A Stochastic Programming Approach to Electric Energy Procurement for Large Consumers

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***Abstract*— This paper provides a technique based on stochastic programming to optimally solve the electricity procurement problem faced by a large consumer. Supply sources include bilateral contracts, a limited amount of self-production and the pool. Risk aversion is explicitly modeled using the CVaR methodology. Results from a realistic case study are provided and analyzed.**

***Index Terms*— Large consumer, electricity procurement, stochastic programming, conditional value-at-risk (CVaR).**

NOTATION

The notation used throughout the paper is stated below for quick reference.

A. Real Variables:

$C_t(\omega)$ Production cost in period t and scenario ω [€].
$P_t^{\mathrm{A}}(\omega)$ Self-produced energy consumed during period t and scenario ω [MWh].
$P_{b,t}^{\mathrm{C}}(\omega)$ Energy purchased from contract b in period t and scenario ω [MWh].
$P_t^{\mathrm{G}}(\omega)$ Energy self-produced by the consumer in period t and scenario ω [MWh].
$P_{i,t}^{\mathrm{G}}(\omega)$ Energy self-produced in block i of the piecewise linear production cost function in period t and scenario ω [MWh].
$P_t^{\mathrm{P}}(\omega)$ Energy purchased from the pool in period t and scenario ω [MWh].
$P_t^{\mathrm{S}}(\omega)$ Self-produced energy sold in the pool in period t and scenario ω [MWh].
ξ Auxiliary variable used to calculate CVaR [€].
$\eta(\cdot)$ Auxiliary variable used to calculate the Kantorovich distance between two probability distributions.
$\mu(\omega)$ Auxiliary variable related to scenario ω and used to calculate CVaR [€].

B. Binary Variables:

$h_b(\omega)$ Binary variable which is equal to 1 if contract b is selected in scenario ω, and 0 otherwise.

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C. Random Variables:

ϵ_t White noise process which represents the error term in the ARIMA model of the pool price in period t [ln(€/MWh)].
λ_t^{P} Price of energy in the pool in period t [€/MWh].

D. Constants:

$A(\omega,k)$ 0/1 constant which is equal to 1 if scenarios ω and $\omega+1$ are equal up to stage k, being 0 otherwise.
F_i Slope of block i of the piecewise linear production cost function [€/MWh].
$H(b)$ Stage in which the decision on the selection of contract b is made.
$P_b^{\mathrm{C,max}}$ Upper limit of energy that can be purchased from contract b in one period [MWh].
P_t^{D} Expected demand in period t [MWh].
P_i^{G} Upper limit of the energy produced in block i of the piecewise linear production cost function [MWh].
$P^{\mathrm{G,max}}$ Upper limit of energy that can be produced by the self-production unit in one period [MWh].
$P_{b,e}^{\max}$ Upper limit of energy that can be purchased from contract b in subset of periods $T_{b,e}$ [MWh].
$P_{b,e}^{\min}$ Lower limit of energy that can be purchased from contract b in subset of periods $T_{b,e}$ [MWh].
$S(t)$ Stage in which the decisions of purchases from contracts are taken in period t.
α Confidence level used in the calculation of CVaR.
β Weighting factor [1/€].
θ_u Parameter related to delay u used in the ARIMA model.
ϵ_{ini} First error term considered in the ARIMA model [ln(€/MWh)].
$\epsilon_t(\omega)$ Error term in period t and scenario ω [ln(€/MWh)].
$\lambda_{b,t}^{\mathrm{C}}(\omega)$ Price of energy for contract b in period t and scenario ω [€/MWh].
$\lambda_{b,t}^{\mathrm{C,fixed}}$ Prefixed price of energy for contract b in period t [€/MWh].
$\lambda_{\mathrm{ini}}^{\mathrm{P}}$ First pool price considered in the ARIMA model [€/MWh].
$\lambda_t^{\mathrm{P}}(\omega)$ Price of energy in the pool in period t and scenario ω [€/MWh].
$\pi(\omega)$ Probability of scenario ω.

ϕ_u Parameter related to delay u used in the ARIMA model.

E. Numbers:

n_{B} Number of bilateral contracts.
n_{E} Number of sets of periods used in the modeling of the contracts.
n_{I} Number of blocks of the piecewise linear approximation of the production cost function.
n_{T} Number of periods.
n_{W} Number of periods comprising a week.
n_{Ω} Number of scenarios after the scenario generation process.
$n_{\Omega'}$ Number of scenarios after the scenario reduction process.

F. Sets:

B Set of bilateral contracts.
B^{M} Set of monthly bilateral contracts.
B_n Set of contracts selected in node n.
N Set of nodes.
T Set of periods.
T_b Set of periods in which contract b is defined.
$T_{b,e}$ Set of periods belonging to subset e in contract b.
Ω Set of scenarios generated in the scenario-generation process.
Ω' Set of preserved scenarios after the scenario reduction process.
Ω^* Set of deleted scenarios in the scenario reduction process.

G. Others:

$c(\omega, \omega')$ Distance between scenarios ω and ω' of a given random variable.
$D_{\mathrm{K}}(\cdot)$ Kantorovich distance of two probability distributions.
Q Probability distribution of a given random variable.

I. INTRODUCTION

This paper considers an electricity market that includes a pool and in which bilateral contracts among producers and consumers can be freely arranged. The pool consists of a day-ahead auction as well as auctions with shorter time horizons, such as control, reserve and balancing auctions. Bilateral contracts can be agreed upon on a daily, weekly or monthly basis, but contracts embracing longer time horizons generally provide more effective hedging against pool price volatility than those spanning shorter time periods. An example of such market arrangement is the electricity market of mainland Spain [1].

This paper considers the perspective of a large consumer that owns a limited self-production facility (e.g. a cogeneration unit), and derives a methodology that allows the consumer to optimally decide its involvement in bilateral contracts, self-production and its participation in the pool. Uncertainty is treated in detail through a stochastic programming framework [2].

The objective pursued is minimizing the expected value of the procurement cost while limiting its volatility (risk) by incorporating risk aversion through the Conditional Value-at-Risk (CVaR) methodology [3], [4].

