

# Applying Multi-stage Stochastic Programming to Investment Planning for Energy Systems

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# Outline



Background on emissions targets

Investment planning models

- Scenarios

- Multi-stage optimization

JuDGE: Julia-based Decomposition for General Expansion

- Problem Structure

- Interface

Case-study

- Assumptions / Simplifications

- Results

Conclusions

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## Energy

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New Zealand has set a target under the Paris Agreement to reduce its greenhouse gas emissions by 30% below 2005 levels by 2030, and to adopt increasingly more ambitious targets in the future.

The Government has asked us to deliver evidence and analysis to a proposed Climate Change Commission. Regarding electricity, the Commission will be expected to make recommendations on:

*Planning for the transition to 100% renewable electricity by 2035 (which includes geothermal) in a normal hydrological year.*

DEEP DIVE

## California strives to nix its natural gas habit without letting the lights go out

The state relies on natural gas for reliability, but won't reach its 100% zero emissions goal without looking to alternatives.

AUTHOR

Herman K. Trabish

**W**hen California's lawmakers mandated 60% renewables by 2030 and targeted zero emissions by 2045 in Senate Bill 100, passed in September, they took on never-before-answered questions about reliability.

<https://www.utilitydive.com>

# What does 100% renewable mean?

- ▶ Permanently shutdown thermal plant?
- ▶ “100% in a **normal hydrological year**.”
- ▶ Keep thermal units, but use only in a low-hydrology year?
  - ▶ e.g. ex-post identification of a “normal” year;
  - ▶ using thermal plant in at most 50% of historical years is very expensive.
- ▶ Real aim of climate policy is to **control average GHG emissions** from electricity generation to below an accepted threshold.
- ▶ Planning for future years involves uncertainty, so we need **stochastic models** which account for:
  - ▶ uncertain long-term demand growth e.g. in electric vehicles;
  - ▶ the fact that capacity plans that affect hydro security of supply.

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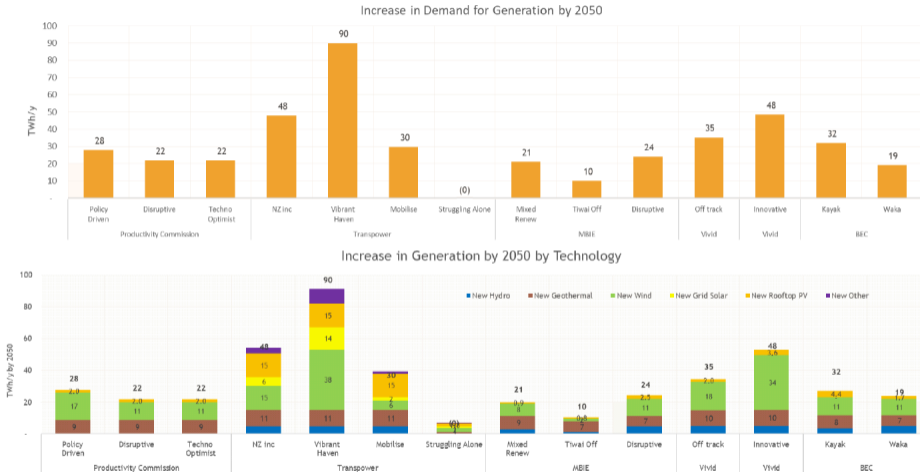
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# Scenarios for the future

## What investments should be made?



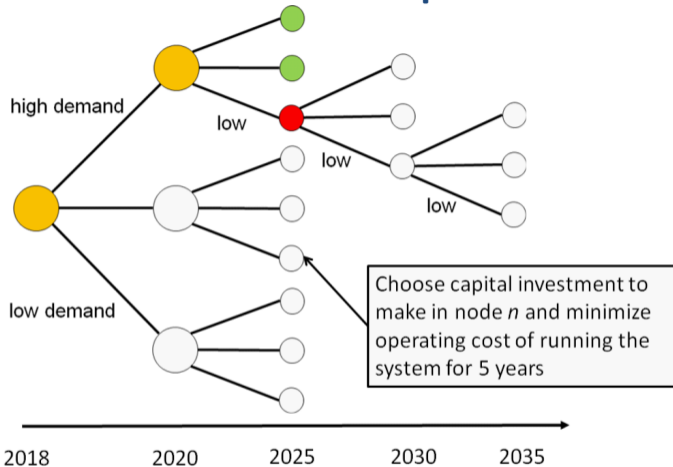
14 scenarios for electricity demand and generation mix in 2050.

There are 14 different 'optimal' plans: which should be implemented, if any?



