If participants are risk-averse, then they may prefer outcomes where they aren't out of pocket in any scenario, even if they lose welfare in expectation. In this section we study the outcome of agent interactions when agents are risk averse with no opportunity to trade risk, and uncover the inefficiencies that transpire as a result. We then move on to completing the risk trade market, and demonstrate that much like in finance [5], when coherent risk measures are used and in presence of a complete market, a martingale measure emerges, leading to an equivalent social planning model that embeds participants' risks and efficiency is restored.

We restrict our attention to law-invariant coherent risk measures because of their natural dual representability (see [20]). Our analysis resembles the analysis of risk-trading conducted by [27] although we restrict our attention to energy-only markets. This restriction allows us to exploit the properties of risk-averse newsvendors established by Choi et al. in [10].

We proceed to remind the reader of the definitions of (1) a coherent risk measure and (2) the risk-neutral competitive equilibrium which corresponds to the social plan implemented by

SLP. These definitions allow us to easily translate the risk-neutral equilibrium defined by SLP's Lagrangian into the following two risk-averse situations: (1) a risk-averse competitive equilibrium without risk instruments, (2) a risk-averse competitive equilibrium with a complete risk market.

Definition 7. A coherent risk measure $\rho : \mathcal{Z} \mapsto \mathbb{R}$ is a function which measures the risk-adjusted disbenefit of a random variable Z in a manner satisfying the four axioms defined by [3].

Each coherent risk measure possesses a dual representation, which is given by:

$$\rho(Z) = \max_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z],$$

where $\mathcal{D}$ is a convex subset of probability measures, refered to as a risk set. For a comprehensive discussion of coherent risk measures and their relationship with convex optimization see [27, 29].

By Kusuoka's Theorem (see [20], [29]) each coherent risk measure can be represented in the following mean-risk form for some risk coefficient κ and some risk set $\mathcal{D}$:

$$\rho(Z) = -\mathbb{E}[Z] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_{\beta}[Z] \mu d\beta \ = -\kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \mathrm{CVaR}_{\beta}[Z] \mu d\beta,$$

where

$$r_{\beta}[Z] = \min_{\eta \in \mathbb{R}} \mathbb{E}[\max((1-\beta)(\eta - Z), \beta(Z - \eta))]$$

is the weighted mean-deviation from the βth quantile,

$$\mathrm{CVaR}_{\beta}[Z] = -\max_{t \in \mathbb{R}} \{t - \frac{1}{\beta} \mathbb{E}[t - Z]_{+}\}$$

is the conditional value at risk, κ is a constant which prices risk by balancing the desirability of maximizing $\mathbb{E}[Z]$ with the undesirability of fluctuations towards the left tail of Z, and $\mathcal{D}$ is a convex subset of probability measures. Note that r_{β}, denoting the mean deviation from the βth quantile of Z, is quite distinct from the deviation costs r_u and r_v, introduced earlier in the stochastic dispatch; this will be clear from the context.

As observed by Philpott et al. in [23], whenever the sample space Ω is finite, at least one of the worst-case probability measures in $\mathcal{D}$ is an extreme point of $\mathcal{D}$. Consequently, in a Sample Average Approximation setting, $\rho(Z)$ is equal to the optimal value of the following linear program:

$$\rho(Z) = \min \quad \theta \text{ s.t. } \ \theta \geq \sum_{\omega} \mathbb{P}_{m\omega} Z_{\omega}, \ \forall m,$$

where $\mathbb{P}_m$ is the measure which corresponds to the mth extreme point of $\mathcal{D}$.

3.1. Risk-averse SDM without risk trading: A risked equilibrium

In this section we study what may eventuate if a stochastic dispatch is utilized when the participating agents are risk averse, as may well be the case in practice. To do so, first we prove that a risked equilibrium exists, and then we proceed to examining the properties of such an equilibrium. We begin by defining SLP's corresponding risk-neutral competitive equilibrium, as it can easily be elicited by decoupling SLP's Lagrangian (see, e.g., [11]).

Definition 8. Given prices $\lambda_{j(i)}(\omega)$ in each scenario ω, each generation agent i determines its dispatch $(x^*, X^*(\omega), U^*(\omega), V^*(\omega))$ by solving the following stochastic optimization problem:

$$\begin{aligned}\text{RNP}(i): \ \max \sum_{\omega} \mathbb{P}(\omega)\Big((\lambda_{j(i)}(\omega) - c_i)X_i(\omega) - r_{u,i}U_i(\omega) - r_{v,i}V_i(\omega)\Big)\\ \text{s.t. } x_i + U_i(\omega) - V_i(\omega) = X_i(\omega), \ \forall \omega \in \Omega,\\ 0 \le X_i(\omega) \le G_i(\omega), \ U_i(\omega), V_i(\omega), x_i \ge 0.\end{aligned}$$

As observed by [33], each generation agent i almost-surely recovers its costs in the long-run, since it can choose the action $(x, X, U, V) = (0, 0, 0, 0)$, which yields a payoff of 0 in each scenario.

Definition 9. Given prices $\lambda_n(\omega)$ in each scenario ω, the ISO determines the optimal distribution of branch flows throughout the network, $F^*(\omega)$, by solving the following optimization problem in order to maximize its cumulative rental, as defined by Philpott and Pritchard in [24]:

$$\text{PISO}(\omega): \ \max \sum_{n} \lambda_n(\omega)\tau_n(F(\omega)) \text{ s.t. } F(\omega) \in \mathcal{F},$$

where we write PISO(ω) to emphasise that the ISO's problem decouples by scenario, meaning the ISO is a wait-and-see agent which chooses the same action under any risk measure. As observed by [24], [33], the ISO recovers its costs in each scenario (i.e. achieves revenue adequacy), since it can choose the action $F(\omega) = 0, \ \forall \omega \in \Omega$ and earn a certain payoff of 0.

Definition 10. Given participant actions $(x, X(\omega), U(\omega), V(\omega), F(\omega))$ and prices $\lambda_n(\omega)$ in each scenario ω, determining whether the actions and prices constitute a risk-neutral competitive equilibrium is equivalent to determining whether
$(x, X(\omega), U(\omega), V(\omega)) \in \arg\max \text{RNP}$, $F(\omega) \in \arg\max \text{PISO}(\omega)$, and whether the participants collective choices of actions and dual prices satisfy the following market clearing condition:

$$0 \le \sum_{i} X_i(\omega) + \tau_n(F(\omega)) - D_n(\omega) \perp \lambda_n(\omega) \ge 0, \ \forall n \in \mathcal{N}, \ \forall \omega \in \Omega. \tag{1}$$

The Condition (1) requires that for each node n and each scenario ω, either supply equals demand or the price of electricity is zero.