A. Motivation, Aim and Solution

A large consumer has the opportunity to procure its electric energy needs through bilateral contracts, self-production and the pool. Signing bilateral contracts reduces the risk associated with the volatility of pool prices usually at the cost of high average prices for the signed contracts. Self-producing also reduces the risk related to pool price. On the other hand, relying mostly on the pool might result in an unacceptable volatility of procurement cost. Hence, the consumer faces a tradeoff between its level of involvement in bilateral contracts, its self-production and its participation in the pool. To resolve such a tradeoff, this paper describes and develops a model that is a multistage stochastic integer programming problem with recourse [2]. This problem is made tractable using scenario-reduction techniques and solved using a commercially available branch-and-cut software. The solution to this problem determines which contracts should be signed among the set of available ones, and the amounts of energy to be purchased from each of the selected contracts. These are *here-and-now* decisions. The solution to the problem also provides the optimal strategy (policy) of pool purchases for each realization of pool prices (constituting the *wait-and-see* decisions of the recourse problem.)

B. Literature Review and Contributions

Although the technical literature is rich on papers addressing the point of view of the producer, i.e., addressing the self-scheduling and bidding problems of generating companies, e.g. [5], [6] and [7], few references are found on how large consumers should procure their electricity consumption. The pioneering work of Daryanian et al. [8], and the recent work of Kirschen [9] deserve special attention. Within a centralized decision framework, in [8] the optimal response of a large consumer to varying electricity spot prices is derived in terms of consumptions and consumption rescheduling. In [9] a detailed analysis and characterization of the decision-making tools that consumers and retailers need to participate in an electricity market are presented. Additionally, in [10], a relevant method for purchase allocation and demand bidding is provided.

The electrical energy procurement problem by a large consumer is treated in [11] without considering risk accruing from uncertainty in the pool prices.

The related problem faced by an industrial consumer managing both electricity and heat (emphasizing heat issues) is addressed by [12] and [13].

The contributions of this paper are:

1) The electricity procurement decision problem faced by a large consumer is formulated as a stochastic programming problem with recourse.

2) Risk aversion is explicitly considered through the CVaR methodology.
3) The stochastic programming problem is made tractable through scenario reduction techniques and solved using a commercial branch-and-cut code.

C. Paper Organization

The rest of this paper is organized as follows. Section II characterizes the uncertainty associated with the considered decision-making problem. It provides a description of the multistage decision framework under uncertainty, specifying both *here-and-now* and *wait-and-see* decisions. Additionally, it characterizes the stochastic variables involved through ARIMA models, builds a scenario tree and reduces the size of this tree to make the associated problem tractable.

Section III formulates mathematically the stochastic decision-making problem as a stochastic programming problem with recourse. The solution technique to tackle this problem is also discussed.

Section IV provides and analyzes results from a realistic case study based on the electricity market of mainland Spain.

Section V provides several relevant conclusions obtained from the study reported in this paper.

Finally, the scenario reduction algorithm used to make the resulting problem tractable is derived in the Appendix.

II. UNCERTAINTY CHARACTERIZATION

A. Decision Framework

A time span of four weeks is considered. At the beginning of the first week, monthly contracts must be selected. Moreover, at the beginning of each week the consumer decides

1) the energy schedule from monthly contracts on the next week,
2) which weekly contracts to sign for that week, and
3) the energy purchased from each weekly contract.

These decisions are named *here-and-now* decisions because they are taken before knowing the realizations of the stochastic variables.

In contrast with the *here-and-now* decisions, the decisions about purchases and sales in the pool and those related to the self-production facility are closer in time to the instant when the values of the stochastic variables are known. Thus, the uncertainty related to these decisions is lower than that related to the decisions taken about bilateral contracts. Therefore, we assume that the values of the stochastic variables are known when decisions about the energy transacted in the pool and self-production have to be made. These decisions, referred to as *wait-and-see* decisions, are different for each realization of the stochastic variables, and thus they are supposed to be made at the end of each week.

Therefore, the decisions are made at the beginning and the end of each week, so that the end and the beginning of two consecutive weeks constitute a unique stage. In this way, the planning horizon of 4 weeks comprises 5 stages, as shown in Fig. 1.

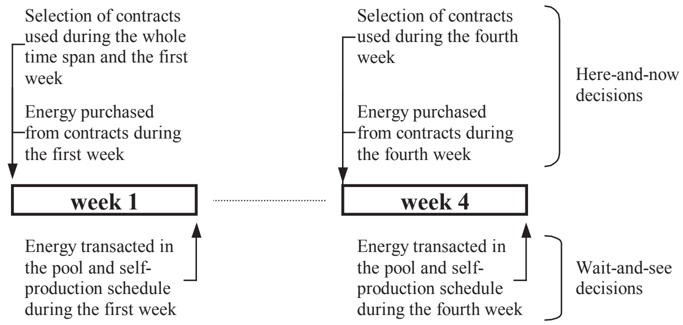

Fig. 1. Decision framework.

Each week has been divided into seven days, and each day into three periods, thus yielding a time span of 84 periods. Each daily period comprises eight hours defined as follows:

$$\text{Day d:}\quad \begin{aligned} \text{Period 1} &= \{1,2,3,4,5,6,7,8\} \\ \text{Period 2} &= \{9,10,15,16,17,18,23,24\} \\ \text{Period 3} &= \{11,12,13,14,19,20,21,22\}. \end{aligned}$$

The hours in each period have been selected depending on their pool price level. Thus, periods 1, 2 and 3 previously defined include the hours with low, medium and high pool prices, respectively.