# Multi-stage stochastic investment problem

## Scenario tree



Options to delay investment when in the yellow nodes until more information accrues. In green nodes invest and in red node don't. We call this multistage model **GEMstone**.<sup>2</sup>

<sup>2</sup>Girardeau P. & Philpott A.B., GEMSTONE: a stochastic GEM (2011)  
<http://www.epoc.org.nz/workshops/ww2011/WW2011-girardeau.pdf>.

# Multi-stage stochastic investment problem

EMERALD



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<sup>3</sup>Downward A. & Philpott A.B. Average-cost infinite-horizon SDDP (2019).

# Multi-stage stochastic investment problem

## EMERALD



- ▶ Emissions Mitigation for Electricity with Reservoir Approximation and Long-term Demand growth.

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# Multi-stage stochastic investment problem

## EMERALD



- ▶ Emissions Mitigation for Electricity with Reservoir Approximation and Long-term Demand growth.
- ▶ Ideally, for each node in the tree, once we choose the generation and transmission investments, we would find a **hydro release policy** which minimizes expected variable costs per year of meeting demand (with possible load shedding).
- ▶ State of the art algorithm for hydro-thermal optimization is **Stochastic Dual Dynamic Programming** (SDDP). [Pereira and Pinto, 1991]
- ▶ SDDP optimizes a stochastic control problem by **estimating the value of water** at each stage, given the the **expected future cost** of meeting demand.

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- ▶ SDDP optimizes a stochastic control problem by **estimating the value of water** at each stage, given the the **expected future cost** of meeting demand.
- ▶ We need a version of SDDP that minimizes **average cost per year**, say  $\mathbb{E}[Z]$ .<sup>3</sup>
- ▶ Unfortunately, it's computationally intractable to embed this within a **stochastic program for investment**.

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# Scenarios for the future

## Hydrology Approximations



In order to maintain tractability in a long-term planning model, we consider four seasons of hydro inflows over a number of years.

We model two types of hydro plant: **run-of-river** and **reservoir storage**.

- ▶ For the run-of-river plants, we modify to available seasonal capacity (MW), based on the type of year.
- ▶ For reservoir storage, at the beginning of the year, we choose hydro storage targets for the end of each season, and ensure that regardless of the inflows, we reach these targets (either by increase or decreasing the output (MWh) from the plant over the season.

# Multistage stochastic investment problem

## Stage problem

$$Q_t(X_{t-1}, \omega) = \min \sum_{p \in \mathcal{P}} C^{p, \omega} (X_t^p - X_{t-1}^p) + \sum_{p \in \mathcal{P}} \sum_{h \in \mathcal{H}} c^{p, \omega} x_h^{p, \omega} + \rho Q_{t+1}(X_{t+1})$$

subject to  $X_t^p \in \mathcal{X}^p$

$$0 \leq x_h^{p, \omega} \leq \alpha_h^p X_h \quad \forall p \in \mathcal{P}, h \in \mathcal{H}$$
$$W_s^p - \sum_{h \in \mathcal{S}} x_h^{p, \omega} - G_s^p(\omega) \leq W_{s \% 4 + 1}^p \quad \forall p \in P_H, s \in \{1, 2, 3, 4\}$$
$$\sum_{p \in \mathcal{P}} x_h^{p, \omega} = d_t(\omega) \quad \forall h \in \mathcal{H}$$

where  $Q_{t+1}(X) = \mathbb{E}_\omega [Q_{t+1}(X_{t+1}, \omega)]$ .

# Multistage stochastic investment problem

## Stage problem



$$\begin{aligned} \text{Cost-to-go}(t) = \min & \quad \text{Investment cost}(t) + \text{Operation Cost}(t) + \text{Cost-to-go}(t + 1) \\ \text{subject to} & \quad (\text{Plant build capacity}) \\ & \quad (\text{Plant availability by period}) \\ & \quad (\text{Seasonal hydro restrictions}) \\ & \quad (\text{Demand balance}) \end{aligned}$$



# Multistage stochastic investment problem

JuDGE



Unfortunately, due to the **lumpiness of investments** in a small market such as New Zealand's, we cannot assume continuous investments, and must consider individual projects, and they will either be chosen to be built in a particular period or not. This involves a number of binary variables, meaning that **SDDP.jl**<sup>4</sup> cannot be utilised.

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# Multistage stochastic investment problem

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We have developed an alternative multi-stage stochastic programming engine in Julia, called **JuDGE**<sup>5</sup> (Julia Decomposition for General Expansion). This implements **Dantzig-Wolfe decomposition** on multistage investment problems described over trees.

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# JuDGE

What type of problems can be modelled in JuDGE?



JuDGE is a Julia/JuMP-based package that enables easy construction of multi-stage stochastic capacity expansion problems.

