Competitive Equilibrium with Risk Averse Agents

Andy Philpott
Electric Power Optimization Centre
University of Auckland.
www.epoc.org.nz
Joint work with Michael Ferris.

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System optimization

Suppose agents $a \in \mathcal{A}$ produce a single good in quantities x_a, $a \in \mathcal{A}$, to meet a demand d. Assume the production cost of each agent is a strictly convex function $C_a(x_a)$. The most efficient production plan solves a system social planning problem

$$\text{P:} \quad \min \sum_{a \in \mathcal{A}} C_a(x_a)$$

$$\text{s.t.} \quad \sum_{a \in \mathcal{A}} x_a \geq d.$$

Lagrange

The solution x to P solves

$$\min \sum_{a\in\mathcal{A}} C_a(x_a) + \pi \left(d - \sum_{a\in\mathcal{A}} x_a \right)$$

where π and x satisfy

$$0 \leq \sum_{a\in\mathcal{A}} x_a - d \perp \pi \geq 0.$$

Note that agents treat π as a fixed parameter when they optimize, so this is not a model for imperfect competition.

Decomposition gives market equilbrium

The ath component x_a of the solution to P solves the agent problem

$$\text{P}(a): \ \max \left(\pi x_a - C_a(x_a)\right)$$

where π and x satisfy equilibrium constraints

$$0 \leq \sum_{a \in \mathcal{A}} x_a - d \perp \pi \geq 0. \quad (1)$$

Each agent a chooses x_a to maximize their own welfare when paid price π, and (1) is called a market clearing condition. This is an example of a MOPEC (Multiple Optimization Problems with Equilibrium Constraints).

Welfare theorems of partial equilibrium

First welfare theorem: Suppose for some π, and each $a \in \mathcal{A}$, that x_a solves the agent problem P(a). If π and x satisfy the market clearing condition then x solves the system planning problem P.

Second welfare theorem: If x solves the system planning problem P then there is some π so that each component x_a solves the agent problem P(a), and π and x satisfy the market clearing condition.

Markets

If correct prices can be determined then we can achieve an optimal solution to a social planning problem that maximizes system welfare. To what extent is this true in markets with risk? We illustrate with toy examples from electricity markets.

- a two-stage market for capacity planning with risk-averse investors.
- a multistage model for energy storage optimization with risk-averse agents.

Summary

Summary

Coherent risk measures
(Artzner et al, 1999)

We assume a finite sample space Ω. Agent a is faced with a random cost or disbenefit $Z_a(\omega)$, $\omega \in \Omega$, and measures its risk using a coherent risk measure ρ_a, so $\rho_a(Z_a)$ can be interpreted as agent a's risk-adjusted disbenefit of Z_a. Any coherent risk measure $\rho(Z)$ has a dual representation

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z]$$

where $\mathcal{D}$ is a convex subset of probability measures on Ω, called the risk set of the coherent risk measure. If $\mathcal{D}$ is polyhedral with known extreme points $\{p^k, k \in \mathcal{K}\}$, then

$$\rho(Z) = \begin{cases} \min & \theta \\ \text{s.t.} & \theta \geq \sum_{\omega \in \Omega} p^k(\omega) Z(\omega), \quad k \in \mathcal{K}. \end{cases}$$

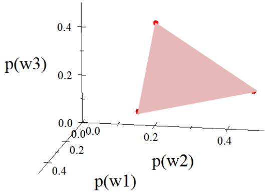
Example: three outcomes

Consider possible cost outcomes

$$Z(\omega_1) < Z(\omega_2) < Z(\omega_3)$$

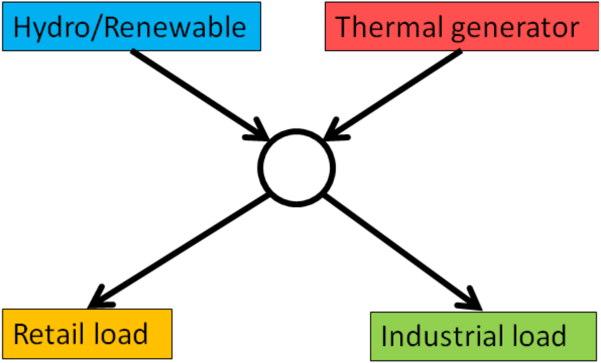
Let

$$\mathcal{D}=\text{conv}\{(\frac{1}{2},\frac{1}{4},\frac{1}{4}),(\frac{1}{4},\frac{1}{2},\frac{1}{4}),(\frac{1}{4},\frac{1}{4},\frac{1}{2})\}$$



