Supply function equilibrium with taxed benefits

A. Downward, A.B. Philpott and K. Ruddell
Electric Power Optimization Centre
University of Auckland

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Abstract

Supply function equilibrium models are used to study electricity market auctions with uncertain demand. We study the effects on supply function equilibrium of a system tax on the observed benefits of suppliers. Such a tax provides an incentive for agents to alter their offers to avoid the tax. We show how this surprisingly can lead to lower prices in equilibrium. The model is extended to a setting in which the agents are taxed on the benefits accruing to them from a transmission line expansion (in order to help fund the line). In these circumstances we study how incentives for agents to alter their bids varies with the relative size of the capacity expansion.

1 Introduction

In electricity market auctions, producers typically submit amounts of generation that they are willing to supply at different prices. These offer curves are then cleared by a system operator in a pool to yield a system marginal price. All generation offered at a price equal or below this market price is dispatched. Each generator is then paid the system marginal price for all the energy they are dispatched. This leads to market rents accruing on infra-marginal offers (those with offer price below the system marginal price).

The offers of each generator may be modeled by a supply curve. In the face of uncertain demand, each agent seeks a curve to maximize expected profit, leading to the concept of supply-function equilibrium (SFE). These models have been applied to the study of electricity market auctions by a number of authors (Green and Newbery, 1992; Holmberg and Newbery, 2010). Although SFE models are not straightforward to work with, and there is a shortage of effective computational procedures to compute asymmetric SFE, these models deal with demand uncertainty in a natural way, a feature that makes them increasingly useful as intermittent renewable generation grows. For computational simplicity, it is customary in SFE models to assume symmetric players with identical costs and capacities, and a market with a price cap. The shock in demand is chosen so that demand exceeds the total supply capacity with some small probability. In these events the market clears at the price cap, and load is shed. Details can be found in the recent survey by Holmberg and Newbery (2010).

In this paper we study the effects on agent behaviour of a system tax levied on the surplus earned by inframarginal rents. Since the true marginal cost functions of the agents are not public knowledge, the rents are computed assuming that the supply function offered represents the agent's marginal cost of supply. The imposition of the tax alters the incentives of the agents in choosing what supply functions to offer to the auction. Their offer curves will adjust in such a way to minimize the tax paid, while not sacrificing too much profit. When electricity demand is deterministic, the agents can anticipate the market clearing price and their dispatch quantity. Given this dispatch point, each agent has an incentive to increase the prices of their inframarginal offers so as to reduce the amount of observed benefit (while maintaining their real benefit). One might then expect all offers to become horizontal at the clearing price up to the anticipated dispatch quantity.

Uncertainty in demand alters this outcome. Agents offering horizontal bids might find if demand is lower than anticipated that they make no money at all. In such circumstances agents do better by offering an increasing supply curve that trades off the amount of tax paid against the need to earn some profit. One might expect this curve to mark up offers to recover the tax through higher prices, however we show that, in equilibrium, this strategy is only applied to the lower end of the supply curve, and at high prices agents discount their offers.

Our study of such a tax is motivated by a proposal mooted by the New Zealand Electricity Authority to charge electricity market participants for transmission upgrades based on the benefits that accrue to them from these upgrades (see NZEA, 2014). Although the details of this 'beneficiary-pays' scheme are still being worked out, some effort has been devoted to estimating these benefits using the software used for dispatching the wholesale market and computing locational maginal prices.

The simplest version of this estimation process works as follows. After the market is dispatched with current transmission assets in place, the benefits of each agent are computed from their bid and offer curves. For a generator this benefit is measured by the rentals earned from inframarginal bids. As we have already remarked, this need not be the true benefit if these bids are marked up above the generators marginal cost. The dispatch software is then run again using the same bids and offers, but with the transmission assets de-rated to their pre-upgrade levels. The benefits for each agent are then computed under this counterfactual and subtracted from the previous estimates. If these are positive then the agents with positive net benefits contribute to the upgrade cost of the transmission system in proportion to these net benefits. A fuller description is provided by NZEA (2012).

The paper is laid out as follows. In section 2, we show how a tax on producer surplus gives rise to a best-response problem that is a convex combination of the best-response problem under uniform and pay-as-bid pricing. We then derive a symmetric equilibrium when such a tax is imposed on two agents at the upstream end of a constrained transmission line. Section 3 deals with the setting when the tax is calculated based on the difference between the actual and counterfactual dispatch from the expansion of the transmission line.

2 Supply function equilibrium

In this paper we confine attention to a symmetric duopoly in which each player chooses a monotone piecewise smooth curve $(q,p) : [0,T] \to \mathbb{R}^2$ to maximize a best-response functional of the form (1). The first-order optimality condition for a profit-maximising curve, given competitors' offers, gives a system of ordinary differential equations which under symmetry of agents yields a single ordinary differential equation that we can solve to find the equilibrium supply functions.

Different market clearing rules may give rise to different profit functionals that bidders seek to maximize. For example, uniform pricing, discriminatory (pay-as-bid) pricing, and taxed producer surplus models all have different objective functionals. When these integral functionals share the property that the integrand is linear in the derivative of the curve, then Theorem 1 gives a uniform set of optimality conditions.

Theorem 1 (Optimality conditions for best response) *Let $f, g : \mathbb{R}^2 \to \mathbb{R}$ be piecewise continuously differentiable functions and $(q,p) : [0,T] \to \mathbb{R}^2$ a continuously differentiable curve. A necessary condition for the curve (q,p) to maximize the functional*

$$\Pi(q,p) = \int_0^T \left(f(q,p) \frac{dq}{dt} + g(q,p) \frac{dp}{dt} \right) dt \tag{1}$$

is that $Z(q,p) = \frac{\partial g}{\partial q} - \frac{\partial f}{\partial p} = 0$ at every point along the curve. Furthermore, if both components of (q,p) are nondecreasing in t then a sufficient condition for optimality is that $Z = 0$ along (q,p) and $\frac{\partial}{\partial q} Z \leq 0$ everywhere.

Proof: The functional Π is a line integral in the (q,p) plane. As this functional is linear in $\frac{dq}{dt}$ and $\frac{dp}{dt}$, we can apply Green's theorem, as in Anderson and Philpott (2002), to obtain the optimal curve. By Green's theorem, the integral of $\left(f(q,p) \frac{dq}{dt} + g(q,p) \frac{dp}{dt}\right)$ around any simple closed curve in the anticlockwise direction in the (q,p) plane is equal to the integral of Z over the area enclosed by the curve. Thus if $Z > 0$ along part of the trajectory (q,p) then (by continuity of Z) there is an improving deviation to the right of the curve as it is traversed. Similarly if $Z < 0$ along part of the trajectory (q,p) then there is an improving deviation to the left of the curve as it is traversed. Thus a maximal curve must have $Z(q,p) = 0$.