To simplify the subsequent analysis, we follow [19] in replacing Condition (1) with a market-clearing agent. The equivalent market clearing agent's problem is defined as follows:

Definition 11. Given participant actions $(x, X(\omega), U(\omega), V(\omega), F(\omega))$, the market-clearing agent in scenario ω solves the following problem to determine the optimal choice of nodal prices:

$$\begin{aligned}\text{MC}(\omega): \ \max - \sum_{n} \lambda_n(\omega)\big(\sum_{i \in T(n)} X_i(\omega) + \tau_n(F(\omega)) - D_n(\omega)\big)\\ \text{s.t. } \lambda_n(\omega) \ge 0, \ \forall n \in \mathcal{N},\end{aligned}$$

where we write MC(ω) to emphasise that the market clearing agent is a wait-and-see agent.

We label the collection of Multiple Optimization Problems with Equilibrium Constraints (MOPEC) defined by the collection of RNP(i), PISO(ω) and $MC(\omega)$ as RNEQ. This definition leads us to the following result:

Proposition 12. *Let SLP satisfy a constraint qualification. If* $(x, X(\omega), U(\omega), V(\omega), F(\omega))$ *solves SLP, then there exists a set of prices* $\lambda_n(\omega)$, $\forall n \in \mathcal{N}$, $\forall \omega \in \Omega$ *such that* $(x, X(\omega), U(\omega), V(\omega), F(\omega), \lambda_n(\omega))$ *solves RNEQ.*

Proof. See Theorem 3 of [23]. □

We now consider the equivalent risk-averse equilibrium, wherein each generation agent i is endowed with a coherent risk measure ρ_i, with a view to show that the risk-averse equilibrium always admits at least one solution. Observe that since the ISO and the market clearing agent do not have first-stage actions, they perform the same action under any coherent risk measure. Therefore, we continue using the problems MC(ω) and PISO(ω) without loss of generality.

The risk-averse generation problem which each agent i solves is defined as follows:

Definition 13. Given prices $\lambda_i(\omega)$ in each scenario ω, generation agent i maximizes its risk-adjusted expected profit by determining the actions $(x, X(\omega), U(\omega), V(\omega))$ which solve the following stochastic optimization problem:

$$
\begin{aligned}
\text{RAP}(i) : \max \quad & \rho_i\Big((\lambda_{j(i)}(\omega) - c_i)X_i(\omega) - r_{u,i}U_i(\omega) - r_{v,i}V_i(\omega)\Big) & \\
\text{s.t.} \quad & x_i + U_i(\omega) - V_i(\omega) = X_i(\omega), & \forall \omega \in \Omega, \\
& 0 \le X_i(\omega) \le G_i(\omega), & \forall \omega \in \Omega, \\
& U_i(\omega), V_i(\omega), x_i \ge 0, & \forall \omega \in \Omega,
\end{aligned}
$$

where ρ_i is a coherent risk measure. Observe that generation agent i almost-surely recovers its risk-adjusted costs in the long-run, since it can choose the action $(x, X, U, V) = (0, 0, 0, 0)$ and earn a certain payoff of 0.

The collection of the problems PISO(ω), RAP(i) and $MC(\omega)$ then defines a risk-averse competitive equilibrium, which we refer to as RAEQ. Our subsequent analysis assumes that RAEQ admits a solution, and therefore requires an existence result. To obtain this existence result, we require the following intermediate lemma:

Lemma 14. *Let generation agent i be a risk-averse price-taking generation agent endowed with the coherent risk measure* ρ_i*. Then agent i's optimization problem, RAP(i), has a closed, convex and bounded strategy set.*

Proof. The constraint $0 \le X_i(\omega) \le G_i(\omega)$ implies generator i's optimal action, x_i^*, satisfies the inequality $0 \le x_i^* \le \max_\omega\{G_i(\omega)\}$, by Proposition 1. Therefore, we can introduce the constraint $0 \le x_i^* \le \max_\omega\{G_i(\omega)\}$ into the problem RAP(i), without loss of optimality. The restrictions on X and x then imply that $0 \le U_i(\omega), V_i(\omega) \le G_i(\omega)$, since $U_i(\omega) = \max(X_i(\omega) - x_i, 0)$ and $V_i(\omega) = \max(x_i - X_i(\omega), 0)$. Therefore, the strategy space RAP(i) is bounded.

The strategy space is also closed and convex, because it is defined by the intersection of a set of linear inequality constraints. □

We also require the following assumption:

Assumption 15. The optimal choice of dual price $\lambda_n(\omega)$ is bounded from above by the Value of Lost Load, or VOLL, for all nodes n and all scenarios ω.

Assumption 15 is common in power system applications; for instance, the NZEM has a price cap of VOLL=\$20,000 per MWh, meaning consumers are willing to curtail their load at a marginal price of \$20,000 per MWh in the short-run (see [30] for a general theory). Moreover, ([33], Lemma 1) establishes that for a fixed pre-commitment setpoint x and set of real-time demand realisations, there exists some price cap such that Assumption 15 holds everywhere except a set of measure 0.

Combining Lemma 14 and Assumption 15 yields the following theorem (c.f. [19]):

Theorem 16. *Suppose that Assumption 15 holds. Then, RAEQ admits a solution.*

Proof. To show this result, we follow the steps of Rosen's theorem (see [28]) in arguing that the following three conditions hold:

1. Each participant's strategy set is non-empty.
2. Each participant's strategy set is a closed, convex and bounded set.
3. Each participant's payoff function is concave in her strategy space, and continuous in all other participants' strategy spaces.

The first statement holds, as each participant can choose the feasible action of setting all their decision variables to 0, and therefore all participants have non-empty strategy sets.

To show that the second statement holds, we consider each class of agent separately. First, the problem PISO(ω)'s strategy space is the closed, convex and bounded set $\mathcal{F}$, which implies that the condition (2) holds for PISO(ω). Second, by Lemma 14, each generation agent i's strategy set is closed, convex and bounded. Finally, by Assumption 15, the market clearing agent's strategy space is a bounded set which can be seen to be closed and convex by inspection. Therefore, Condition (2) holds for all participants in RAEQ.

The third statement holds for all generation agents, as their decision variables are continuous and they are endowed with coherent risk measures, i.e., convex risk measures where positive homogeneity also holds. Consequently, their payoff functions are concave with respect to maximization. Similarly, the third statement holds for the ISO and market-clearing agents, as they solve wait-and-see optimization problems by choosing continuous decision variables from convex strategy sets.

Therefore, it follows from Rosen's Theorem (see [28]) that RAEQ admits a solution. □

Remark 17. Existence does not imply uniqueness.
Theorem 16 shows that, with a price cap of VOLL, there exists a set of prices which clear the market when the participants are risk-averse. However, Theorem 16 does not imply that this set of prices is unique. Indeed, [13] provides examples of energy-only pool markets with risk averse generators which admit multiple equilibria.

