Taxation and supply-function equilibrium

Andy Philpott
Electric Power Optimization Centre
www.epoc.org.nz

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Abstract

We consider the effect that a tax on observed profits has on supplier strategies in supply-function equilibrium. In some circumstances such a tax can make supply offers more competitive, decrease prices, and give greater efficiency.

1 Introduction

The supply function equilibrium (SFE) is a natural concept to use in studying electricity pool markets. Here generators submit supply schedules in the form of increasing offer curves to an auction that dispatches generation to those suppliers offering at the lowest prices. The first use of SFE in this context was by Green and Newbery [6] in a study of the England and Wales electricity pool. The paper [6] was based on the theory of supply function equilibrium laid out by Klemperer and Meyer [11]. This theory has been extended to inelastic demand and pay-as-bid auctions by several authors, notably [1],[2],[3],[4],[5],[7],[12],[14]. As well as [6], a number of authors have applied SFE in a practical setting. For example [10] and [13] use the SFE model to study generator behaviour in the Texas electricity pool. The recent survey paper by Holmberg and Newbery [9] gives a good overview of the state of the art in SFE models.

In this paper we examine a supply-function equilibrium model in circumstances where suppliers are taxed on their profits. At first sight, paying a (non-progressive) tax that is a fixed proportion of profits will not alter agent behaviour. Each supplier will still want to maximize after-tax profit, which will be achieved by maximizing profit before tax. The profit in any market outcome can be estimated by computing the difference between clearing price and marginal cost at each level of production and integrating over all production levels below the cleared level. Taxation can be used as a mechanism to redistribute wealth, or to extract payment for assets (like transmission lines) that are shared between market participants. A tax on profits can be seen as an approximation of a model in which the beneficiaries of transmission investments contribute to their cost in proportion to the benefits (profit) accrued.

In some settings, like electricity markets, suppliers offer supply curves to the market that in equilibrium are marked up over marginal cost. Here the offer curves are revealed to the market auctioneer, but the true marginal costs are private information. In such circumstances the auctioneer can estimate supplier profits based on the difference between clearing price and offered price, and tax this *observed profit* at a fixed proportion. Since the offer curve is a choice of the supplier, it can affect the observed profit by its offer, without having too much effect on its actual profit. In the simplest case where demand is certain, a supplier might increase the price on *infra-marginal* units, i.e. units that have offered below the clearing price, and make observed profit very small without affecting the actual profit.

When demand is uncertain, increasing offer prices on inframarginal units must be done carefully since higher offer prices might decrease the probability of being dispatched. In this setting one can use the theory of market distribution functions to derive a SFE that illustrates the incentive to increase prices on inframarginal offers. This analysis draws heavily on the market distribution theory of both uniform-price auctions [3] and pay-as-bid auctions [2]. Our model shows that increasing taxes on suppliers can make them more competitive and reduce deadweight losses that arise from the exercise of market power.

The paper is laid out as follows. In the next section we review supply function equilibrium through the lens of market distribution functions. This is used to derive optimality conditions for suppliers who are taxed a fixed proportion of their observed profit. We show that pure-strategy SFE can be computed as long as the tax is not too high. In the extreme case where the tax is 100%, it is easy to see that the payment mechanism becomes a pay-as-bid scheme. It is well known [2] that pure-strategy SFE occur very rarely in these auctions and mixed strategies prevail. Section 3 deals with the conditions for supply function equilibrium in symmetric duopoly when demand is inelastic and additive demand shocks have a uniform distribution. In section 4 we repeat the analysis for a symmetric duopoly facing a linear demand curve with a uniform demand shock. The last section concludes the paper.

2 Supply function equilibrium

As shown in [3] the optimal offer curve $p(q)$ for a generator with cost $C(q)$ facing a market distribution function $\psi(q,p)$ will maximize

$$\Pi = \int (qp - C(q))d\psi(q,p).$$

The market distribution function $\psi(q,p)$ defines the probability that a supplier is not fully dispatched if they offer the quantity q at price p. It can be interpreted as the measure of residual demand curves that pass below and to the left of the point (q,p). Suppose we treat p as function of q. Then

$$\Pi = \int_0^{q_m} (qp(q) - C(q)) \left(\frac{\partial \psi(q,p)}{\partial p} p'(q) + \frac{\partial \psi(q,p)}{\partial q} \right) dq$$

The Euler-Lagrange equation that $p(q)$ must satisfy to minimize a general functional

$$\int_0^{q_m} H(q,p,p')dq$$

is

$$Z(q,p) = \frac{d}{dq} H_{p'} - H_p = 0.$$

In the case where the functional is Π we obtain

$$H_{p'} = (qp(q) - C(q)) \frac{\partial \psi(q,p)}{\partial p}$$

$$H_p = q \left(\frac{\partial \psi(q,p)}{\partial p} p'(q) + \frac{\partial \psi(q,p)}{\partial q} \right) + (qp(q) - C(q))(\psi_{pp} p'(q) + \psi_{qp}),$$

and

$$\frac{d}{dq} H_{p'} = (p + qp'(q) - C'(q)) \frac{\partial \psi(q,p)}{\partial p} + (qp(q) - C(q))(\psi_{pp} p'(q) + \psi_{qp}).$$

This gives

$$\frac{d}{dq} H_{p'} - H_p = (p - C'(q)) \frac{\partial \psi(q,p)}{\partial p} - q \frac{\partial \psi(q,p)}{\partial q}$$

which can be identified with the standard Z function of [3].