Now suppose $Z = 0$ along (q,p) and $\frac{\partial}{\partial q} Z \leq 0$ everywhere. If a candidate curve is nondecreasing and $\frac{\partial}{\partial q} Z \leq 0$ everywhere, then there can be no region to the right of the curve on which Z has a positive integral and so no improving deviations exist to the right of the curve as it is traversed. Similarly on the left there are no regions for which Z has a negative integral, so no improving deviations exist to the left. It follows that (q,p) is a global maximum. ∎

Note that the condition $Z = 0$ is equivalent to the Euler-Lagrange equation $\frac{d}{dp} F_{\dot{q}} - F_q = 0$ from the calculus of variations when the offer curve is modeled as a supply function $q(p)$. In this case, replacing t by p gives

$$F = f(q,p) \frac{dq}{dt} + g(q,p) \frac{dp}{dt} = f(q,p) \frac{dq}{dp} + g(q,p).$$

Then

$$\begin{aligned} \frac{d}{dp} F_{\dot{q}} - F_q &= \frac{d}{dp} f(q,p) - \frac{\partial f}{\partial q} \frac{dq}{dp} - \frac{\partial g}{\partial q} \\ &= \frac{\partial f}{\partial p} + \frac{\partial f}{\partial q} \frac{dq}{dp} - \frac{\partial f}{\partial q} \frac{dq}{dp} - \frac{\partial g}{\partial q} \\ &= -Z. \end{aligned}$$

In a uniform-price auction, the expected payoff to a firm offering a curve $(q(t), p(t))$ is

$$\begin{aligned} \Pi^U &= \int_0^T (qp - C(q))\, d\psi(q,p) \\ &= \int_0^T (qp - C(q)) \left(\frac{dq}{dt} \psi_q + \frac{dp}{dt} \psi_p \right) dt, \end{aligned} \tag{2}$$

where $C(q)$ is the firm's cost to produce quantity q and $\psi(q,p)$ is the market distribution function (see Anderson and Philpott, 2002), which gives the probability that a supplier is not fully dispatched if they offer the quantity q at price p. It can be interpreted as the measure of residual demand curves that pass below and to the left of the point (q,p). The integrand in Π^U is clearly linear in $\frac{dq}{dt}$ and $\frac{dp}{dt}$, so we can compute the Z function as in Anderson and Philpott (2002):

$$\begin{aligned} Z^U(q,p) &= \frac{\partial\left(qp - C(q)\right)\psi_p}{\partial q} - \frac{\partial\left(qp - C(q)\right)\psi_q}{\partial p} \\ &= \left(p - C'(q)\right)\psi_p - q\psi_q, \end{aligned}$$

as the cross terms containing ψ_{qp} cancel.

In general, ψ can be discontinuous though it must be monotone and finite and hence of bounded variation. In most cases of a discontinuity in ψ the sufficient condition of theorem 1 need only be modified to '$Z = 0$ along the curve and $\frac{\partial}{\partial q}Z \le 0$ almost everywhere.' There is one important case where we can no longer rely on theorem 1; when more than one firm offers a perfectly elastic segment, the Π^U functional (2) is unable to represent dispatch rationing rules (pro-rata on the margin, used by Holmberg (2008), for instance). By this we mean that the probability of dispatch in the interior of such a segment is not a function of just the price-quantity point, but also depends on the enpoints of the perfectly elastic segments in play. When such a situation occurs, we must take the best-response problem as a piecewise calculus of variations problem, with an explicit profit function outside the integral for dispatch rationed over the perfectly elastic segment. We present an example in section 3.3.1.

In a discriminatory-price (pay-as-bid) auction (Anderson et al. (2013)), the expected payoff is

$$\begin{aligned} \Pi^D &:= \int_0^T \left(\int_0^t \left[p(u) - C'(q(u))\right]\frac{dq}{du}\,du\right) d\psi\left(q(t), p(t)\right) \qquad (3) \\ &= \int_0^T \left[\int_u^T d\psi\left(q(t), p(t)\right)\right]\left(p - C'(q)\right)\frac{dq}{du}\,du \\ &= \int_0^T \left[\psi\left(q(T), p(T)\right) - \psi\left(q(u), p(u)\right)\right]\left[p(u) - C'(q(u))\right]\frac{dq}{du}du \\ &= \int_0^T \left[p - C'(q)\right]\left[1 - \psi(q,p)\right]\frac{dq}{dt}\,dt. \end{aligned}$$

Again this is linear in $\frac{dp}{dt}$ and $\frac{dq}{dt}$ and theorem 1 holds with

$$\begin{aligned} Z^D &= -\frac{\partial\left(p - C'(q)\right)\left(1 - \psi(q,p)\right)}{\partial p} \\ &= \left(p - C'(q)\right)\psi_p - \left(1 - \psi(q,p)\right). \end{aligned}$$

Suppose that some fraction $\alpha \in (0,1)$ of the observed producer surplus earned by a generator is paid as tax. If the market clears at quantity q for a generator at price π then the generator receives

$$\begin{aligned} R(q,\pi) &= q\pi - C(q) - \alpha\int_0^q \left(\pi - p(t)\right)dt \\ &= q\pi - C(q) - \alpha q\pi + \alpha\int_0^q p(t)\,dt = (1-\alpha)\left(q\pi - C(q)\right) + \alpha\left(\int_0^q p(t)\,dt - C(q)\right). \end{aligned}$$

This is a convex combination of uniform and pay-as-bid pricing with multiplier α. Thus the total payoff will be

$$\Pi^A = (1-\alpha)\,\Pi^U + \alpha\Pi^D. \qquad (4)$$

We can write down the optimality conditions for the problem faced by a generator maximizing Π^A. These use the scalar field defined by $Z^A(q,p) = (1-\alpha) Z^U + \alpha Z^D$. Thus

$$\begin{aligned} Z^A(q,p) &= (1-\alpha)\left(\left(p - C'(q)\right)\psi_p - q\psi_q\right) + \alpha\left(\left(p - C'(q)\right)\psi_p - \left(1 - \psi(q,p)\right)\right) \\ &= \left(p - C'(q)\right)\psi_p - (1-\alpha) q\psi_q - \alpha\left(1 - \psi(q,p)\right). \end{aligned}$$

2.1 Example

We illustrate the above analysis with a simple example with two symmetric agents, located at node 1 of the two-node network shown in figure 1. Here there is a price-taking, random and price-inelastic demand ε at node 2. The line connecting the two nodes has capacity K, which is less than the maximum demand at node 2. For the time being there is no demand at node 1. This is the simplest possible model of a transmission-constrained network.

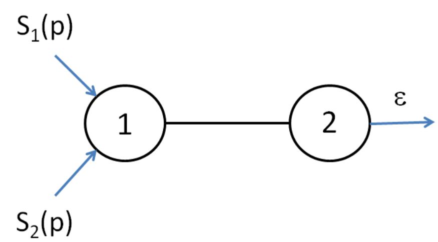

Figure 1: Symmetric equilibrium example. Line has capacity K.