## What type of problems can be modelled in JuDGE?

JuDGE is a Julia/JuMP-based package that enables easy construction of multi-stage stochastic capacity expansion problems.

- ▶  $\mathcal{N}$  is the set of nodes in the scenario tree;
- ▶  $\phi_n$  the probability of the state of the world  $n$  occurring;
- ▶  $\mathcal{P}_n$  the set of nodes on the path to (and including) node  $n$ ;
- ▶  $m$  is the number of binary expansion variables;
- ▶  $z_n$  are the binary variables for the expansions;
- ▶  $y_n$  is the variable vector for stage-problem  $n$ ;
- ▶  $\mathcal{Y}_n$  is the stage-problem feasibility set.

$$\begin{aligned} \min_{y,z} \quad & \sum_{n \in \mathcal{N}} \phi_n (c_n^\top z_n + q_n^\top y_n) \\ \text{s.t.} \quad & A_n y_n \leq b + D \sum_{h \in \mathcal{P}_n} z_h, \quad \forall n \in \mathcal{N}, \\ & y_n \in \mathcal{Y}_n, \quad \forall n \in \mathcal{N}, \\ & \sum_{h \in \mathcal{P}_n} z_h \leq \mathbf{1}^m, \quad \forall n \in \mathcal{N}, \\ & z_n \in \{0, 1\}^m, \quad \forall n \in \mathcal{N}. \end{aligned}$$

## What type of problems can be modelled in JuDGE?

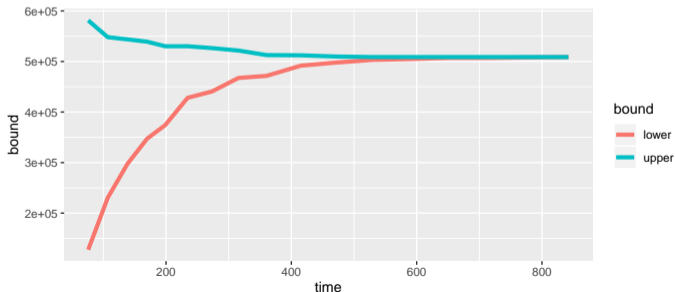
JuDGE is a Julia/JuMP-based package that enables easy construction of multi-stage stochastic capacity expansion problems.

JuDGE implements a **Dantzig-Wolfe decomposition** of the problem by automatically constructing a master problem that handles the investment decisions, and generates columns from the stage-problems.

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JuDGE has several benefits compared to formulating a multi-stage stochastic investment problem directly.

- ▶ It separates the structure of the scenario tree from the stage-problem.
- ▶ Decomposition allows problems to be solved that would otherwise be too large.
- ▶ We still compute lower and upper bounds, like a MIP.



The interface to JuDGE involves:

- ▶ defining the tree, which enables custom data to be stored at each node (the tree can be custom-built, or specified in terms of depth, degree. To construct a binary tree you type:

```
mytree = buildtree(depth=5,degree=2)
m = JuDGEModel(mytree)
```

- ▶ Investments are defined as custom JuMP variables:

```
JuDGEexpansions!(m) do sp
    @expansion(sp,bags[1:2])
end
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    bags = expansion[:bags]
    @expression(master, sum(n.data.cost[o]*bags[o] for o in 1:2))
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JuDGEsubproblem!(m) do sp,n,expansion
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- ▶ finally we solve the problem:

```
JuDGEsolve!(m,GurobiSolver(OutputFlag=0)) do time, iter, lb, ub
    println(lb," ",ub," ",time)
    if time > 10 || iter > 200
        return true
    end
    if (ub - lb) < 0.001
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# New Zealand Electricity Market Overview

## Generation Locations

### NON-RENEWABLE



#### FOSSIL FUELS

Includes coal, oil and natural gas. The energy comes from the fossilised remains of plants and animals from millions of years ago.

### RENEWABLE



#### HYDRO ENERGY

Energy created by falling water



#### WIND ENERGY

Energy from the force of wind



#### GEOTHERMAL ENERGY

Energy from underground steam



#### SOLAR ENERGY

Energy from the sun



#### BIOENERGY

Fuel/Energy from waste materials



#### MARINE ENERGY

Energy from tidal movements and waves



We will now present a case-study inspired by New Zealand.