Suppose that some fraction $\lambda \in (0,1)$ of the profit earned by a generator is paid as tax. When the market clears at quantity q for a generator at price π then the generator receives

$$\begin{aligned} & q\pi - C(q) - \lambda \int_0^q (\pi - p(t))dt \\ = \; & q\pi - C(q) - \lambda q\pi + \lambda \int_0^q p(t))dt = (1-\lambda)(q\pi - C(q)) + \lambda(\int_0^q p(t)dt - C(q)). \end{aligned}$$

This is a convex combination of uniform and pay-as-bid pricing with multiplier λ. Thus the total payoff will be

$$\Pi = (1-\lambda)\int (qp - C(q))d\psi(q,p) + \lambda \int (p - C'(q))\,(1-\psi(q,p))dq.$$

We can write down the optimality conditions for the problem faced by a generator maximizing Π. These use the scalar field defined by $Z(q,p) = \frac{d}{dq}H_{p'} - H_p$. Thus

$$\begin{aligned} Z(q,p) &= (1-\lambda)((p - C'(q))\psi_p - q\psi_q) - \lambda(1-\psi(q,p) - (p - C'\left(q\right))\psi_p) \\ &= (p - C'(q))\psi_p - (1-\lambda)q\psi_q - \lambda(1-\psi(q,p)) \end{aligned}$$

The first-order conditions are given by $Z(q,p) = 0$ and global optimality is guaranteed for a monotonic solution to $Z(q,p) = 0$ if $\frac{\partial}{\partial q}Z(q,p) \leq 0$.

3 Symmetric duopoly for inelastic demand

We use the optimality conditions to look for an equilibrium in symmetric duopoly. Suppose the other player offers a smooth supply function $S(p)$, and demand has cumulative probability distribution function F. Then

$$\begin{aligned} \psi(q,p) &= \Pr[h < q + S(p)] \\ &= F(q + S(p)) \end{aligned}$$

and

$$\begin{aligned} Z(q,p) &= (p - C'(q))\psi_p - (1-\lambda)q\psi_q - \lambda(1-\psi(q,p)) \\ &= (p - C'(q))S'(p)f(q+S(p)) - (1-\lambda)qf(q+S(p)) - \lambda(1-F(q+S(p))) \end{aligned}$$

Thus $Z = 0$ is equivalent to

$$(p - C'(q))S'(p) = (1-\lambda)q + \lambda\frac{(1-F(q+S(p))}{f(q+S(p))}. \tag{1}$$

The global optimality conditions (see [2]) are

$$\begin{aligned} (p - C'(q))S'(p) - (1-\lambda)q - \lambda\tfrac{(1-F(q+S(p))}{f(q+S(p))} &\geq 0, \quad q < S(p) \\ (p - C'(q))S'(p) - (1-\lambda)q - \lambda\tfrac{(1-F(q+S(p))}{f(q+S(p))} &= 0, \quad q = S(p) \\ (p - C'(q))S'(p) - (1-\lambda)q - \lambda\tfrac{(1-F(q+S(p))}{f(q+S(p))} &\leq 0, \quad q > S(p) \end{aligned}$$

These can be guaranteed by $\frac{\partial}{\partial q}Z(q,p) \leq 0$ which amounts to

$$-C''(q)S'(p) - (1-\lambda) - \lambda\left[\frac{(1-F(q+S(p))}{f(q+S(p))}\right]_q \leq 0.$$

3.1 Examples

Suppose $F(t) = t$ is uniform on $[0, 1]$ and $\lambda \leq \frac{1}{2}$. Assume a symmetric duopoly where each generator has capacity $\frac{1}{2}$. Then $C'''(q)S'(p) \geq 2\lambda - 1$ guarantees that

$$\begin{aligned} -C''(q)S'(p) - (1-\lambda) - \lambda \left[\frac{(1 - F(q + S(p))}{f(q + S(p))} \right]_q &= -C''(q)S'(p) - (1-\lambda) + \lambda \\ &\leq 0 \end{aligned}$$

The first order condition is then enough to give a supply-function equilibrium. Replacing q by $S(p)$ in (1) yields

$$pS'(p) - (1-\lambda)S(p) - \lambda(1 - 2S(p)) = 0$$

This differential equation can be solved using an integrating factor, whereby

$$pS'(p) + (3\lambda - 1)S(p) = \lambda$$
$$p^{3\lambda-1}S'(p) + (3\lambda - 1)p^{3\lambda-2}S(p) = \lambda p^{3\lambda-2}$$
$$\left(p^{3\lambda-1}S(p)\right)' = \lambda p^{3\lambda-2}$$

$$\begin{aligned} S(p) &= ap^{1-3\lambda} + p^{1-3\lambda}\frac{\lambda}{3\lambda - 1}p^{3\lambda-1} \\ S(p) &= \frac{\lambda}{3\lambda - 1} + ap^{1-3\lambda} \end{aligned}$$

A unique equilibrium can be found by imposing a price cap P at which both generators offer their capacity [7].

