Risk trading in capacity equilibrium models

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Outline

1. Competitive capacity equilibrium problems
   - RN competitive capacity equilibrium problems
   - Risk aversion and risk markets
   - Risky competitive capacity equilibrium problems
   - example of RN competitive capacity equilibria
2. Risky Design Games
   - Coherent risk measures
   - Risky design games with complete markets
   - Application to risky competitive capacity equilibrium
3. RN and risky Cournot capacity equilibrium problems
   - Risky Cournot capacity equilibrium with complete market
   - Example of competitive and Cournot capacity equilibria
4. Conclusion & references

Stochastic capacity equilibria under risk aversion for uncertain competitive or Cournot spot market

Electricity capacity expansion is kind of stochastic equilibrium

Stage 0. "Open Loop" Investment Today

- Agents invest in capacity and financial hedges
  - ▷ Agents are risk averse (risk neutrality is special case)
- Agents take a two or multi stage view of their cost/profit but not how their investments affect others

Stage 1. Spot Market in Uncertain Tomorrow

- Scenarios for different fuel & C prices, weather (demand) etc.
- Competitive spot market: Perfect Competition
  - ▷ Gencos set production given price, but don't act strategically
  - ▷ Price P_ω that clears market is endogenous to equilibrium
- Cournot spot market allows strategic production levels
  - ▷ Price is a known function of total quantity in market
  - ▷ Gencos see their effect on price but not on others'quantities

Why study stochastic equilibria under risk?

Gencos use long term capacity equilibrium models

- Forecast power prices, perhaps stochastically, eg, TIMES
- Put financial value of a commercial or physical investment
- Typically modelling (perfectly) competitive spot markets

Policy makers & regulators consume outputs of equilibrium models

- Forecast evolution of markets re supply and demand
- Assess economic in/efficiency of existing or proposed markets
- Typically interested in market power, eg, Cournot type

Managers are risk averse: Risk management in power industry

- Financial, eg, forward contracts or more exotic products
  Meridian-Genesis swaption
- Operational, eg, demand-side management by large consumers
- Strategic, eg, vertical integration of retailer and genco

What this talk doesn't cover

This presentation could be extended, in various ways, to discuss

- Multi stage capacity equilibrium problems problems
- Any number of technologies for gencos and consumers/retailers
- Capacity equilibrium in a multi commodity markets
- Electricity markets with congested transmission — LMPs

The analysis won't extend naturally to nonconvexity, eg,

- Nonconvex investment or production cost
- "Closed loop" models: Strategic capacity decisions, Stackelberg games, multi leader multi follower games, EPECs

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Notation for competitive spot market scenario ω given x

Given capacity x, spot market equilibrium in scenario $\omega = 1, \ldots, \Omega$:

Genco optimises production Y_ω given cap. x, price P_ω

$$\mathbf{V}_{\mathbf{G}\omega}(\mathbf{x}, \mathbf{P}_\omega) \; := \; \min_{Y_\omega} C_\omega\big(Y_\omega\big) \; - \; P_\omega Y_\omega \quad \text{s.t.} \quad 0 \le Y_\omega \le x$$

where $C_\omega(y) :=$ convex cost of producing quantity y

Retailer optimises consumption Q_ω given price P_ω

$$\mathbf{V}_{\mathbf{R}\omega}(\mathbf{P}_\omega) \; := \; \min_{Q_\omega} P_\omega Q_\omega - U_\omega(Q_\omega) \quad \text{s.t.} \quad Q_\omega \ge 0$$

where $U_\omega : \mathbb{R} \to \mathbb{R}$ is concave utility of consumption

Price of electricity P_ω clears spot market:

$$0 \; \le \; Y_\omega - Q_\omega \quad \perp \quad P_\omega \; \ge \; 0$$

Risk Neutral (RN) competitive capacity equilibrium problem

Suppose agents are **Risk Neutral** via probability $\Theta = (\Theta_\omega)_\omega$
Stage 0 is capacity investment with convex investment cost $I(x)$
Stage 1 is stochastic spot market as above

