Mixed Integer Dynamic Approximation Scheme

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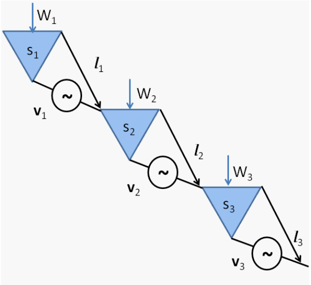
Motivation: daily hydro generation in a river chain



Given hourly electricity prices $\pi(t)$ the generator arranges releases $v_i(t)$ and spill $l_i(t)$ of water to maximize revenue $\sum_t \pi(t) \sum_i g_i(v_i(t))$ while respecting the water flow constraints of the river chain. Here g_i converts water flow into power.

Autoregressive price model and random inflows

Given initial state $\mathbf{x}_0$, we seek an optimal policy yielding $V_1(\mathbf{x}_0)$, where

$$\begin{aligned} V_t(\mathbf{x}) &= \mathbb{E}_{\xi_t}\left[\max_{u \in U(x)} \{r_t(\mathbf{x}, \mathbf{u}, \boldsymbol{\xi}_t) + V_{t+1}(f_t(\mathbf{x}, \mathbf{u}, \boldsymbol{\xi}_t))\}\right], \\ V_{T+1}(\mathbf{x}) &= R(\mathbf{x}) \end{aligned}$$

$$\mathbf{x} = \begin{bmatrix} s & \pi \end{bmatrix}^\top, \ \mathbf{u} = \begin{bmatrix} v & l \end{bmatrix}^\top, \ \boldsymbol{\xi}_t = \begin{bmatrix} \omega_t & \eta_t \end{bmatrix}^\top$$

$$f_t(s, \pi, v, l, \omega_t, \eta_t) = \begin{bmatrix} s_t - v_t - l_t + \omega_t \\ \alpha_t \pi_t + (1 - \alpha_t) b_t + \eta_t \end{bmatrix},$$

$$r_t(s, \pi, v, l, \omega_t, \eta_t) = \pi \sum_i g_i(v).$$

How to solve this

- Stochastic programming (Fleten and Kristoffersen, 2007)
- Backward recursion (Pritchard and Zakeri, 2003)
- SDDP (Pereira and Pinto, 1991, P., Dallagi, Gallet, 2013)
- ADDP (Löhndorf et al, 2013)
- Linear decision rules (Braathen et al, 2013)
- SDDP with MIPs (Zhou, Ahmed, Sun, 2016)

Our approach

- Stochastic control using approximate dynamic programming.
- SDDP constructs cutting-plane outer approximation of convex or concave value functions.
- MIDAS: similar methodology to (approximately) solve stochastic optimal control problems with nonconvex value functions (e.g from AR model of price).
- Solves stage problems using mixed integer programming.
- Convergence requires monotonicity and continuity of $V_t(\mathbf{x})$.

Summary

Summary

Dynamic programming formulation

Given initial state x_0, we seek an optimal policy yielding $V_1(x_0)$, where

$$\begin{aligned} V_t(x) &= \mathbb{E}_{\xi_t}\left[\max_{u \in U(x)} \{r_t(x,u,\xi_t) + V_{t+1}(f_t(x,u,\xi_t))\}\right] \\ V_{T+1}(x) &= R(x). \end{aligned}$$

Here $V_t(x)$ denotes the maximum expected reward from the beginning of stage t onwards, given the state is x, and we take action u_t after observing the random disturbance ξ_t. We assume that $R(x)$ is continuous, and $U(x)$ is sufficiently regular so that V_t is continuous if V_{t+1} is.
Given $\varepsilon > 0$, there is some δ so that for all $t = 1, 2, \ldots, T+1$,

$$\|x - y\|_\infty < \delta \Rightarrow |V_t(x) - V_t(y)| < \varepsilon.$$

Outer approximation of continuous monotonic functions

Given a continuous nondecreasing function $Q(x) \leq M$, and a finite set of values