B. Random Variables

As stated above, the pool price is treated as a stochastic variable, which is characterized by an ARIMA model [14], [15]. Using a time series model, it is possible to represent adequately the probability distribution of a stochastic variable through the generation of multiple realizations of it. The set of different realizations of the stochastic variable can be structured to build a scenario tree. A scenario tree constitutes a discrete and finite approximation of the probability distribution of the stochastic variable. The ARIMA model below is used to generate a scenario tree to represent the market-clearing prices of December 2004 of the electricity market of mainland Spain [1]:

$$\begin{aligned} &\left(1-\phi_1 B^1-\phi_2 B^2\right)\left(1-\phi_3 B^3-\phi_{24} B^{24}-\phi_{27} B^{27}\right) \\ &\left(1-\phi_{21} B^{21}-\phi_{42} B^{42}\right)\left(1-B^3\right)\left(1-B^{21}\right) ln\left(\lambda_t^{\mathrm{P}}\right)= \\ &\qquad\left(1-\theta_3 B^3\right)\left(1-\theta_{21} B^{21}\right)\epsilon_t \quad \forall t \in T \end{aligned} \tag{1}$$

where B^u is the backshift operator, which if applied to λ_t^{P} renders:

$$B^u \lambda_t^{\mathrm{P}} = \lambda_{t-u}^{\mathrm{P}}, \tag{2}$$

and ϵ_t is the error term, which is assumed to be a series obtained randomly from a normal distribution with zero mean and constant variance, σ^2; that is, a white noise process. The standard deviation of ϵ_t, σ, is obtained at the end of the estimation phase of the ARIMA model. For model (1), $\sigma = 0.1186$. The parameters contained in (1) are listed in Table I. The above ARIMA model shows that λ_t^{P} depends on $\left\{\lambda_{\mathrm{ini}}^{\mathrm{P}}, \ldots, \lambda_{t-1}^{\mathrm{P}}\right\}$ and $\{\epsilon_{\mathrm{ini}}, \ldots, \epsilon_t\}$, where $\lambda_{\mathrm{ini}}^{\mathrm{P}}$ and ϵ_{ini} are, respectively, the first pool price and error term considered in the estimation phase of the construction of the ARIMA model.

C. Scenario Tree

A scenario tree is a set of nodes and branches used in models of decision-making under uncertainty. The nodes represent states of the "world" at a particular instant, being the points where decisions are taken. Each node has only one predecessor and can have several successors. The first node is called the root node and, in this work, it corresponds to the beginning of the first week of the planning horizon. In the root node, the first-stage decisions are taken. In the same way, the nodes placed in the next stage represent the points where the second-stage decisions are taken, and so on. The number of nodes in the last stage equals the number of scenarios. These nodes are denominated leaves. In a stochastic scenario tree, the branches are different realizations of the stochastic variables. Considering only uncertainty in the pool prices, the branches leaving the root node are different realizations of the pool price during the first week. Analogously, the branches connecting nodes placed in the second and third stages represent realizations of the pool prices during the second week, and so on. Therefore, according to the time discretization presented in II-A, each one of these branches represents a set of 21 pool prices. Fig. 2 shows an example of scenario tree.

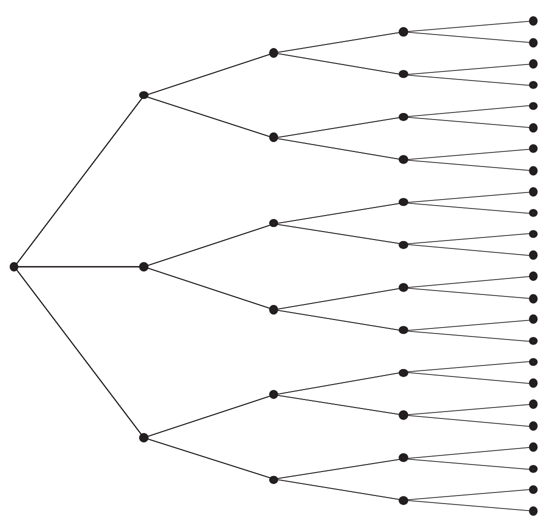

Fig. 2. Scenario tree example.

The multiple realizations of λ_t^{P} required to build a scenario tree are obtained by assigning different values to the error term ϵ_t. For every period $t \in T$ and every scenario $\omega \in \Omega$ a random value of $\epsilon_t(\omega)$ with a $N(0,\sigma)$ distribution is generated and, through the ARIMA model (1), $\lambda_t^{\mathrm{P}}(\omega)$ is obtained. It should be noted that if two scenarios ω and ω' are coincident in the same branch in period t, the relation $\lambda_t^{\mathrm{P}}(\omega) = \lambda_t^{\mathrm{P}}(\omega')$ must be enforced. The algorithm used to generate the scenario tree is depicted in Fig. 3.

TABLE I
PARAMETERS OF THE ARIMA MODEL (1)

$\phi_1 = 0.7257$	$\phi_{21} = -0.1134$	$\phi_{42} = -0.0976$
$\phi_2 = -0.1603$	$\phi_{24} = 0.0661$	$\theta_3 = 0.7491$
$\phi_3 = 0.1206$	$\phi_{27} = -0.0852$	$\theta_{21} = 0.8304$

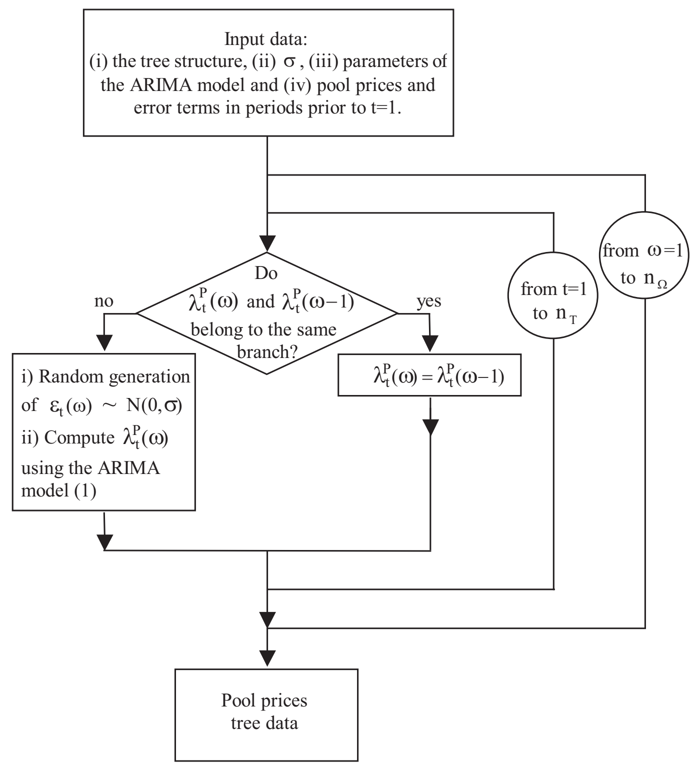

Fig. 3. Scenario generation process.