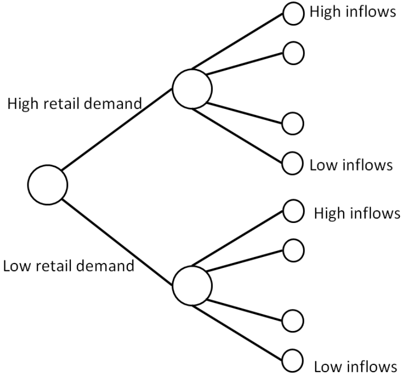
$$\rho(Z) = \max_{\mu\in\mathcal{D}} \mathbb{E}_\mu[Z] = \frac{1}{4}Z(\omega_1) + \frac{1}{4}Z(\omega_2) + \frac{1}{2}Z(\omega_3)$$

Example toy problem



In state of the world ω, industrial load buys power at the wholesale price $\pi(\omega)$, and values it at V. A retailer buys electricity at $\pi(\omega)$ and sells it to consumer at a fixed price R.

We consider eight equally likely scenarios



In our example there are eight equally likely scenarios. Demand is either high or low, and varying inflow levels influence hydro generation capacity for that outcome.

Our example risk measure: eight equally likely outcomes

Assume everyone uses the same coherent risk measure

$$\rho(Z) = 0.5\mathbb{E}[Z] + 0.5\mathrm{AVaR}_{0.75}(Z).$$

This has risk set

$$\begin{aligned}\mathcal{D} \quad = \quad & \mathrm{conv}\{(\frac{5}{16},\frac{5}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16}),\\ & (\frac{5}{16},\frac{1}{16},\frac{5}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16}),\\ & (\frac{5}{16},\frac{1}{16},\frac{1}{16},\frac{5}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16}),\\ & \ldots,\\ & (\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{5}{16},\frac{5}{16})\}.\end{aligned}$$

$$\rho(Z) = \max_{\mu \in \mathcal{D}} \sum_{i=1}^{8} \mu_i Z(\omega_i).$$

Risk-averse system optimization problem

Choose capacity x_g and energy $u_g(\omega)$ for each generator g, to meet retailer load $d(\omega)$ (value R) and industrial load $e(\omega)$ (value V) to minimize risk-adjusted social disbenefit.

$$
\begin{array}{llrcl}
\text{SP:} & \min & \sum_g K_g(x_g) & + & \rho(Z(\omega)) \\
 & \text{s.t.} & u_g(\omega) & \leq & x_g \phi_g(\omega), \quad \text{[inflow uncertainty]} \\
 & & y(\omega) & \leq & d(\omega), \quad \text{[retail load]} \\
 & & s(\omega) & \leq & e(\omega), \quad \text{[industrial load]} \\
 & & \sum_g u_g(\omega) + y(\omega) + s(\omega) & = & d(\omega) + e(\omega), \quad \text{[total load]} \\
 & & Z(\omega) & = & \sum_g C_g(u_g(\omega)) \\
 & & & & -V(e(\omega) - s(\omega)) \\
 & & & & -R(d(\omega) - y(\omega)) \\
 & & & & +Py(\omega) \\
 & & x_g, u_g(\omega), y(\omega), s(\omega) & \geq & 0.
\end{array}
$$

System optimum: 731 MW thermal, 600 MW hydro

Welfare ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Welfare
Thermal	-72.07	-72.76	-73.05	-73.08	393.52	392.02	-72.75	-72.90	-14.72
Hydro	21.44	-51.07	-104.35	-102.67	214.24	238.57	-98.97	-97.93	-50.55
Industry	164.38	211.49	230.61	230.65	158.46	205.35	229.16	229.35	184.43
Retailer	300.20	363.72	414.92	415.15	-314.84	-220.26	617.43	618.37	3.39
System	413.95	451.37	468.13	470.04	451.37	615.67	674.87	676.89	480.22

Benefits (welfare) for each agent from a social planning solution. This is not a competitive equilibrium. Yellow cells identify the two worst scenarios for the system. The right-hand column shows the result of evaluating the policy of each agent using its risk set. The orange figure 480.22 is risk-adjusted total social welfare.