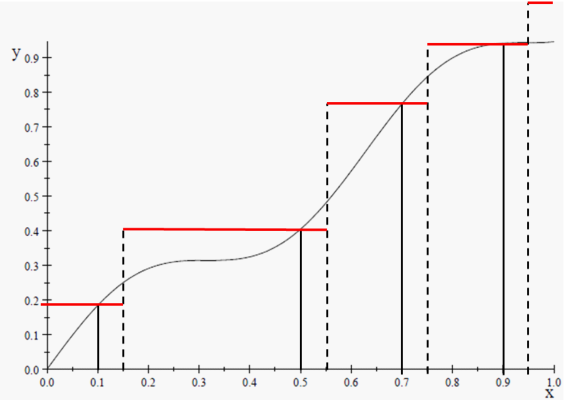
$$Q(x^h) = q^h, \ h = 1, 2, \ldots, H,$$

approximate $Q(x)$ by a piecewise constant function $Q^H(x)$ so that for every x

$$Q(x) \leq Q^H(x) + \varepsilon$$

- q^h is a real number and Q^H is a function;
- $Q(x)$ is assumed monotonic to guarantee that $Q(x) \leq Q^H(x) + \varepsilon$ for every x.

Example



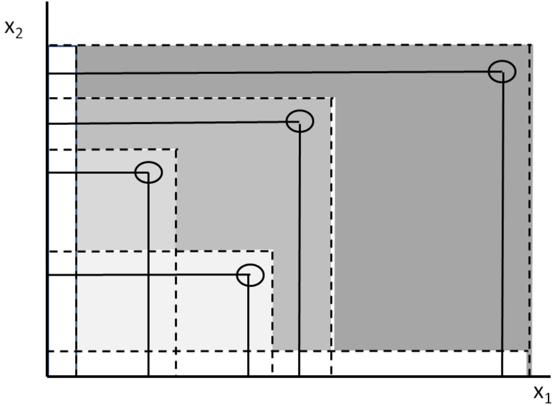
Approximation of $Q(x) = x + 0.1\sin(10x)$ at points $x^h = 0.1, 0.5, 0.7, 0.9$, and $\delta = 0.05$. $Q^H(x)$ shown in red is upper semicontinuous, and is an upper bound on $Q(x) - \varepsilon$.

MIP approximates a continuous monotonic function

Assume that

$$
\begin{array}{llcll}
Q^H(x) = \max & \varphi & & & \\
\text{s.t.} & \varphi & \leq & q^h + M(1 - y^h), & h = 1, 2, \ldots, H, \\
& x_k & \geq & x_k^h z_k^h + \delta, & k = 1, 2, \ldots, n, \\
& & & & h = 1, 2, \ldots, H, \\
& \sum_{k=1}^n z_k^h & = & 1 - y^h, & h = 1, 2, \ldots, H, \\
& y^h & \in & \{0, 1\}, & h = 1, 2, \ldots, H, \\
& z_k^h & \in & \{0, 1\}, & k = 1, 2, \ldots, n, \\
& & & & h = 1, 2, \ldots, H.
\end{array}
$$

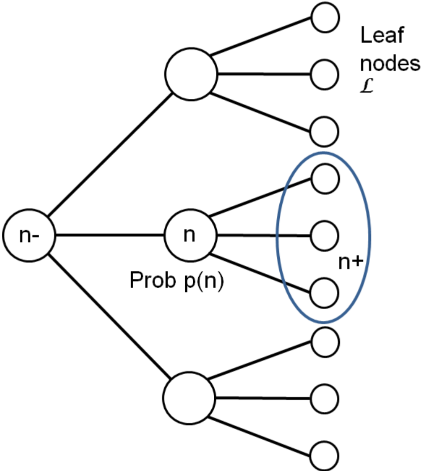
Example in two dimensions



Contour plot of $Q^H(x)$ when $H = 4$. Circled points are x^h, $h = 1, 2, 3, 4$. Darker shading indicates increasing values of $Q^H(x)$ that equals $Q(x^h)$ in each region containing x^h, $h = 1, 2, 3, 4$. $Q^H(x)$ defined only when $x \geq \delta e$.

