Appendix 2: Determining the correct value for δ

A key question in our analysis is to determine the correct value of δ in a network setting with transmission losses and constraints and step-function stacks. We attempt to shed some light on this in this appendix.

Single Nodes

To study this problem we make some assumptions. For simplicity we begin with a single node market. We shall assume that the errors in load are unbiased and that the errors in prices that come from the load errors are unbiased. (It would be an interesting project to study whether this assumption is correct. Offer stacks might be chosen deliberately so that dispatches occur close to the upper end of each tranche, which would give an upward bias to prices under our assumptions on load errors.) Under our assumptions, the bias in payments is a direct consequence of a positive correlation between loads and prices.

Let D be the (random) load at the node, and $\pi(D)$ the (random) price that pertains at the node. We shall assume throughout this document that the dispatch problem is a linear program in which the offer curves are step functions. This means that $\pi(D)$ is a nondecreasing step function, so it is natural in this setting to assume that prices and loads are correlated. Our assumption on the distribution of D is that $E[D] = d$, the true load, and $E[\pi(D)] = \pi$, the true nodal price. The range of D will typically be small, and so it is also natural to assume that $D \geq 0$ and that there is some (sufficiently large) perturbation Δ so that $\pi(D - \Delta) \leq \pi$ for all D.

Proposition 1 *Suppose at a single node that load and spot price has a strictly positive correlation, and that errors in load and price have zero expectation. Then payments by loads are biased upwards.*

Proof. The excess payment in expectation is

$$E[\pi(D)D] - \pi d = E[\pi(D)D] - E[\pi(D]E[D] = cov(D, \pi(D)) > 0$$

because of a strictly positive correlation. ■

Can we always find a δ to make the expected payment $E[\pi(D - \delta)D]$ match up with πd, i.e. what is required exactly? When the offer curves are step functions, our analysis breaks down when the errors have a discrete distribution since there could be a nonzero probability that an error in demand placed demand exactly on a tranche boundary. However, as long as the error distribution can be represented by a density function, then we can show that there exists δ to make the expected payment match up with what is required in the case of no errors.

Proposition 2 *Suppose at a single node that load and spot price has a strictly positive correlation, and that errors in load and price have zero expectation, where the errors in load have a density. Then there exists δ so that*

$$E[\pi(D - \delta)D] = \pi d.$$

Proof. Let D have density f, and consider the function

$$\begin{aligned} g(z) &= E[\pi(D-z)D] \\ &= \int_{-\infty}^{+\infty} \pi(x-z)xf(x)dx \end{aligned}$$

Now $g(z)$ is continuous, even though the step function $\pi(y)$ has (possibly) some jump discontinuities. Now by Proposition 1 $g(0) > \pi d$, and there is some Δ with $\pi(D-\Delta) \leq \pi$ for all D. Thus $g(\Delta) \leq \pi d$. If $g(\Delta) = \pi d$, then $\delta = \Delta$ gives the result, otherwise $g(\Delta) < \pi d$, and so there is some $\bar{z} \in (0, \Delta)$ with $g(\bar{z}) = \pi d$. Setting $\delta = \bar{z}$ gives the result. ■

Transmission Networks

The discussion above has applied only to a single node. It is tempting to suppose that it may be applied at any node of a transmission system. This can be done with some care since the measurement errors in the load (the vector $\mathbf{D}$) now have a multi-variate distribution. Moreover even if these errors are independent, any error in load at one node (possibly) affects the shadow prices at all other nodes, so the prices $\pi_i(\mathbf{D})$ are not independent

It is therefore not clear that a single run of the dispatch model with perturbed loads will yield a set of (random) prices that give the exact payments in expectation for every node. (The resolution of this problem amounts to showing that n simultaneous nonlinear equations in n variables has a solution. It is very unlikely that this is the case for all choices of data.)

Suppose, on the other hand that a single node i, say, wished to pay the correct amount for its power. It is possible to select a value of δ_i so that decreasing the (random) load D_i at node i by δ_i (and not changing any other loads) will alter the marginal distribution of the price π_i by enough so that $E[\pi_i(\mathbf{D} - \delta_i e_i)D_i] = \pi d$.

Observe that computing such δ_i for each node and perturbing all nodal loads by δ_i simultaneuously is likely to lead to an underestimate of the true payments. So to apply the procedure in practice means that each node i requires its own re-solve of SPD with its load decreased by δ_i (and all others left unchanged). If node i then pays the (random) amount $\pi_i(\mathbf{D} - \delta_i e_i)D_i$, on average it will pay the true amount $\pi_i d_i$. Of course if this is done for every node then the dispatch model must be solved many times for each trading period. This would seem to indicate a need to limit the application of this to only some trading periods and some nodes.