D. Scenario Tree Reduction

The size of the scenario tree obtained by running the scenario generation process is typically very large, resulting in an optimization model that is intractable. To regain tractability, we endeavor to reduce the number of scenarios while still retaining the essential features of the scenario tree. We seek a tractable scenario tree that yields an optimal solution that is close in value to the solution of the original problem.

There has been much attention paid to this problem in the academic literature, and it is still an area of active research [16]. In two-stage stochastic programming problems it is possible to reduce a large scenario tree to a simpler tree that is close to the original tree when measured by a so-called *probability distance*. Under mild conditions on the problem data, it can be shown that the optimal value of the simpler problem will be close to the value of the solution to the original problem if the scenario trees are close in the probability metric. Such stability results are no longer valid for multistage stochastic programming problems, and require the introduction of a *filtration distance* [16] that essentially measures how close the branching structures of the two trees are.

The most common probability distance used in stochastic optimization is the *Kantorovich distance*, $D_K(\cdot)$, defined between two probability distributions Q and Q' by

$$D_K(Q,Q') = \inf_{\eta}\{\int_{\Omega\times\Omega} c(\omega,\omega')\eta(d\omega,d\omega') : \int_{\Omega}\eta(\cdot,d\omega') = Q, \quad \int_{\Omega}\eta(d\omega,\cdot) = Q'\}, \tag{3}$$

where c is a nonnegative, continuous, symmetric, cost function and the infimum is taken over all joint probability distributions defined on $\Omega\times\Omega$. If

$$c(\omega,\omega') = \|\omega-\omega'\|^r \tag{4}$$

this gives the Wasserstein metric of order r, that can be shown to have some appealing properties when approximating stochastic optimization problems [17].

In the current context, where Q and Q' are finite distributions corresponding to an initial set Ω of scenarios and a reduced set $\Omega'\subset\Omega$, we define

$$c(\omega,\omega') = \sum_{t=1}^{n_T} |\lambda_t^P(\omega) - \lambda_t^P(\omega')|. \tag{5}$$

This can be shown [18] to give the Kantorovich distance

$$D_K(Q,Q') = \sum_{\omega\in\Omega\setminus\Omega'} \pi(\omega) \min_{\omega'\in\Omega'} \left(\sum_{t=1}^{n_T} |\lambda_t^P(\omega) - \lambda_t^P(\omega')|\right) \tag{6}$$

which is attained by assigning the probabilities $\pi(\omega)$ of all scenarios $\omega\in\Omega\setminus\Omega'$ to the "closest" scenario ω' in the remaining set Ω'.

As outlined in [19], this formula can be used to derive several heuristics for generating scenario trees that are close to an original tree in the Kantorovich metric. We have chosen to implement the *fast-forward* algorithm as described in [19] and implemented as SCENRED in GAMS [20]. This algorithm is an iterative greedy process starting with an empty tree. In each iteration, from the set of non-selected scenarios, the scenario which minimizes the Kantorovich distance between the reduced and original trees is selected. The algorithm stops if either a specified number of scenarios or a certain Kantorovich distance is reached. Finally, the probabilities of the selected scenarios are updated. In the Appendix, the steps of this algorithm are described in detail.

We conclude this section by recalling that the scenario reduction technique we have used is only a heuristic, with no known performance guarantees. The reduced scenario tree generated by the fast-forward algorithm is not guaranteed to be the closest in the Kantorovich metric to the original tree (over all reduced trees of the same cardinality). Moreover, without a bound on the filtration distance we have no guarantee that the reduced tree will give a good approximation to the optimal value of the original problem. Nevertheless the empirical results reported in the literature (e.g. in [19]) indicate that the reduced trees defined by the fast-forward algorithm perform well in practice.

III. Optimization Framework

A. Problem Formulation

The mathematical formulation of the deterministic equivalent of the stochastic problem with recourse faced by the large consumer is stated below:

Minimize:

$$\sum_{\omega\in\Omega'}\pi(\omega)\Bigg(\sum_{b\in B}\sum_{t\in T_b}\lambda_{b,t}^{\mathrm{C}}(\omega)P_{b,t}^{\mathrm{C}}(\omega) + \sum_{t\in T}\lambda_t^{\mathrm{P}}(\omega)\Big(P_t^{\mathrm{P}}(\omega)-P_t^{\mathrm{S}}(\omega)\Big) + \sum_{t\in T}C_t(\omega)\Bigg) + \beta\left(\xi + \frac{1}{1-\alpha}\sum_{\omega\in\Omega'}\pi(\omega)\mu(\omega)\right) \tag{7}$$

Subject to:

1) CVaR constraints:

$$\sum_{b\in B}\sum_{t\in T_b}\lambda_{b,t}^{\mathrm{C}}(\omega)P_{b,t}^{\mathrm{C}}(\omega) + \sum_{t\in T}\lambda_t^{\mathrm{P}}(\omega)\Big(P_t^{\mathrm{P}}(\omega)-P_t^{\mathrm{S}}(\omega)\Big) + \sum_{t\in T}C_t(\omega) - \xi - \mu(\omega) \le 0; \quad \forall\omega\in\Omega' \tag{8}$$

$$\mu(\omega)\ge 0; \quad \forall\omega\in\Omega' \tag{9}$$

2) Contract constraints:

$$0\le P_{b,t}^{\mathrm{C}}(\omega)\le P_b^{\mathrm{C,max}}; \quad \forall b\in B, \forall t\in T_b, \forall\omega\in\Omega' \tag{10}$$

$$P_{b,t}^{\mathrm{C}}(\omega) = 0; \quad \forall b\in B, \forall t\in T\setminus T_b, \forall\omega\in\Omega' \tag{11}$$

$$P_{b,e}^{\min}h_b(\omega) \le \sum_{t\in T_{b,e}} P_{b,t}^{\mathrm{C}}(\omega) \le P_{b,e}^{\max}h_b(\omega); \quad e=1,\ldots,n_{\mathrm{E}}; \forall b\in B, \forall\omega\in\Omega' \tag{12}$$