Summary

Multistage algorithm uses a scenario tree



Tree formulation of stochastic control problem

$$
\begin{array}{lll}
\text{MSPT:} & \max & \sum_{n\in\mathcal{N}\setminus\{0\}} p(n) r_n(x_{n-}, u_n) + \sum_{n\in\mathcal{L}} p(n) R(x_n) \\
& \text{s.t.} & x_n = f_{n-}(x_{n-}, u_n, \xi_n), \\
& & x_0 = \overline{x}, \\
& & u_n \in U(x_n), \\
& & x_n \in X_n.
\end{array}
$$

DP recursion is:

$$
\begin{aligned}
V_n(x_n) &= \sum_{m\in n+} \frac{p(m)}{p(n)} \max_{u\in U(x_n)} \left\{ r_m(x_n, u) + V_m\left(f_n(x_n, u, \xi_m)\right)\right\} \\
V_n(x_n) &= R(x_n),\ n \in \mathcal{L},
\end{aligned}
$$

where we seek a policy that maximizes $V_0(x_0)$.

MIDAS algorithm

1. Set $Q_n^1(x) = M$, for every $n \in \mathcal{N} \setminus \mathcal{L}$;
2. For $H = 1, 2, \ldots,$
   - set $Q_n^H(x) = R(x)$, for every $n \in \mathcal{L}$;
   - perform a forward pass then a backward pass.

Forward pass

Set $x_0^H = x_0$, and $n = 0$. While $n \notin \mathcal{L}$:

1. Sample $m \in n+$ to give ξ_m^H;
2. Solve $\max_{u \in U(x_n^H)} \left\{ r_m(x_n^H, u) + Q_m^H(f_n(x_n^H, u, \xi_m^H)) \right\}$ to give u_m^H;
3. If $\left\| f_n(x_n^H, u_m^H, \xi_m^H) - x_m^h \right\|_\infty < \delta$ for $h < H$ then set $x_m^{H+1} = x_m^h$, else set $x_m^{H+1} = f_n(x_n^H, u_m^H, \xi_m^H)$;
4. Set $n = m$.

Backward pass

For the particular node $n \in \mathcal{L}$ at the end of forward pass update $Q_n^H(x)$ to $Q_n^{H+1}(x)$ by adding $q_n^{H+1} = R(x_n^{H+1})$ at point x_n^{H+1}.
While $n > 0$

1. Set $n = n-$;
2. Compute

$$\varphi_n = \sum_{m \in n+} \frac{p(m)}{p(n)} \max_{u \in U(x_n^H)} \left\{ r_m(x_n^{H+1}, u) + Q_m^{H+1}(f_n(x_n^{H+1}, u, \xi_m)) \right\}$$

3. Update $Q_n^H(x)$ to $Q_n^{H+1}(x)$ by adding $q_n^{H+1} = \varphi_n$ at point x_n^{H+1};

Sampling property

FPSP: For each $n \in \mathcal{L}$, with probability 1

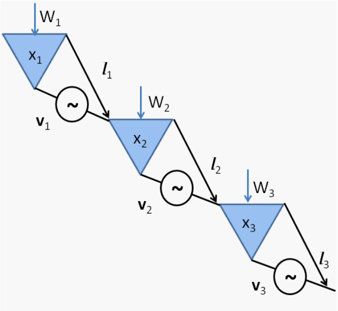
$$\left|\left\{H : \xi_n^H = \xi_n\right\}\right| = \infty.$$

Theorem

If step 1 of forward pass satisfies FPSP then sampled MIDAS converges almost surely to a $(T+1)\varepsilon$-optimal solution to MSPT.

Summary

Recall hydro generation in a river chain



Given future electricity prices $\pi(t)$ the generator arranges releases $v_i(t)$ and spill $l_i(t)$ of water to maximize revenue $\sum_t \pi(t) \sum_i g_i(v_i(t))$ while respecting the water flow constraints of the river chain.