Summary

Competitive Risk-Averse Generation Expansion (CRAGE)

(Kok, Philpott, Zakeri, 2018, de Maere d'Aertrycke et al, 2017)

- Given energy prices $\pi(\omega)$, each agent solves a two-stage risk-averse stochastic programming problem:
  - Stage 1: choose generation capacity for each generator.
  - Stage 2: Offer all generation capacity at short-run marginal cost in the spot market and earn market revenue at prices $\pi(\omega)$.
- Each agent uses the same coherent risk measure ρ and minimizes risk-adjusted disbenefit.
- If generation quantities clear the energy markets in each ω then we have a competitive equilibrium.
- A GAMS MOPEC model coded under the EMP system (Ferris et al 2009) and solved using PATH (Dirkse and Ferris, 1995).

Agent problem (generator)

Given electricity wholesale price $\pi(\omega)$, generator $g \in (h, t)$ chooses capacity expansion x, and sells generation $u_g(\omega)$ to solve

$$
\begin{array}{llll}
\text{GP}(g): & \min & K_g\,(x_g) & + \quad \rho(Z_g) \\
& \text{s.t.} & u_g(\omega) & \leq \quad x\phi_g(\omega), \\
& & Z_g(\omega) & = \quad -\pi(\omega)u_g(\omega) + C(u_g(\omega)), \\
& & x_g\,, u_g(\omega) & \geq \quad 0.
\end{array}
$$

Agent problem (retailer)

Given retail demand $d(\omega)$, compensation penalty P, retail price R, and electricity wholesale price $\pi(\omega)$, purchase $d(\omega) - y(\omega)$ to solve

$$
\begin{aligned}
\text{RP:} \quad \min \quad & \rho(Z_r) \\
\text{s.t.} \quad Z_r(\omega) &= (\pi(\omega) - R)(d(\omega) - y(\omega)) + Py(\omega), \\
y(\omega) &\leq d(\omega), \\
y(\omega) &\geq 0.
\end{aligned}
$$

Agent problem (industrial)

Given industrial demand $e(\omega)$, industrial value of electricity V, and electricity wholesale price $\pi(\omega)$, purchase $e(\omega) - s(\omega)$ to solve

$$\begin{aligned} \text{IP:} \quad \min \quad & \rho(Z_i) \\ \text{s.t.} \quad Z_i(\omega) &= (\pi(\omega) - V)(e(\omega) - s(\omega)) \\ s(\omega) &\leq e(\omega), \\ s(\omega) &\geq 0. \end{aligned}$$

Competitive risked equilibrium conditions

$$(x_g, u_g) \in \arg\min \mathrm{GP}(g),\ g \in (h, t),$$

$$y \in \arg\min \mathrm{RP},$$

$$s \in \arg\min \mathrm{IP},$$

$$0 \leq \sum_g u_g(\omega) + y(\omega) + s(\omega) - d(\omega) - e(\omega) \perp \pi(\omega) \geq 0.$$

Risk-averse equilibrium: 650 MW thermal, 440 MW hydro

Welfare ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Welfare
Thermal	-42.76	-64.20	-64.85	-64.92	351.65	349.88	109.58	-44.03	0.70
Hydro	24.29	66.64	-51.28	-73.41	158.74	278.74	145.08	33.10	5.20
Industry	158.92	164.67	224.14	230.58	157.73	158.74	205.79	206.09	173.28
Retailer	273.70	300.72	397.53	414.79	-316.74	-314.13	221.95	506.20	-64.97
System	414.15	467.84	505.54	507.03	351.37	473.22	682.40	701.36	447.81

Benefits (welfare) for each agent for the competitive equilibrium. Yellow cells identify the two worst scenarios. The right-hand column shows the result of evaluating the policy of each agent using its risk set. The orange figure 447.81 is risk-adjusted system welfare.

System optimum: 731 MW thermal, 600 MW hydro

Welfare ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Welfare
Thermal	-72.07	-72.76	-73.05	-73.08	393.52	392.02	-72.75	-72.90	-14.72
Hydro	21.44	-51.07	-104.35	-102.67	214.24	238.57	-98.97	-97.93	-50.55
Industry	164.38	211.49	230.61	230.65	158.46	205.35	229.16	229.35	184.43
Retailer	300.20	363.72	414.92	415.15	-314.84	-220.26	617.43	618.37	3.39
System	413.95	451.37	468.13	470.04	451.37	615.67	674.87	676.89	480.22

Benefits (welfare) for each agent. This is not a competitive equilibrium. Yellow cells identify the two worst scenarios for the system. The right-hand column shows the result of evaluating the policy of each agent using its risk set. The orange figure 480.22 is risk-adjusted total social welfare.

