A Survey of Utilization of Optimization for Generation in Wholesale Electricity Markets

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February 8, 2010

Abstract

While operations research is utilized across all sectors of wholesale electricity markets, it is most widely and intensely used in the generation sector. We review the operations of a wholesale electricity market and provide a detailed treatment of optimization in the generation sector.

1 Introduction

The first example of energy market concepts and privatization of electric power systems took place in Chile in the early 1980s. The idea behind the Chilean model was to bring rationality and transparency to the operations of the power system that would ultimately be reflected in power prices. Other rationales for the eventuation of electricity markets include better reliability (e.g. in the case of Argentinean electricity market,) and signaling appropriate levels of investment in infrastructure in the energy sector. Today many countries and jurisdictions rely on electricity markets to meet their electricity needs. These include UK, Australia, New Zealand, the Nordpool market consisting of Scandinavian countries, Brazil and other South American countries as well as many jurisdictions in the US.

Electricity markets typically consist of five main sectors: the system operator, generators, consumers, distributors, and regulatory bodies overseeing the regulations and operations of the market. While operations research is utilized in every one of these sectors on a day to day basis, to keep this article to a reasonable length and be informative, we will only discuss the first two sectors. In what follows we will describe the operation of an electricity market. We then discuss each of the generation sector and describe how operations research is utilized within that sector.

2 Market operation and the system operator

Electricity is not a storable commodity. It is injected into a transmission grid at certain nodes of that transmission grid often referred to as grid injection points (GIPs), and flows through the grid complying with physical constraints. Electricity is withdrawn at grid exit points (GXPs) and delivered to consumers. Due to the physical constraints on the flow of electricity, in all electricity markets the dispatch of the generation of electricity is left to an independent system operator (ISO). In most electricity markets, an additional function of the ISO is to determine the price of electricity at different nodes of the transmission network.

Typically in a wholesale electricity market, in each period of the day, each generator offers in generation quantities for each of its plants (possibly located at different GIPs), at certain prices. In its most general form, the generation offers are supply functions (also known as offer curves) denoted $p = S(q)$, where $S(q)$ is the marginal price of producing quantity q. These supply offers are collected by the ISO. The ISO also estimates the demand over that period. The ISO then solves a side constrained network optimization problem where the objective is to minimize the total cost of production of electricity. The constraints of this optimization problem reflect that demand must be met at every node of the network, and that physical flow constraints such as transmission line capacities and Kirchhoff's laws must be complied with. Often reactive power modelling is left out of the ISO's dispatch problem and the problem is in fact a direct current equivalent load flow model [1, 2]. A general model for the ISO's economic dispatch problem (EDP) is formulated below.

$$
\begin{array}{llllll}
\text{EDP:} & \text{minimize} & \sum_i \sum_{m\in\mathcal{O}(i)} \int_0^{q_m} C_m(x)dx & & & \\
& \text{s.t.} & g_i(y) + \sum_{m\in\mathcal{O}(i)} q_m & = \; D_i, & i \in \mathcal{N}, & [\pi_i] \\
& & q_m \in Q_m, \quad m \in \mathcal{O}(i), & & i \in \mathcal{N}, & \\
& & y \in Y. & & &
\end{array}
$$

We use i as the index for the nodes in the transmission grid. We use m as the index for the generators and $\mathcal{O}(i)$ indicates the set of all generators located at node i. Generator m can supply quantity q_m and the demand at node i is denoted by D_i. Q_m indicates the capacity of generator m.