Example: AR1 price model for river chain

$$\mathbf{x}_{t+1} = f_t(\mathbf{x}, \mathbf{u}, \xi_t),$$

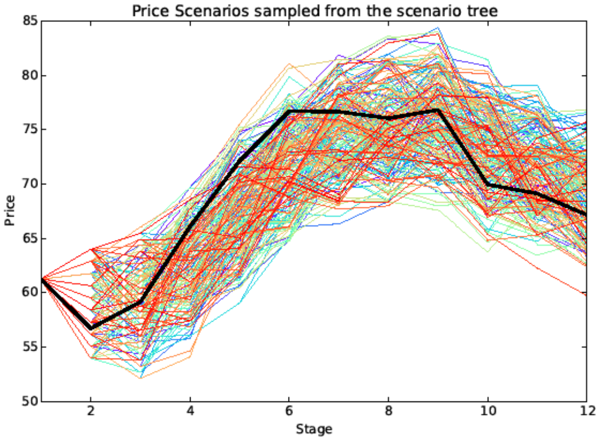
$$\begin{bmatrix} s_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} s_t - v_t - l_t + \omega_t \\ \alpha_t \pi_t + (1 - \alpha_t) b_t + \eta_t \end{bmatrix},$$

Here ω_t is (random) reservoir inflow, η_t is error term for price model, so $\xi_t = \begin{bmatrix} \omega_t & \eta_t \end{bmatrix}^\top$ and $\mathbf{u} = \begin{bmatrix} v & l \end{bmatrix}^\top$ release and spill. Reward in stage t is

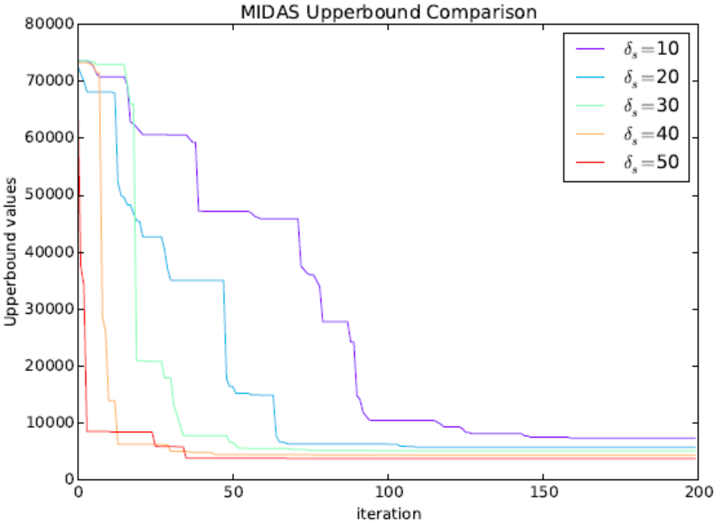
$$r_t(s, \pi, v, l, \omega_t, \eta_t) = \pi \sum_i g_i(v),$$

from released energy $g(v)$ sold at price π, and $U(\mathbf{x}) = \{(v, l) : v \in U_0,\ v + l \in [0, s]\}$.

