On revenue adequacy of financial transmission right auctions

Andy Philpott *

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1 Introduction

It is well known (and demonstrated in [1]) that revenue adequacy of financial transmission rights in nodal electricity markets is guaranteed through the *simultaneous feasibility test* for the general case of economic dispatch problems when the transmission constraints are convex. This short note explores this result when applied to financial transmission rights between hubs.

Consider the nodal economic dispatch problem given by

$$\begin{array}{lll} \text{SPD: minimize} & \sum_i \sum_{j\in O(i)} c_j x_j & \\ \text{subject to} & g_i(f) + \sum_{j\in O(i)} x_j = d_i, \quad i = 1,2,\cdots,n & [\pi_i] \\ & x \in X & \\ & f \in U. & \end{array}$$

Here,

x_j is the level of dispatch of tranche $j \in O(i)$ where $O(i)$ indicates the set of offered tranches at node i of the transmission network.

c_j is the offer price (therefore the cost to the system) of tranche $j \in O(i)$.

d_i is the demand at node i.

f denotes the vector of flows on the transmission network links.

π_i is the nodal price at node i.

$g_i(f)$ is a concave function that gives the amount of power flow entering node i when link flows are f. The function g takes account of any power losses in the network.

X is a convex set that defines the tranche levels.

*Electric Power Optimization Centre, University of Auckland

We assume that the set of flows f lies in a convex set U that encapsulates any line capacities and other electrical constraints such as the loop flow constraints.

The total constraint and loss rentals from this dispatch problem are defined by

$$\sum_i \pi_i (d_i - \sum_{j \in O(i)} x_j)$$

which can be shown (see [1]) to be equal to $\sum_i \pi_i g_i(f)$.

2 Rentals without losses

An alternative formulation of SPD without losses is

$$\begin{array}{lll} \text{SPD: minimize} & \sum_i \sum_{j \in O(i)} c_j x_j & \\ \text{subject to} & e_i^\top Af \geq d_i - \sum_{j \in O(i)} x_j \quad i = 1, 2, \cdots, n & [\pi_i] \\ & x \in X, & \\ & f \in F, & \\ & -K_k \leq f_k \leq K_k, & [\sigma_k, \rho_k] \end{array}$$

Here we replace $g_i(f)$ by $e_i^\top Af$ where A is the node-arc incidence matrix of the network, defined by

$$a_{ik} = \begin{cases} -1, & \text{if arc } k \text{ is oriented away from node } i, \\ 1, & \text{if arc } k \text{ is oriented towards node } i, \\ 0, & \text{otherwise,} \end{cases}$$

and decompose U into $\{f : f \in F, -K \leq f \leq K\}$. Here F can be thought of as the set of loop-flow constraints for instance.

The new formulation has Lagrangian.

$$\begin{aligned} \mathcal{L}(x, f) &= \sum_i \sum_{j \in O(i)} c_j x_j + \sum_i \pi_i (d_i - \sum_{j \in O(i)} x_j - e_i^\top Af) \\ &\quad + \rho^\top (f - K) + \sigma^\top (-K - f) \end{aligned}$$

Suppose (x^*, f^*) is an optimal solution to SPD. Then assuming that a constraint quailification holds, the Lagrangian duality theorem implies that there exist Lagrange multipliers $\pi_i, \sigma_k, \rho_k \geq 0$ with

$$\mathcal{L}(x^*, f^*) \leq \mathcal{L}(x, f), \text{ for all } x \in X,\ f \in F.$$

Observe that by complementary slackness we have

$$\sum_i \pi_i (d_i - \sum_{j \in O(i)} x_j^* - e_i^\top A f^*) = 0 \tag{1}$$

and

$$\rho^\top (f^* - K) + \sigma^\top (-K - f^*) = 0 \tag{2}$$

and optimizing $\mathcal{L}(x, f)$ over $f \in F$ gives for every $f \in F$,

$$-\sum_i \pi_i e_i^\top A f^* + \sum_k \rho_k f_k^* - \sum_k \sigma_k f_k^* \leq -\sum_i \pi_i e_i^\top A f + \rho^\top f - \sigma^\top f \tag{3}$$