Here the components of vector x measure the dispatch of each generator and the components of the vector y measure the flow of power in each transmission line. We denote the flow in the directed line from i to k by y_{ik}, where by convention we assume $i < k$. (A negative value of y_{ik} denotes flow in the direction from k to i.) It is required that this vector lies in the convex set Y, which means that each component satisfies the thermal limits on each line, and satisfies loop flow constraints that are required by Kirchhoff's Law. The function $g_i(y)$ defines the amount of power arriving at node i for a given choice of y. This notation enables different loss functions to be modelled. For example, if there are no line losses then we obtain

$$g_i(y) = \sum_{k<i} y_{ki} - \sum_{k>i} y_{ik}.$$

With quadratic losses we obtain

$$g_i(y) = \sum_{k<i} y_{ki} - \sum_{k>i} y_{ik} - \sum_{k<i} \frac{1}{2} r_{ki} y_{ki}^2 - \sum_{k>i} \frac{1}{2} r_{ik} y_{ik}^2.$$

As indicated above, one of the functions of the ISO is to set the price. The price of electricity is determined as the shadow price π_i of the node balance constraints above that indicate demand must be met at all nodes. This price is the system cost of meeting one more unit of demand at node i. This method of determining the electricity price is sometimes referred to as locational marginal pricing (LMP). New Zealand and the PJM market in the US are examples of electricity markets with LMP.

It is worth noting that some wholesale electricity markets operate by assuming that demand and supply are located at the same node and trading takes place in that one node. This means that a single price of electricity is arrived at. Nevertheless, in order to ensure that the demand is met at all nodes and that the flow complies with physical constraints, a balancing market would follow in real time where the residuals of the single node market are traded. The UK wholesale electricity market is an example of a single node market.

2.1 unit commitment

While the economic dispatch problem described so far minimizes the total cost of generation based on the generators' offers, it is concerned with cost minimization over a single period. Besides the marginal cost of running generation units, other costs such as start up and shut down costs and constraints such as minimum up and down time for the generation units may need to be considered. In some markets these concerns are left to the generators. They are to figure these costs and constraints and reflect them in the way they offer their generation into the market. Other markets, such as the PJM have unit commitment constraints embedded and the economic dispatch problem runs over a time horizon spanning a day. For a comprehensive discussion on the unit commitment problem and the problem of market design embedding unit commitment see [3].

3 Generators

The generation sector in wholesale electricity markets usually consists of coal or gas fired thermal, nuclear power, hydro-electric, and/or other renewable (such as wind or solar powered) generation. As mentioned above, generators in a wholesale electricity market are required to submit an offer curve for each of their generation units. Therefore the main question faced by any generator in a wholesale electricity market is how to offer into the market in such a way as to maximize its return. Before we explore this problem further we must make a distinction between two different types of generators, the so-called price makers and price takers. We should also note that since the future is uncertain and neither the competitor offers and nor the demand is known, in absence of any risk attitude, the generators will be maximizing their expected profits.

3.1 Price-taker generators

A generator is defined to be a price-taker if the (nodal) price of power is not affected by the generation strategy of the generator. A true price taker is likely to have a relatively small capacity for generation (relative to the rest of the market) with correspondingly little storage capacity. Therefore the typical time horizon for the optimization problem is short to medium term. Here the only significant uncertainty is in the price process. Other factors such as the price of thermal fuel and inflows may be treated as deterministic as they are well forecasted over the short time horizon. If the electricity prices were independent from period to period then the optimal policy would be to submit a stack so that dispatch is ensured if the price is above expectation and water is saved for the subsequent period if the price is below expectation. However this independence assumption does not hold for electricity prices.

Pritchard and Zakeri [4] have developed a time inhomogeneous Markov model for electricity prices (at a node), where the transition probabilities depend on the time of day and the time of week. They use this price process to set up and solve a stochastic dynamic program in order to determine the release policies for operating a river chain, so that the expected profit of a price-taker hydro-electric generator is maximized.

In a subsequent paper, Pritchard et. al. [5] relax the assumption of limited storage. Here they consider a model where weekly expected releases for a river chain are determined based on a long term model for electricity prices. This expected release, along with a standard deviation on release is then passed to another optimization problem with a much shorter time horizon that focuses on a particular week. They then construct period to period offer stacks for the week in question using dynamic programming. This model integrates the long and and short term plans and allows for target release levels (the expected release each week) as well as allowing deviations from this target should an opportunity present itself in the shorter time horizon model to take advantage of favorable prices.