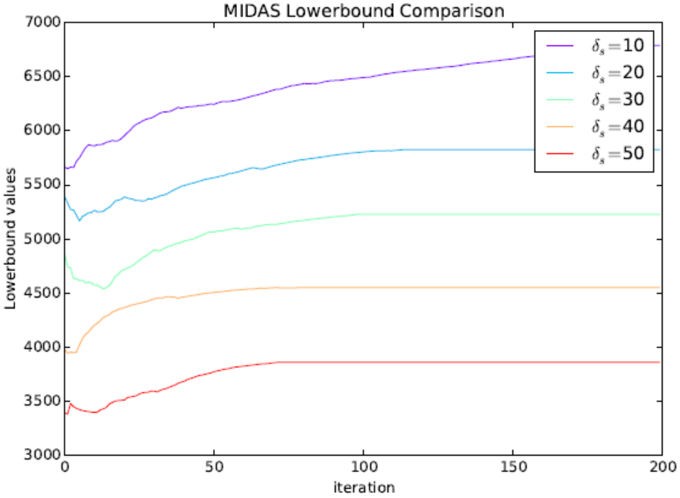
Price scenarios sampled from AR1 model



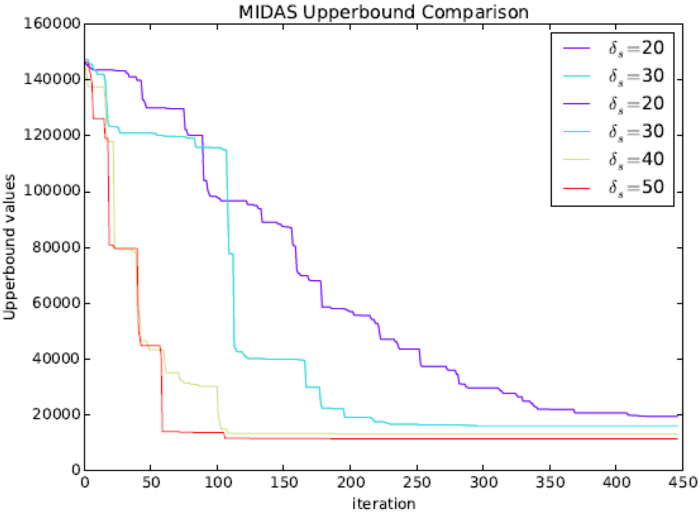
Epsilon upper bounds for single reservoir problem



Estimated policy payoffs for different delta



Epsilon upper bounds for two-reservoir problem



Summary

Conclusions

- SDDP has proved very successful in hydrothermal scheduling;
- Sampled trajectories reduce DP computation;
- Concave value functions a limitation;
- MIDAS is an attempt to extend these features to more general stochastic dynamic programs in the hope of making them tractable by solving small MIPs;
- Convergence wp1 can be shown for continuous monotonic value functions.

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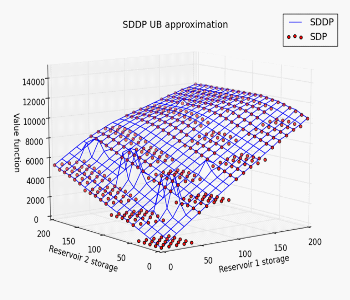
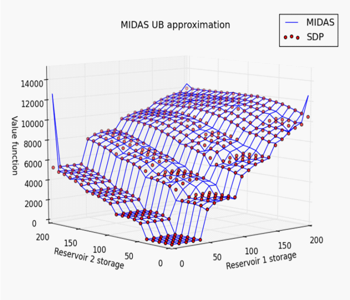
SDDP and MIDAS for stochastic MIPs





True value function shown in red. SDDP and MIDAS give different outer approximations when true value function is not concave.

SDDP and MIDAS

Solved & simulated offer policy of MIDAS & SDDP for range of initial storage levels

MIDAS policy: On average 98% of optimal value

SDDP policy: On average 93% of optimal value

	Mean	**Median**	**Upper quartile**	**Lower quartile**
SDDP	92.92	88.22	81.18	96.35
MIDAS	97.93	99.06	97.18	99.67

The one-dimensional case

$$
\begin{array}{llrcll}
V_H(x) = \max & & \varphi & & & \\
\text{s.t.} & & \varphi & \leq & V^h + Mz^h, & h = 1, 2, \ldots, H, \\
& & x & \geq & x^h z^h + \delta, & h = 1, 2, \ldots, H, \\
& & z^h & \in & \{0, 1\}, & h = 1, 2, \ldots, H.
\end{array}
$$