3) Self-production constraints:

$$0\le P_t^{\mathrm{G}}(\omega)\le P^{\mathrm{G,max}}; \quad \forall t\in T, \forall\omega\in\Omega' \tag{13}$$

$$P_t^{\mathrm{G}}(\omega) = \sum_{i=1}^{n_{\mathrm{I}}} P_{i,t}^{\mathrm{G}}(\omega); \quad \forall t\in T, \forall\omega\in\Omega' \tag{14}$$

$$0\le P_{1,t}^{\mathrm{G}}(\omega)\le P_1^{\mathrm{G}}; \quad \forall t\in T, \forall\omega\in\Omega' \tag{15}$$

$$0\le P_{i,t}^{\mathrm{G}}(\omega)\le P_i^{\mathrm{G}} - P_{i-1}^{\mathrm{G}}; \quad i=2,\ldots,n_{\mathrm{I}}-1; \forall t\in T, \forall\omega\in\Omega' \tag{16}$$

$$0\le P_{n_{\mathrm{I}},t}^{\mathrm{G}}(\omega)\le P^{\mathrm{G,max}} - P_{n_{\mathrm{I}}-1}^{\mathrm{G}}; \quad \forall t\in T, \forall\omega\in\Omega' \tag{17}$$

$$C_t(\omega) = \sum_{i=1}^{n_{\mathrm{I}}} F_i P_{i,t}^{\mathrm{G}}(\omega); \quad \forall t\in T, \forall\omega\in\Omega' \tag{18}$$

$$P_t^{\mathrm{G}}(\omega) = P_t^{\mathrm{A}}(\omega) + P_t^{\mathrm{S}}(\omega); \quad \forall t\in T, \forall\omega\in\Omega' \tag{19}$$

4) Demand constraints:

$$P_t^{\mathrm{A}}(\omega) + P_t^{\mathrm{P}}(\omega) + \sum_{b\in B} P_{b,t}^{\mathrm{C}}(\omega) = P_t^{\mathrm{D}}; \quad \forall t\in T, \forall\omega\in\Omega' \tag{20}$$

5) Nonanticipativity constraints:

$$h_b(\omega) = h_b(\omega+1);$$
$$\forall b \in B, \omega = 1, \ldots, n_{\Omega'} - 1; if\, A(\omega, H(b)) = 1 \quad (21)$$

$$P^{\mathrm{C}}_{b,t}(\omega) = P^{\mathrm{C}}_{b,t}(\omega+1);$$
$$\forall b \in B, \omega = 1, \ldots, n_{\Omega'} - 1; if\, A(\omega, S(t)) = 1 \quad (22)$$

6) Constraints on variables:

$$h_b(\omega) \in \{0,1\}; \quad \forall b \in B, \forall \omega \in \Omega' \quad (23)$$

$$P^{\mathrm{A}}_t(\omega), P^{\mathrm{P}}_t(\omega), P^{\mathrm{S}}_t(\omega) \geq 0; \quad \forall t \in T, \forall \omega \in \Omega'. \quad (24)$$

Note that in the formulation above

$$\lambda^{\mathrm{C}}_{b,t}(\omega) = \frac{\lambda^{\mathrm{C,fixed}}_{b,t} + \lambda^{\mathrm{P}}_t(\omega)}{2};$$
$$\forall b \in B, \forall t \in T_b, \forall \omega \in \Omega'. \quad (25)$$

Problem (7)-(24) is explained below.

1) Objective function and CVaR constraints: The objective function (7) comprises the expected cost of the electrical procurement of the consumer and a penalized risk measure. The expected cost includes (i) the expected cost of buying from bilateral contracts, (ii) the expected net cost of buying from the pool (purchase cost minus revenue from selling), and (iii) the expected cost incurred by the self-production facility. The expected cost is calculated as the sum over all scenarios of the cost in each scenario multiplied by its probability.

The risk measure included in this work is the conditional value-at-risk at α confidence level (α-CVaR) [3]. For a discrete cost distribution, α-CVaR is approximately the expected cost of the $(1-\alpha)100\%$ scenarios with greater cost. In [4] a linear formulation of α-CVaR is provided. Let $\mathrm{DCF}(\omega), \omega \in \Omega'$, be a discrete cost distribution, α-CVaR is the result of the following optimization problem:

$$\text{Minimize}_{\xi,\mu(\omega)} \qquad \xi + \frac{1}{1-\alpha} \sum_{\omega \in \Omega'} \pi(\omega)\mu(\omega) \quad (26)$$

Subject to:

$$\mathrm{DCF}(\omega) - \xi - \mu(\omega) \leq 0; \quad \forall \omega \in \Omega' \quad (27)$$

$$\mu(\omega) \geq 0; \quad \forall \omega \in \Omega' \quad (28)$$

The optimal value of ξ, ξ^*, represents the smallest value such that the probability that the cost exceeds or equals ξ^* is less than or equal to $1-\alpha$. Also, ξ^* is known as the value-at-risk (VaR). On the other hand, $\mu(\omega)$ is the difference between the cost of scenario ω and VaR. Constraints (27) and (28) are equivalent to (8) and (9). The objective function (26) corresponds to the last term of (7).

The weighting factor β in (7), $\beta \in [0, \infty)$, models the tradeoff between expected procurement cost and risk, and so depends on the preferences of the consumer. A conservative consumer prefers minimizing risk while its demand is satisfied, so it chooses a large value of β to increase the weight of the risk measure in (7). On the other hand, another consumer might be willing to assume higher risk in the hope of obtaining a lower cost, so its selected value for β would be close to 0. A detailed discussion on how to obtain appropriate values for the weighting factor β is beyond the scope of this paper.

2) Contract constraints: In this work, volume contracts have been considered. The total energy consumed from a volume contract must satisfy pre-specified upper and lower limits. The planning horizon of each contract is usually divided into several subsets of periods depending on the pool prices. For example, if the planning horizon of contract b is divided into n_{E} subsets of periods, then

$$\bigcup_{e=1,\ldots,n_{\mathrm{E}}} T_{b,e} = T_b \quad \forall b \in B \quad (29)$$

where $T_{b,e}, e = 1, \ldots, n_{\mathrm{E}}$, are subsets of periods for contract b. In this work, four subsets of periods, i.e., $n_{\mathrm{E}} = 4$, have been defined for each contract, namely *valley* (V), *shoulder* (S), *peak* (P) and *weekend* (W). Fig. 4 shows the distribution of subsets of periods in each week. Taking into account these definitions, constraints (10) set the limits of the energy consumed from every contract in every period. Constraints (11) state that it is not possible to purchase energy outside of the planning horizon of each contract. Constraints (12) set the upper and lower limits for the energy consumed from contracts in each subset of periods.