Example 1: Suppose $\lambda = \frac{1}{2}$ and $P = 4$, and each generator has capacity $\frac{1}{2}$. Then

$$S(p) = \frac{\lambda}{3\lambda - 1} + ap^{1-3\lambda}$$
$$\left(\frac{a}{q-1}\right)^2 = p$$

To pass through $(\frac{1}{2}, 4)$ we choose $a = 1$.

Example 2: Suppose $\lambda = \frac{1}{4}$ and $P = 4$, and each generator has capacity $\frac{1}{2}$. Then the solution through $(\frac{1}{2}, 4)$ is

$$q = -1 + \frac{3}{2\sqrt{2}}p^{\frac{1}{4}}$$
$$p = \frac{64}{81}(1 + q)^4$$

These solutions are shown in Figure 1.

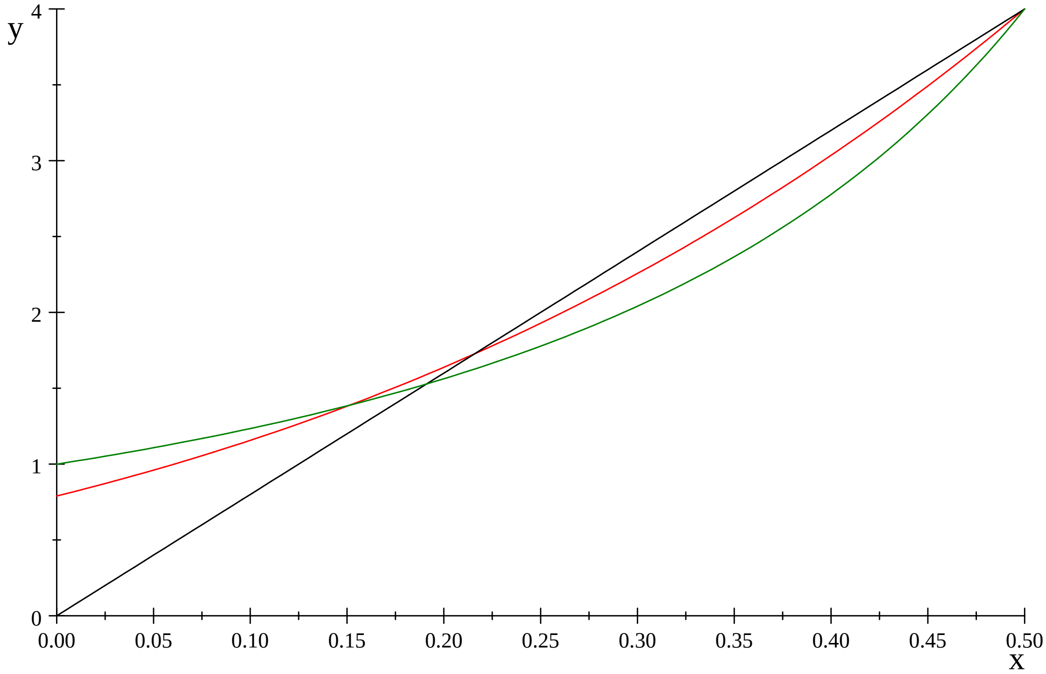


Figure 1: Plot of supply-function equilibrium solutions in a duopoly when inelastic demand is uniformly distributed on [0,1] and each player has capacity 0.5. The red curve is an equilibrium when each supplier must pay 25% of his surplus (above the red curve) as a tax. The green curve gives the equilibrium offer for a 33% tax.

3.2 Welfare calculations.

We can compute the changes in welfare of each agent from the change in equilibrium. Consider first the case where $\lambda = 0$, and there is a price cap at 4. In perfect competition each generator would offer at price zero and earn no profit. The welfare of the consumers in demand realization h is $4h$, and so the expected competitive total welfare is

$$W = \int_0^1 4h dh = 2$$

In a supply function equilibrium, both players offer linear supply functions $q = \frac{p}{8}$. Thus the total supply is $S(p) = \frac{p}{4}$. When demand is h the clearing price is $4h$, so the total generator profit assuming zero marginal cost is $4h^2$. The expected total generator profit is then

$$\begin{aligned} G &= \int_0^1 4h^2 dh \\ &= \frac{4}{3} \end{aligned}$$

The consumer welfare (assuming a maximum valuation of 4), is

$$\begin{aligned} C &= \int_0^1 h(4 - 4h) dh \\ &= \frac{2}{3} \end{aligned}$$

Expected total welfare of 2 is divided $\frac{2}{3}$ to the demand and $\frac{2}{3}$ to each generator. The *observed* profit of each generator in demand realization h is defined by the area above their curve, namely the integral of the clearing price at their dispatch $\frac{h}{2}$ minus the offered price at quantity q from $q = 0$ to $\frac{h}{2}$.

$$\begin{aligned} G(h) &= \int_0^{\frac{h}{2}} \left(8\frac{h}{2} - 8q \right) dq \\ &= h^2 \end{aligned}$$

The expected observed profit for both suppliers is then $2 \int_0^1 h^2 dh = \frac{2}{3}$, or half their actual profit.