Contracts for differences

A contract for differences has a payout of $\pi(\omega)$ in outcome ω. Industrial and retail agents can hedge the risk of high prices in stage 2 by buying Q_i (Q_r) contracts at stage 1 at a market contract price f. In scenario ω, this increases their short run profits by $Q_i(\pi(\omega) - f)$ and $Q_r(\pi(\omega) - f)$ respectively. A generator can purchase Q_g contracts at price f to increase their short run profit in scenario ω by $Q_g(\pi(\omega) - f)$. Typically $Q_g \leq 0$, and $Q_i \geq 0$, $Q_r \geq 0$. The market clearing condition for contract trading is

$$0 \leq -\sum_g Q_g - Q_r - Q_i \perp f \geq 0.$$

Competitive risked equilibrium conditions

$$(Q_g, x, u) \in \arg\min \mathrm{GP}(g),\ g \in (h, t),$$

$$(Q_r, y) \in \arg\min \mathrm{RP},$$

$$(Q_i, s) \in \arg\min \mathrm{IP},$$

$$0 \leq -\sum_g Q_g - Q_r - Q_i \perp f \geq 0,$$

$$0 \leq \sum_g u_g(\omega) + y(\omega) + s(\omega) - d(\omega) - e(\omega) \perp \pi(\omega) \geq 0.$$

Risk-averse equilibrium: 650 MW thermal, 440 MW hydro

Welfare ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Welfare
Thermal	-42.76	-64.20	-64.85	-64.92	351.65	349.88	109.58	-44.03	0.70
Hydro	24.29	66.64	-51.28	-73.41	158.74	278.74	145.08	33.10	5.20
Industry	158.92	164.67	224.14	230.58	157.73	158.74	205.79	206.09	173.28
Retailer	273.70	300.72	397.53	414.79	-316.74	-314.13	221.95	506.20	-64.97
System	414.15	467.84	505.54	507.03	351.37	473.22	682.40	701.36	447.81

Benefits (welfare) for each agent for the competitive equilibrium. Yellow cells identify the two worst scenarios. The right-hand column shows the result of evaluating the policy of each agent using its risk set. The orange figure 447.81 is risk-adjusted system welfare.

With contracts: 750 MW thermal, 600 MW hydro

Welfare ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Welfare
Thermal	-126.55	30.25	93.88	93.99	28.90	164.40	28.90	89.83	0.81
Hydro	-9.36	10.41	-5.41	-3.65	-5.41	102.58	52.09	-1.46	5.04
Industry	170.16	199.95	212.04	212.06	199.69	225.10	199.69	211.24	194.33
Retailer	377.78	208.85	165.72	165.73	238.43	133.83	392.27	375.37	203.51
System	412.03	449.46	466.22	468.13	461.61	625.91	672.96	674.98	479.83

Benefits (welfare) for each agent for the equilibrium with contracts. Yellow cells identify the two worst scenarios. The right-hand column shows the result of evaluating the policy of each agent using its risk set. The orange figure 479.83 is risk-adjusted system welfare.

System optimum: 731 MW thermal, 600 MW hydro

Welfare ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Welfare
Thermal	-72.07	-72.76	-73.05	-73.08	393.52	392.02	-72.75	-72.90	-14.72
Hydro	21.44	-51.07	-104.35	-102.67	214.24	238.57	-98.97	-97.93	-50.55
Industry	164.38	211.49	230.61	230.65	158.46	205.35	229.16	229.35	184.43
Retailer	300.20	363.72	414.92	415.15	-314.84	-220.26	617.43	618.37	3.39
System	413.95	451.37	468.13	470.04	451.37	615.67	674.87	676.89	480.22

Benefits (welfare) for each agent. This is not a competitive equilibrium. Yellow cells identify the two worst scenarios for the system. The right-hand column shows the result of evaluating the policy of each agent using its risk set. The orange figure 480.22 is risk-adjusted total social welfare.

