Carbon charges in electricity systems may increase emissions.

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Abstract

We examine the effect of introducing a carbon charge on electricity generation. We model this by way of a two generator Cournot game over a two node electricity network. We find that within the electricity system, emissions of carbon dioxide can increase after a carbon charge is introduced.

Keywords: electricity market, Cournot, strategic behaviour, equilibrium, climate policy, carbon tax.
1 Introduction

Over the past decade, the climate change debate has moved away from whether global warming has been caused by anthropogenic CO$_2$ emissions, and is now focusing on the design of policy instruments to reduce such emissions. The favoured options are to either introduce a tax on activities that generate emissions, or to form an emissions trading (cap-and-trade) scheme whereby emitters buy credits from those who offset their emissions, as described by Green et al. (2007). In a cap-and-trade scheme, one is guaranteed (or at least hopes for) a reduction of emissions to the cap. Whereas with a tax, one expects emissions to be reduced, as they incur higher costs, as discussed by Metcalf (2007). Another possible carbon reduction policy is the introduction of carbon performance standards (see Beamon et al. (2001)), whereby maximum emissions levels are specified for new generation investments.

In this paper, we show that if one confines attention to the electricity system in isolation, then CO$_2$ charges (either from a tax or from the need to buy permits from outside the system) may lead to an increase in CO$_2$ emissions by the electricity generators.

It has been demonstrated that the reduction of market power can lead to an increase in negative social or environmental outcomes (see for example
Lipsey and Lancaster (1957), Gopinath and Wu (1999) and Fullerton and Metcalf (2002)). Moreover, Buchanan (1969) shows, in a market with a monopolist firm, that in the absence of pollution, increasing the output of the firm increases social welfare; however, when pollution is a concern, it may be beneficial to reduce output. Thus it is unclear a priori whether a tax on pollution will increase or decrease welfare.

The counter-intuitive result that we discuss in this paper occurs because, rather than solely discouraging fossil-fuel use, the imposition of a carbon charge can also reduce market power. In fact, as we will demonstrate, the reduction of market power may be the dominant effect, leading to an increase in emissions. Although our example is somewhat contrived, it highlights the need for care to be taken when predicting outcomes of such policy instruments when they are applied to electricity systems.

We model the electricity market as Cournot game with linear demand functions. In a Cournot game, each player (generator) has some degree of market power; this assumption may not be valid in electricity markets with many generators behaving competitively. However, in small electricity markets with few generators, for example New Zealand, some degree of market power has been observed (Wolak 2009). Also note that Cournot models of electricity markets have some potential drawbacks relating to existence and uniqueness of equilibria as discussed by Borenstein et al. (2000). However due to their relative tractability over other models (for example supply function models) they are still widely used in modelling strategic behaviour in
electricity markets, (see for example Metzler et al. (2003), Neuhoff et al. (2005)).

2 Model

A diagram of the electricity system that we consider is shown in Figure 1. Here, there are two nodes connected by a line of capacity $K$. At each node, $i$, there is a generator, with a constant marginal fuel cost, $c_i$, injecting a quantity of electricity, $q_i$. The nodal prices are calculated from inverse demand curves of the form: $p_i = a_i - b_i y_i$, whereby $a_i$ and $b_i$ are positive constants and $y_i$ is the satisfied demand at node $i$. We shall assume that generator 1 is a coal-fired power plant and generator 2 is a gas-turbine plant, and their emissions per megawatt are $\gamma_1$ and $\gamma_2$ tonnes of CO$_2$ respectively. The tax charged to generators is $\$\alpha$ per tonne of CO$_2$ emitted. This means that the tax that generator $i$ would pay is $\$\alpha \gamma_i$ per megawatt. We model the system as a one-shot Cournot game in which the generators are assumed to act with full rationality, maximizing their profits.