Suppose we now impose a tax by choosing $\lambda = \frac{1}{4}$. Then in equilibrium, each generator offers

$$q = -1 + \frac{3}{2\sqrt{2}} p^{\frac{1}{4}} \tag{2}$$

or

$$p = \frac{64}{81}(1+q)^4. \tag{3}$$

The (before-tax) *observed* profit in demand realization h of each generator if they offer this curve is defined by the area above their curve, namely the integral of the clearing price at their dispatch $\frac{h}{2}$ minus the offered price at quantity q from $q = 0$ to $\frac{h}{2}$. This gives

$$\begin{aligned} G(h) &= \int_0^{\frac{h}{2}} \left(\frac{64}{81}(1+\frac{h}{2})^4 - \frac{64}{81}(1+q)^4 \right) dq \\ &= \frac{4}{405} h^2 \left(2h^3 + 15h^2 + 40h + 40 \right) \end{aligned}$$

The (before-tax) expected observed profit for both suppliers is then

$$2 \int_0^1 \frac{4}{405} h^2 \left(2h^3 + 15h^2 + 40h + 40 \right) dh = \frac{128}{243} = 0.527 < \frac{2}{3}$$

which is the total observed profit under a linear supply curve offer. The new offer is arranged to reduce the tax while maintaining a healthy profit. The total tax paid is then $\lambda \frac{128}{243} = \frac{32}{243} < \frac{1}{6}$, the tax collected when linear curves are offered.

Since their costs are zero, the before-tax *actual* profit in demand realization h of each generator if they offer the optimal curve is $\frac{h}{2}\frac{64}{81}(1+\frac{h}{2})^4$. The total expected before-tax *actual* profit for both suppliers is then

$$2 \int_0^1 \frac{h}{2}\frac{64}{81}(1+\frac{h}{2})^4 dh = \frac{1586}{1215}$$

The total expected after-tax *actual* profit for both suppliers is

$$T = \frac{1586}{1215} - \frac{32}{243} = \frac{1426}{1215}. \tag{4}$$

Recall that

$$\psi(q,p) = \begin{cases} 0, & q + S(p) \le 0 \\ q + S(p), & 0 < q + S(p) < 1 \\ 1, & q + S(p) \ge 1 \end{cases}$$

so

$$\psi(q, p(q)) = 2q$$

and

$$\begin{aligned} d\psi(q,p) &= \left(\frac{\partial \psi(q,p)}{\partial q} + \frac{\partial \psi(q,p)}{\partial p} \frac{dp(q)}{dq} \right) dq \\ &= (1 + S'(p)p'(q))\, dq \\ &= 2dq \end{aligned}$$

and so the after-tax profit is

$$\begin{aligned}
\Pi &= (1-\lambda)\int (qp - C(q))d\psi(q,p) + \lambda \int (p - C'(q))\,(1-\psi(q,p))dq \\
&= \frac{3}{4}\int (qp - C(q))d\psi(q,p) + \frac{1}{4}\int_0^{\frac{1}{2}} p(q)(1-\psi(q,p))dq \\
&= \frac{3}{4}\int (q\frac{64}{81}(1+q)^4)d\psi(q,p) + \frac{1}{4}\int_0^{\frac{1}{2}} \frac{64}{81}(1+q)^4(1-\psi(q,p))dq \\
&= \frac{3}{4}\int_0^{\frac{1}{2}} (2q\frac{64}{81}(1+q)^4)dq + \frac{1}{4}\int_0^{\frac{1}{2}} \left(\frac{64}{81}(1+q)^4(1-2q)\right) dq \\
&= \frac{713}{1215}
\end{aligned}$$

which is half the figure T we computed in (4) as expected.

The welfare of consumers is slightly improved by the tax. Without the tax, generators offer linear supply functions, and the consumer welfare is $\frac{2}{3}$. When a tax is imposed, the generators change their offers, and the price under demand realization h is $\frac{64}{81}(1+\frac{h}{2})^4$. We can then compute the expected total welfare for consumers as

$$\begin{aligned}
C &= \int_0^1 h(4 - \frac{64}{81}(1+\frac{h}{2})^4)dh \\
&= \frac{844}{1215} > \frac{2}{3}
\end{aligned}$$

The total welfare is then the sum of consumer welfare, generator profit, and tax giving

$$\frac{844}{1215} + \frac{1426}{1215} + \frac{32}{243} = 2.$$

In summary if each generator offers a linear supply curve (as they would in an untaxed equilibrium) then they each earn $\frac{2}{3}$ before tax and $\frac{7}{12}$ after tax, after paying $\frac{1}{12}$ in tax on observed profit of $\frac{1}{3}$. If they instead offer the curve (2) then each generator will appear to earn a profit of $\frac{64}{243}$ but in fact will earn $\frac{793}{1215}$. They will then pay less tax of $\frac{16}{243}$ and each retain a profit of $\frac{793}{1215} - \frac{16}{243} = \frac{713}{1215} > \frac{7}{12}$. The total welfare is 2, and so consumers' welfare increases from $\frac{2}{3}$ to $2-(2) = \frac{844}{1215}$. The total welfare is consumer welfare plus generator welfare plus tax, giving

$$\frac{844}{1215} + (2)\frac{713}{1215} + (2)\frac{16}{243} = 2$$

So the reaction of the suppliers after the imposition of the tax is to offer to improve their welfare and minimize the tax. The effect of this is to transfer some wealth to consumers.