Arrow-Debreu securities

An Arrow-Debreu security for outcome $\omega \in \Omega$ in stage 2 is a contract that has a payout of 1 in outcome ω. We denote the price of such a contract in stage 1 by $\mu(\omega)$.

A market for risk is complete if there is an Arrow-Debreu security for every $\omega \in \Omega$. Instead of contracts for differences, suppose that each agent buys $W_a(\omega)$ Arrow-Debreu securities at stage one, costing $\sum_{\omega\in\Omega} \mu(\omega) W_a(\omega)$, to receive return $W_a(\omega)$ in outcome ω in stage 2.

Risked equilibrium with Arrow-Debreu securities

$$(W_g, x, u) \in \arg\min \mathrm{GP}(g),\ g \in (h, t),$$

$$(W_r, y) \in \arg\min \mathrm{RP},$$

$$(W_i, s) \in \arg\min \mathrm{IP},$$

$$0 \leq -\sum_g W_g(\omega) - W_r(\omega) - W_i(\omega) \perp \mu(\omega) \geq 0,$$

$$0 \leq \sum_g u_g(\omega) + y(\omega) + s(\omega) - d(\omega) - e(\omega) \perp \pi(\omega) \geq 0.$$

Risked equilibrium with Arrow-Debreu securities

Welfare ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Welfare
Thermal	-14.40	-14.40	-14.40	-14.40	-14.40	-14.40	-14.40	-14.40	-14.40
Hydro	-42.20	-4.78	-4.78	-4.78	-4.78	159.52	-4.78	-4.78	-6.20
Industry	192.48	192.48	209.24	211.15	192.48	192.48	216.29	216.48	197.68
Retailer	278.07	278.07	278.07	278.07	278.07	278.07	477.76	479.59	303.15
System	413.95	451.37	468.13	470.04	451.37	615.67	674.87	676.89	480.22

The system optimal solution (731 MW thermal, 600 MW hydro) corresponds to a competitive equilibrium when Arrow Debreu securities are traded.

System optimum: 731 MW thermal, 600 MW hydro

Welfare ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Welfare
Thermal	-72.07	-72.76	-73.05	-73.08	393.52	392.02	-72.75	-72.90	-14.72
Hydro	21.44	-51.07	-104.35	-102.67	214.24	238.57	-98.97	-97.93	-50.55
Industry	164.38	211.49	230.61	230.65	158.46	205.35	229.16	229.35	184.43
Retailer	300.20	363.72	414.92	415.15	-314.84	-220.26	617.43	618.37	3.39
System	413.95	451.37	468.13	470.04	451.37	615.67	674.87	676.89	480.22

Agent welfare outcomes from the social planning solution (731 MW thermal, 600 MW hydro).

The difference is the income from trade

AD Income ($M)	Low retail demand				High retail demand				Risk Adj
Inflows	50%	75%	125%	150%	50%	75%	125%	150%	Income
Thermal	57.67	58.36	58.65	58.68	-407.92	-406.42	58.35	58.50	0.00
Hydro	-63.64	46.29	99.57	97.89	-219.01	-79.05	94.19	93.15	0.00
Industry	28.10	-19.01	-21.36	-19.50	34.02	-12.87	-12.87	-12.87	0.00
Retailer	-22.13	-85.64	-136.85	-137.08	592.92	498.33	-139.67	-138.78	0.00
Total	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The additional income from trading Arrow-Debreu securities is zero in risk-adjusted terms but aligns the risks for each agent with the system risk.

Summary of risk-averse equilibrium

		No contracts	Contracts	AD securities
Expansion decisions	Thermal	650.00	750.00	730.86
	Hydro	440.00	600.00	600.00
Risk adjusted welfare ($M)	Thermal	0.70	0.81	-14.40
	Hydro	5.20	5.04	-6.20
	Industry	173.28	194.33	197.68
	Retail	-64.97	203.51	303.15
	System	447.81	479.83	480.22

Risk-adjusted benefits (welfare) for each agent, and risk-adjusted social welfare for the system for each equilibrium.

What have we learned?

- We solve a risk-averse two-stage system capacity expansion problem that minimizes risk-adjusted social disbenefit.
- The optimal solution has more invested capacity and higher risk-adjusted welfare than a risk-averse competitive equilibrium.
- Agents in the risk-averse competitive equilibrium will invest more, and create more system welfare if they can trade risk using contracts.
- The socially optimal solution for the system can be recovered if there is complete market of Arrow-Debreu securities.