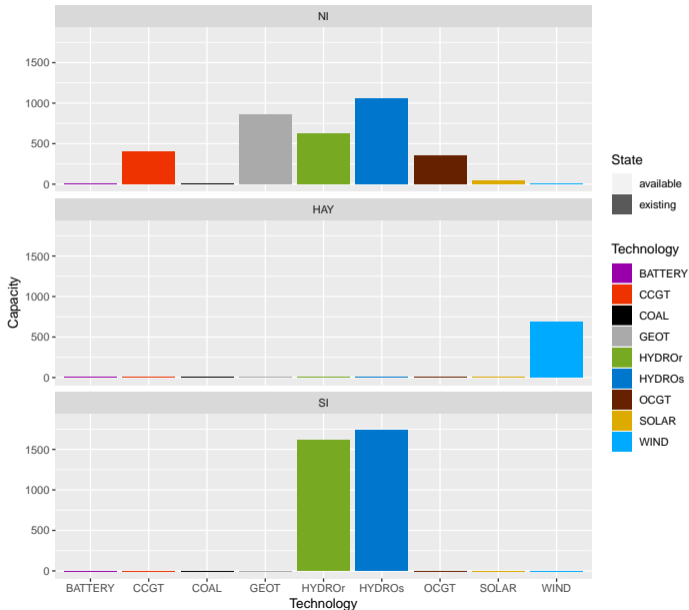
- ▶ We model New Zealand as three nodes.
- ▶ We will explore both carbon-price trajectories, and explicit restrictions on non-renewables.
- ▶ We use 30 load blocks / wind scenarios, for each of the four seasons.
- ▶ We use 13 historical years to calibrate seasonal hydrological inflows.
- ▶ Data is based on Michael Ferris and Andy Philpott's two-stage renewables model.<sup>6</sup>

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<sup>6</sup>Ferris M.C. & Philpott A.B. 100% Renewables with Storage (2019)  
<http://www.epoc.org.nz/papers/100PercentOperResv10.pdf>.

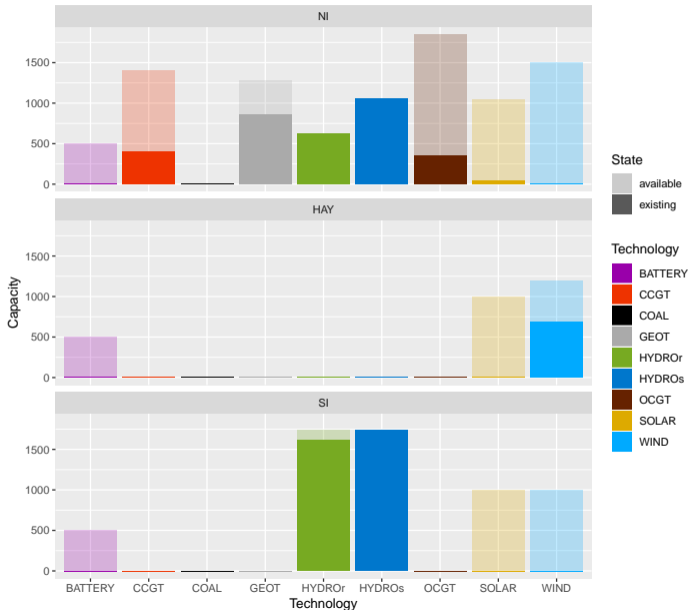
# Model Inputs

## Technologies



# Model Inputs

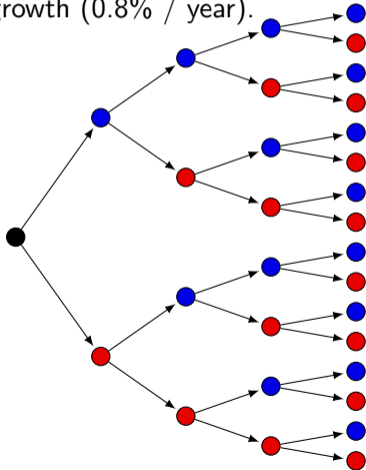
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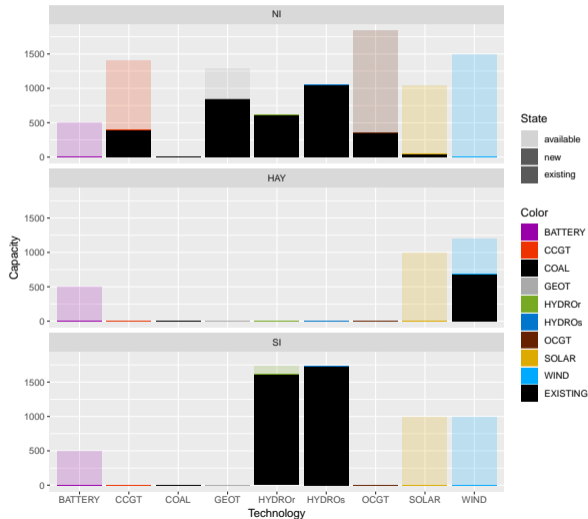
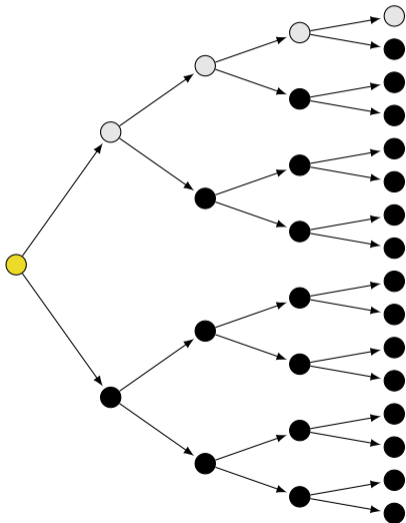
## Scenario Tree