4 Symmetric duopoly for elastic demand

We now consider a model in which demand is elastic and defined by a demand curve $D(p)$. We assume that generators have no capacity constraints. Recall the optimality conditions for a taxed equilibrium are given by

$$Z(q,p) = (p - C'(q))\psi_p - (1-\lambda)q\psi_q - \lambda(1 - \psi(q,p)) = 0$$

Assuming an additive demand shock h with cumulative distribution F, we get

$$\begin{aligned} \psi(q,p) &= \Pr[h < q + S(p) - D(p)] \\ &= F(q + S(p) - D(p)). \end{aligned}$$

Thus

$$\begin{aligned} Z(q,p) &= (p - C'(q))\psi_p - (1-\lambda)q\psi_q - \lambda(1 - \psi(q,p)) \\ &= (p - C'(q))(S'(p) - D'(p)f(q + S(p) - D(p)) \\ &\quad -(1-\lambda)qf(q + S(p) - D(p)) - \lambda(1 - F(q + S(p) - D(p))) \\ &= 0 \end{aligned}$$

gives

$$(p - C'(q))(S'(p) - D'(p)) = (1-\lambda)q + \lambda\frac{1 - F(q + S(p) - D(p))}{f(q + S(p) - D(p))}.$$

The second order conditions are

$$\begin{aligned} (p - C'(q))(S'(p) - D'(p)) - (1-\lambda)q - \lambda\tfrac{1-F(q+S(p)-D(p))}{f(q+S(p)-D(p))} &\geq 0, \quad q < S(p) \\ (p - C'(q))(S'(p) - D'(p)) - (1-\lambda)q - \lambda\tfrac{1-F(q+S(p)-D(p))}{f(q+S(p)-D(p))} &= 0, \quad q = S(p) \\ (p - C'(q))(S'(p) - D'(p)) - (1-\lambda)q - \lambda\tfrac{1-F(q+S(p)-D(p))}{f(q+S(p)-D(p))} &\leq 0, \quad q > S(p) \end{aligned}$$

These can be guaranteed by $\frac{\partial}{\partial q}Z(q,p) \leq 0$ which amounts to

$$-C''(q)(S'(p) - D'(p)) - (1-\lambda) - \lambda\left[\frac{1 - F(q + S(p) - D(p))}{f(q + S(p) - D(p))}\right]_q \leq 0$$

When F is uniform this gives

$$C''(q)S'(p) \geq 2\lambda - 1$$

which is guaranteed by $\lambda \leq \frac{1}{2}$.

4.1 Monopoly response

Recall

$$Z(q,p) = (p - C'(q))\psi_p - (1-\lambda)q\psi_q - \lambda(1-\psi(q,p)).$$

We use this to investigate the monopoly response of a supplier in an example. Suppose $\lambda = \frac{1}{4}$ and $C(q) = 0$, $D(p) = 1-p$, and the cumulative distribution of demand shock t is $F(t) = \frac{h}{2} + \frac{1}{2}$, $h \in [-1,1]$

$$p\psi_p - (1-\lambda)q\psi_q - \lambda(1-\psi(q,p))$$

Let

$$\begin{aligned}\psi(q,p) &= \frac{1}{2}(q + S(p) - D(p) + 1) \\ &= \frac{1}{2}(q+p)\end{aligned}$$

Then

$$\begin{aligned}Z(q,p) &= p\psi_p - (1-\lambda)q\psi_q - \lambda(1-\psi(q,p)) \\ &= \frac{1}{2}p - (1-\lambda)q\frac{1}{2} - \lambda(1 - \frac{q+p}{2}) = 0 \\ p &= \frac{1}{\lambda+1}(q + 2\lambda - 2q\lambda)\end{aligned}$$

If $\lambda = 0$, then we choose $p = q$.

If $\lambda = \frac{1}{4}$, then we choose $p = \frac{2}{5}q + \frac{2}{5}$.Thus $C''(q)(S'(p) - D'(p)) \geq 2\lambda - 1$ guarantees that

$$\begin{aligned}&-C''(q)(S'(p) - D'(p)) - (1-\lambda) - \lambda\left[\frac{1 - F(q + S(p) - D(p))}{f(q + S(p) - D(p))}\right]_q \\ =\ &-C''(q)(S'(p) - D'(p)) - (1-\lambda) + \lambda \\ \leq\ &0\end{aligned}$$

which ensures a global optimium.