We proceed to consider RAP(i)'s first-order optimality condition, with a view to obtain insight into the relationship between x^* and $X^*(\omega)$. To do so, we change perspective and assume that Ω represents the true distribution of uncertainty. Consequently, the below results hold for the true distribution of uncertainty, while solutions to RAEQ constitute Sample Average Approximation (SAA) estimators of the solution for the true distribution. However, SAA estimators for variational inequalities converge exponentially as we increase the sample size (see [32]). Therefore, the below results also hold for RAEQ, provided our sample of the underlying distribution is sufficiently rich.

We also need to clarify our understanding of the remuneration process. To see this, consider the auctioneer's price-setting problem MC(ω) in scenario ω, and assume that the optimal choice of dual price is $VOLL > \lambda_n^*(\omega) > 0$ for some node n. Then, the corresponding DC-load-flow constraint must be met exactly, because otherwise the unique optimal choice of nodal price is VOLL. Therefore, we have that any $\lambda_n(\omega) \in [0, VOLL]$ is an optimal choice of dual price at this node, with all such choices providing the auctioneer with a payoff of 0. That is, the remuneration scheme suggested by RAEQ provides highly degenerate dual prices. Consequently, we assume that participants are dispatched and remunerated in the same manner as SLP, although they may be risk-averse when making their pre-commitment decision. In this context, each generation agent i solves a risk-averse newsvendor problem to determine their pre-commitment behaviour.

This observation allows us to characterize the impact of a generator's risk-aversion on their pre-commitment behaviour in the following proposition:

Proposition 18. *Let generator i be risk-averse and endowed with the coherent risk measure $\rho : \mathcal{Z} \mapsto \mathbb{R}$, which has the following Kusuoka representation:*

$$\rho[Z] = -\mathbb{E}[Z] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z] \mu d\beta.$$

Then, for a given set of second stage dispatches $X_i^*(\omega)$, generator i makes the pre-commitment decision x_i^*, where:

$$x_i^* = F^{-1}_{X_i^*(\omega)}\Big(\frac{r_{u,i}}{(r_{u,i} + r_{v,i})(1 + \kappa(1 - \bar{\beta}))}\Big),$$
$$\bar{\beta} = \int_0^1 \mu^{RN} \beta d\beta;\ \kappa \in [0, \frac{1}{\bar{\beta}}].$$

Proof. See Appendix A.1. □

Proposition 18 indicates that increasing generator i's risk coefficient $\kappa(1 - \bar{\beta})$ from its risk-neutral level $\kappa(1 - \bar{\beta}) = 1(1 - 1) = 0$ results in generator i decreasing their pre-commitment set-point from its risk-neutral level. We formalize this observation in the following corollary to Proposition 18.

Corollary 19. *Let generator i be endowed with the coherent risk measure ρ_i. Then, for a given set of second stage dispatches $X_i^*(\omega)$, generator i makes the pre-commitment decision x_i^*, which is bounded from above by the following expression:*

$$x_i^* = F^{-1}_{X_i^*(\omega)}\Big(\frac{r_{u,i}}{(r_{u,i} + r_{v,i})(1 + \kappa(1 - \bar{\beta}))}\Big) \leq F^{-1}_{X_i^*(\omega)}\Big(\frac{r_{u,i}}{(r_{u,i} + r_{v,i})(1 + 1(1 - 1))}\Big) = x_{i,RN}^*,$$

where $x_{i,RN}^*$ is generator i's risk-neutral choice of pre-commitment decision for the set of second-stage dispatches $X_i^*(\omega)$.

Proof. The proof follows a simple application of Proposition 18. □

The analysis in the previous sections indicates that modifying the total pre-commitment magnitude impacts the payoffs to the market participants. Consequently, a pertinent question is "what is the impact of generator risk-aversion on the generator's expected payoff?". We provide a lower bound on this quantity in the following proposition:

Proposition 20. *Let generator i's risk-aversion be represented by the risk measure ρ_i, which has a Kusuoka representation such that $\bar{\beta}_i := \int_0^1 \mu^{RN} \beta_i d\beta_i$, $\kappa_i \in [0, \frac{1}{\bar{\beta}_i}]$, and combine these quantities by defining $\alpha_i := \frac{1}{1+\kappa_i(1-\bar{\beta}_i)}$. Then generator i's expected profit is at least $(1-\alpha_i) r_{u,i} x_i^*$. This quantity is 0 if generator i is risk-neutral and positive otherwise.*

Proof. Proposition 18 shows that the quantity $\alpha_i := \frac{1}{\kappa_i - \kappa_i \beta_i}$ summarizes the relationship between generator i's pre-commitment and its production, since x_i^* is a $\frac{\alpha_i r_{u,i}}{r_{u,i}+r_{v,i}}$ quantile of the distribution of $X_i^*(\omega)$. Therefore, $X_i^*(\omega) \leq x_i^*$ with probability $\frac{\alpha_i r_{u,i}}{r_{u,i}+r_{v,i}}$. Applying Proposition 2 then reveals that generator i receives a payoff of at least $-r_{v,i}x_i^*$ with probability $\frac{\alpha_i r_{u,i}}{r_{u,i}+r_{v,i}}$, and at least $r_{u,i}x_i^*$ with probability $\frac{(1-\alpha_i)r_{u,i}+r_{v,i}}{r_{u,i}+r_{v,i}}$. Computing the expected payoff then yields the result. □

Proposition 20 suggests that generator risk-aversion results in a lower pre-commitment magnitude than the optimal risk-neutral setpoint, which reduces cumulative system welfare. Moreover, Proposition 20 indicates that generators have an incentive to behave in a risk-averse manner, as doing so increases their expected profit.

To see that this situation can also arise in a dispatch mechanism such as SDM, observe that generators can express their risk-aversion by inflating the relative magnitude of $r_{v,i}$, their marginal cost of deviating downward, in order that the auctioneer dispatches them at a lower pre-commitment setpoint. Indeed, a recent numerical study [16] confirms our finding, by demonstrating that in a two-market stochastic equilibrium where generators are endowed with the CVaR risk criterion, generators prefer to pre-commit less generation when they are more risk-averse

Fortunately, [27] provides a means to extend SDM to cope with risk-aversion: introducing an auxiliary financial market wherein generators and the ISO can trade risk. If generators and the ISO are endowed with intersecting risk sets, then trading Arrow-Debreu securities causes each participant's effective risk-aversion to decrease to the least risk-averse participant's risk-aversion, leaving only residual risk. In this case, each generator's pre-commitment decision is equivalent to the decision made by a risk-averse system optimizer who uses the *least* risk averse agent's risk set as its own [27].

While a fully liquid risk market is unattainable in reality, a recent numerical study [12] demonstrates that, in a capacity expansion setting, partially liquid risk markets yield similar outcomes to fully liquid risk markets where participants trade Arrow-Debreu securities. This observation justifies our consideration of fully liquid risk markets.

3.2. Risk-averse SDM with risk trading: A risk-averse social planner

In this section, we extend our preceding analysis to consider a stochastic energy-only market where participants trade Arrow-Debreu securities on an exchange. We begin by defining the market clearing problem.