In this tree, we model 2020–2050; each node represents 6 years of the electricity system. Blue nodes correspond to high demand growth (1.6% / year), and red nodes correspond to low demand growth (0.8% / year).



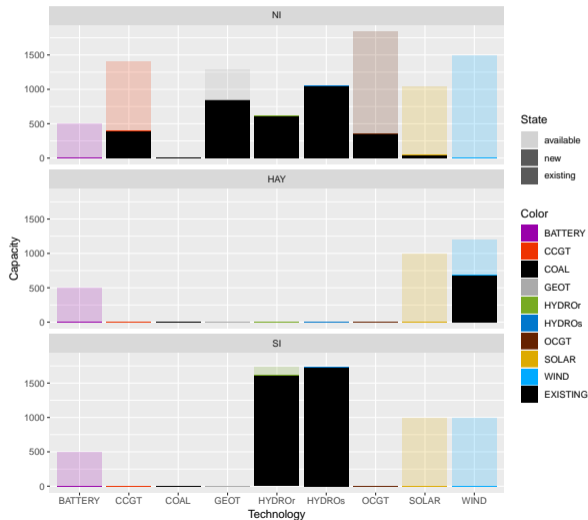
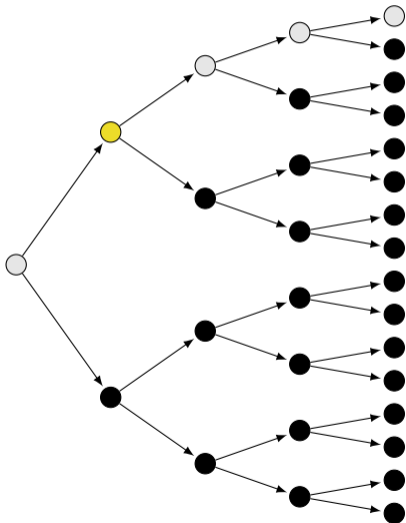
# Results

## High Demand Trajectory



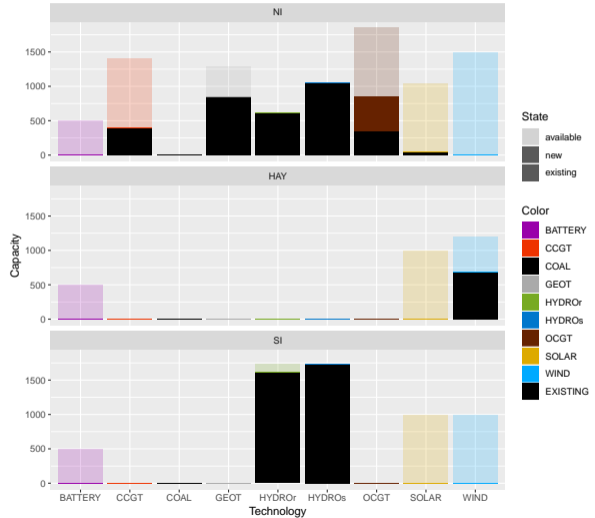
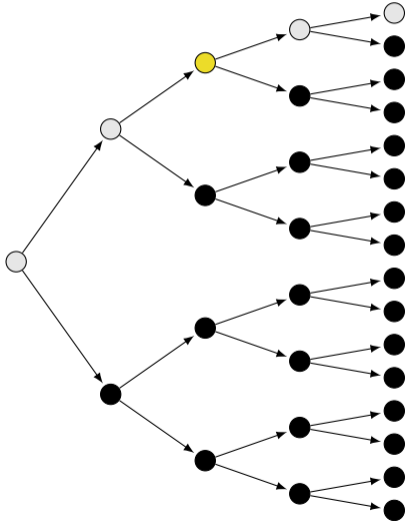
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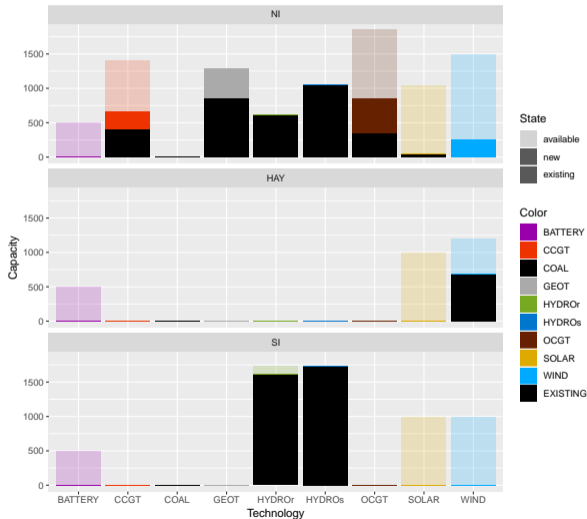
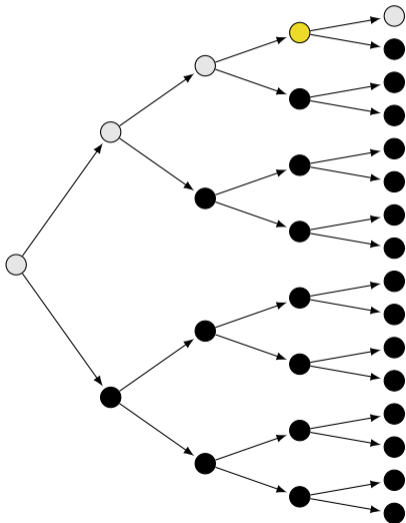
## High Demand Trajectory