4.2 Duopoly

Suppose $C(q) = 0$, $D(p) = 1-p$, with an additive demand shock of h, where the cumulative distribution of h is $F(h) = \frac{h}{2} + \frac{1}{2}$, $h \in [-1,1]$. The equilibrium condition is

$$p(S'(p) - D'(p)) - (1-\lambda)q - \lambda\frac{1 - \frac{q+S(p)-D(p)}{2} - \frac{1}{2}}{\frac{1}{2}} = 0$$

which setting $q = S(p)$ gives

$$\begin{aligned} p(S'(p) - D'(p)) - (1-\lambda)S(p) - \lambda\frac{1-(2S(p)-D(p))}{1} &= 0 \\ p(S'(p) - D'(p)) + (3\lambda - 1)S(p) - \lambda(1 + D(p)) &= 0 \end{aligned}$$

$$\begin{aligned} p(S'(p) - D'(p)) - (1-\lambda)S(p) - \lambda\frac{1-(2S(p)-D(p))}{1} &= 0 \\ p(S'(p) - D'(p)) + (3\lambda - 1)S(p) - \lambda(1 + D(p)) &= 0 \end{aligned}$$

$$pS'(p) + (3\lambda - 1)S(p) = \lambda(1 + D(p)) + pD'(p)$$

This has solution given by

$$p^{3\lambda-1}S'(p) + (3\lambda-1)p^{3\lambda-2}S(p) = \lambda p^{3\lambda-2}(1 + D(p)) + p^{3\lambda-1}D'(p)$$

$$\begin{aligned} \left(p^{3\lambda-1}S(p)\right)' &= \lambda p^{3\lambda-2}(1 + D(p)) + p^{3\lambda-1}D'(p) \\ S(p) &= ap^{1-3\lambda} + p^{1-3\lambda}\int \left(\lambda p^{3\lambda-2}(1 + D(p)) + p^{3\lambda-1}D'(p)\right) dp \end{aligned}$$

Example 1 Suppose $\lambda = 0$ and $D(p) = 1 - p$. This gives the differntial equation

$$\left(p^{-1}S(p)\right)' = +p^{-1}D'(p)$$

which has solution

$$\begin{aligned} p^{-1}S(p) &= A - \ln p \\ S(p) &= Ap - p\ln p \end{aligned}$$

Different choices of A give rise to different candidate equilibrium curves, as shown in Figure 2.

Some of these curves cannot give supply-function equilibria since they become too steep and bend back. This behaviour is admissible as long as it occurs in regions of (q, p) space where there is no likelihood of being dispatched. Observe also that “ironing” curves vertically in this setting will not result in equilibrium unless the suppliers are at capacity, since we show above that the optimal response to a vertical curve is a sloping monopoly supply curve, not a vertical curve.

The least competitive equilibrium will consist of the highest curve in Figure 2 that does not bend back. This is the curve that is vertical at the point where the Cournot line $p = q$ meets the maximum demand curve (shown in Figure 2 as

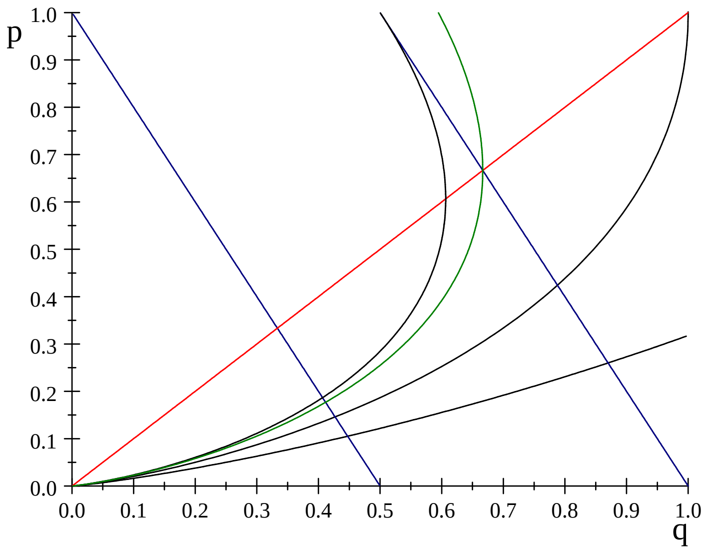


Figure 2: SFE candidates for elastic demand. The least competitive equilibrium passes vertically through the maximum demand where it intersects the Cournot line.

a green curve). Since $D(p) = 2 - p$, we have $p = 2 - 2q$, where q is the dispatch of each generator. This occurs where $p = q = \frac{2}{3}$. This defines a by

$$a\frac{2}{3} - \frac{2}{3}\ln\frac{2}{3} = \frac{2}{3}$$

giving

$$S(p) = (1 + \ln\frac{2}{3})p - p\ln p.$$

Now consider nonzero lambda with $D(p) = 1 - p$. In the region where the generators are dispatched we have

$$p^{3\lambda-1}S'(p) + (3\lambda - 1)p^{3\lambda-2}S(p) = \lambda p^{3\lambda-2}(1 + D(p)) + p^{3\lambda-1}D'(p)$$

$$\begin{aligned}
\left(p^{3\lambda-1}S(p)\right)' &= \lambda p^{3\lambda-2}(1 + D(p)) + p^{3\lambda-1}D'(p) \\
S(p) &= ap^{1-3\lambda} + p^{1-3\lambda}\int\left(\lambda p^{3\lambda-2}(1 + D(p)) + p^{3\lambda-1}D'(p)\right)dp \\
S(p) &= ap^{1-3\lambda} - \frac{1}{3}\frac{\left(6\lambda^2 + p\,(\lambda+1)\,(1-3\lambda)\right)}{\lambda\,(1-3\lambda)}
\end{aligned}$$