Genco sets investment x & views production $Y = (Y_\omega)_\omega$ via

$$\begin{array}{ll} \min\limits_{x,Y} & I(x) + \mathbb{E}_{\Theta}\Big[C_\omega(Y_\omega) - P_\omega Y_\omega\Big] \\ \text{s.t.} & x \in \mathcal{X},\ 0 \le Y_\omega \le x \text{ for all } \omega \end{array}$$

This is standard 2 stage stochastic program with recourse

Retailer sets consumption Q_ω in each spot scenario as before

Price of electricity P_ω clears spot market in each scenario

Outline

Risk Aversion in capacity equilibrium problems

We are interested in affect on equilibrium of agents' behaviour in

1. **risk aversion**
2. **risk trading**, or hedging via financial products

Take an agent with a stochastic cost $Z = (Z_\omega)$, e.g., Genco has uncertain production cost in Stage 1, $Z_G = (V_{G\omega}(x, P_\omega))$.

- Suppose probability Θ is agreed and fixed, whatever Z is
- Instead of assessing Z as $\mathbb{E}_\Theta[Z]$, or $\mathbb{E}_\Theta[V_{G\omega}(x, P_\omega)]$, the agent may have a utility function $\mathbf{r}(Z)$
- **Risk averse** means higher cost: $\mathbf{r}(Z) \geq \mathbb{E}_\Theta[Z]$ for all possible Z, with equality if $Z_\omega =$ constant independent of ω.

Risk trading means buying a set of financial securities / products / contracts $\mathbf{W} = (W_\omega)$ that pay out depending on ω:

- Genco with cost Z_G wants W with $r(Z_G - \mathbf{W}) \leq r(Z_G)$
- What contracts W are available? What would any W cost?

Some background on risk markets

Let's say Genco buys W_G to reduce $r_G(Z_G)$ to $r_G(Z_G - W_G)$
Likewise, Retailer buys W_R to reduce $r_R(Z_R)$ to $r_R(Z_R - W_R)$

In a competitive risk market, the price $P^{\mathrm{r}} = (P^{\mathrm{r}}_\omega)$ emerges

- Any contract W costs $P^{\mathrm{r}}[W] = \sum_\omega P^{\mathrm{r}}_\omega W_\omega$
- Agents optimize, e.g., Genco minimizes total cost

$$\min_{W_G} \quad P^{\mathrm{r}}[W] + r_G(Z_G - W_G) \quad \text{s.t. } W_G \in \mathcal{W}$$

- $\mathcal{W}$ is a subspace of traded risk products
  - ▷ Complete case: $\mathcal{W}$ = space of all uncertainties $\mathcal{Z} = \mathbb{R}^\Omega$
  - ▷ Incomplete case: $0 \in \mathcal{W}$ proper closed convex subset of $\mathcal{Z}$
- At equilibrium, P^{r} clears spot market: $W_G + W_R = 0$

For equilibrium to exist, risk measures should

- Be nice convex functions
- "Match" so that agents don't bet infinitely against each other

Outline

Risky competitive capacity equilibrium problem

Stage 0: Capacity investment & risk trading (may be in/complete)
Stage 1: Stochastic spot market.

Genco invests in capacity x & risk W_G, plans production Y via

$$\begin{array}{ll} \min\limits_{x,Y,W_G} & I(x) + P^{\mathrm{r}}[W_G] + r_G\Big(C_\omega(Y_\omega) - P_\omega Y_\omega - W_{G\omega}\Big) \\ \text{s.t.} & x \in \mathcal{X}, W_G \in \mathcal{W},\ 0 \le Y_\omega \le x \text{ for all } \omega \end{array}$$

Retailer trades risk W_R, plans consumption Q via

$$\begin{array}{ll} \min\limits_{Q,W_R} & P^{\mathrm{r}}[W_R] + r_R\Big(P_\omega Y_\omega - U_\omega(Q_\omega) - W_{R\omega}\Big) \\ \text{s.t.} & W_R \in \mathcal{W},\ 0 \le Q_\omega \text{ for all } \omega \end{array}$$

Price of electricity P_ω clears spot market in each scenario

Price of risk P^{r} clears risk market $W_G + W_R = 0$

Reformulating and solving risky capacity problems

In general, ie, for $\{0\} \subsetneq \mathcal{W} \subsetneq \mathcal{Z}$, reformulate risky competitive capacity equilibrium via KKT conditions. Solve the "all-KKT" problem as a large complementarity problem, possibly conic.