Theorems
(Ralph and Smeers, 2015)

Suppose agents have risk sets $\mathcal{D}_a$ with $\mathcal{D}_s = \bigcap_{a\in\mathcal{A}} \mathcal{D}_a \neq \emptyset$, and there is a complete market for Arrow-Debreu securities.

Theorem

If $\{\bar{\pi}(\omega), \omega \in \Omega\}$, and $\{\bar{\mu}(\omega), \omega \in \Omega\}$ give a competitive risked equilibrium $\{(W_g, x, u),(W_r, y),(W_i, s)\}$ then $\{(x, u),y,\ s\}$ solves the risked social planning problem SP with social risk measure ρ_s corresponding to the social risk set $\mathcal{D}_s$.

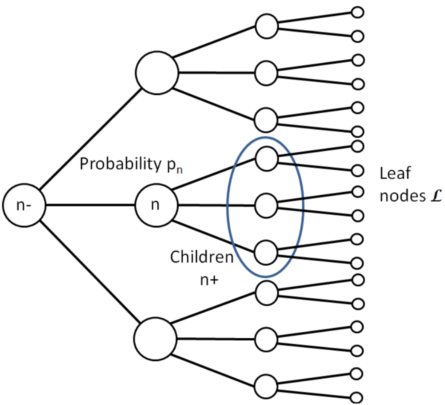
Theorem

If $\{(x, u),y,\ s\}$ solves the risked social planning problem SP with social risk set $\mathcal{D}_s$ then there exists prices $\{\bar{\pi}(\omega), \omega \in \Omega\}$, and $\{\bar{\mu}(\omega), \omega \in \Omega\}$ and trades in Arrow-Debreu securities so that $\{(W_g, x, u),(W_r, y),(W_i, s)\}$ is a competitive risked equilibrium.

Summary

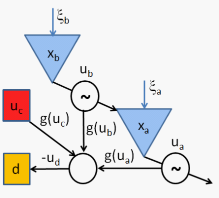
1. Introduction
2. Capacity planning under risk
3. Risked equilibrium
4. Multistage risked equilibrium

A scenario tree represents uncertain outcomes



Scenario tree with node set $\mathcal{N}$. At each $n \in \mathcal{N}$ there is a realization of a random variable $\xi_a(n)$. The parent node of node n is denoted n_-, which is also the root node 0 in this case.

Example: Optimization and equilibrium with storage



Storage (i.e. batteries, hydroelectric reservoirs, pumped storage) adds dynamics. We have a storage state variable x_a for agent a affected by controls (e.g. reservoir releases, u, possibly from other agents) and random disturbances ξ (e.g. inflows).

$$x_a(n) \leq x_a(n_-) + \sum_{b \in \mathcal{A}} T_{ab} u_b(n) + \xi_a(n), \quad a \in \mathcal{A}, n \in \mathcal{N}.$$

In each node $n \in \mathcal{N}$, agents produce electricity to meet demand (that is also treated as an agent).

Nested risk measure

Any set of controls adapted to the scenario tree gives disbenefits $Z(m)$, $m \in \mathcal{N} \backslash \{0\}$. It also results in a future risk-adjusted disbenefit $\theta(n) = \bar{\theta}(n)$ in node $n \in \mathcal{L}$, that can be computed or estimated. For each node $n \in \mathcal{N}$ we assume a polyhedral risk set $\mathcal{D}(n)$ with known extreme points $\left\{p^k(m), m \in \mathcal{M}(n), k \in \mathcal{K}(n)\right\}$. The future risk-adjusted disbenefit $\theta(n)$ in node $n \notin \mathcal{L}$ is defined recursively:

$$\begin{aligned} \theta(n) &= \sup_{\mu \in \mathcal{D}(n)} \sum_{m \in \mathcal{M}(n)} \mu(m)(Z(m) + \theta(m)) \\ &= \begin{cases} \min & \theta \\ \text{s.t.} & \theta \geq \sum_{m \in \mathcal{M}(n)} p^k(m)(Z(m) + \theta(m)), \quad k \in \mathcal{K}(n). \end{cases} \end{aligned}$$