Now suppose that for some matrix Y, F is of the form $\{f : Yf = 0\}$. This would be the case for loop-flow constraints for example. Then $-\sum_i \pi_i e_i^\top A f + \rho^\top f - \sigma^\top f$ is unbounded below as a function of f, unless

$$-\sum_i \pi_i e_i^\top A f + \rho^\top f - \sigma^\top f = 0$$

for every $f \in F$. (We require this to ensure that (3) is true). In particular

$$-\sum_i \pi_i e_i^\top A f^* + \rho^\top f^* - \sigma^\top f^* = 0.$$

Now using (2) we have

$$-\sum_i \pi_i e_i^\top A f^* + \rho^\top K + \sigma^\top K = 0.$$

and so (1) yields

$$\begin{aligned} \sum_i \pi_i (d_i - \sum_{j \in O(i)} x_j^*) &= \sum_i e_i^\top A f^* & (4) \\ &= \rho^\top K + \sigma^\top K & (5) \end{aligned}$$

Thus we have the following result.

Proposition 1 *Suppose there are no losses and the branch flows f satisfy the constraints $Yf = 0$, $-K \leq f \leq K$ for some matrix Y. Then the transmission rentals are $\rho^\top K + \sigma^\top K$, where ρ and σ are the Lagrange multipliers of the line capacity constraints at the optimal economic dispatch.*

3 Rentals with losses

Now consider the case with losses defined by a concave function $g_i(f)$. Here g_i could be piecewise linear as in SPD, or

$$g_i(f) = \sum_{k\in F(i)} -f_k - \frac{r}{2}f_k^2 + \sum_{k\in T(i)} f_k - \frac{r}{2}f_k^2$$

where $F(i)$ is the set of arcs coming from node i and $T(i)$ is the set of arcs going to node i. In demonstrating the previous lemma we used

$$-\sum_i \pi_i e_i^\top A f^* + \sum_k \rho_k f_k^* - \sum_k \sigma_k f_k^* \leq -\sum_i \pi_i e_i^\top A f + \rho^\top f - \sigma^\top f$$

which becomes

$$-\sum_i \pi_i g_i(f^*) + \sum_k \rho_k f_k^* - \sum_k \sigma_k f_k^* \leq -\sum_i \pi_i g_i(f) + \rho^\top f - \sigma^\top f$$

for any $f \in F$. Since $0 \in F$, we obtain

$$-\sum_i \pi_i g_i(f^*) + \sum_k \rho_k f_k^* - \sum_k \sigma_k f_k^* \leq 0$$

Thus

$$\begin{aligned}\sum_i \pi_i(d_i - \sum_{j\in O(i)} x_j^*) &= \sum_i \pi_i g_i(f^*) \\ &\geq \rho^\top K + \sigma^\top K\end{aligned}$$

so the rental pool with losses will exceed the "no-loss congestion rentals" as we would expect.

4 Financial transmission rights

A *financial transmission right* (FTR) can be thought of as a vector h of nodal injections and offtakes, where $h_i < 0$ for injections and $h_i > 0$ for offtakes. The coupon payment from this vector is

$$\sum_i \pi_i h_i.$$

A *balanced FTR* of 1MW from node j to node k has only two nonzero components -1 at node j and +1 at node k.

5 Lossless FTRs

We will assume for the moment that there are no losses in the network. This means that $g(y)$ is linear (not piecewise linear concave as we would have with losses). A *static hub-to-hub FTR* is defined by two sets of nodes A and B, and fixed weights a_i, $i \in A$ and b_i, $i \in B$, where

$$\sum_{i \in A} a_i = \sum_{i \in B} b_i = 1.$$

Note that the weights are fixed, and are not necessarily proportional to observed generation (or demand), but could be chosen by the seller of the FTR to be proportional to their time averages. We do not require $a_i \geq 0$ or $b_i \geq 0$, but this will usually be the case in practice. The coupon payment for a static hub-to-hub FTR from A to B is the volume of the FTR multiplied by

$$\sum_{i \in B} b_i \pi_i - \sum_{i \in A} a_i \pi_i.$$

How many of these FTR contracts can we sell at auction? Observe that a single static hub-to-hub FTR is equivalent to an FTR in the original network with injections $-a_i$ and offtakes b_i. If we want to sell Q MW of such contracts then to ensure revenue adequacy we must ensure that they are simultaneously feasible, i.e. there exists a vector y such that

$$\begin{array}{lll} \text{SFT:} & g_i(y) = -Qa_i, & i \in A, \\ & g_i(y) = Qb_i, & i \in B, \\ & y \in U. & \end{array}$$