We require the following definition:

Definition 21. *An Arrow-Debreu security is a contract which charges the price π_ω to receive a payoff of 1 in scenario ω. We let $W_{i\omega}$ denote the bundle of Arrow-Debreu securities held by agent i (see [27]).*

We also require the following notation:

- θ_i is generator i's risk-adjusted payoff.
- θ_k is the ISO's risk-adjusted payoff.
- $W_{i\omega}$ is the quantity of Arrow-Debreu securities purchased by generator i in scenario ω.
- $W_{k\omega}$ is the quantity of Arrow-Debreu securities purchased by the ISO in scenario ω.

- $\mathbb{P}_{im\omega}$ is the probability measure corresponding to the mth extreme point of generator i's risk set.
- $\mathbb{P}_{km\omega}$ is the probability measure corresponding to the mth extreme point of the ISO's risk set.

Assume that each generator submits the same offers as in SDM, that all generators and the ISO submit their risk sets before the market is cleared, and that the intersection of the participants' risk sets is non-empty. Then, clearing the market is equivalent to minimizing cumulative risk-adjusted disutility [27], i.e., solving the following risk-averse stochastic program:

$$
\begin{aligned}
\text{RASLP:} \quad & \min \sum_i \theta_i + \theta_k \\
\text{s.t.} \quad & \theta_i \geq \sum_\omega \mathbb{P}_{im\omega}(c_i X_i(\omega) + r_{u,i} U_i(\omega) + r_{v,i} V_{i,\omega} - W_{i\omega}), && \forall i, \forall m, \\
& \theta_k + \sum_\omega \mathbb{P}_{km\omega} W_{k\omega} \geq 0, && \forall m, \\
& \sum_{i \in \mathcal{T}(n)} X_i(\omega) + \tau_n(F(\omega)) \geq D_n(\omega), && \forall \omega \in \Omega, \\
& \sum_i W_{i\omega} + W_{k\omega} = 0, && \forall \omega \in \Omega, \qquad [\pi_\omega], \\
& x + U(\omega) - V(\omega) = X(\omega), && \forall \omega \in \Omega, \\
& F(\omega) \in \mathcal{F}, && \forall \omega \in \Omega, \\
& 0 \leq X(\omega) \leq G(\omega), \; U(\omega), V(\omega), x \geq 0, && \forall \omega \in \Omega,
\end{aligned}
$$

where we follow [23] in enumerating the extreme points of each generator's risk set in order to express the market-clearing problem as a single linear optimization problem.

After solving RASLP, participants are remunerated for their dispatch in the same manner as SDM, and participants are remunerated with the term $W_{i\hat{\omega}} - \sum_\omega \pi_\omega W_{i\omega}$ in scenario $\hat{\omega}$ for their financial instruments as per [27].

As noted by [27], the dual prices of the Arrow-Debreu securities, π_ω, correspond to the system optimizer's risk-adjusted probability measure. The equivalence between the dual prices and the worst-case probability measure allows us to rewrite the system optimization objective function with the following objective:

$$
\min_{x,u,v} \rho(c^\top X(\omega) + r_u^\top U(\omega) + r_v^\top V(\omega)),
$$

where ρ is the coherent risk measure coupled with its dual risk set $\mathcal{D}$. If $\mathcal{D} = \{\mathbb{P}(\omega)\}$ then (1) there exists a risk-neutral agent which absorbs all risk in the market, (2) the Arrow-Debreu securities are priced at $\mathbb{P}(\omega)$, and (3) there is no residual system risk (see [27] or [23]).

Interestingly, unlike the risk-averse competitive equilibrium studied in the previous section, it is straightforward to elicit verifiable conditions for existence and uniqueness of a risk-averse competitive equilibrium in the presence of risk trading. Existence can be verified by solving the system optimization problem. Moreover, if $\mathcal{F}$ is a polyehdral set, uniqueness can be verified by solving RASLP, and subsequently maximizing the sum of the non-basic variables with reduced cost 0, subject to fixing all other non-basic variables at their bound-if the auxillary LP has an optimal value of 0 then the stochastic market clearing problem has a unique solution, and if not then the auxillary system optimization problem elicits a second risk-averse competitive equilibrium (c.f. Exercise 3.9 of [4]).

Our main interest in this paper is determining the impact of the existence of financial instruments on the pre-commitment setpoint. Consequently, we change our perspective and assume that Ω represents the true distribution of uncertainty. Strictly speaking, the optimal solution to RASLP constitutes an SAA estimator of the optimal solution for the true distribution. However, SAA estimators are known to converge almost surely to the optimal solution for the underlying distribution (see [29]). Therefore, the below results hold almost surely true for solutions to RASLP, wherever the sample of the underlying distribution is sufficiently rich.

We proceed to consider the system optimization's first-order optimality condition with respect to each generator i. As we are considering a system optimization problem rather than an individual generator's problem, our objective is risk-adjusted expected fuel cost minimization rather than risk-adjusted expected profit maximization, and we are risk-averse to scenarios with high fuel plus deviation costs rather than scenarios with low nodal prices. This observation leads to the following proposition:

Proposition 22. *Suppose that the system is risk-averse and endowed with the law invariant coherent risk measure $\rho : \mathcal{Z} \mapsto \mathbb{R}$, which has the Kusuoka representation:*

$$\rho[Z] = -\mathbb{E}[Z] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z] \mu d\beta.$$

Then, for a given dispatch policy $X^*(\omega)$, each generator makes the pre-commitment decision to produce x_i^*, where:

$$x_i^* = F^{-1}_{X_i^*(\omega)}\Big(\frac{r_{u,i} + (r_{u,i} + r_{v,i})(\kappa - \kappa\bar{\beta})}{(r_{u,i} + r_{v,i})(1 + \kappa(1 - \bar{\beta}))}\Big),$$
$$\bar{\beta} = \int_0^1 \mu^{RN} \beta d\beta, \ \ \kappa \in [0, \frac{1}{\bar{\beta}}].$$

Proof. See Appendix A.2. □

We remind the reader that the dispatch policy $X^*(\omega)$ obtained from RASLP and used in Proposition 22 is not necessarily the same dispatch policy as that obtained from RAEQ and used in Proposition 18. In particular, both dispatch policies are functions of (1) the problem data and (2) their (respective and possibly different) pre-commitment setpoints.