Suppose there is a proportional tax α imposed on the observed surplus of each agent. We apply the optimality conditions of the previous section to look for an equilibrium in symmetric duopoly. Suppose the other player offers a piecewise smooth supply function $S(p)$ and demand has cumulative probability distribution function F. Then

$$\begin{aligned} \psi(q,p) &= \Pr\left[\varepsilon < q + S(p)\right] \\ &= F\left(q + S(p)\right) \end{aligned}$$

and

$$\begin{aligned} Z^A(q,p) &= \left(p - C'(q)\right)\psi_p - (1-\alpha) q\psi_q - \alpha\left(1 - \psi(q,p)\right) \\ &= \left(p - C'(q)\right) S'(p) f\left(q + S(p)\right) - (1-\alpha) q f\left(q + S(p)\right) - \alpha\left(1 - F\left(q + S(p)\right)\right). \end{aligned} \quad (5)$$

Here $f = F'$ is the probability density of the demand shock. When this is strictly positive we can divide (5) through by $f(q + S(p))$ to obtain

$$\hat{Z}^A(q,p) := \left(p - C'(q)\right) S'(p) - (1-\alpha) q - \alpha \frac{1 - F\left(q + S(p)\right)}{f\left(q + S(p)\right)}. \quad (6)$$

By theorem 1, the curve defined implicitly by $\hat{Z}^A(q,p) = 0$ is a profit-maximising response if it is monotone and $\frac{\partial}{\partial q}\hat{Z}^A(q,p) \leq 0$ for all p and q, i.e.

$$-C''(q) S'(p) - (1-\alpha) - \alpha \frac{\partial}{\partial q}\left[\frac{1 - F\left(q + S(p)\right)}{f\left(q + S(p)\right)}\right] \leq 0. \quad (7)$$

Holmberg (2009) refers to the term $G(x) = \frac{1-F(x)}{f(x)}$ as the "inverse hazard rate" of the distribution. In his model, for pure pay-as-bid pricing, $\alpha = 1$ and so for constant marginal costs, it is necessary that $G' \geq 0$ for (7) to hold. This restricts the analysis to probability distributions that decay faster than the exponential distribution, which has $G' = 0$. If the tax rate α is less than 1 then we have many more reasonable probability distributions of the demand shock for which (7) holds. For instance, as shown in the example below, if $\alpha < \frac{1}{2}$, then (7) holds for a uniform distribution.

We now choose some specific problem data to illustrate the equilibrium. Suppose that the demand shock is uniformly distributed on $[0, \bar{\varepsilon}]$ and $\alpha < \frac{1}{2}$. Assume that the line capacity K is infinitesimally smaller than $\bar{\varepsilon}$. Suppose that marginal costs for each agent are the same and are constant ($C' = c$). Then

$$\begin{aligned}\frac{\partial}{\partial q}\hat{Z}^A(q,p) &= -(1-\alpha) - \alpha\frac{\partial}{\partial q}\left[\frac{1-(q+S(p))/\bar{\varepsilon}}{1/\bar{\varepsilon}}\right] \\ &= 2\alpha - 1 \\ &< 0\end{aligned}$$

and so solving $Z^A = 0$ gives a symmetric equilibrium. If we set $q(p) = S(p) = Q(p)$, then the condition $Z^A = 0$ gives

$$Q'(p) = \frac{(1-3\alpha)Q}{p-c} + \frac{\alpha\bar{\varepsilon}}{p-c}.$$

This is a first order linear ODE which can be solved using an integrating factor to give

$$Q(p) = k(p-c)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha},$$

where k is a constant of integration that can be chosen to satisfy an endpoint condition. As the line has a capacity that binds at the highest levels of demand, there exists a unique endpoint condition $q(\bar{p}) = \frac{K}{2} \approx \frac{\bar{\varepsilon}}{2}$ for which no profitable deviation is possible (see Holmberg, 2008).

We can compute the changes in welfare of each agent from the change in equilibrium. Suppose $K = 10$ and $\bar{\varepsilon} = 10$, and consider first the case where $\alpha = 0$, there is a price cap at $\bar{p} = 6$ and constant marginal costs of $c = 1$. In perfect competition each generator would offer at price equal to marginal cost and earn no profit. However, in our supply function game the equilibrium curve S has equation

$$q(p) = p - 1.$$

It is simpler to write the integrals in terms of the inverse

$$p(q) = q + 1.$$

As there are two firms, the total supply is $2S(p) = 2(p-1)$, and as the market clears when supply equals demand $2S(p) = D(p,\varepsilon) = \varepsilon$, we can write the market price as a function of demand as $p(\varepsilon) = \frac{1}{2}\varepsilon + 1$. The expected consumer surplus (assuming all consumers value electricity at $\bar{p}$) is

$$\begin{aligned}CS &= \int_0^{\bar{\varepsilon}} \varepsilon(\bar{p} - p(\varepsilon))\, f(\varepsilon)\, d\varepsilon \\ &= 8.3333.\end{aligned}$$

The expected producer revenue is the firm's objective function $\Pi^U = 8.3333$. Their expected observed surplus however, is 4.1666. If a tax is applied to this curve, the firms each pay α of their perceived surplus, so if $\alpha = \frac{1}{4}$, then each firm pays 1.0416 in tax, leaving net profit of 7.2917. As shown in figure 2, the tax gives an incentive for firms to change the shape of their offer curve. Our firms will settle on a new equilibrium curve S^A which has equation

$$q(p) = \frac{15}{\sqrt[4]{5}}(p-1)^{\frac{1}{4}} - 10$$

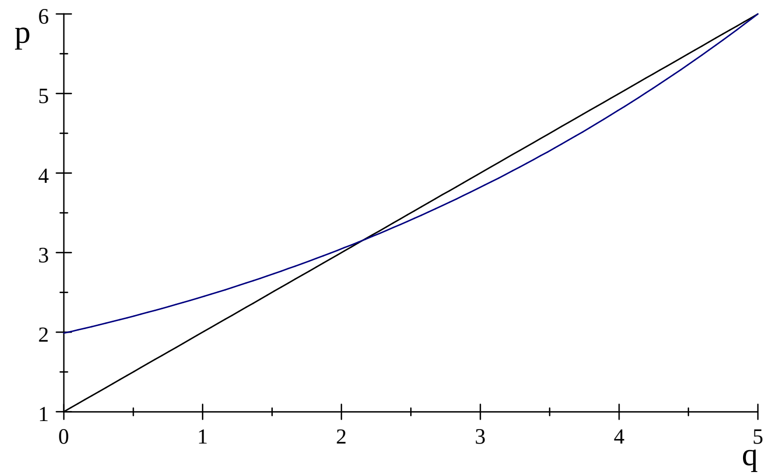

Figure 2: Equilibrium supply curves for no tax (black) and a 25% tax on perceived surplus (blue).

with inverse

$$p(q) = \frac{5}{15^4}(q+10)^4 + 1.$$

The expected consumer surplus is 8.6831; the expected producer profit, before tax, is 8.1584; and the producer surplus perceived by the market operator is 3.2922. Each producer pays taxes of 0.8231 and so earns 7.3354 net profit.

Table 1 summarises the changes in producer and consumer surplus between the untaxed and taxed scenarios. The overall effect of the tax is a small transfer of welfare from producers to consumers. Though higher prices are charged at times of low demand, this is offset by lower prices higher up the offer curves. Note that social surplus (the sum of consumer and producer surpluses $CS + 2\Pi^U$) does not change with the introduction of the tax; this is because demand is inelastic. Also note that expected consumer surplus actually rises once firms adjust to the tax, as the new equilibrium SFE is more competitive for the higher demand realisations.

Curve	α	CS	Π^U	Π^A	Tax per firm	Social Surplus
S	0	8.3333	8.3333	8.3333	0	25
S	0.25	8.3333	8.3333	7.2917	1.0416	25
S^A	0.25	8.6831	8.1584	7.3354	0.8231	25

Table 1: Benefits and taxes under a producer-surplus tax.