For $\mathcal{W} = \{0\}$, ie, no risk trading, this simplifies: omit Ws entirely. Fewer variables but still solved via all-KKT.

For complete case $\mathcal{W} = \mathcal{Z}$, a welfare-style simplification is possible

Theorem

[Ehrenmann-Smeers-11] [Philpott-etal-16] [deMaere-etal-17] Risky competitive capacity equilibrium with complete market $\Leftrightarrow$ risk averse System Planner minimising net cost, or maximising welfare

Outline

Small example of RN competitive capacity equilibria

One producer, annualized plant CAPEX $I = 90$ €/kW, operating cost $C = 60$ €/MWh over $\tau = 8760$ hours pa

RN case has 5 equally likely scenarios, $\Theta = (1/5, \ldots, 1/5)$

RN Genco solves

$$\min_{x,Y} Ix + \tau\, \mathbb{E}_\Theta\left[(C - P_\omega)Y_\omega\right] \text{ s.t. } 0 \le Y_\omega \le x.$$

Retailer has quadratic utility $U_\omega(Q_\omega) = A_\omega Q_\omega - \frac{B}{2}Q_\omega^2$

Linear demand intercepts: $(A_\omega) = (300, 350, 400, 450, 500)$

RN Retailer solves

$$\min_{Q} \tau\, \mathbb{E}_\Theta\left[P_\omega Q_\omega - A_\omega Q_\omega + \frac{B}{2}Q_\omega^2\right].$$

RN competitive capacity equilibrium: $x = 389MW$ with

Scenario ω	1	2	3	4	5	$\mathbb{E}_\Theta[\,]$
Q [MWh]	240	290	340	389	389	330
P [€/MWh]	60	60	60	61	111	70
Invest. margin [€/kW]	-90	-90	-90	-84	354	0

Small example of risky competitive capacity equilibria

Compare RN equilibrium to equilibria in three risky cases where details of risk aversion are given later

- Complete: $\mathcal{W} = \mathcal{Z} = \mathbb{R}^5$, all uncertainties are priced
- 1 product: $\mathcal{W}$ is 1-dimensional subspace
- 0 products: $\mathcal{W} = \{0\}$

Risk trading	Capacity	Mean Welfare	Mean Quantity	Mean Price	Mean Invest. margin
(RN)	389	491	330	70	0
Complete	349	388	314	87	142
1 product	348	387	313	87	144
0 products	339	376	309	90	176

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Coherent risk measures (CRMs)

[Artzner-etal-99] characterise a CRM r as **worst case expectation**

- $r(Z) = \max_{\Pi \in \mathcal{D}} \mathbb{E}_\Pi[Z]$ for any cost $Z \in \mathbb{R}^K$ . . . risk averse
  - ▷ $\mathcal{D}$ is nonempty closed convex set of probability measures
  - ▷ $\mathcal{D}$ is **risk set** of r
  - ▷ If $\mathcal{D}$ contains base or "physical" probability Θ as interior point then $r(Z) > \mathbb{E}_\Theta[Z]$ unless Z = constant
- CVaR/AVaR/E Tail Loss CRM in finance & optimization
  - ▷ Polyhedral, modelled via LP [Rock-Uryas-00]
  - ▷ Puts more probabilistic weight on bad outcomes
- Good Deal CRM adapted from finance
  - ▷ Conic risk set expands variance, CRM modelled as SOCP [Druenne-etal-11]
  - ▷ Derived from finance, calibrated via Sharpe ratio
  - ▷ Can be expressed via conic optimization
  - ▷ In above Example, agents have same Good Deal CRM

Outline

Risky design games

Consider agents $i = 1$ or 2. If $i = 1$, take $-i = 2$ and vice versa.