Proposition 22 has the following interpretation: although the real-time price $\lambda(\omega)$ is a maximally monotone operator with respect to realised demand, meaning risk-averse generators place additional emphasis on high-wind scenarios and reduce their pre-commitment from a risk-neutral setpoint, Arrow-Debreu securities re-align generators incentives, causing them to view high-wind scenarios favourably, and increase their pre-commitment magnitude from its risk-neutral setpoint. We formalize this observation in the following corollary to Proposition 22:

Corollary 23. *Let the system be endowed with the coherent risk measure ρ. Then, for a given second stage dispatch policy $X^*(\omega)$, each generator makes the pre-commitment decision x_i^*, which* *is bounded from below by the following expression:*

$$x_i^* = F^{-1}_{X_i^*(\omega)}\Big(\frac{r_{u,i} + (r_{u,i} + r_{v,i})(\kappa - \kappa\bar{\beta})}{(r_{u,i} + r_{v,i})(1 + \kappa(1 - \bar{\beta}))}\Big) \geq F^{-1}_{X_i^*(\omega)}\Big(\frac{r_{u,i} + (r_{u,i} + r_{v,i})(1(1-1))}{(r_{u,i} + r_{v,i})(1 + 1(1 - 1))}\Big) = x^*_{i,RN},$$

where $x^*_{i,RN}$ is generator i's risk-neutral pre-commitment decision.

We remind the reader that the second-stage dispatches from SLP and RASLP are distinct, meaning we cannot make a direct comparison between x_i^* and $x^*_{i,RN}$. However, recalling that the optimal real-time dispatch is continuous in the pre-commitment decision x, Corollary 23 applies when the least risk-averse participant's behaviour is sufficiently close to risk-neutrality that the second-stage dispatches under SLP and RASLP are identical. Consequently, the above corollaries can be thought of as risk-averse sensitivity analysis results.

The analysis in the previous sections suggests that increasing the total amount of pre-commitment decreases expected nodal prices. Consequently, a pertinent question is "does an auxiliary risk market remove the positive relationship between a generator's risk-aversion and its expected payoff?".

Proposition 24. *Let the system be risk-averse with risk measure ρ, which has a Kusuoka representation such that $\bar{\beta} := \int_0^1 \mu^{RN}\beta d\beta$, $\kappa \in [0, \frac{1}{\bar{\beta}}]$, and combine these two quantities by defining $\alpha := \frac{1}{1+\kappa(1-\bar{\beta})}$. Then, generator i's expected profit is at least $-(1-\alpha)r_{v,i}x_i^*$. This quantity is 0 when the least risk-averse agent is risk-neutral and is negative otherwise.*

Proof. Proposition 22 shows that the quantity $\alpha_i := \frac{1}{\kappa_i - \kappa_i\beta_i}$ summarizes the relationship between generation agent i's pre-commitment and its production, since x_i^* is a $\frac{r_{u,i}+(1-\alpha_i)r_{u,i}}{r_{u,i}+r_{v,i}}$ quantile of the distribution of $X_i^*(\omega)$. Therefore, $X_i(\omega)^* \leq x_i^*$ with probability $\frac{r_{u,i}+(1-\alpha_i)r_{u,i}}{r_{u,i}+r_{v,i}}$. Applying Proposition 2 then reveals that each generation agent i receives a payoff of at least $-r_{v,i}x_i^*$ with probability $\frac{r_{u,i}+(1-\alpha)r_{v,i}}{r_{u,i}+r_{v,i}}$ and receives a payoff of at least $r_{u,i}x_i^*$ with probability $\frac{\alpha r_{v,i}}{r_{u,i}+r_{v,i}}$. Computing the expected payoff then yields the result. □

The above analysis might appear to suggest that expected cost-recovery is not guaranteed in RASLP. However, this is not the case, as the above analysis does not include payoffs from the auxiliary risk market. Moreover, by comparison with the feasible choice of non-participation in both markets, which has a certain payoff of 0 under any coherent risk measure, we can show that risk-averse generators recover their risk-adjusted costs *in expectation.* However, profits from the auxiliary risk market are derived by assuming risk, unlike the situation described in Proposition 20.

4. Risk-averse numerical results

The theoretical results in the previous section indicate that, for a given set of real-time dispatches, if generators can trade risk via contracts then their pre-commitment setpoint depends on the market risk and becomes higher as the market risk increases. One might be tempted to infer that as the market risk increases, each generator's pre-commitment also increases. However, this is not necessarily the case. Indeed, a more risk-averse system might procure generation from more flexible but more expensive sources in order to hedge against low wind scenarios, in which case cheaper and less flexible generation units may pre-commit less generation. In this section, we demonstrate that this situation arises in the six-node network introduced by Pritchard et al. [26] and studied by [34], [11]. We use the problem data described in [11] but reproduce it here in order that this paper is self-containing.

4.1. A six node example under the CVaR *criterion*

Consider a transmission network with the topology depicted in Figure 1, which comprises two inflexible thermal generators who cannot ramp up or down in the second stage, two flexible hydro generators who can ramp up or down at marginal costs of 35 and 20, and two intermittent wind generators who can ramp up or down without incurring a deviation cost. The output capacity of each generator and their marginal cost of generation are indicated by the notation "X@\$Y". The two wind generators independently draw an output capacity from the sample space $\{30, 50, 60, 70, 90\}$ with each realization occurring with equal probability, resulting in 25 scenarios each having probability 0.04. There is a single deterministic and inflexible consumer who requires 264 units of generation in each scenario, and a transmission constraint dictates that up to 150 units can be transmitted from node A to node B or vice versa. The lines are assumed to obey DC load-flow constraints imposed by Kirchhoff's voltage laws and have equal reactances, meaning $\frac{1}{6}$ of the power generated by the hydro generators flows via the constrained line, and $\frac{1}{3}$ of the power generated by Wind 2 flows via the constrained line. We prevent dual degeneracy by imposing quadratic losses on all transmission lines, with a loss coefficient of 10^{-8}. The market is cleared by minimizing the risk-adjusted fuel plus deviation cost, where the system optimizer is endowed with the β-CVaR risk measure for several different values of β.