3 Line capacity expansion

We now consider a model in which the transmission line is expanded from capacity J to capacity K, and a proportional tax on observed benefits is levied to recover the costs of the line expansion. The model is again a simple two-node network as shown in Figure 3, with symmetric players at one node, and an inelastic demand shock ε at the downstream end of a line.

Suppose player 2 offers a supply function $S_2(p)$. If player 1 offers quantity q at price p then the market distribution function is just the probability that either the total quantity offered $q + S_2(p)$ exceeds the line capacity K or that the combined offers of the two firms $q+S_2(p)$ at price p exceeds the demand shock ε;

$$\psi(q,p) = \Pr[q + S_2(p) > \min(K,\varepsilon)].$$

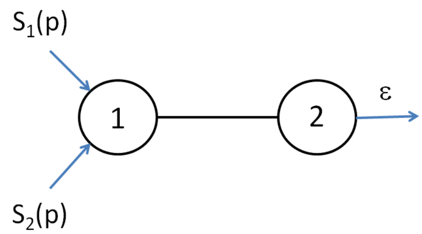

Figure 3: Network example. Line is expanded from J to K.

If the demand shock is uniformly distributed on $[0,\bar{\varepsilon}]$, then we obtain the following piecewise definition for ψ:

$$\psi\left(q,p\right)=\begin{cases}\frac{q+S_{2}(p)}{\bar{\varepsilon}} & \text{if } q<K-S_{2}\left(p\right)\\ 1 & \text{if } q\geq K-S_{2}\left(p\right).\end{cases} \tag{8}$$

The partial derivatives are $\psi_q=\frac{1}{\bar{\varepsilon}}$ and $\psi_p=\frac{S_2'}{\bar{\varepsilon}}=S_2'\psi_q$ when $q\leq K-S_2(p)$ and both are zero otherwise. There is a jump in the value of $\psi(q,p)$ lying on the curve $q=K-S_2\left(p\right)$.

3.1 Payoffs

We now look at the payoffs. The actual pre-tax producer profit if a generator is dispatched $\theta\left(\varepsilon\right)$ at price $\pi\left(\varepsilon\right)$ under demand realisation ε is

$$P\left(\varepsilon\right)=\pi\left(\varepsilon\right)\theta\left(\varepsilon\right)-C\left(\theta\left(\varepsilon\right)\right).$$

The system operator assumes that the submitted curve is marginal cost and observes a different surplus. The observed surplus is

$$\sigma\left(\varepsilon\right)=\int_{0}^{t(\varepsilon)}\left(\pi\left(\varepsilon\right)-p(t)\right)\frac{dq}{dt}dt,$$

where $t\left(\varepsilon\right)$ satisfies $(q(t),p\left(t\right))=\left(\theta\left(\varepsilon\right),\pi\left(\varepsilon\right)\right)$. Integrating by parts gives

$$\begin{aligned}\sigma\left(\varepsilon\right) &= \left[(\pi\left(\varepsilon\right)-p(t))q\left(t\right)\right]_{t=0}^{t(\varepsilon)}-\int_{0}^{t(\varepsilon)}-q\left(t\right)\frac{dp}{dt}dt\\ &= \int_{0}^{t(\varepsilon)}q\left(t\right)\frac{dp}{dt}dt.\end{aligned}$$

Taking the expectation of this surplus over all demand outcomes gives

$$\begin{aligned}\mathbb{E}[\sigma] &= \int_{0}^{\bar{\varepsilon}}\int_{0}^{t(\varepsilon)}q\left(t\right)\frac{dp}{dt}dt\,f\left(\varepsilon\right)\,d\varepsilon\\ &= \int_{0}^{T}\left(\int_{\varepsilon(t)}^{\bar{\varepsilon}}f\left(h\right)\,dh\right)q\left(t\right)\frac{dp}{dt}\,dt.\end{aligned}$$

But

$$\begin{aligned}\int_{\varepsilon(t)}^{\bar{\varepsilon}}f\left(h\right)dh &= \Pr\left(h>\varepsilon\left(t\right)\right)\\ &= 1-\psi\left(q(t),p\left(t\right)\right)\end{aligned}$$

so

$$\mathbb{E}[\sigma] = \int_0^T (1 - \psi(q(t), p(t)))\, q(t) \frac{dp}{dt}\, dt.$$

The clearing price π depends on the demand realisation ε and the network configuration, so the two network configurations give two price functions $\pi(\varepsilon)$ and $\hat{\pi}(\varepsilon)$. These in turn give rise to two distinct realisations of producer surplus $\sigma(\varepsilon)$ and $\hat{\sigma}(\varepsilon)$. There is some ambiguity in the computation of the producer surplus if the offers of two agents are perfectly elastic (vertical). We assume when the market clears on these vertical segments for some demand ε that it does so at some well-defined price $\pi(\varepsilon)$ between the endpoints p_- and p_+ of the vertical segment.

For a given level of demand, our firm will be dispatched at one price and quantity in the actual and a different price and quantity in the counterfactual network. The market distribution functions give the distributions of these price-quantity pairs relative to the firm's offer. The firm pays a portion α of the difference between the perceived surplus in the actual network and counterfactual network cases; i.e. the tax is

$$\alpha(\sigma(\varepsilon) - \hat{\sigma}(\varepsilon)).$$

This gives profit net of tax of

$$R(\varepsilon) = P(\varepsilon) - \alpha(\sigma(\varepsilon) - \hat{\sigma}(\varepsilon)).$$

The generator then constructs an offer curve to maximize this tax-adjusted profit. Since $R(\varepsilon)$ is the linear combination of three terms, we can express the expectation as a linear combination of the individual expectations. Here $P(\varepsilon)$ and $\sigma(\varepsilon)$ are evaluated using the market distribution function ψ assuming a full line capacity, whereas $\hat{\sigma}(\varepsilon)$ is evaluated using the counterfactual market distribution function ϕ assuming the unexpanded capacity. The expected profit over the entire supply curve is

$$\begin{aligned}
\Pi^L &= \mathbb{E}[P] - \alpha(\mathbb{E}[\sigma] - \mathbb{E}[\hat{\sigma}]) \\
&= \int_0^T (pq - C(q)) \left(\frac{dp}{dt}\psi_p + \frac{dq}{dt}\psi_q\right) dt - \alpha\left(\int_0^T q[1 - \psi(q,p)] \frac{dp}{dt}\, dt - \int_0^T q[1 - \phi(q,p)] \frac{dp}{dt}\, dt\right) \\
&= \int_0^T \left((pq - C(q)) \left(\frac{dp}{dt}\psi_p + \frac{dq}{dt}\psi_q\right) - \alpha q(\phi(q,p) - \psi(q,p)) \frac{dp}{dt}\right) dt.
\end{aligned}$$

The resulting Z function is

$$Z^L = (p - C'(q))\psi_p - q\psi_q - \alpha(q(\phi_q - \psi_q) + \phi - \psi). \tag{9}$$

In our model with a one-dimensional shock in the downstream node, $[\phi - \psi]$ is non-zero only when

$$J < q + S_2(p) \leq K,$$

in which case $\phi = 1$. Hence our functional Z^L can be thought of as piecewise defined; equal to Z^U when $\phi - \psi = 0$ and Z^A otherwise.