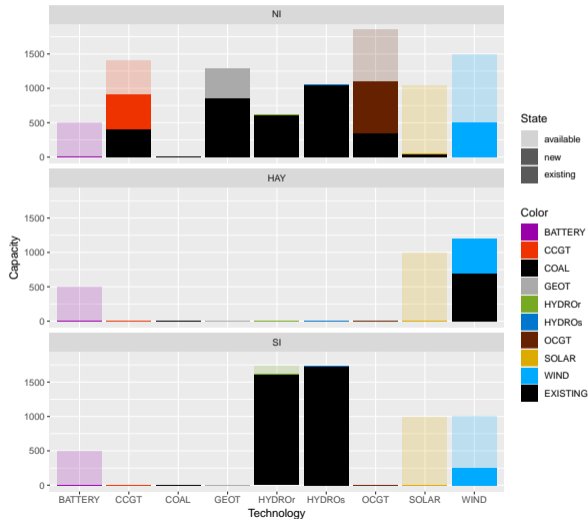
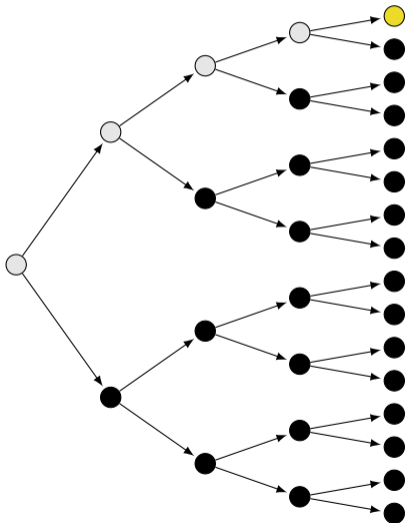
# Results

## High Demand Trajectory



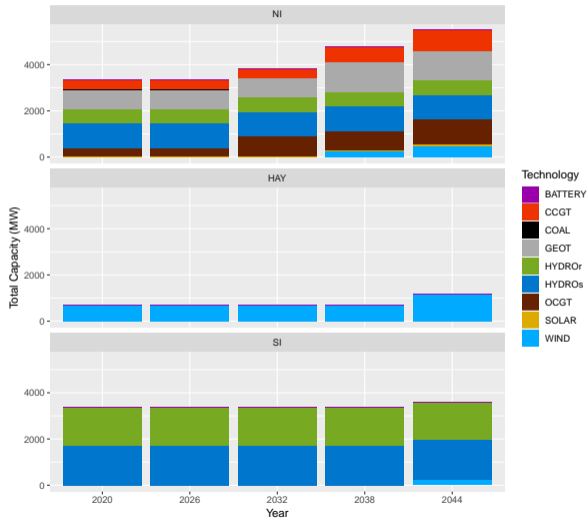
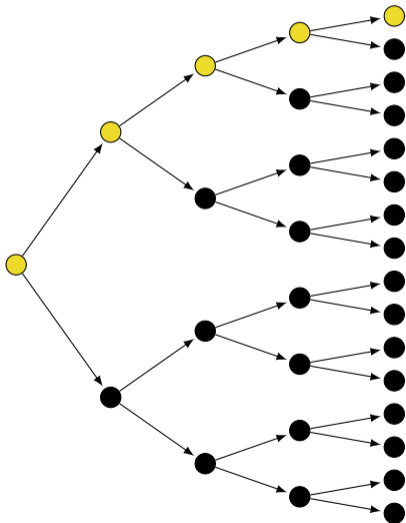
# Results