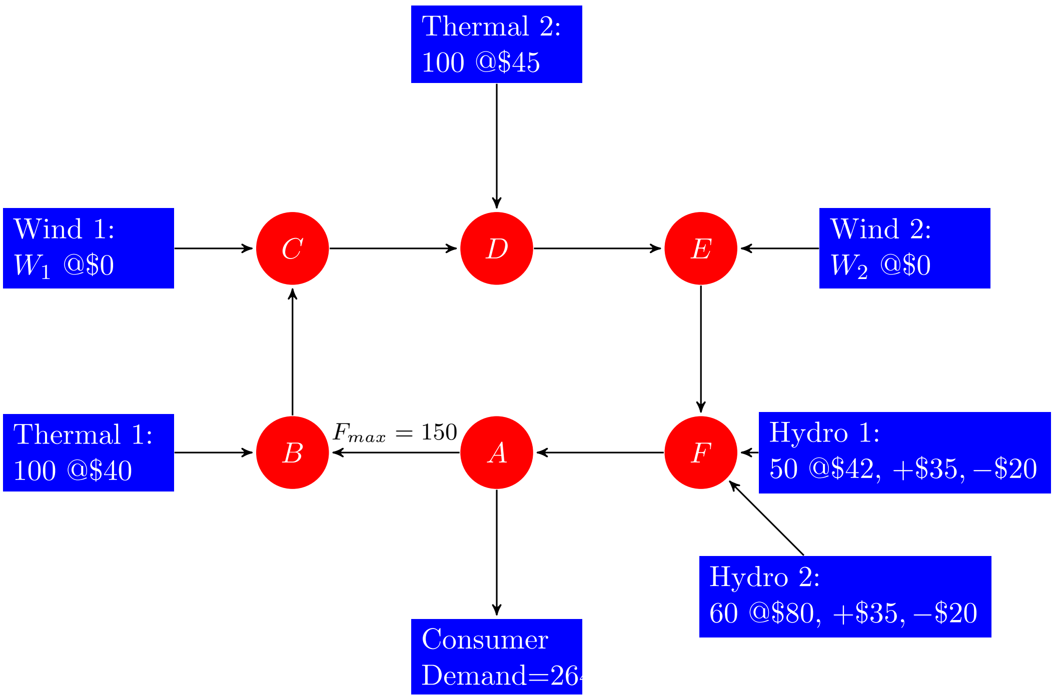

Figure 1: The six node example studied in [26], [34] and [11].

Table 1 depicts the pre-commitment behaviour of each deterministic generator as the market becomes more risk-averse, which corresponds to the least risk-averse generation agent in the market becoming more risk-averse. Observe that the system never cumulatively pre-commits less generation with higher residual market risk. However, the distribution of second-stage dispatches changes with market risk, causing Hydro 1 to pre-commit less generation at moderate risk-aversion levels than when the system is risk-neutral. This observation justifies our previous remark that increased system risk does not cause each generator's pre-commitment to increase.

Table 1: The pre-commitment behaviour of each deterministic generator in Figure 1 by risk.

Gen	β coefficient in CVaR ($\beta = 0$ is risk-neutral)									
(MW)	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Thermal 1	79	94	84	79	79	79	74	74	76.5	74
Thermal 2	75	40	50	55	55	45	50	50	45	40
Hydro 1	50	50	50	40	40	42.5	40	40	37.5	40
Hydro 2	0	0	0	0	0	0	0	0	0	0
Cumulative	204	184	184	174	174	166.5	164	164	159	154

Corollary 5 demonstrates that increases in pre-commitment decrease expected nodal prices. Combining this finding with Table 1 suggests that forward contracts induce lower expected nodal prices in the presence of generator risk-aversion. The reader may appreciate that this phenomena is similar to that observed by Allaz and Vila in [1], who show that forward contracts lower nodal prices in an oligopolistic setting where risk-neutral participants exercise market power. In further support of this idea, Appendix B demonstrates that a similar phenomena arises in a full-scale model of the New Zealand Electricity Market.

5. Conclusions

Our results demonstrate that generator risk-aversion tends to result in less pre-commitment and higher expected nodal prices; an outcome which causes higher consumer costs and higher generator profits. Alternatively, introducing a complete set of forward contracts results in more pre-commitment and depressed expected nodal prices; a result which causes better consumer outcomes, worse generator outcomes and improved system efficiency in the presence of generator risk-aversion.

Our results suggest that wind power futures may be necessary to account for the market imperfections introduced by intermittent generation, particularly in the presence of risk aversion. This observation was previously made by Gersema and Wozabal in [14], who considered a two-agent equilibrium model with intermittency. Indeed, the European Energy Exchange (EEX) recently introduced wind power futures to mitigate the market imperfections introduced by intermittency.

Acknowledgements

The authors are grateful to Andy Philpott for stimulating discussions on the New Zealand Electricity Market. Ryan Cory-Wright is grateful to the Department of Engineering Science at the University of Auckland, the Energy Centre at the University of Auckland and the Energy Education Trust of New Zealand for their financial support.

Appendix A Proofs of Propositions

A.1 Proof of Proposition 18

To show this result, we model an arbitrary generator as a risk-averse newsvendor by using the notation in [10], and we convert to the notation used in the main body of this paper ex-post. This choice maintains consistency with the newsvendor literature, because conventional newsvendor models assume the cost of stocking a product is incurred in the first stage, whilst we assume that the cost of stocking a product is incurred in the second stage, and modify our deviation costs accordingly. Our approach can be viewed as a generalization of that taken in Section 5 of [10], as we include the possibility that newsvendors might back-order in the second stage and incur an additional cost for doing so (i.e. ramp up their plant's production at a marginal cost of $c_i + r_{u,i}$), while the analysis conducted by Choi et al. in [10] precludes the possibility that $X_i^*(\omega) > x_i^*$.

We require the following terms:

- e is the marginal emergency order cost.
- s is the marginal salvage value.
- p is the marginal sale price.
- c is the marginal ordering cost.
- x is the initial order quantity.
- D is the stochastic demand.
- $y_+ = \max(y, 0)$ is the positive component of y.
- $\Pi(x, D)$ is the newsvendor's profit with initial stock x and demand D.
- ρ is a law-invariant coherent risk measure.

We follow [9] in defining the newsvendor's profit function as:

$$\begin{aligned}\Pi(x, D) &= pD - cx + s(x - D)_+ - e(D - x)_+,\\ &= (p - e)D + (e - c)x - (e - s)(x - D)_+,\\ &= (e - c)x + Z_+.\end{aligned}$$

Using Definition 7, we invoke Kusuoka's Theorem (see [20]) to represent the law-invariant coherent risk measure ρ via the following expression:

$$\rho[Z] = -\mathbb{E}[Z] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z] \mu d\beta,$$

where $r_\beta[Z] = \min_{\eta \in \mathbb{R}} \mathbb{E}[\max((1 - \beta)(\eta - Z), \beta(Z - \eta))] = \beta(\mathrm{CVaR}_\beta[Z] + \mathbb{E}[Z])$.

The above expression permits a representation of the newsvendor's risk-adjusted profit via the following function:

$$\begin{aligned}\rho(\Pi(x, D)) &= -\mathbb{E}[\Pi(x, D)] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[\Pi(x, D)] \mu d\beta,\\ &= -(e - c)x - \mathbb{E}[Z_+] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z_+] \mu d\beta,\end{aligned}$$

as $(e-c)x$ is invariant and $r_\beta[Z+a] = r_\beta[Z]$ for nonrandom a. Moving $\mathbb{E}[Z_+]$ within the integral and using the substitution
$r_\beta[Z] = \beta(\mathrm{CVaR}_\beta[Z] + \mathbb{E}[Z])$, provides the following expression:

$$\rho(\Pi(x,D)) = -(e-c)x + \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \Big(\mathbb{E}[Z_+](\kappa\beta - 1) + \kappa\beta \mathrm{CVaR}_\beta[Z_+] \Big) \mu d\beta.$$