3.2 Example

We now consider an example. The base levels of parameters are as follows:

$\bar{\varepsilon} = 10$	maximum shock
$c = 1$	marginal cost
$K = 8$	enlarged line capacity
$J = 2$	restricted line capacity
$\alpha = \frac{1}{4}$	tax rate
$\bar{p} = 6$	price cap

The first order condition for an SFE is

$$(p - C')\,\psi_p - q\psi_q = 0 \text{ for } q < J - S_2(p) \tag{10}$$

$$(p - C')\,\psi_p - (1-\alpha)\,q\psi_q - \alpha\,(1 - \psi(q,p)) = 0 \text{ for } q > J - S_2(p)\,. \tag{11}$$

Here, as in the previous example,

$$\psi(q,p) = \begin{cases} \frac{q+S_2(p)}{\bar{\varepsilon}} & \text{if } q \leq K - S_2(p) \\ 1 & \text{if } q > K - S_2(p)\,, \end{cases}$$

so

$$\psi_p = \frac{S_2'(p)}{\bar{\varepsilon}} \text{ and } \psi_q = \frac{1}{\bar{\varepsilon}}.$$

Replacing $S_2(p)$ and q by $Q(p)$ in (11) yields

$$(p - c)\,Q'(p) - (1-\alpha)\,Q(p) - \alpha\,(\bar{\varepsilon} - 2Q(p)) = 0,$$

which can be solved using an integrating factor, whereby

$$Q(p) = k\,(p - c)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha}. \tag{12}$$

To pass through the price cap we require

$$k\,(p - c)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha} = \frac{K}{2}$$

$$k = \left(\frac{K}{2} + \frac{\alpha\bar{\varepsilon}}{1-3\alpha}\right)(\bar{p} - c)^{3\alpha-1}\,.$$

The equation $Z = 0$ gives an ordinary differential equation for $q < J - S_2(p)$ with general solution

$$q(p) = k\,(p - c)\,. \tag{13}$$

For continuity of the curve, we choose $k = \frac{J}{2(p^*-c)}$, where p^* solves $Q(p) = \frac{J}{2}$ in (12). In section 3.3 we show that this is the only correct choice for k. Our equilibrium candidate is thus

$$Q(p) = \begin{cases} \left(\frac{K}{2} + \frac{\alpha\bar{\varepsilon}}{1-3\alpha}\right)\left(\frac{p-c}{\bar{p}-c}\right)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha} & \text{if } \; p \geq p^* \\ \frac{J}{2}\frac{p-c}{p^*-c} & \text{if } \; p < p^*. \end{cases} \tag{14}$$

Observe that the exponent of $\left(\frac{p-c}{\bar{p}-c}\right)$ vanishes when $\alpha = \frac{1}{3}$. In that case the differential equation

$$(p - c)S'(p) + (3\alpha - 1)Q(p) = \alpha\bar{\varepsilon}$$

becomes

$$Q'(p) = \frac{\alpha\bar{\varepsilon}}{(p - c)},$$

so the symmetric equilibrium supply functions are

$$Q(p) = \alpha\bar{\varepsilon}\log\frac{p-c}{\bar{p}-c} + \frac{K}{2} \text{ for } Q > \frac{J}{2}.$$

The supply-function equilibria for two different choices of α are plotted in figure 4 below.

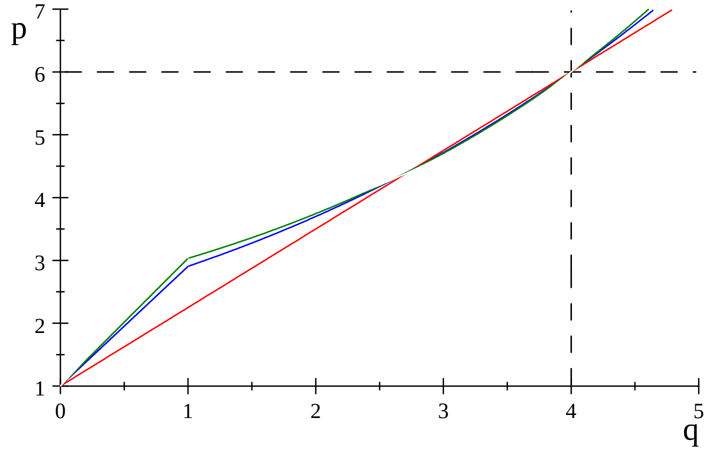

Figure 4: Plot of untaxed equilibrium offer (red) and taxed equilibrium offers when maximum demand is 10 and $\alpha = \frac{1}{4}$ (blue) and $\alpha = \frac{1}{3}$ (green).

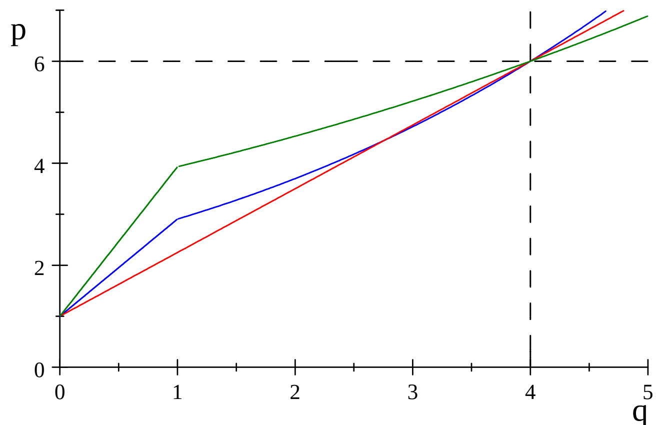

Figure 5: Plot of untaxed equilibrium offer (red) and taxed equilibrium offer (blue) when the maximum demand is 10. The equilibrium offer when the maximum demand is 20 is shown in green.

The degree to which the taxed equilibrium is marked up above the untaxed equilibrium depends on the range of the demand shock. If the range of the demand shock is large, then there is a high probability that the expanded line will be congested, which even so provides significant benefits compared with the unexpanded line. This means that the equilibrium offers try to avoid taxation of these by flattening the offer curve. This can be observed in Figure 5.

We finish this example by computing equilibria for $\alpha = \frac{1}{4}$, $\bar{\varepsilon} = 10$, and different values of J. These are shown in Figure 6. Observe that for small increases in line capacity (from $J = 6$ to $K = 8$) the magenta and red curves almost coincide, so there is minimal change in offer strategy to avoid the tax.

3.2.1 Welfare calculations

We may calculate consumer and producer welfare for different levels of tax. Taking the base level parameters defined above and repeating the analysis of the previous section, we obtain the values of Table 2. Again, the total surplus does not change.