Conejo et. al. [6] develop a model that addresses similar questions pertaining to a thermal generator. The maximum output of a thermal unit is not available instantaneously. Thermal generators are bound by their ramp rate characteristics that dictate how the generator's available output rises towards it's maximum output as a function of time. They must also comply with minimum up and down constraints that constitute once a unit is turned off it may not be turned on again until some time has elapsed and vice versa. Conejo et. al. develop a mixed integer linear program (MILP) that produces a bidding strategy for a price-taker thermal generator over the course of a day given a price forecast for electricity prices that day. This bidding strategy maximizes the generator's expected profit. It should be noted that while most models in the literature assume the generators are risk neutral and aim to maximize expected profits, there has been some research addressing production schedules in presence of a risk attitude; see e.g. [7]. For further reading on price-taker generation optimization see also [8, 9, 10].

3.2 Price-maker generators

Due to economies of scale and the spatial distribution of consumers, almost all electricity markets are oligopolies. Therefore the most natural setting for a model is to assume that the offer strategies of a generator influence the (nodal) price of electricity. In such a setting, the fundamental question for the generators raised above needs to be revisited. The price of electricity is no longer exogenous hence the profit optimization problem faced by the generator now needs to take into account how the actions of the generator influence the price of electricity. This problem was first addressed in a paper by Klemperer and Meyer [11] who were interested in modeling an oligopoly facing uncertain demand, where each firm bids a supply function as its strategy. This is in contrast to previous models in the economics literature where firms were restricted to strategize over their quantities only (Cournot models) or their prices only (Bertrand models) and allows a firm to adapt better to an uncertain environment. Green and Newbery address the same question but in the context of the British spot market [12].

To begin, let us assume that there are only two generators supplying the market (i.e. we are dealing with a duopoly) and suppose that the offer curve of the competitor is given by $q = S(p)$. Let us also assume that the demand curve is given by $q = D(p)$, that is the market will absorb quantity q if the price is p. For their analysis, Klemperer and Meyer use the concept of the residual demand curve faced by the generator. Consider the curve given by $q = D(p) - S(p)$. This determines what quantity must be offered into the market if we desire the price to be p based on the demand curve and the competitor's offer strategy. The inverse of this curve describes how the price is influenced by the quantity we offer and is referred to as the residual demand curve. With this information at hand, it is now easy to optimize the profits of the generator in question (see Figure 1).

Recall that Klemperer and Meyer point out that supply functions allow a firm to adapt better to an uncertain environment. If there are multiple possible residual demand curves that a generator may face, the supply function response may allow selecting a point on each of these residual demand curves that would optimize the generator's profit given that that residual demand curve has realized. This is referred to as a strong supply function response (see Figure 2). A number of papers construct the residual demand curve by simulating the (single node) market and explicitly building the supply function response, see e.g. [13] and [14]. In [13] the residual demand curve takes on a step function form and the authors develop a non-linear integer programming model of the generator's revenue optimization problem. They develop a combined coordinate search, branch and bound method to solve this problem. Torre et. al. exploit the nature of the previous problem to develop a more efficient solution method in [14].

In a sequence of papers Anderson and Philpott have also addressed the profit maximization problem of a price maker generator under various assumptions. In [15] they assume that a price-maker generator knows its competitors' offer curves, but is faced with uncertain demand. They first establish the existence of a strong supply function response, for such a generator, that would be optimal for any realization of the uncertain demand. This strong