Observe that the β quantile of Z_+ must be lower than in the risk-neutral case. In the risk-neutral case, the optimal choice of x is the $\frac{c-s}{e-s}$ quantile of D (see Proposition 1), which corresponds to equality between the βth quantile of Z_+ and $(p-s)D$. In the risk-averse case, the βth quantile of Z_+ is (equal or) lower and is therefore equal to $(p-s)D - (e-s)x$ for some x and some D. This observation allows us to define the partial derivatives of the expectation and CVaR terms within $\rho(\Pi(x,D)$ as follows:

$$\begin{aligned}
\frac{\partial \mathbb{E}[Z_+]}{\partial x} &= -(e-s)\mathbb{P}(x > D), \\
\frac{\partial \mathrm{CVaR}_\beta[Z_+]}{\partial x} &= -\frac{\partial}{\partial x}\Big\{ (p-s)D - (e-s)x - \frac{1}{\beta}\mathbb{E}[(p-s)D - (e-s)x - Z_+] \Big\}, \\
&= (e-s) - \frac{1}{\beta}(e-s) + \frac{1}{\beta}(e-s)\mathbb{P}(x > D), \\
&= (e-s)(1 - \frac{1}{\beta}) + \mathbb{P}(x > D)(e-s)\frac{1}{\beta}.
\end{aligned}$$

Now, assume that the supremum over the risk set D is uniquely attained at the measure $\hat{\mu}$; then we have the following first-order optimality condition:

$$\begin{aligned}
\frac{\partial \rho(\Pi(x,D))}{\partial x} &= -(e-c) \\
&+ \int_{\beta=0}^{\beta=1} \Big((e-s)(1+\kappa-\kappa\beta)\mathbb{P}(x > D) - \kappa(e-s)(1-\beta) \Big) \hat{\mu} d\beta, \\
&= -(e-c) + \mathbb{P}(x > D)(e-s)\Big(1 + \kappa - \kappa\big(\int_{\beta=0}^{\beta=1} \beta\hat{\mu} d\beta\big)\Big) \\
&- \kappa(e-s)(1 - (\int_{\beta=0}^{\beta=1} \beta\hat{\mu} d\beta)).
\end{aligned}$$

Setting this condition to 0 and re-arranging for $\mathbb{P}(x > D)$ yields:

$$\mathbb{P}(x > D) = \frac{(e-c) + \kappa(e-s)(1-\bar{\beta})}{(e-s)(1+\kappa-\kappa\bar{\beta})},$$

where $\bar{\beta} = \int_0^1 \mu^{RN} \beta d\beta$ is the expected value of the risk-averse probabilities with respect to the risk-neutral probabilities.

Netting against 1 to find $\mathbb{P}(x \leq D)$ then yields:

$$\mathbb{P}(x \leq D) = \frac{(c-s)}{(e-s)(1+\kappa-\kappa\bar{\beta})}.$$

To convert to our notation, observe that $r_u = c - s$ and $r_v = e - c$, giving $r_u + r_v = e - s$. Therefore, we have that:

$$\mathbb{P}(x \leq D) = \frac{r_u}{(r_u + r_v)(1 + \kappa - \kappa\bar{\beta})},$$

as required. Note that the corresponding quantile is not necessarily unique. $\square$

A.2 Proof of Proposition 22

To show this result, we invoke the observation made by [33] that for a given set of second-stage dispatches $X^*(\omega)$, the system solves a newsvendor problem in order to determine the pre-commitment setpoint which minimizes the term

$$\mathbb{E}[r_{u,i}U_i(\omega) + r_{v,i}V_i(\omega)]$$

for each generator i. Consequently, we use the same notation as in the proof of Proposition 18. We require $p = 0$, as we are considering a system optimization problem and any revenue accrued by a generator is provided by the ISO. Therefore, the system's residual cost with respect to a particular generator's pre-commitment decision, $\Pi(x, D)$, is defined by the following expression:

$$\begin{aligned}
\Pi(x, D) &= -cx + s(x - D)_+ - e(D - x)_+, \\
&= -eD + (e - c)x - (e - s)(x - D)_+, \\
&= (e - c)x + Z_+.
\end{aligned}$$

By following the steps outlined in Appendix A.1, we obtain the following risk-adjusted profit function:

$$\rho(\Pi(x, D)) = -(e - c)x - \mathbb{E}[Z_+] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z_+]\mu d\beta.$$

Now observe that since $p = 0$, the risk-neutral critical fractile becomes $-eD$. Since $s(x - D) - ex$ gives a lower system cost than $-eD$ we therefore have that the critical fractile within each CVaR term becomes equal to $-eD$. This situation is similar to Section 5.3 of [10], although we include emergency holding costs. Consequently, the partial derivatives of the terms which constitute ρ become:

$$\begin{aligned}
\frac{\partial \mathbb{E}[Z_+]}{\partial x} &= -(e - s)\mathbb{P}(x > D), \\
\frac{\partial \mathrm{CVaR}_\beta[Z_+]}{\partial x} &= -\frac{\partial}{\partial x}\Big\{ -eD - \frac{1}{\beta}\mathbb{E}[-eD - Z_+]\Big\} = \frac{1}{\beta}(e - s)\mathbb{P}(x > D).
\end{aligned}$$

Now assume that $\mu = \hat{\mu}$ is the unique optimal pdf. Then, we have the following first-order condition:

$$\begin{aligned}
\frac{\partial \rho(\Pi(x, D))}{\partial x} &= -(e - c) + \int_{\beta=0}^{\beta=1} \Big((e - s)(1 + \kappa - \kappa\beta)\mathbb{P}(x > D)\Big)\hat{\mu}d\beta, \\
&= -(e - c) + \mathbb{P}(x > D)(e - s)\Big(1 + \kappa - \kappa \int_{\beta=0}^{\beta=1} \beta\hat{\mu}d\beta\Big).
\end{aligned}$$

Setting this condition to 0 and re-arranging for $\mathbb{P}(x > D)$ yields:

$$\mathbb{P}(x > D) = \frac{(e-c)}{(e-s)(1+\kappa-\kappa\bar{\beta})},$$

where $\bar{\beta} = \int_0^1 \mu^{RN}\beta d\beta$ is the expected value of the risk-averse probabilities with respect to the risk-neutral probabilities.
Netting against 1 to find $\mathbb{P}(x \leq D)$ then yields:

$$\mathbb{P}(x \leq D) = \frac{(c-s)+(e-s)(\kappa-\kappa\bar{\beta})}{(e-s)(1+\kappa-\kappa\bar{\beta})}.$$

To convert to our notation, observe that $r_u = c - s$ and $r_v = e - c$, giving $r_u + r_v = e - s$. Therefore, we have that:

$$\mathbb{P}(x \leq D) = \frac{r_u + (r_u+r_v)(\kappa-\kappa\bar{\beta})}{(r_u+r_v)(1+\kappa-\kappa\bar{\beta})},$$

as required. Note that the corresponding quantile is not necessarily unique. □

Appendix B Risk-averse stochastic dispatch in the New Zealand Electricity Market

In this appendix, we introduce the NZEM and the software used to clear it, and investigate the relationship between system risk-aversion and generator pre-commitment.