Consider (x^h, V^h), $h = 1, 2, \ldots, H$. The variable $z^h = 1$ picks out all the x^h lying at or below $x - \delta$. These do not constrain φ. The x^h lying strictly above $x - \delta$ have $z^h = 0$. Thus

$$V_H(x) = \min\{V^h : x^h > x - \delta\}.$$

Explanation of approximate stage problem 1

If $w_h = 0$ for some h then $\sum_k z_k^h = 1$, so $z_k^h = 1$ for exactly one k. The constraint

$$x_k \geq x_k^h z_k^h + \delta$$

means that there is at least one dimension k with $x_k > x_k^h$. So x is somewhere in the complement of the rectangular box $R = \{y \mid y \leq x^h\}$. Since $w_h = 0$, we have

$$\varphi \leq v_h + M$$

so the value v^h from top right-hand corner of R does not bound φ.

Explanation of approximate stage problem 2

If $x \leq x^h$ for $h \in \mathcal{G} \subseteq \{1, 2, \ldots, H\}$, then for each $h \in \mathcal{G}$ we must have every $z_k^h = 0$, so $\sum_k z_k^h = 0$. It follows from

$$\sum_{k=1}^{n} z_k^h \quad = \quad 1 - w_h,$$

that $w_h = 1$, $h \in \mathcal{G}$, so the constraints become

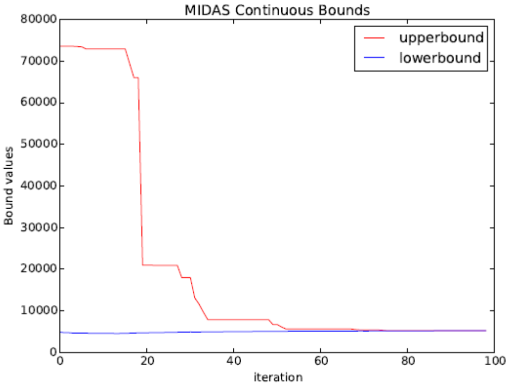
$$\begin{array}{ll} \varphi \leq v_h, & h \in \mathcal{G}, \\ x_k \geq 0 + \delta, & k = 1, \ldots, n, \end{array}$$

so this will result in

$$\varphi \leq \min\{v_h \mid h \in \mathcal{G}\},$$

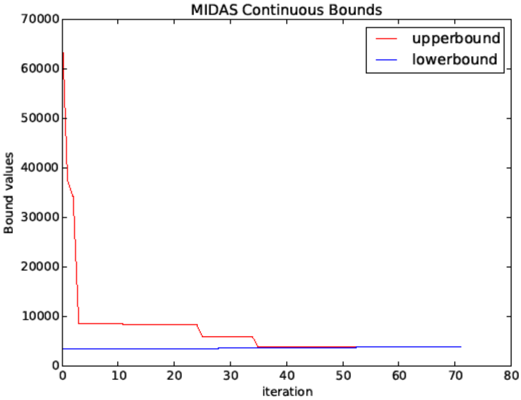
as desired.

Epsilon upper and lower bounds for single reservoir problem



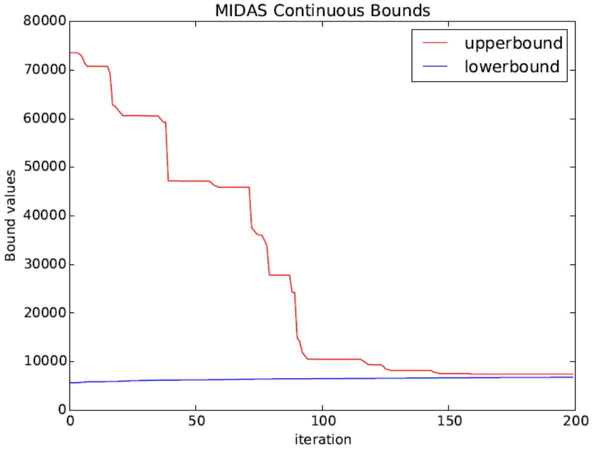
With $\delta = 30$

Epsilon upper and lower bounds for single reservoir problem



With $\delta = 50$

Epsilon upper and lower bounds for single reservoir problem



With $\delta = 10$