We see a slight decrease in consumer surplus as the very slight discounting at the top of the

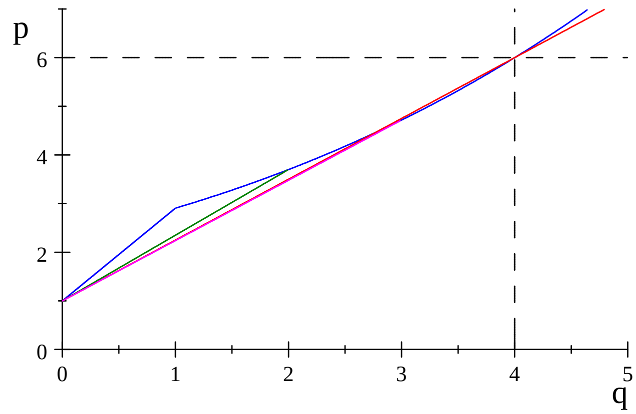

Figure 6: Plot of untaxed equilibrium offer (red) and taxed equilibrium offer when $J = 2$ (blue), $J = 4$ (green), $J = 6$ (magenta).

Curve	α	CS	Π^U	Π^A	Tax per firm	Social Surplus
S	0	5.3333	9.3333	9.3333	0	24
S	0.25	5.3333	9.3333	8.3020	1.0313	24
S^A	0.25	5.0152	9.4924	8.6428	0.8496	24

Table 2: Benefits and taxes under a tariff on line-expansion benefits.

offer curve is not sufficient to offset the heavy markups around $q = \frac{J}{2}$. For small expansions in line capacity this effect diminishes.

We can measure the change in consumer surplus, profits and tax collected as the magnitude of the line expansion varies. If we keep K constant at 8 and vary J from 0 to K , we cover a range of scenarios, from a completely new line to a miniscule (zero) increase in line capacity. The change in welfare after the tariff is imposed depends on the size of the counterfactual line J, as well as the probability of line congestion $1 - \frac{K}{\bar{\varepsilon}}$. The plots for a low probability of line congestion ($\bar{\varepsilon} = 10$, giving 20% probability) are shown in figures 7, 8, and 9. Solid curves represent values pertaining to equilibria where producers take the tax into account and dashed curves measure the same thing for equilibria where agents ignore the tax. Note that when $J \approx K = 8$, the mark-down effect dominates so that there is actually a reduction in price levels in the post-tariff SFE, leading to a slight gain in consumer surplus and slight reductions in producer profits and transmission charges collected, compared to the equilibrium when no tariff is charged.
Corresponding plots for a high probability of line congestion ($\bar{\varepsilon} = 20$, giving 60% probability) are shown in figures 10, 11, and 12. Note that the SFE under the tariff are now marked up sufficiently that the expected prices are higher for all possible levels of counterfactual line capacity.

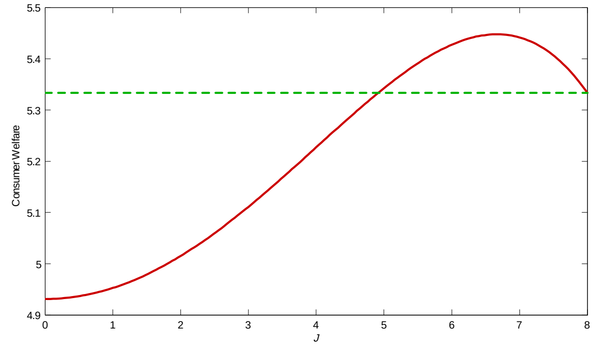

Figure 7: Consumer surplus as J varies when probability of line congestion is small.

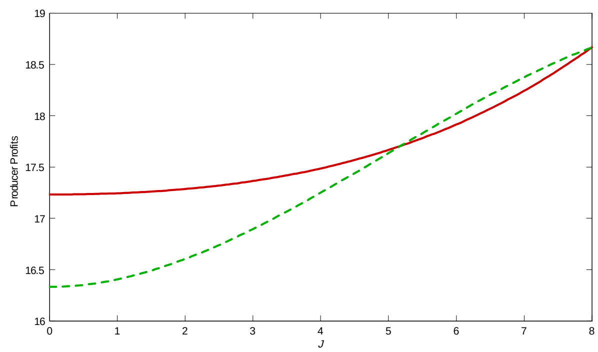

Figure 8: Producer profits as J varies when probability of line congestion is small.

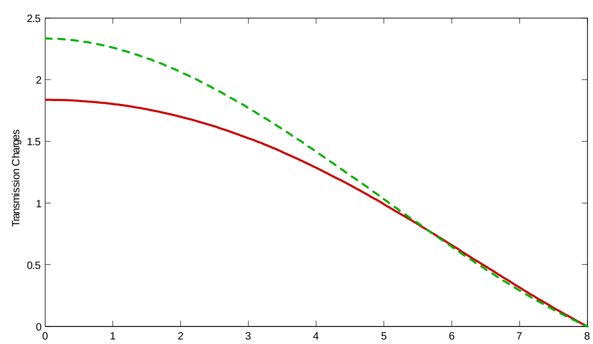

Figure 9: Transmission charges as J varies when probability of line congestion is small.

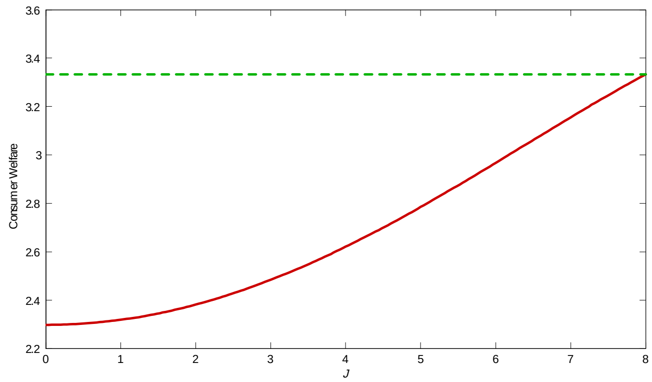

Figure 10: Consumer surplus as J varies when probability of line congestion is large.

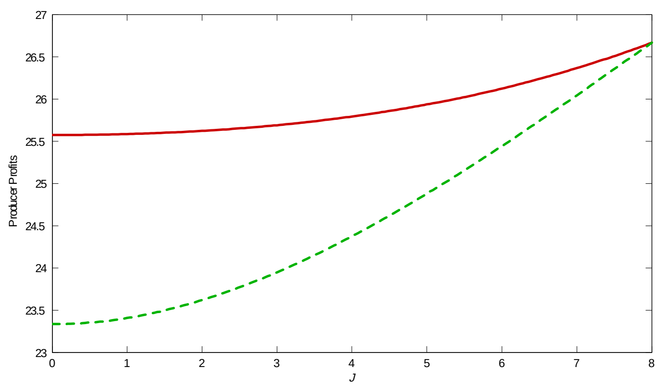

Figure 11: Producer net profit as J varies when probability of line congestion is large.

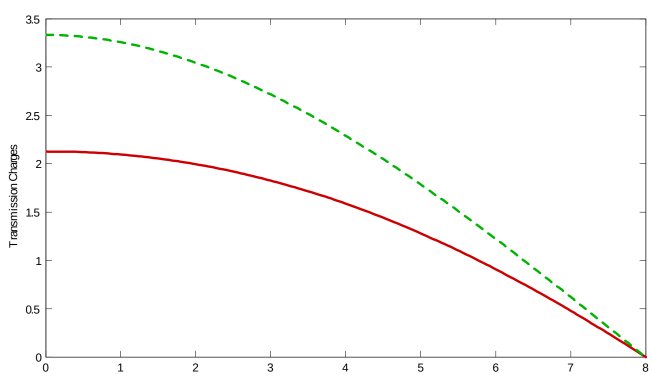

Figure 12: Transmission charges as J varies when probability of line congestion is large.