The least competitive supply curve will be vertical when it passes through the maximum demand at some price P. Thus we require that P satisfies

$$\begin{aligned} 2S(P) &= 2-P \\ S'(P) &= 0 \end{aligned}$$

This gives

$$(1-3\lambda)aP^{-3\lambda} - \frac{1}{3}\frac{(\lambda+1)}{\lambda} = 0$$

$$P^{-3\lambda} = \frac{1}{3a}\frac{(\lambda+1)}{\lambda(1-3\lambda)}$$

so

$$a = \frac{1}{3}\frac{(\lambda+1)}{\lambda(1-3\lambda)}P^{3\lambda}$$

$$\begin{aligned} 2S(P) &= 2aP^{1-3\lambda} - \frac{2}{3}\frac{\left(6\lambda^2 + P(\lambda+1)(1-3\lambda)\right)}{\lambda(1-3\lambda)} \\ &= \frac{2}{3}\frac{(\lambda+1)}{\lambda(1-3\lambda)}P - \frac{2}{3}\frac{\left(6\lambda^2 + P(\lambda+1)(1-3\lambda)\right)}{\lambda(1-3\lambda)} \\ &= 2-P \end{aligned}$$

This gives

$$P = \frac{2(1-\lambda)}{3-\lambda}$$

and so

$$a = \frac{1}{3}\frac{(\lambda+1)}{\lambda(1-3\lambda)}\left(\frac{2(1-\lambda)}{3-\lambda}\right)^{3\lambda}.$$

The least competitive supply-function equilibrium is then

$$S(p) = \frac{1}{3}\frac{(\lambda+1)}{\lambda(1-3\lambda)}\left(\frac{2(1-\lambda)}{3-\lambda}\right)^{3\lambda}p^{1-3\lambda} - \frac{1}{3}\frac{\left(6\lambda^2 + p(\lambda+1)(1-3\lambda)\right)}{\lambda(1-3\lambda)}.$$

Example 2 Suppose $\lambda = \frac{1}{4}$. The least competitive supply-function equilibrium is

$$\frac{20}{33}\sqrt[4]{33}\sqrt[4]{72}\sqrt[4]{p} - \frac{5}{3}p - 2.$$

This is shown in Figure 3 as a solid curve. The price at which this curve becomes vertical is less than the corresponding price if there is no tax.

One can also see from Figure 3 that the tax has increased the competitiveness at high demand outcomes. This has the effect of reducing peak prices and

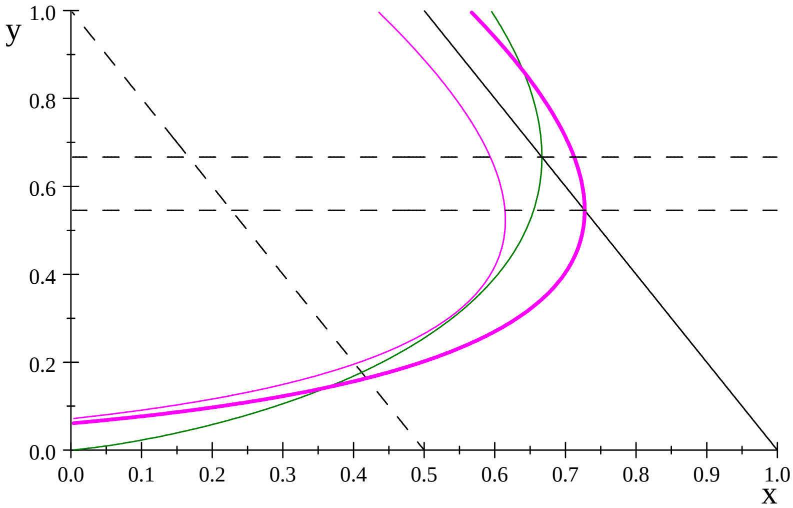


Figure 3: Supply-function equilibria with elastic demand. The green curve is the least competitive equilibrium without a 25% tax. The magenta curves are equilibria with a 25% tax. The solid curve is the least competitive.

increasing offpeak prices. Without a tax, the clearing price under demand shock h is $p(h)$, which solves

$$2S(p) = 1 + h - p$$

$$2(1 + \ln \frac{2}{3})p - 2p \ln p = 1 + h - p$$

If $\lambda = \frac{1}{4}$ then $P(h)$ solves

$$2\left(\frac{20}{33}\sqrt[4]{33}\sqrt[4]{72}\sqrt[4]{p} - \frac{5}{3}p - 2\right) = 1 + h - p,$$