Agent i:

- Invests x_i, e.g., capacity, in a risky asset, e.g., cost or $-$profit stream from a plant: $\Xi_i(x_i, x_{-i}) = (\Xi_{i\omega}(x_i, x_{-i}))$
  - Chooses "design" x_i in closed convex "strategy" set $X_i \subset \mathbb{R}^{n_i}$
  - $x_i \mapsto \Xi_{i\omega}(x_i, x_{-i})$ is convex in each scenario ω
  - $I_i(x_i) =$ convex cost of design
- May be risk neutral, via $\mathbb{E}_\Theta[\cdot]$
- May be risk averse, via CRM $r_i = \sigma_{\mathcal{D}_i}$, and trade risk

The following will be used by a hypothetical System Planner:

- $\mathcal{D}_\mathbf{0} := \mathcal{D}_\mathbf{1} \cap \mathcal{D}_\mathbf{2}$ has nonempty interior relative to set of all probability measures
- $\mathbf{r_0} := \sigma_{\mathcal{D}_\mathbf{0}}(\cdot)$ is called **System CRM**

Design games and equilibrium problems

In **RN design game**, each agent i decides its design x_i via

$$\min_{x_i} \quad I_i(x_i) + \mathbb{E}_\Theta\left[\Xi_i(x_i, x_{-i})\right] \quad \text{s.t.} \quad x_i \in X_i\,.$$

This is an example of a stochastic Nash noncooperative game

In **risky design equilibrium problem** with in/complete market:

- Each agent i decides on (x_i, W_i) via

$$\min_{x_i, W_i} I_i(x_i) + P^{\mathrm{r}}[W_i] + r_i\big(\Xi_i(x_i, x_{-i}) - W_i\big) \quad \text{s.t.} \quad x_i \in X_i\,,\ W_i \in \mathcal{W}.$$

- Price of risk P^{r} clears risk market: $W_1 + W_2 = 0$

Theorem (R-Smeers-15)

In a complete risk market:
$(x_1, x_2, P^{\mathrm{r}})$ with some (W_1, W_2) is a risky design equilibrium
$\iff$ (i) (x_1, x_2) is equilibrium of RN design game with $\Theta = P^{\mathrm{r}}$
and (ii) $\Pi = P^{\mathrm{r}}$ solves

$$\max_{\Pi} \mathbb{E}_\Pi[\Xi_1(x_1, x_2) + \Xi_2(x_2, x_1)] \quad \text{s.t.} \quad \Pi \in \mathcal{D}_0$$

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1. Recall risky competitive capacity equilibrium problem with complete market

I. Genco invests in capacity x & risk W_G, plans production Y via

$$\begin{array}{ll} \min\limits_{x,Y,W_G} & I(x) + P^{\mathrm{r}}[W_G] + r_G\Big(C_\omega(Y_\omega) - P_\omega Y_\omega - W_{G\omega}\Big) \\ \text{s.t.} & x \in \mathcal{X},\ 0 \le Y_\omega \le x \text{ for all } \omega \end{array}$$

II. Retailer trades risk W_R, plans consumption Q via

$$\begin{array}{ll} \min\limits_{Q,W_R} & P^{\mathrm{r}}[W_R] + r_R\Big(P_\omega Y_\omega - U_\omega(Q_\omega) - W_{R\omega}\Big) \\ \text{s.t.} & 0 \le Q_\omega \text{ for all } \omega \end{array}$$

III. Price of electricity P_ω clears spot market in each scenario

IV. Price of risk P^{r} clears risk market $W_G + W_R = 0$

2. Apply risky design equilibrium theorem to I, II, IV

Risky competitive capacity equilibrium with complete market $\Leftrightarrow$

I′. RN Genco invests in capacity x, plans production Y via

$$\begin{aligned} \min_{x,Y} \quad & I(x) + \mathbb{E}_\Pi\Big[C_\omega(Y_\omega) - P_\omega Y_\omega\Big] \\ \text{s.t.} \quad & x \in \mathcal{X},\ 0 \le Y_\omega \le x \text{ for all } \omega \end{aligned}$$

II′. RN Retailer plans consumption Q via

$$\begin{aligned} \min_{Q} \quad & \mathbb{E}_\Pi\Big[P_\omega Y_\omega - U_\omega(Q_\omega)\Big] \\ \text{s.t.} \quad & 0 \le Q_\omega \text{ for all } \omega \end{aligned}$$