A multistage risked optimization problem with storage
(Philpott, Ferris, Wets, 2016, Ferris and Philpott, 2018)

$$\text{SO}(\mathcal{D}): \min_{u,x,\theta} \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta(0)$$

$$\text{s.t. } \theta(n) \geq \sum_{m\in n_+} p^k(m)\left(\sum_{a\in\mathcal{A}} C_a(u_a(m)) + \theta(m)\right),$$

$$k \in \mathcal{K}(n),\ n \in \mathcal{N}\setminus\mathcal{L},$$

$$x_a(n) \leq x_a(n_-) + \sum_{b\in\mathcal{A}} T_{ab}u_b(n) + \xi_a(n), \quad a\in\mathcal{A}, n\in\mathcal{N},$$

$$\sum_{a\in\mathcal{A}} g_a(u_a(n)) \geq 0 \quad n\in\mathcal{N},$$

$$\theta(n) = -\sum_{a\in\mathcal{A}} V_a(x_a(n)), \quad n\in\mathcal{N},$$

$$u_a(n)\in\mathcal{U}_a, \quad x_a(n)\in\mathcal{X}_a, \quad n\in\mathcal{N}, \quad a\in\mathcal{A}.$$

New definitions for multistage

Definition

For $n \in \mathcal{N} \setminus \mathcal{L}$ the social planning risk set is

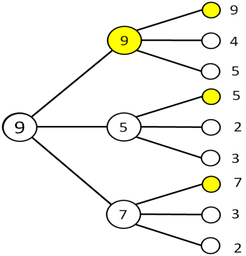
$$\mathcal{D}_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n).$$

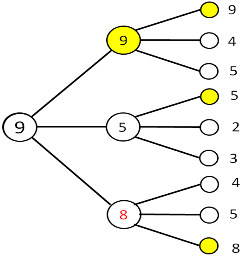
Definition

Given any node $n \in \mathcal{N} \setminus \mathcal{L}$, an Arrow-Debreu security for node $m \in n_+$ is a contract that charges a price $\mu(m)$ in node n to receive a payment of 1 in node $m \in n_+$.

Time consistency

For some risk measures (e.g. AVaR or worst-case) the risk sets have extreme points with zero components. In nested form, these might give optimal policies that are not time consistent.





Disbenefits from two optimal plans using nested worst-case risk measure, giving optimal solutions with disbenefit 9. Changes in the actions in the bottom subtree will still give an optimal risk-averse policy for the full tree as long as disbenefits don't exceed 9, but this may not be optimal in the subtree.

Some assumptions

Assumption 1: All risk sets $\mathcal{D}_a(n)$ lie strictly inside the positive orthant, implying time-consistent policies.

Assumption 2: (Complete risk markets) At every node $n \in N \setminus L$, there is an Arrow-Debreu security for each child node $m \in n_+$.

Assumption 3: For $n \in N \setminus L$, $\mathcal{D}_s(n) \neq \emptyset$, i.e.

$$\bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n) \neq \emptyset.$$

Multistage risked agent optimization

$\text{AO}_a(\pi, \alpha, \mu, \mathcal{D}_a)$:

$$\begin{aligned}
\min_{u_a, x_a, W_a, \theta_a} \quad & Z_a(0; u, x, W) + \theta_a(0) \\
\text{s.t.} \quad & \theta_a(n) \geq \sum_{m \in n_+} p_a^k(m)(Z_a(m; u, x, W) - W_a(m) + \theta_a(m)) \\
& \qquad k \in K_a(n),\ n \in \mathcal{N} \setminus \mathcal{L}, \\
& \theta_a(n) = -V_a(x_a(n)), \quad n \in \mathcal{L}, \\
& u_a(n) \in \mathcal{U}_a,\ x_a(n) \in \mathcal{X}_a, \quad n \in \mathcal{N},
\end{aligned}$$

$$\begin{aligned}
Z_a(n; u, x, W) = {} & C_a(u_a(n)) - \pi(n) g_a(u_a(n)) \\
& + \alpha_a(n) \left(x_a(n) - x_a(n_-) - \xi_a(n) \right) \\
& - \sum_{b \in \mathcal{A}} \alpha_b(n) T_{ba} u_a(n) + \sum_{m \in n_+} \mu(m) W_a(m), \quad n \in \mathcal{N}.
\end{aligned}$$