	Periods																				
	Monday			Tuesday			Wednesday			Thursday			Friday			Saturday			Sunday		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
V	×			×			×			×			×								
S		×			×			×			×			×							
P			×			×			×			×			×						
W																×	×	×	×	×	×

V: Valley; S: Shoulder; P: Peak; W: Weekend

Fig. 4. Weekly distribution of subsets of periods.

The above contracting format is motivated by industry practice in Spain. However, note that a different format can be considered and simpler purchaser settings are possible, e.g., buying a quantity of energy at constant power and price during a given time period.

3) Self-production constraints: Constraints (13) bound the production of the self-production unit by the upper limit of energy that can be produced during one period, which is equal to the capacity of the unit multiplied by the duration of the period. Constraints (14)-(18) are required to model the piecewise linear production cost of the self-production unit [5]. Constraints (19) state that the energy generated by the self-production facility can be used either to satisfy the demand of the consumer or to be sold in the pool.

4) Demand constraints: Constraints (20) enforce that the demand must be satisfied in each period of the planning horizon.

5) Nonanticipativity constraints: Constraints (21) and (22) model the nonanticipativity constraints for $h_b(\omega)$ and $P^{\mathrm{C}}_{b,t}(\omega)$, respectively. Nonanticipativity constraints enforce that if the realizations of the stochastic variables are equal in two scenarios ω and ω' up to stage k, then the value of the *here-and-now* decisions in stage k must be the same. For computational implementation, these variables are made scenario independent.

6) Constraints on variables and others: Constraints (23) and (24) constitute variable declarations. Finally, the expressions (25) define the final price of energy purchased from bilateral contracts as the average of the pool prices and a prefixed price (contracts for differences).

B. Solution Procedure

Problem (7)-(24) is a mixed-integer linear optimization problem that can be solved by commercially available branch-and-cut software [21]-[23].

Table II provides the size of problem (7)-(24) expressed as the number of binary variables, real variables and constraints.

TABLE II
COMPUTATIONAL SIZE OF PROBLEM (7)-(25)

# of binary variables	$n_{\mathrm{B}} n_{\Omega'}$
# of real variables	$n_{\Omega'}\left(n_{\mathrm{T}}(n_{\mathrm{B}}+n_{\mathrm{I}}+5)+1\right)+1$
# of constraints	$n_{\Omega'}\big(n_{\mathrm{T}}(n_{\mathrm{B}}+n_{\mathrm{I}}+5)+2n_{\mathrm{B}}n_{\mathrm{E}}+1+$ $\sum_{n\in N}\big(\sum_{b\in B_n}(n_{\Omega',n}-1)+$ $\sum_{b\in\{B_n\bigcup B^{\mathrm{M}}\}} n_{\mathrm{W}}(n_{\Omega',\mathrm{n}}-1)\big)$

IV. CASE STUDY

The performance of the proposed decision-making approach is illustrated through a case study based on the electricity market of mainland Spain [1]. A time series of 11 months has been used to estimate pool prices for December 2004. Fig. 5 shows the pool prices in all of the scenarios for the 84 periods considered. The bold line in Fig. 5 corresponds to the expected pool prices. We assume that the consumer has an accurate forecast of its demand, which is considered deterministic and is plotted in Fig. 6.

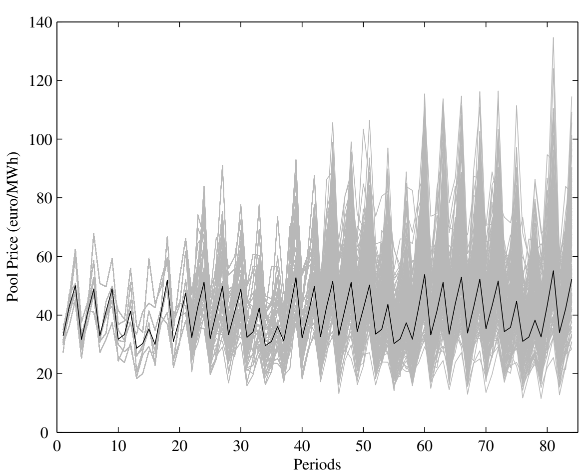

Fig. 5. Pool prices.

The consumer has the possibility of signing two monthly bilateral contracts as well as four weekly contracts, one per week. The energy consumption limits of each contract are reported in Table III. For the sake of conciseness, both monthly contracts are identical in terms of energy limits. Analogously, the four weekly contracts share the same consumption limits. Price data of the monthly and weekly contracts are provided in Tables IV and V, respectively.

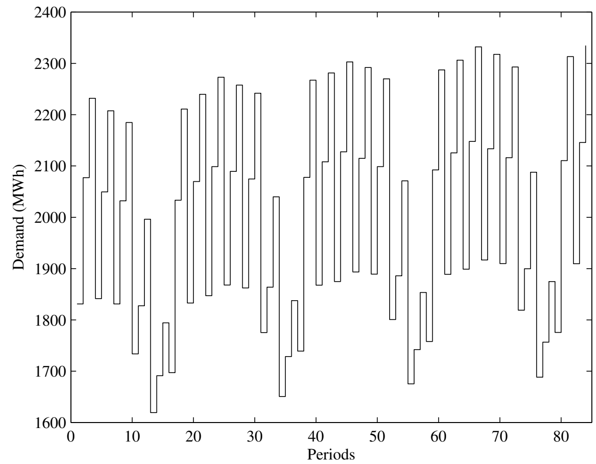

Fig. 6. Consumer demand.