3.3 Other candidate SFE

We now examine whether other choices for the slope k in (13) lead to curves that are symmetric supply function equilibria.

3.3.1 Ruling out perfectly elastic segments

To show that a curve with a perfectly elastic segment at the transition to the line charge is not a symmetric equilibrium, we present a profitable deviation. Suppose our competitor plays a perfectly elastic segment from q^- to $q^+ = \frac{J}{2}$ at the price p^*, i.e.

$$S_2(p) = \begin{cases} q^- \frac{p-c}{p^*-c} & \text{if} \quad p < p^* \\ \left(\frac{K}{2} + \frac{\alpha\bar{\varepsilon}}{1-3\alpha}\right)\left(\frac{p-c}{\bar{p}-c}\right)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha} & \text{if} \quad p \geq p^*, \end{cases}$$

with $q^- < \frac{J}{2}$. This is shown in Figure 13.

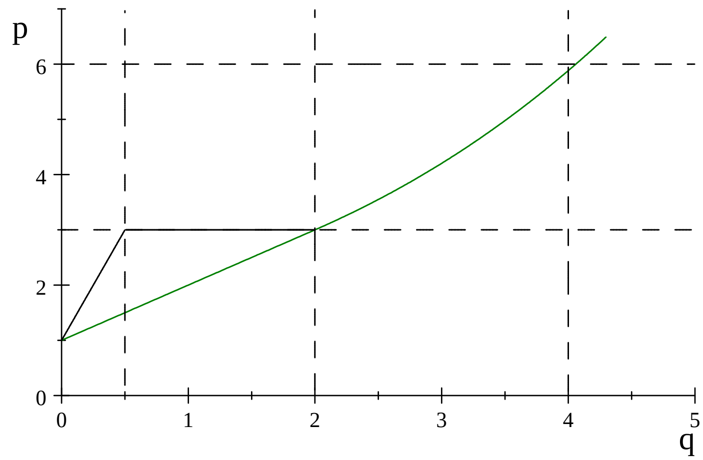

Figure 13: Plausible deviation to the equilibrium supply curve with a perfectly elastic segment. Here $q^- = 0.5$, $q^+ = \frac{J}{2} = 2$, and $p^* = 3$.

Suppose, for the sake of illustration, that when two firms have perfectly elastic segments at the same price, dispatch is by pro-rata on the margin. This means that if their horizontal segments are identical then they share demand equally. Suppose we offer the curve $S(p) = S_2(p)$. Our expected payoff is

$$\begin{aligned} \Pi^U(S) = & \int_c^{p^*} (p-c)\, q \left(\psi_q \frac{dq}{dp} + \psi_p\right) dp \\ & + \int_{2q^-}^{J} (p^*-c)\left(q^- + \frac{(\varepsilon - 2q^-)(q^+ - q^-)}{J - 2q^-}\right) f(\varepsilon)\, d\varepsilon \\ & + \int_{p^*}^{\bar{p}} (p-c)\, q \left(\psi_q \frac{dq}{dp} + \psi_p\right) dp. \end{aligned} \tag{15}$$

For uniformly distributed demand on $[0, \bar{\varepsilon}]$, the second term becomes (on substituting $2q^+$ for J)

$$\int_{2q^-}^{2q^+} (p^*-c)\left(q^- + \frac{(\varepsilon - 2q^-)(q^+ - q^-)}{2q^+ - 2q^-}\right)\frac{1}{\bar{\varepsilon}} d\varepsilon = \frac{p^*-c}{\bar{\varepsilon}}\left(q^+ - q^-\right)\left(q^+ + q^-\right).$$

If we undercut our competitor with a perfectly elastic segment from q^- to $q^+ = \frac{J}{2}$ at $p^* - \delta$, then the expected payoff is

$$\begin{aligned}\Pi^U(S_\delta) = &\int_c^{p^*-\delta} (p-c)\, q \left(\psi_q \frac{dq}{dp} + \psi_p\right) dp \\ &+ \int_{q^-}^{q^+} (p^* - \delta - c)\, q\, \psi_q\, dq + \Pr[p = p^*]\, q^+ (p^* - c) + o(\delta) \\ &+ \int_{p*}^{\overline{p}} (p-c)\, q \left(\psi_q \frac{dq}{dp} + \psi_p\right) dp.\end{aligned}$$

It is only the terms on the second line that differ from those in $\Pi^U(S)$ in (15) by an amount that doesn't vanish with δ. The first is

$$\begin{aligned}\int_{q^-}^{q^+} (p^* - \delta - c)\, q\, \psi_q\, dq &= \int_{q^-}^{q^+} (p^* - \delta - c)\, q \frac{1}{\bar{\varepsilon}}\, dq \\ &= \frac{p^* - c}{\bar{\varepsilon}} \left(q^+ - q^-\right) \frac{q^+ + q^-}{2}\end{aligned}$$

and the second is

$$\Pr[p = p^*]\, q^+ (p^* - c) = \frac{q^+ - q^-}{\bar{\varepsilon}} q^+ (p^* - c).$$

Therefore the gain made by offering S_δ is the difference

$$\begin{aligned}\Pi^U(S_\delta) - \Pi^U(S) &= \frac{p^* - c}{\bar{\varepsilon}} \left(q^+ - q^-\right) \left(\frac{3}{2} q^+ + \frac{1}{2} q^-\right) - \frac{p^* - c}{\bar{\varepsilon}} \left(q^+ - q^-\right)\left(q^+ + q^-\right) + o(\delta) \\ &= \frac{p^* - c}{2\bar{\varepsilon}} \left(q^+ - q^-\right)^2 + o(\delta) > 0\end{aligned}$$

for sufficiently small δ. This undercutting argument above can be applied to almost any sharing rule (see Holmberg, 2008) to show that perfectly elastic segments cannot occur in symmetric equilibrium in the untaxed portion of the curve.

One may use a similar argument to rule out perfectly elastic segments in the taxed portion of the curve too. We can write the discriminatory-pricing payoff from matching the competitor's perfectly elastic segment as

$$\begin{aligned}\Pi^D(S) = &\int_c^{p^*} (p-c)\left(1 - \psi(q,p)\right) dp \\ &+ \int_{2q^-}^{J} (p^* - c)\left(1 - F(\varepsilon)\right) \frac{q^+ - q^-}{J - 2q^-}\, d\varepsilon \\ &+ \int_{p^*}^{\overline{p}} (p-c)\left(1 - \psi(q,p)\right) dp,\end{aligned}$$

and then use (4) to calculate the positive advantage of undercutting perfectly elastic segments when a tax is applied.

3.3.2 Ruling out perfectly inelastic segments

Now consider a curve with a perfectly inelastic (vertical) segment at $q^* = \frac{J}{2}$, like that shown in figure 14.