III. Price of electricity P_ω clears spot market in each scenario

IV′. Price of risk $\Pi = P^{\mathrm{r}}$ solves

$$\max_\Pi \mathbb{E}_\Pi\big[C_\omega(Y_\omega) - U_\omega(Q_\omega)\big] \quad \text{s.t.} \quad \Pi \in \mathcal{D}_0 := \mathcal{D}_G \cap \mathcal{D}_R$$

3. Apply classical RN welfare theory to I′, II′, III

Risky competitive capacity equilibrium with complete market $\Leftrightarrow$

I″. RN System Planner invests in capacity x, plans Y & Q via

$$\begin{array}{ll} \min\limits_{x,Y,Q} & I(x) + \mathbb{E}_\Pi\Big[C_\omega(Y_\omega) - U_\omega(Q_\omega)\Big] \\ \text{s.t.} & x \in \mathcal{X},\ 0 \le Y_\omega \le x,\ 0 \le Q_\omega \text{ for all } \omega \end{array}$$

IV′. Price of risk $\Pi = P^{\mathrm{r}}$ solves

$$\max_\Pi \mathbb{E}_\Pi\big[C_\omega(Y_\omega) - U_\omega(Q_\omega)\big] \quad \text{s.t.} \quad \Pi \in \mathcal{D}_0 := \mathcal{D}_G \cap \mathcal{D}_R$$

$\Longleftrightarrow$

Risk averse System Planner invests in capacity x, plans Y & Q via CRM $r_0 = \sigma_{\mathcal{D}_0}$ and

$$\begin{array}{ll} \min\limits_{x,Y,Q} & I(x) + r_0\Big(C_\omega(Y_\omega) - U_\omega(Q_\omega)\Big) \\ \text{s.t.} & x \in \mathcal{X},\ 0 \le Y_\omega \le x,\ 0 \le Q_\omega \text{ for all } \omega \end{array}$$

Outline

Notation for Cournot spot market scenario ω given x_i

For Genco $i = 1, 2$:

- $I_i(x)$ & $C_{i\omega}(y_i)$ are convex investment & production costs
- $p^{\mathrm{C}}_\omega(q)$ = market price for total quantity q
  - ▷ Assume $q \mapsto p^{\mathrm{C}}_\omega(q + q_0)q$ is concave given $q, q_0 \geq 0$
  - ▷ Hence Genco's net cost or −profit is convex in quantity
- **Genco** i **sets quantity** y_i, given x_i and y_{-i}, via convex problem

$$\min_y C_{i\omega}(y_i) - p^{\mathrm{C}}_\omega(y_1 + y_2)y_i \quad \text{s.t.} \quad 0 \leq y \leq x_i$$

Consumer $i = 3$, is a quantity-taker.

- Utility in spot market scenario ω is $U^C_\omega(Q_\omega) := \int_0^{Q_\omega} p^{\mathrm{C}}_\omega(q)dq$
- **Consumer** has surplus

$$p^{\mathrm{C}}(y_1 + y_2)(y_1 + y_2) - U^C_\omega(y_1 + y_2).$$

RN Cournot capacity equilibrium problems

Write $C_i(Y_i)$ for $\left(C_{i\omega}(Y_{i\omega})\right)$ and $p^{\mathrm{C}}(Y_i)Y_i$ for $\left(p^{\mathrm{C}}(Y_{i\omega})Y_{i\omega}\right)$ etc.

RN Cournot capacity equilibrium problem for $i = 1, 2$:

- **Genco** $i = 1, 2$ invests in capacity x_i, plans production Y_i

$$\begin{array}{ll} \min\limits_{x_i, Y_i} & I_i(x_i) + \mathbb{E}_\Pi\left[C_i(Y_i) - p^{\mathrm{C}}(Y_1 + Y_2)Y_i\right] \\ \text{s.t.} & x_i \in X_i,\ 0 \le Y_{i\omega} \le x_i \text{ for all } \omega, \end{array}$$

Remark. When price $p^{\mathrm{C}}(\cdot)$ is linear, RN Cournot capacity equilibrium problem has an equivalent optimization formulation. (This is more relevant for computation than economics.)