TABLE III
ENERGY CONSUMPTION LIMITS OF THE BILATERAL CONTRACTS (MWh)

	Monthly contracts		Weekly contracts	
	$P_{b,e}^{\min}$	$P_{b,e}^{\max}$	$P_{b,e}^{\min}$	$P_{b,e}^{\max}$
V	3540	7870	550	1250
S	6125	12360	1000	2310
P	7525	16850	1425	3365
W	2125	4880	150	550

TABLE IV
PRICES OF THE MONTHLY CONTRACTS (€/MWh)

	Week # (monthly contract 1)				Week # (monthly contract 2)			
	1	2	3	4	1	2	3	4
V	42.0	43.0	44.0	45.0	42.0	43.0	44.0	45.0
S	52.8	53.8	54.8	55.8	55.2	56.2	57.2	58.2
P	62.4	63.4	64.4	65.4	64.8	65.8	66.8	67.8
W	44.4	45.4	46.4	47.4	43.2	44.2	45.2	46.2

The consumer owns a 100-MW self-production unit. Since each time period comprises 8 hours, the maximum energy that can be produced in each period, $P^{\mathrm{G,max}}$, is equal to 800 MWh. Table VI lists the data of the 3-piece linear cost function considered.

In order to build the scenario tree, 7 branches leave each node, thus yielding a decision tree with $7^4 = 2401$ scenarios. After applying the scenario reduction technique mentioned in II-D and described the Appendix, the resulting tree contains just 200 scenarios. The relative distance between the original tree and reduced trees with fewer scenarios is depicted in Fig. 7. The relative distance is defined as the Kantorovich distance between the original and reduced probability distributions

TABLE V

PRICES OF THE WEEKLY CONTRACTS (€/MWh)

	Week 1	Week 2	Week 3	Week 4
V	44.0	46.2	48.0	49.2
S	57.4	59.7	60.0	61.2
P	66.5	67.2	69.0	69.8
W	45.0	48.6	50.4	51.6

TABLE VI

PRODUCTION COST DATA OF THE COGENERATION UNIT

P_1^{G} (MWh)	P_2^{G} (MWh)	F_1 (€/MWh)	F_2 (€/MWh)	F_3 (€/MWh)
160	480	33	36	39

divided by the Kantorovich distance between a 1-scenario tree and the original distribution, where the distance between two trees is calculated with (6). As can be observed, a tree with 200 scenarios is an appropriate choice. This result is corroborated by Fig. 8, which shows the evolution of the optimal expected cost with the number of scenarios for $\beta = 0$ and $\beta = 5$. Similar curves have been obtained for other values of the risk parameter β. For all the simulations, a confidence level of $\alpha = 0.95$ has been used in the calculation of CVaR.

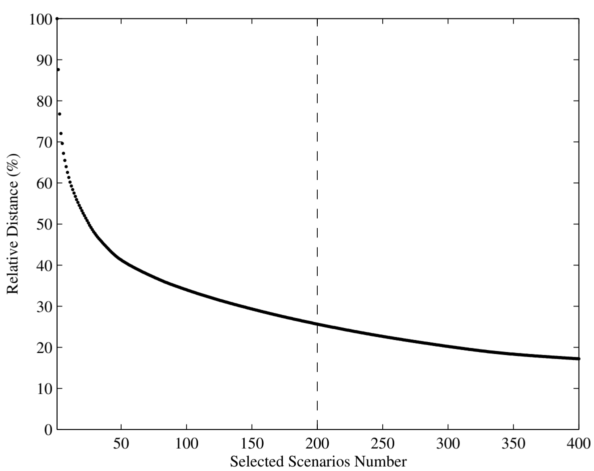

Fig. 7. Relative distance between the original tree and reduced trees.

The resulting problem, characterized by 326288 constraints, 236602 real variables, and 1200 binary variables, has been solved for different values of the weighting factor β using CPLEX 9.0 under GAMS [20]. Fig. 9 provides the efficient frontier, i.e., the plot of expected cost versus CVaR for different values of β. The expected cost ranges from 6.55 million € if risk is ignored ($\beta = 0$), to 6.68 million € if risk is accounted for ($\beta = 5$). The CPU time required to solve problem (7)-(24) for different values of β with a Dell PowerEdge 6600 with 2 processors at 1.60 GHz and 2 GB of RAM memory was less than 130 seconds.

Fig. 10 illustrates the energy procurement of the consumer and the use of bilateral contracts for different values of β. The energy represented in each sector of the pie charts is the weighted average over all of the scenarios. As expected, the

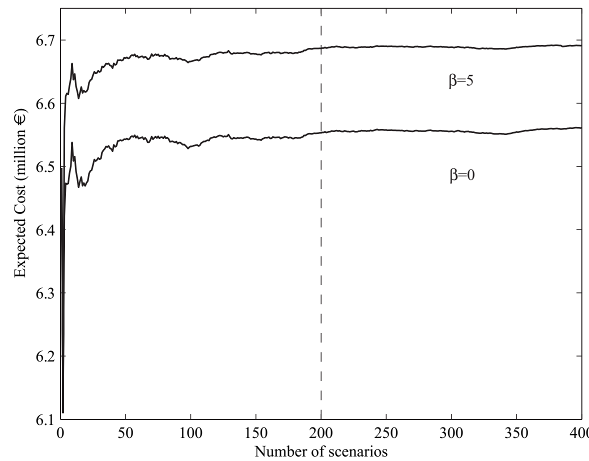

Fig. 8. Expected cost as a function of the number of scenarios considered.

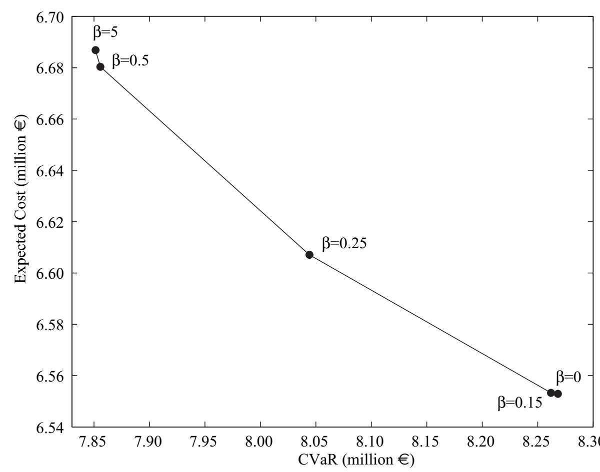

Fig. 9. Expected cost versus CVaR.

share of bilateral contracts considerably increases with β in order to hedge against risk exposure to pool prices volatility. With $\beta = 0$ only weekly contracts for weeks 3 and 4 are signed representing a 1.1% of the total energy consumed. For $\beta = 0.25$, 14.5% of the energy consumed is procured through bilateral contracts, mostly from monthly contract 1. Finally, for $\beta = 5$ the volume of energy purchased from contracts rises up to 28.2%, making it necessary to sign monthly contract 2 and weekly contract 2. Note that weekly contract 1 is not signed due to its high price. It is also remarkable that the volume of energy self-produced by the consumer keeps at a stable share around 20% and experiences a slight drop with β due to the relatively high minimum consumption limits of bilateral contracts.