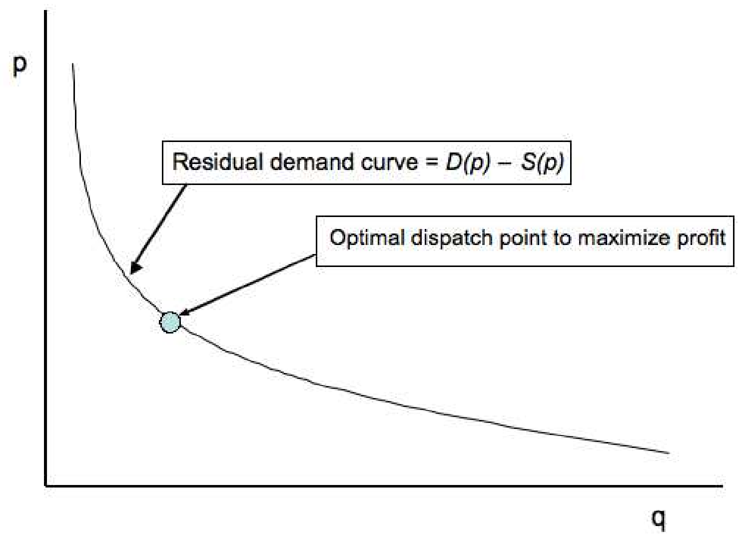

Figure 1: Optimal point for a generator to get dispatched along a residual demand curve.

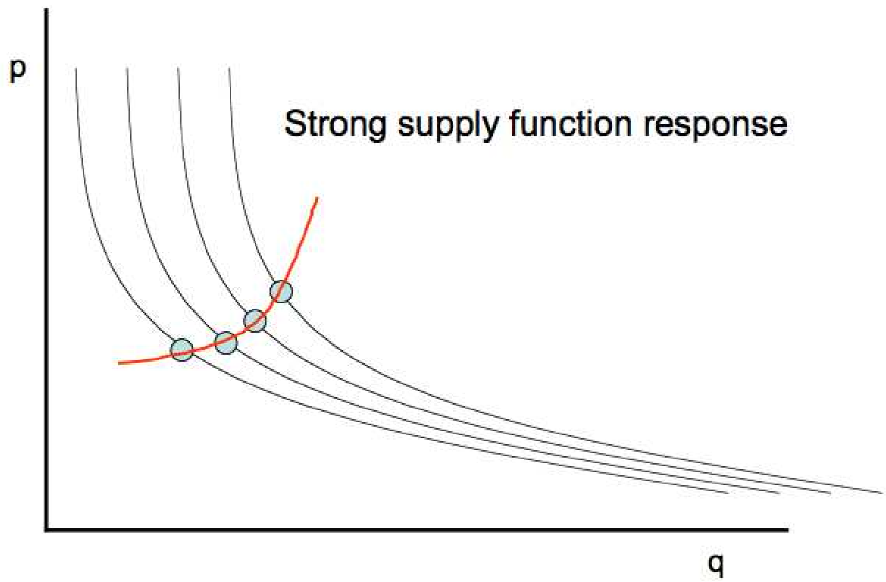

Figure 2: Building a strong supply function response from a distribution of residual demand curves.

supply function response is guaranteed to exist when the generation costs of the generator in question are increasing and convex, and the competitor offers are log concave. They discuss a procedure where the true aggregate offer stack of the competitors is approximated by a log concave function. Note that this (aggregate) offer stack would be a step function in almost all real world electricity markets. They construct a strong supply function response S_g, for the generator in question. Subsequently they approximate S_g in order to comply with market rules. Finally they provide bounds on the performance of such an offer strategy.

In [16] Anderson and Philpott generalize their model by allowing uncertainty not only in the demand but also allow the competitor offers to be unknown. They introduce the concept of a market distribution function $\psi(q, p)$ pertaining to a specific generator at a specific transmission node. They define $\psi(q, p)$ to be the probability of not being fully dispatched if the generator submits a quantity q at price p. Let $R(q, p)$ denote the or profit that the generator makes if it is dispatched q at a clearing price of p. They demonstrate that if the generator submits the curve s and the pertinent market distribution function ψ is continuous then the expected profit of the company is given by

$$V(s) = \int_s R(q, p) d\psi(q, p).$$

They proceed to provide conditions that guarantee (local) optimality of an offer stack s that would maximize $V(s)$. To address the question of estimating the market distribution function see [17] and [18].