In the auction where bidder k bids β_k for a quantity q_k of such FTRs, we would seek to maximize the benefit from selling Q such instruments by solving

$$\begin{array}{lll} \text{FTR: maximize} & \sum_k \beta_k q_k & \\ & \sum_k q_k = Q, & \\ & g_i(y) = -Qa_i, & i \in A, \\ & g_i(y) = Qb_i, & i \in B, \\ & y \in U. & \end{array}$$

If we have an existing fixed rental allocation of Q_{mn} MW corresponding to a price difference between two nodes m and n, say, then we may remove these from the FTR auction. (These are also equivalent to what are called reserved transmission rights.) In the case where $m \in A$ and $n \in B$ we get

$$\begin{array}{lll} \text{FTR: maximize} & \sum_k \beta_k q_k & \\ & \sum_k q_k = Q & \\ & g_i(y) = -Qa_i, & i \in A \setminus \{m\}, \\ & g_i(y) = Qb_i, & i \in B \setminus \{n\}, \\ & g_m(y) = -Qa_m - Q_{mn} & \\ & g_n(y) = Qb_n + Q_{mn} & \\ & y \in U. & \end{array}$$

In the case where A is in the South Island, and B is in the North Island and mn is the HVDC link, if Q_{mn} is the capacity of the HVDC link then any solution to FTR cannot have have $Q > 0$ without incurring a risk of being revenue inadequate.

A special case of this setup is when A and B are single nodes, say Benmore and Otahuhu (and m and n are not in A and not in B respectively) and we get

$$\begin{array}{ll} \text{FTR: maximize} & \sum_k \beta_k q_k \\ & \sum_k q_k = Q \\ & g_A(y) = -Q, \\ & g_B(y) = Q, \\ & g_m(y) = -Q_{mn}, \\ & g_n(y) = Q_{mn}, \\ & y \in U. \end{array}$$

6 FTRs with losses

As shown in [1], simultaneous feasibility still ensures revenue adequacy in the presence of losses, as long as negative prices do not occur. Under this assumption, simultaneous feasibility of Q static hub-to-hub FTRs amounts to the existence of a vector (y, z) satisfying

$$\begin{array}{lll} \text{SFT:} & g_i(y) - z_i = -Qa_i, & i \in A, \\ & g_i(y) - z_i = Qb_i, & i \in B, \\ & z \geq 0, \ y \in U. & \end{array}$$

This means that there is free disposal of excess supply at any node.

If A and B were to partition the nodes of the original network into two sets joined by a single lossy line k, then our definition of a static hub-to-hub FTR will not suffice as by definition of g,

$$\sum_{i \in B} g_i(y) + \sum_{i \in A} g_i(y) = -\text{the loss on line } k$$

which is negative for any nonzero Q, but adding the constraints of SFT gives

$$\sum_{i \in B} g_i(y) + \sum_{i \in A} g_i(y) = \sum_{i \in A \cup B} z_i \geq 0$$

which is a contradiction. If we wish to preserve revenue adequacy via simultaneous feasibility then any hub-to-hub FTR must account for losses. This means that we need to choose the weights to account for line losses. In particular we need

$$\sum_{i \in B} b_i < \sum_{i \in A} a_i.$$

What is the appropriate choice of weights? Suppose in the example of the single lossy line, that the FTR applies to periods when the line has marginal losses of 10%. This means that if an agent sends $\sum_{i \in A} a_i$ MW from A to B, then 10% will be lost in transmission, and B expects to receive $0.9 \sum_{i \in A} a_i$. The transmission rental from this flow is then

$$\sum_{i \in B} \pi_i \left(0.9 a_i\right) - \sum_{i \in A} \pi_i a_i$$

So choosing $b_i = 0.9 a_i$ will provide the appropriate FTR for this hedge.

There is no reason why many different choices of scale factor could not be made corresponding to marginal losses under different congestion conditions. In other words there would be a collection of different FTR contracts for sale for different types of periods. On the other hand, having too many contracts for sale must be traded off against liquidity.

References

[1] A. Philpott and G. Pritchard. Financial transmission rights in convex pool markets. *Operations Research Letters*, 32(2):109–113, 2004.