![Figure 1: Two node electricity network.](image)

| $c_1$/MW | $q_1$ | $|f| \leq K$ | $q_2$ | $c_2$/MW |
|-----------|-------|----------------|-------|----------|
| $y_1$     |       |                | $y_2$ |          |
In our model the optimal dispatch of electricity is determined by a system
operator (ISO) who determines the prices and power flow on the line, with
the objective to maximize total welfare. The formulation for this problem is
shown below:

$$\max \quad a_1 y_1 - \frac{1}{2} b_1 y_1^2 + a_2 y_2 - \frac{1}{2} b_2 y_2^2$$

s.t.  \quad y_1 + f = q_1 \\
\quad y_2 - f = q_2 \\
\quad -K \leq f \leq K \tag{1}$$

The game proceeds as follows: in the first stage all generators simulta-
neously commit to a generation level for a given period; in the second stage
the system operator solves the dispatch problem (above) to determine line
flows and demand levels (and hence nodal prices) that maximize total wel-
fare. Since this is a two stage game the generators are able to anticipate
the prices chosen by the system operator in the second stage. The ge nera-
tors here are said to possess full-rationality, since they can anticipate how
their actions affect congestion in the network; whereas in bounded-rationality
models (see (Yao, Oren, and Adler 2008)), the system operator problem is
solved at the same time the generators make their generation decisions. This
changes the structure of the game considerably, allowing it to be solved as a
convex quadratic program.

Because we are particularly interested in the effect that the presence
of transmission has on the impact of carbon charges on the strategic be-
haviour of generators, we employ the full rationality paradigm in our model. Transmission constraints in the system operator’s dispatch problem lead to generators facing non-concave profit maximization problems. For example generator 1’s profit optimization problem is as follows:

$$\max q_1 (p_1 - c_1 - \alpha \gamma_1)$$

s.t. $$q_1 \geq 0,$$

where $$p_1$$ is determined by the dispatch problem in (1).

In the next section we investigate an example of a two-node network, both before and after a carbon charge is applied.

3 Example

Now consider an example in which $$K = 125$$MW and the inverse demand curves at the nodes are, $$p_1 = 125 - \frac{5}{16}y_1$$ and $$p_2 = 250 - \frac{1}{2}y_2$$, respectively. We have a coal plant at node 1 with a marginal fuel cost of $40 / MWh, whereas at node 2 we have a gas plant with a marginal fuel cost of $50 /MWh. The rate of emissions of two plants are $$\gamma_1 = 1.0$$ and $$\gamma_2 = 0.4$$, respectively; this corresponds to a higher CO2 charge per megawatt on the coal plant. In this example we will illustrate a possible effect of a carbon tax on Cournot equilibrium solutions, by considering two situations: one without a carbon
tax ($\alpha = 0$), and one with a carbon tax ($\alpha = 26$).

We will first consider the case where there is no CO$_2$ charge. The best response correspondences (see Vives (1999), and the Appendix) of the two plants are shown in Figure 2 below.

![Figure 2: Best response correspondences: $\alpha = 0$.](image)

In this case, at equilibrium, the gas plant has an incentive to withhold electricity, rather than try to compete with the less expensive coal plant, because the marginal cost of gas is higher than the cost of coal. This leads to an equilibrium where both generators act as local monopolists, with the line congested towards node 2. The equilibrium result is shown in Figure 3. The nodal prices are $p_1 = $102.03 and $p_2 = $118.75; these prices differ due to the line congestion. (Note that if the capacity of the line had been infinitely large then a duopoly equilibrium would result with a price at both nodes of $87.69.)
However, if the generators are subject to a CO$_2$ charge, the effective marginal cost of generation for the gas plant is now lower than that of the coal plant. From the updated best response correspondences in Figure 3, we can see that the equilibrium has shifted. In fact, at the new equilibrium, the line is no longer congested; this is because the coal plant, facing significantly higher costs, reduces its generation, alleviating congestion on the line, which gives incentive for the gas plant to increase its generation.

The updated equilibrium details are given in Figure 4 below. Note that since there is no congestion, the prices at both nodes are the same: $p_1 = p_2 =$
$99.83; this equilibrium is equivalent to a single node duopoly equilibrium.

Figure 4: With carbon charge, $\alpha = 26$.