The work described thus far only deals with generators that are located at a single node of the market or alternatively assumes that the wholesale market is a single node market. As noted in section 2 however, most wholesale electricity markets use locational marginal pricing where the price of electricity is different from node to node. To capture the effects of the transmission network, a generator must look at the variations in the prices from the dispatch problem EDP as a function of how it offers into the market. The revenue optimization problem is now posed as a bilevel program, or a mathematical program with equilibrium constraints (MPEC) and becomes a non-convex optimization problem.

$$\begin{array}{llllll}
\text{maximize} & R(x,\pi) & & & & \\
\text{s.t.} & (x,\pi) \in & \text{arg min} & \sum_i \sum_{m\in\mathcal{O}(i)} \int_0^{q_m} C_m(x)dx & & \\
 & \text{s.t.} & & g_i(y) + \sum_{m\in\mathcal{O}(i)} q_m & = \; D_i, & i \in \mathcal{N}, \\
 & & & q_m \in Q_m, \quad m \in \mathcal{O}(i), & & i \in \mathcal{N}, \\
 & & & y \in Y. & &
\end{array}$$

Here x denotes the vector of quantities dispatched at each node if the generator offered at that node (or is 0 if the generator in question does not own generation at a particular node), and π is the vector of electricity prices. Note that the inner optimization problem, namely the economic dispatch problem EDP can be replaced with its necessary and sufficient conditions for optimality as it is a convex problem (see e.g. chapter 4 of [19]). In this case, the reformulation is referred to as an MPEC [20]. Several papers have addressed this problem and have developed techniques to produce local or global optimal solutions for it. In [21], Fampa et. al. develop a stochastic version of the above problem in which they consider various demand and market clearing scenarios. This results in a number of follow-on economic dispatch problems. They propose some heuristic methods for solving this problem. Pritchard also considers a stochastic version of the above problem in [22]. In his model, the generator in question only owns generation assets at a single node of the transmission network. He proposes a stochastic dynamic program to solve the generator optimization problem. It should be noted that the literature surveyed for price-maker generators is only concerned with short to medium term time horizons, where a reasonable distribution for competitor behaviour and demand, or a reasonable estimate of the market distribution function is available to a generator.

4 Other electricity sectors

The focus of this short article has been on the generation side of wholesale electricity markets. Operations research is utilized in every sector of the electricity market and in this section we provide a very brief overview of how it is utilized by the demand side and the regulators.

Consumers of electricity can be loosely classified as major or minor users. Major users of electricity are typically large industrial users who have the ability to observe electricity prices in real time. In many wholesale electricity markets, such users can also bid into the electricity market by specifying a demand curve where they indicate their willingness to consume at different prices. These consumers are similar to price-maker generators in that they the ability to alter the price of electricity (through the amount that they consume). Frequently they are not only able to withdraw electricity from a GXP, but they may be in a position to produce their own power through a gas or diesel generator say, or they may have flexibility to reduce their production of goods which translates to reduction in consumption of electricity. Gomez-Villalva and Ramos [23] have developed a mixed integer linear program for such an industrial consumer in a wholesale electricity market.

For some major consumers it may also be possible to shift the electricity usage from one period to another, albeit at a cost. Large industrial chillers may be able to cool their contents at earlier hours of the morning to lower degrees simply because the price of electricity is lower in those off peak periods. The cooler may be able to retain the low temperatures reasonably well for instance and hence reduce their electricity consumption in the peak morning hours thereby reducing their total cost of consumption. This is referred to as load shifting. Middelberg et. al. [24] study an optimal control model for load shifting with application to energy management of a colliery.

Regulators are interested in designing electricity markets so that they are as competitive as possible and an efficient and reliable supply is accomplished. They are also interested in diagnosing any ill functioning, such as abuse of power by generating firms and introducing regulation that would prevent such behaviour. They are therefore interested in models of steady-state behaviour of the market. There is a vast amount of literature on such models ranging from models that derive Nash-Cournot equilibria and supply function equilibria to agent based simulation models. For some references see [25], [26], [11], [27], [28], [29], [30], [31], [32], [33].

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