## High Demand Trajectory



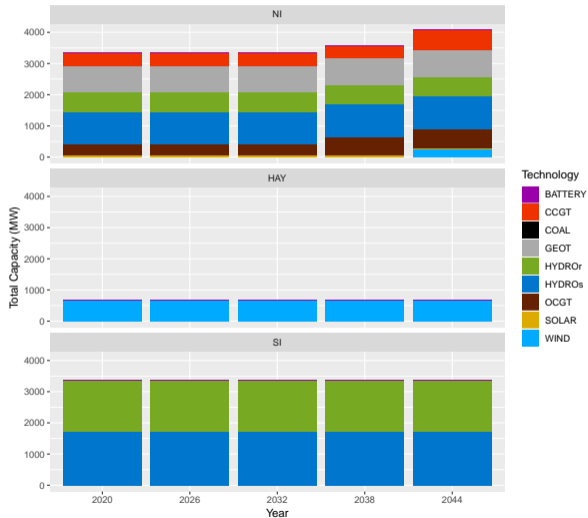
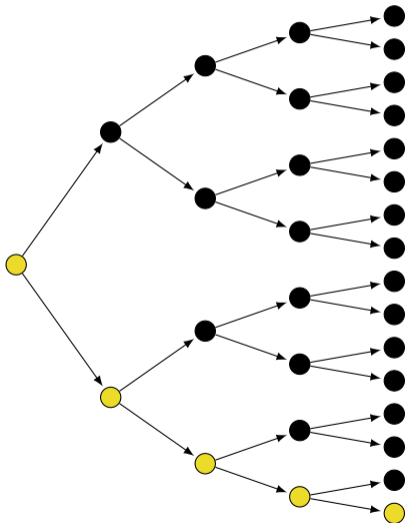
# Results

## High Demand Trajectory



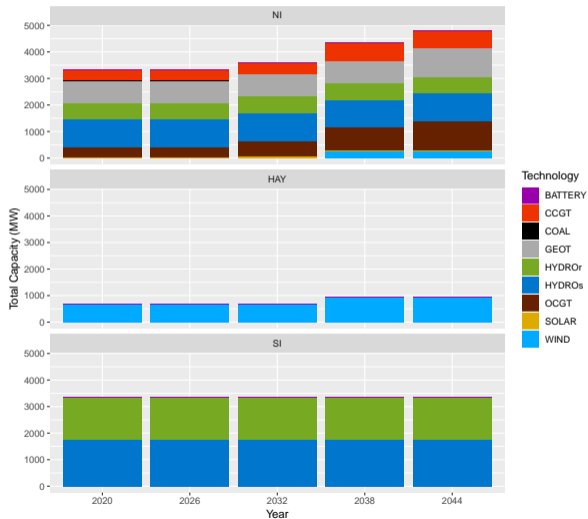
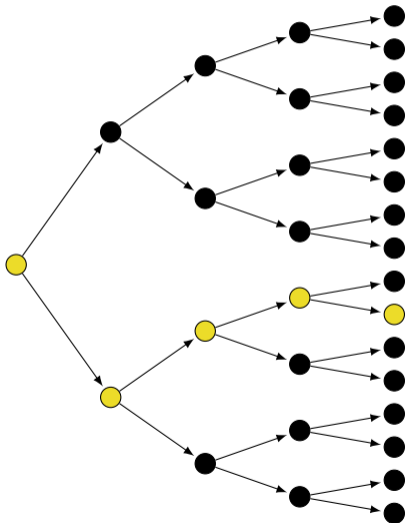
# Results