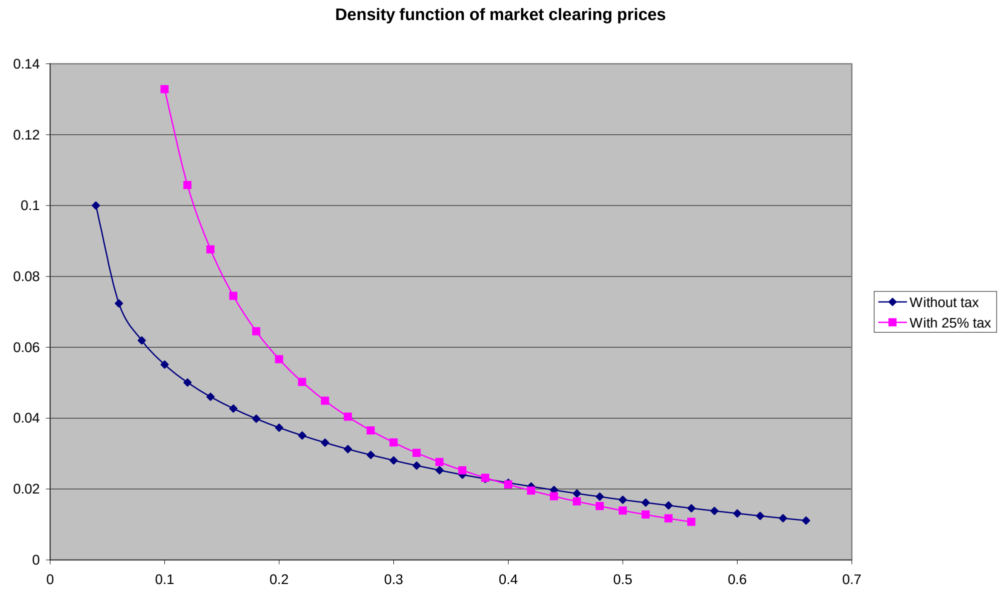
so the least competitive supply-function equilibrium is:

$$S(p) = \frac{20}{33}\sqrt[4]{33}\sqrt[4]{72}\sqrt[4]{p} - \frac{5}{3}p - 2$$

Let us compare this with the least competitive equilibrium when $\lambda = 0$. This is

$$S(p) = (1 + \ln \frac{2}{3})p - p \ln p$$

It is difficult to compare these analytically. When $\lambda = \frac{1}{4}$ we can compare the distribution of clearing prices under each equilibrium.



Probability density function of market equilibrium clearing prices with and without a 25% tax on observed benefits.

One can see that the tax decreases the range of price outcomes, and increases the probability of low prices, except possibly for very low demand outcomes. The consumer welfare in demand realization h is the area under the demand curve above the clearing price $p(h)$. This is

$$W(h) = \frac{(1 + h - p(h))^2}{2}$$

The expected consumer welfare is

$$W = \int_{-1}^{1} W(h) \frac{1}{2} dh.$$

When the demand shock realization is h, the demand met by each supplier is

$$\frac{1 + h - p(h)}{2}.$$

Since the cost of supply is zero, the actual profit of each supplier when the demand shock realization is h is then

$$\Pi(h) = p(h) \frac{1 + h - p(h)}{2}$$

The expected welfare of each supplier is then

$$\Pi = \int_{-1}^{1} \Pi(h) \frac{1}{2} dh.$$

Since the actual cost of generation is zero, the perfectly competitive solution would meet all demand at zero price 0. Thus the deadweight loss in any demand shock realization h is the area of the triangle to the right and below the dispatch point. This has height $p(h)$ and base $p(h)$. The expected deadweight loss is then

$$L = \int_{-1}^{1} \frac{p(h)^2}{2} \frac{1}{2} dh.$$

Total welfare is

$$T = \int_{-1}^{1} \frac{(1+h)^2}{2} . \frac{1}{2} dh = \frac{2}{3}.$$

We can compute expected welfare values numerically in the two cases where $\lambda = 0$ and $\lambda = \frac{1}{4}$. Numerical approximations of these values are shown in Figure 4.

Tax level	0	25%
Suppliers gross profit	0.239	0.216
Consumer welfare	0.382	0.427
Deadweight loss	0.040	0.029

Figure 4: Results for SFE with and without a 25% tax. The taxation collected is not counted in these figures. Both columns add up to a total welfare and deadweight loss before tax is deducted of $\frac{2}{3}$.

The expected gross profit for both suppliers in this example decreases from 0.239 without the tax to 0.216 with the tax at 25%.

5 Conclusions

The supply-function equilibrium models outlined in this paper show that taxes imposed on electricity generators do not necessarily lead to less competitive outcomes. It is interesting to speculate whether these results remain true for increases in the number of players, asymmetry in suppliers, and contracting. Since they lead to lower overall welfare for suppliers, one might conjecture that some recovery of these losses will be achieved possibly by some out-of-market mechanism. However in equilibrium it does not appear to be optimal to markup offer curves to recover lost profits from avoiding tax.

One motivation for this paper is the "beneficiary-pays" transmission charging regime, in which the increase in benefits from a transmission asset are estimated by measuring the difference in observed profits with and without the asset.

Beneficairies are then charged in proportion to this increase in benefits. The tax we investigate in this paper has some similarities with the "beneficiary-pays" transmission charging regime, and we conjecture that the incentives on supplier behaviour are likely to be the same, namely a flattening of optimal supply curves to decrease the observed benefits.

It is possible that a more direct analysis of the "beneficiary-pays" transmission charging regime can be carried out. The simplest case of this would involve the computation of a supply-function equilibrium in a two-node network, with varying line capacity, and a tax imposed on the increase in welfare that extra line capacity confers on the suppliers assuming that their offer does not change in the low-capacity case. Transmission constraints substantially complicate the calculation of supply-function equilibria (see [8], [14]) and so even in this simple case one can imagine that results are not straightforward to obtain.

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