B.1 Experimental Setup

The New Zealand Electricity Market (NZEM) is an energy-only pool market where generators and retailers submit bids to buy and sell electricity in each half hour trade period. Bids consist of up to five tranches (marginal-price generation-quantity pairs) per participant, and are permitted between 36 and 2 hours before each trade period commences. Transpower, the system operator then clears the market using software called Scheduling, Pricing and Dispatch (SPD, see [2]).

Vectorised Scheduling Pricing and Dispatch (vSPD) is a publicly available replica of SPD, which is written in GAMS and allows each trade period from 2004 onwards to be independently replicated (see [22]). We back-test SDM in the NZEM by modifying vSPD to clear two-hour-ahead pre-commitment markets using a nonanticipative forecast, and clear the real-time markets using realised wind.

Generating wind scenarios via quantile regression

To generate nonanticipative ensemble forecasts, we model the geographically adjacent Tararua, Te Apati and Te Rere Hau wind farms in the Central North Island as one wind farm with a maximum output capacity of 300 MW, the geographically adjacent West Wind and Mill Creek wind farms as a second wind farm with a maximum output capacity of 200 MW, assume the two wind farms are conditionally independent, and treat the remaining wind farms as deterministic negative demand. Our approach models 500 MW of the 690 MW of wind production in New Zealand at an affordable computational cost. We generate splines corresponding to the 1st, 5th, 15th, ..., 95th, 99th quantiles of the conditional distribution of future wind at both locations using techniques developed by Pritchard [25] and implemented in the R package Quantreg (see [18]). The spline coefficients are determined using historical wind farm data from 2011 – 2013 (see [22]). Wind scenarios in SDM correspond to the outer product of the two sets of splines, and each occur with probability $\frac{1}{144}$.

Modelling the marginal deviation costs

The generator offer stacks in vSPD do not provide the marginal deviation costs. Therefore, we estimate them in the same manner as [17], i.e., assume they are of the form

$$\mathrm{r_{u,i}} = \frac{\mathrm{K}}{\text{Ramp up rate}}, \ \mathrm{r_{v,i}} = \frac{\mathrm{K}}{\text{Ramp down rate}},$$

where the ramp rates are dictated by the parameter i_TradePeriodOfferParameter within vSPD, and K depends on market conditions such as the degree to which hydro generators are risk-averse to dry winters.

To determine K, observe that in the NZEM in 2014—2015, reserve prices are on the order of \$2 per MW, ramp rates for hydro plants are on the order of 100 MW per hour and ramp rates for thermal plants are on the order of 5 MW per hour (see [22]). Consequently, K = 10 provides marginal deviation costs of \$0.1 per MW for hydro generators and \$2 per MW for thermal generators, while K = 100 provides marginal deviation costs of \$1 per MW for hydro generators and \$20 per MW for thermal generators. Therefore, setting K = 10 and K = 100 in two separate experiments provides bounds on the cumulative savings.

B.2 Numerical Results

To see how a fully liquid risk market impacts generator pre-commitment, we modify SDM's objective function to a β-CVaR risk measure for ten different values of β (by an abuse of notation we refer to a worst-case risk measure as $\beta = 1$, although we implement it via a minimax LP) and solve for the pre-commitment policies for the first 7 days of 2014. Table 2 depicts the additional pre-commitment procured for ten distinct values of β. For the 335 trade periods considered, enlarging the system risk set never results in a smaller amount of generation being procured, which is consistent with our findings in Proposition 22. Consequently, by our numerical results, residual market risk decreases expected real-time nodal prices and benefits consumers at the expense of generators in the NZEM. This is the opposite situation to when no risk-trading instruments exists. When computing Table 2, we omit trade period 34 of 2 January, because line congestion makes the problem infeasible in some scenarios, making the worst-case optimal penalty violation cost non-zero and meaning the equality $x + U - V = X$ is violated when $\beta = 1$.

Table 2: Additional pre-commitment procured by a risk-averse planner, 1-7 Jan 2014, K = 10.

Gen	β coefficient in CVaR ($\beta = 1$ is a worst-case risk measure)									
(MW)	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Max	397.01	335.43	234.36	187.11	136.09	101.41	78.62	60.04	39.68	21.77
3rd Qu.	175.22	125.82	96.25	77.92	62.50	49.54	37.86	29.51	19.00	9.78
Mean	137.28	103.37	78.76	63.13	49.54	38.22	29.48	21.80	14.57	7.20
Median	108.15	77.77	59.60	47.38	38.67	30.03	23.98	17.93	11.89	6.05
1st Qu.	78.35	59.16	45.11	35.31	28.61	22.48	17.81	13.30	9.18	4.37
Min	18.13	3.08	1.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00

To determine the impact of system risk-aversion on real-time nodal prices, we back-test the risk-averse pre-commitment policies on realised wind generation

Tables 3 and 4 depict the real-time nodal prices at Haywards (HAY2201) and Benmore (BEN2201); the two reference nodes on either end of the High Voltage Direct Current, or HVDC, cable which connects New Zealand's two main islands. Observe that although additional market risk does not depress nodal prices in each trade period, it does depress expected nodal prices.

Table 3: Haywards nodal prices with a risk-averse planner, 1-7 Jan 2014, K = 10.

Price	β coefficient in CVaR ($\beta = 1$ is a worst-case risk measure)									
($ per MWh)	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Max	71.77	71.77	71.77	71.77	71.77	71.77	71.77	71.77	71.77	71.77
3rd Qu.	42.45	42.45	42.44	42.45	42.44	42.470	42.49	42.50	42.50	42.50
Mean	29.46	29.47	29.51	29.56	29.59	29.66	29.74	29.84	29.88	29.94
Median	35.06	35.06	34.76	35.06	34.76	36.03	36.01	36.03	36.01	36.19
1st Qu.	4.47	4.47	4.47	4.47	4.47	4.47	4.47	4.48	4.60	4.60
Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 4: Benmore nodal prices with a risk-averse planner, 1-7 Jan 2014, K = 10.

Price	β coefficient in CVaR ($\beta = 1$ is a worst-case risk measure)									
($ per MWh)	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Max	66.02	66.02	66.02	66.02	66.02	66.02	66.02	66.03	66.02	66.02
3rd Qu.	40.04	40.03	40.03	40.04	40.03	40.06	40.07	40.07	40.07	40.07
Mean	27.52	27.56	27.58	27.62	27.66	27.72	27.79	27.89	27.93	27.98
Median	33.10	32.71	32.71	33.10	32.71	34.02	33.99	34.02	33.99	34.17
1st Qu.	4.11	4.11	4.11	4.11	4.11	4.11	4.11	4.12	4.33	4.32
Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

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