Looking at the welfare for these two situations, we see in Table 1 that the producer welfare drops considerably when the carbon tax is applied. This is because the tax has increased their marginal cost. The consumer welfare, however, has increased; this is due to the generators changing from acting as local monopolists to competing in a duopoly, leading to lower electricity prices. The congestion rent has dropped to zero, because the line joining the nodes is no longer congested, after the tax is applied.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 0$</th>
<th>$\alpha = 26$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer Welfare</td>
<td>$21,766$</td>
<td>$14,032$</td>
</tr>
<tr>
<td>Consumer Welfare</td>
<td>$18,071$</td>
<td>$23,566$</td>
</tr>
<tr>
<td>Congestion Rents</td>
<td>$2,090$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Table 1: Comparison of welfare.

Table 2 shows that the total emission of CO$_2$ increases when the carbon charge is applied. The CO$_2$ emissions increase 1.74% at equilibrium when the tax is introduced. The total amount of tax collected in the equilibrium is $6,705.
\begin{tabular}{|c|c|c|}
\hline
 & $\alpha = 0$ & $\alpha = 26$ \\
\hline
CO$_2$ Emissions & 253.5 t & 257.9 t \\
Tax Collected & $0$ & $6,705$ \\
\hline
\end{tabular}

Table 2: Comparison of emissions and tax revenue.

As the carbon tax is just a transfer of wealth from producers to the
government, we will include the collected tax in the total welfare. Table
3 shows that the total welfare has in fact increased after the tax has been
introduced.

\begin{tabular}{|c|c|c|}
\hline
 & $\alpha = 0$ & $\alpha = 26$ \\
\hline
Total Welfare & $41,927$ & $44,304$ \\
\hline
\end{tabular}

Table 3: Total welfare.

We now summarize the result: the application of a tax on emissions of
carbon dioxide has increased generation costs, which predictably decreases
the profits of generators. However, it has unexpectedly lowered electricity
prices, increasing the welfare accruing to consumers, and it has eliminated
congestion in the electricity grid. The most interesting result is that the to-
tal emissions from the generators have increased. The cause of this apparent
paradox is the combination of the strategic behaviour of the generators and
the transmission constraint. Together these conspire to produce two different
Nash equilibria, with the more competitive one arising from a regime of car-
bon charges. In the absence of strategic behaviour, or without a transmission
constraint, this paradox would not occur.
However, one must be careful when interpreting this result. If we consider the welfare associated with the emissions, assuming that the carbon charge is set at the social marginal damages, we can calculate that the increase in emissions of 4.4 tonnes per hour reduces welfare by only $114.4 per hour; this reduction in welfare is less than the increase in welfare from the alleviation of congestion in the network, leading to an overall increase in welfare.

This example highlights the need for care when predicting the effect that CO$_2$ policy instruments will have on electricity markets where transmission constraints may be binding.

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**References**


Borenstein, S., J. Bushnell, and S. Stoft (2000). The competitive effects of transmission capacity in a deregulated electricity industry. *RAND*


Metzler, C., B. Hobbs, and J.-S. Pang (2003, June). Nash-Cournot equilibria in power markets on a linearized DC network with arbitrage:
Formulations and properties. *Networks and Spatial Economics* 3(2), 123–150.


Appendix

Best Response Correspondences

In order to derive the best response correspondences, we first formulate the optimization problem of each generator, as a function of the other generator’s action. For example, the following mathematical program with complementarity constraints (MPCC) is the profit maximization problem for generator 1 as a function of $q_2$ (Ralph and Smeers 2006).

$$
\Pi_1(q_2) := \max_{q_1} \; q_1 \left( p_1 - c_1 - \alpha \gamma_1 \right) \\
\text{s.t.} \quad y_1 + f = q_1 \\
\quad y_2 - f = q_2 \\
\quad a_1 - b_1 y_1 - p_1 = 0 \\
\quad a_2 - b_2 y_2 - p_2 = 0 \\
\quad p_1 - p_2 + \eta_1 - \eta_2 = 0 \\
\quad 0 \leq K - f \perp \eta_1 \geq 0 \\
\quad 0 \leq K + f \perp \eta_2 \geq 0
$$

The constraints of the above problem are the KKT conditions of the dispatch problem (1) (Kuhn 2006); this set of constraints is non-convex, as it contains orthogonality conditions.

The best response correspondence for generator 1 can be generated by parametrically solving the above problem as a function of $q_2$. This same method can then be used to find the best response of generator 2.