Multistage risk-trading optimization equilibrium

A multistage risk-trading optimization equilibrium $\mathrm{RTOE}(\mathcal{D}_{\mathcal{A}})$ is a stochastic process of prices $\{\pi(n), n \in \mathcal{N}\}$, $\{\alpha_a(n), a \in \mathcal{A}, n \in \mathcal{N}\}$, $\{\mu(n), n \in \mathcal{N} \setminus \{0\}\}$, and a corresponding collection of actions, $\{u_a(n), n \in \mathcal{N}\}, \{W_a(n), n \in \mathcal{N} \setminus \{0\}\}$ with the property that $(u_a, x_a, W_a, \theta_a)$ solves the problem $\mathrm{AO}_a(\pi, \alpha, \mu, \mathcal{D}_a)$, and

$$0 \leq \pi(n) \quad \perp \quad \sum_{a \in \mathcal{A}} g_a(u_a(n)) \geq 0, \quad n \in \mathcal{N}, \quad \text{[energy market]}$$

$$0 \leq \alpha_a(n) \quad \perp \quad -x_a(n) + x_a(n_-) + \sum_{b \in \mathcal{A}} T_{ab} u_b(n) + \xi_a(n) \geq 0, \quad a \in \mathcal{A}, n \in \mathcal{N}, \quad \text{[water market]}$$

$$0 \leq \mu(n) \quad \perp \quad -\sum_{a \in \mathcal{A}} W_a(n) \geq 0, \quad n \in \mathcal{N} \setminus \{0\} \quad \text{[risk market]}.$$

First welfare theorem
[Ferris and P, 2018]

Suppose Assumptions 1 and 2 hold, and consider a set of agents $a \in \mathcal{A}$, each endowed with a polyhedral node-dependent risk set $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$ satisfying Assumption 3.

Theorem

If $\{\bar{\pi}(n), n \in \mathcal{N}\}$, $\{\bar{\alpha}_a(n), a \in \mathcal{A}, n \in \mathcal{N}\}$, *and* $\{\bar{\mu}(n), n \in \mathcal{N} \setminus \{0\}\}$ *form a multistage risk-trading optimization equilibrium* $RTOE(\mathcal{D}_{\mathcal{A}})$ *in which agent a solves* $AO_a(\pi, \alpha, \mu, \mathcal{D}_a)$ *with a policy defined by* $(\bar{u}_a(\cdot), \bar{x}_a(\cdot), \bar{\theta}_a(\cdot))$ *together with a policy of trading Arrow-Debreu securities defined by* $\{\bar{W}_a(n), n \in \mathcal{N} \setminus \{0\}\}$, *then* $(\bar{u}, \bar{x}, \bar{\theta})$ *is a solution to* $SO(\mathcal{D}_s)$ *where* $D_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$ *and* $\bar{\theta}(n) = \sum_{a \in \mathcal{A}} \bar{\theta}_a(n)$.

Second welfare theorem
[P. Ferris, Wets, 2016]

Suppose Assumptions 1 and 2 hold, and consider a set of agents $a \in \mathcal{A}$, each endowed with a polyhedral node-dependent risk set $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$ satisfying Assumption 3.

Theorem

Let (u, θ^s) be a solution to $SO(\mathcal{D}_s)$ with risk sets $D_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$, giving rise to Lagrange multipliers $\{\alpha_a(n), a \in \mathcal{A}, n \in \mathcal{N}\}$ (for storage) and $\{\pi(n), n \in \mathcal{N}\}$ (for energy). Then there exists $\mu(n), n \in \mathcal{N} \setminus \{0\}$ so that the prices $\{\pi(n), n \in \mathcal{N}\}$, $\{\alpha_a(n), a \in \mathcal{A}, n \in \mathcal{N}\}$ and actions $\{u_a(n), n \in \mathcal{N}\}$, $\{W_a(n), n \in \mathcal{N} \setminus \{0\}\}$ form a multistage risk-trading optimization equilibrium $RTOE(\mathcal{D}_{\mathcal{A}})$.

Some practical implications of this theory

- Helps to understand why investment seen in the market may not match what the planner hopes for.
  - e.g. current work in NZ is focused on government policy aiming for 100% renewable electricity.
- Regulators can predict performance of complete, perfectly competitive risked markets with storage, using risk-averse multistage optimization (e.g. risk-averse SDDP).
  - market monitoring of imperfect competition becomes feasible for hydro-dominated systems.
- System operators can try to emulate outcomes of a complete perfectly competitive risked market using centrally operated software (e.g. NEWAVE in Brazil).

The End

THE END

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