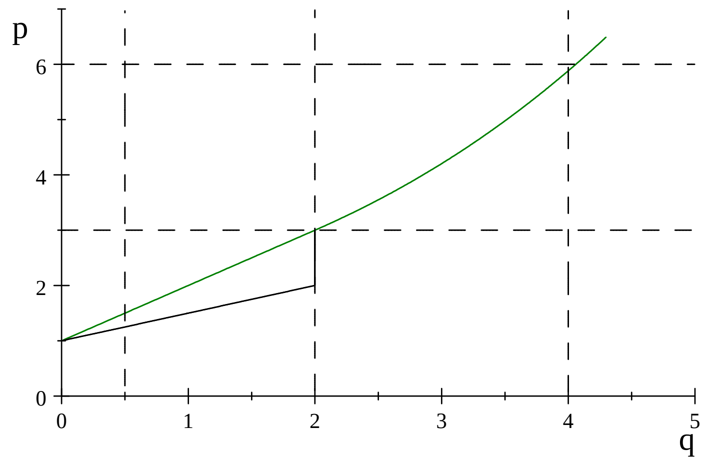

Figure 14: Plausible deviation to the equilibrium supply curve with perfectly inelastic segment. Here $q^* = \frac{J}{2} = 2$, $p_- = 2$ and $p_+ = 3$.

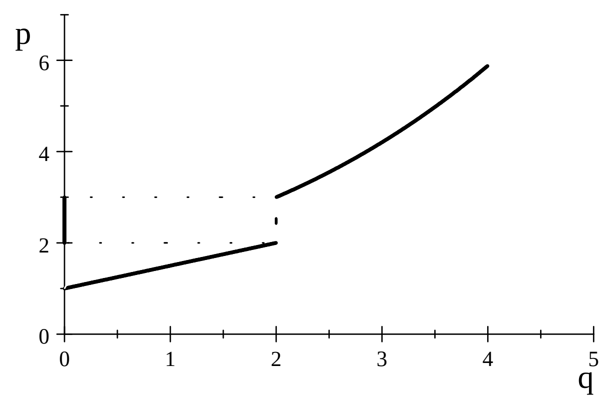

Figure 15: Locus of $Z = 0$ when competitor plays an inelastic segment at $q = 2$ from $p = 2$ to $p = 3$. Z is positive to the left of $Z = 0$ and negative to its right. Z is not defined at $p = 2$ or $p = 3$, but is negative for all $p \in (2, 3)$.

A plot of $Z = 0$ is shown in figure 15. One can observe that the set of points where $Z = 0$ is not monotone or even connected. Although Z is not defined everywhere, ψ is continuous and ψ_p is piecewise continuous (in contrast to the case of elastic segments where ψ has a point mass at p^*). We proceed to show that there exists a profitable deviation.

Disregard for a moment the effects of taxation; this puts us back in the uniform-price regime. Playing against the continuous supply function

$$S_2(p) = \begin{cases} k_2(p-c) & \text{if} \quad p < p_- \\ \frac{J}{2} & \text{if} \quad p \in [p_-, p_+] \\ \left(\frac{K}{2} + \frac{\alpha\bar{\varepsilon}}{1-3\alpha}\right)\left(\frac{p-c}{\bar{p}-c}\right)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha} & \text{if} \quad p > p_+, \end{cases}$$

the payoff to player 1 is given by

$$\Pi(S, S_2) = \int_c^{p_-} (p-c)\, q(p) \left(k_2 + \frac{dq}{dp}\right) \frac{1}{\bar{\varepsilon}}\, dp + \int_{p_-}^{p_+} (p-c)\, q(p) \frac{dq}{dp} \frac{1}{\bar{\varepsilon}}\, dp + V,$$

where V is the payoff over scenarios where the network is constrained in the counterfactual scenario.

If we respond with an identical supply function, then we receive a payoff of

$$\begin{aligned}\Pi\left(S_2, S_2\right) &= \int_c^{p_-} (p-c)^2 \frac{2k_2^2}{\bar{\varepsilon}}\, dp + \int_{p_-}^{p_+} 0\ dp + V \\ &= \frac{(p_- - c)^3}{3\bar{\varepsilon}} 2k_2^2 + V.\end{aligned}$$

If however we play the supply function from the symmetric equilibrium (14), with $k_1 < k_2$ chosen so that $k_1\left(p_+ - c\right) = \frac{J}{2}$, then we receive

$$\begin{aligned}\Pi\left(S_1, S_2\right) &= \int_c^{p_-} (p-c)^2 k_1 \frac{k_2 + k_1}{\bar{\varepsilon}}\, dp + \int_{p_-}^{p_+} (p-c)^2 \frac{k_1^2}{\bar{\varepsilon}}\, dp + V \\ &= \frac{1}{3\bar{\varepsilon}} \left((p_- - c)^3 k_1 k_2 + (p_+ - c)^3 k_1^2 \right) + V.\end{aligned}$$

Using the relation $k_1\left(p_+ - c\right) = k_2\left(p_- - c\right) = \frac{J}{2}$, we see that the difference of these,

$$\Pi\left(S_1, S_2\right) - \Pi\left(S_2, S_2\right) = \frac{k_1 k_2}{3\bar{\varepsilon}} \left(p_- - c\right) \left[\left(p_+ - c\right) - \left(p_- - c\right) \right]^2,$$

is strictly positive. Hence $S_2(p)$ cannot be part of a symmetric SFE. Without a tax on the benefits of line expansion, the amount paid for supply of the quantity $\frac{J}{2}$ is inconsequential as the probability of being dispatched that exact quantity is precisely zero.

If we now bring the tax back into consideration, the price paid for supply of the quantity $\frac{J}{2}$ becomes important, as this will be the counterfactual price for all dispatches above this level, and so it will receive a strictly positive point mass when calculating the expectation. We have only assumed that this price lies in the interval $[p_-, p_+]$. Players will prefer this price to be as high as possible, since the benefit charges decrease as the counterfactual price rises. The curve S_1 assures this price will be at the upper end of the interval; this gives an additional incentive to choose offer curve S_1 over S_2.

4 Conclusion

This work has examined the incentives of firms to adjust their offering strategies (in equilibrium) as a charge is applied as a percentage of either perceived profits (where the regulator believes that the firm offers at marginal cost), or perceived benefits of an investment in transmission assets (e.g. a line capacity upgrade). In a deterministic setting one may think that there would be an incentive to conceal one's perceived benefits by increasing the offers up to the dispatch point. However in a setting where the dispatch point is not known in advance (uncertain residual demand), we have shown that a balance must be struck between concealing the benefits and maximizing the (untaxed) profit. This new balance does not always exhibit higher mark-ups than the un-taxed regime.

In regions of quantity-price space where the tax applies, producers' optimize functionals that are a convex combination of uniform and pay-as-bid profit functionals. For a tax rate below a certain threshold a symmetric SFE exists that, compared to the equilibrium without the tax, has generally higher markups at low offer quantities but possibly smaller markups near the capacity constraint. We showed that there is no incentive for firms, in equilibrium, to play strategies that contain vertical or horizontal segments at the point where the line congests in the counter-factual dispatch. Therefore the equilibrium we found under a line-expansion charge is unique.

We discovered a counter-intuitive effect of the 'beneficiary-pays' charge in a duopoly setting. When the size of the line upgrade is small – and the probability of line-congestion is low – the consumer surplus can *increase* when the charge is applied, since firms submit offer curves that are strictly lower than the untaxed curves. Moreover, due to the competition firms in fact receive a lower profit and actually pay more tax than they would under the un-taxed equilibrium.

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