## Low Demand Trajectory



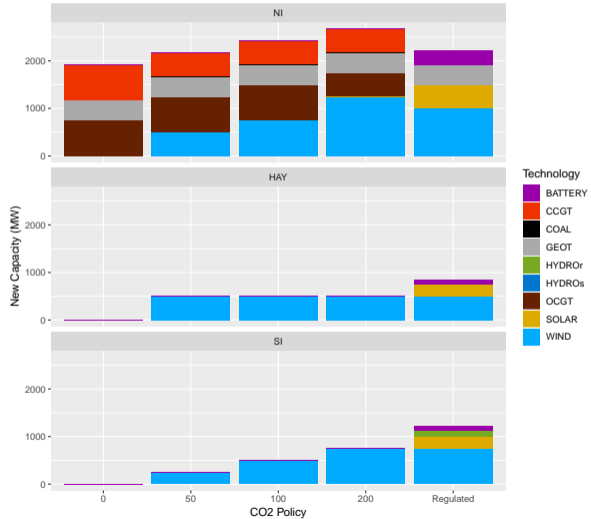
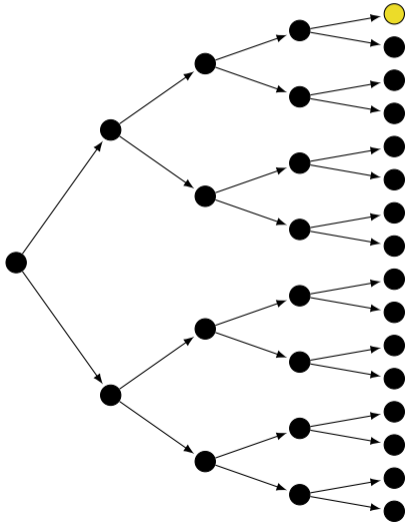
# Results

## Medium Demand Trajectory



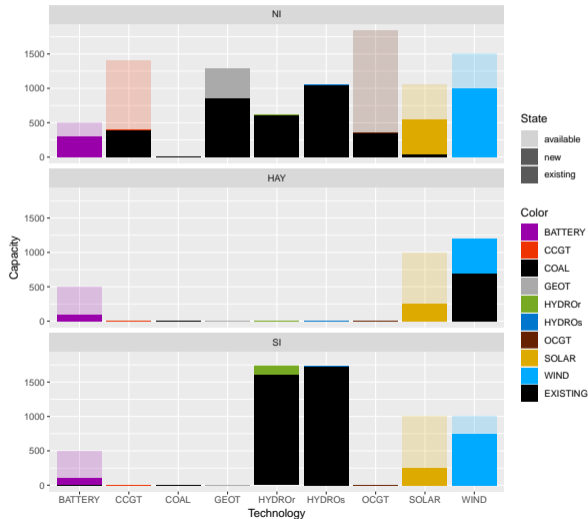
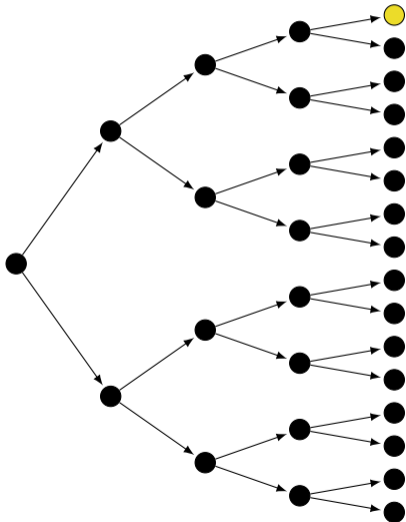
# Results

## Carbon Policy Comparison (Investments)



# Results

## Carbon Policy: 100% Renewables



# Outline



Background on emissions targets

Investment planning models

- Scenarios

- Multi-stage optimization

JuDGE: Julia-based Decomposition for General Expansion

- Problem Structure

- Interface

Case-study

- Assumptions / Simplifications

- Results

Conclusions



# Conclusions

## Summary



- ▶ We have developed a Julia package, JuDGE, that enables easy modelling of multistage stochastic binary capacity expansion problems, with mixed-integer stage-problems.
- ▶ We have applied JuDGE to the problem of decarbonising New Zealand's electricity sector, creating a model called EMERALD.
- ▶ We find that directly regulating the use of non-renewable technologies leads to the deployment of solar and battery technology, which would not be optimal with reasonable prices for CO<sub>2</sub> emissions.

# Conclusions

## Summary



- ▶ We have developed a Julia package, JuDGE, that enables easy modelling of multistage stochastic binary capacity expansion problems, with mixed-integer stage-problems.
- ▶ We have applied JuDGE to the problem of decarbonising New Zealand's electricity sector, creating a model called EMERALD.
- ▶ We find that directly regulating the use of non-renewable technologies leads to the deployment of solar and battery technology, which would not be optimal with reasonable prices for CO<sub>2</sub> emissions.
- ▶ We plan to continue to make enhancements to JuDGE; including: upgrading to Julia 1.\*, adding multi-threading, and extending the types of models that can be solved.
- ▶ We need to further calibrate the EMERALD model enhancing the detail captured in the stage-problem, and the size of the tree.

**Thanks for your attention.**

**Any questions?**

JuDGE.jl Julia Library <https://github.com/reganbaucke/JuDGE.jl>

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