V. CONCLUSIONS

This paper provides a methodology for energy procurement of a large consumer based on stochastic programming. Risk aversion is modeled through the CVaR technique. Scenarios

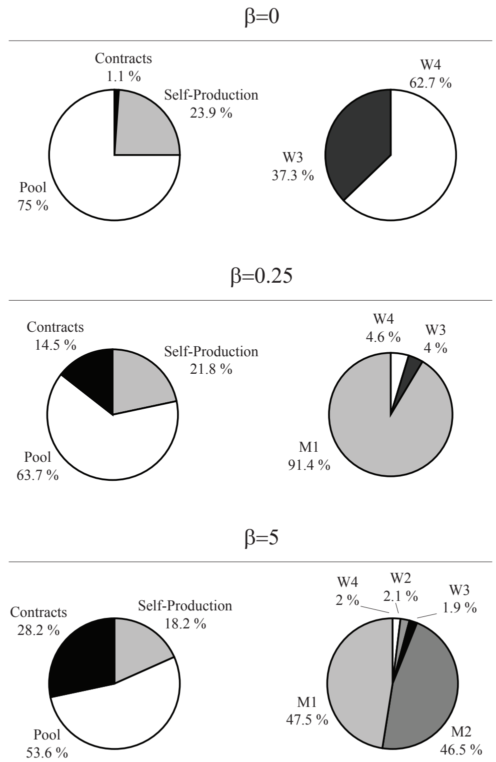

Fig. 10. Electricity procurement and use of contracts.

are reduced through an appropriate algorithm to make tractable the resulting mixed-integer linear programming problem that is solved using a commercial branch-and-cut solver. The validity of this methodology is assessed through a realistic case study. Its usefulness is particularly clear to resolve the tradeoff of minimum expected cost versus maximum risk.

Appendix

The objective of the scenario reduction algorithm is to find a new discrete probability distribution, Q', that minimizes the Kantorovich distance to the original probability distribution Q, as formulated in (6). The resulting Q' is supported comprising a subset of scenarios from the original set. This subset (of "labeled" scenarios) is constructed recursively by starting with the single scenario that is the closest in total Kantorovich distance to all the others, labeling it, and then labeling the next single scenario that minimizes the total Kantorovich distance from unlabeled scenarios to it and other labeled scenarios.

The algorithm proceeds as follows [19]:

Step 0)

Set

$$\Omega_0^* = \Omega$$

Step 1)

Compute

$$d_{\omega,\omega'}^{[1]} = c(\omega,\omega'), \quad \forall \omega,\omega' \in \Omega_0^*$$

$$D_\omega^{[1]} = \sum_{\substack{\omega' \in \Omega_0^* \\ \omega' \neq \omega}} \pi(\omega') d_{\omega,\omega'}^{[1]}, \quad \forall \omega \in \Omega_0^*$$

Set

$$\omega^{[1]} : \omega^{[1]} \in \arg\min_{\omega \in \Omega_0^*} D_\omega^{[1]}$$

$$\Omega_1^* = \Omega_0^* - \{\omega^{[1]}\}$$

Step ν)

Compute

$$d_{\omega,\omega'}^{[\nu]} = \min\Big\{ d_{\omega,\omega'}^{[\nu-1]}, d_{\omega,\omega^{[\nu-1]}}^{[\nu-1]} \Big\} \quad \forall \omega,\omega' \in \Omega_{\nu-1}^*$$

$$D_\omega^{[\nu]} = \sum_{\substack{\omega' \in \Omega_{\nu-1}^* \\ \omega' \neq \omega}} \pi(\omega') d_{\omega,\omega'}^{[\nu]}, \quad \forall \omega \in \Omega_{\nu-1}^*$$

Set

$$\omega^{[\nu]} : \omega^{[\nu]} \in \arg\min_{\omega \in \Omega_{\nu-1}^*} D_\omega^{[\nu]}$$

$$\Omega_\nu^* = \Omega_{\nu-1}^* - \{\omega^{[\nu]}\}$$

Step $n_{\Omega'}$+1)

Compute the probabilities for the preserved scenarios:

$$\overline{\pi}(\omega) = \pi(\omega) + \sum_{\omega' \in \Omega(\omega)} \pi(\omega'), \quad \omega \in \Omega \setminus \Omega^*$$

where

$$\Omega(\omega) = \Big\{ \omega' \in \Omega^* : \ \omega = j(\omega') \Big\}, \quad \forall \omega = \Omega \setminus \Omega^*$$

$$j(\omega') \in \arg\min_{\omega \notin \Omega^*} c(\omega,\omega'), \quad \forall \omega' \in \Omega^*$$

In iteration ν, the algorithm selects the scenario $\omega^{[\nu]}$ which minimizes the distance $D_\omega^{[\nu]}$ between the two probability distributions Q and Q'. The reduced probability distribution Q' is formed by $n_{\Omega'}$ scenarios, whose probabilities are computed in step $n_{\Omega'}$+1. The probability of each preserved scenario $\overline{\pi}(\omega), \forall \omega \in \Omega \setminus \Omega^*$, is equal to the sum of its probability in Q and the probabilities of deleted scenarios that are closest to it.

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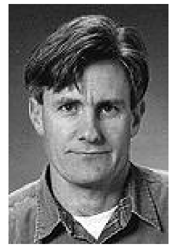

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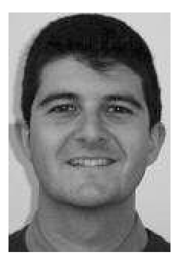

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