Risky Cournot capacity equilibrium problem

Risky Cournot capacity equilibrium problem with in/complete risk market:

- **Genco** $i = 1, 2$ invests capacity x_i & risk W_i, plans prod. Y_i

$$\begin{array}{ll} \min\limits_{x_i, Y_i, W_i} & I_i(x_i) + P^{\mathrm{r}}[W_i] + r_i\big(C_i(Y_i) - p^{\mathrm{C}}(Y_1 + Y_2)Y_i - W_i\big) \\ \text{s.t.} & x_i \in X_i,\ W_i \in \mathcal{W},\ \ 0 \le Y_{i\omega} \le x_i \text{ for all } \omega, \end{array}$$

- **Consumer** $i = 3$ invests in risk W_3

$$\min_{W_N} \quad P^{\mathrm{r}}[W_N] + r_3\big(p^{\mathrm{C}}_{\omega}(Y_1 + Y_2)(Y_1 + Y_2) - U(Y_1 + Y_2) - W_3\big),$$

- **Price of risk** P^{r} clears risk market $W_1 + W_2 + W_3 = 0$

Outline

Risky Cournot capacity equilibrium with complete risk market

Assume the system risk set $\mathcal{D}_0 := \mathcal{D}_1 \cap \mathcal{D}_2 \cap \mathcal{D}_3$ has interior relative to the set of all probabilities.

The prior theorem on risky design games gives:

Theorem

Under a complete financial market:
(x_1, Y_1), (x_2, Y_2), P^{r}, *with some* (W_1, W_2, W_3), *is a risky Cournot capacity equilibrium*
$\iff$ *(i)* (x_1, Y_1), (x_2, Y_2), *is a RN Cournot capacity equilibrium and (ii)* $\Pi = P^{\mathrm{r}}$ *solves*

$$\max_{\Pi} \mathbb{E}_{\Pi}\big[C_1(Y_1) + C_2(Y_2) - U(Y_1 + Y_2)\big] \quad \text{s.t.} \quad \Pi \in \mathcal{D}_0$$

This gives a Nash game, simpler than the risky equilibrium model.
It is particularly simple when $p^{\mathrm{C}}(\cdot)$ is linear...

Outline

Example of RN and risky Cournot capacity equilibria. 1.

Cournot spot market price is $p^{\mathrm{C}}_\omega(y) = A_\omega - By$.

RN Cournot capacity equilibrium problem

- 2 symmetric Gencos
- **Genco** i invests in capacity x_i, plans production Y_i

$$\min_{x_i, Y_i} Ix_i + \tau \mathbb{E}_\Theta\left[\big(C - A_\omega + B(Y_{1,\omega} + Y_{2,\omega})\big) Y_{i,\omega}\right] \quad \text{s.t.} \quad 0 \le Y_{i,\omega} \le x_i$$

- where I, C, A, B and Θ are as previously.

Example of RN and risky Cournot capacity equilibria 2.

Risky Cournot capacity equilibrium problem with in/complete risk market:

- 2 symmetric gencos; $r_1 = r_2 = r_3$, good deal risk measure.
- **Genco** i invests in capacity x_i & risk W_i, plans prod. Y_i

$$\begin{array}{ll} \min_{x_i, Y_i, W_i} & I x_i + P^{\mathrm{r}}[W_i] + \; r_i\Big(\tau\big(C - A_\omega + B(Y_{,\omega} + Y_{2,\omega})\big)Y_{i,\omega} - W_{i\omega}\Big) \\ \text{s.t.} & W_i \in \mathcal{W},\ 0 \le Y_{i,\omega} \le x_i \text{ for all } \omega \end{array}$$

- **Consumer** also trades risk W_3

$$\min_{W_3} \quad P^{\mathrm{r}}[W_3] + \; r_3\Big(- \tfrac{\tau B}{2}(Y_{1,\omega} + Y_{2,\omega})^2 - W_{3\omega}\Big) \text{ s.t. } W_3 \in \mathcal{W}$$

- **Price of risk** P^{r} clears risk market $W_1 + W_2 + W_3 = 0$

Comparison of RN & risky, competitive & Cournot capacity equilibria

Risk attitude	Spot market	Financial market	Welfare [M€]	Capacity [MW]	Strategic margin [€/kW]	Risk margin [€/kW]
Neutral	Compet.	-	491	389	-	0
Neutral	Cournot	-	436	259	841	0
Averse	Compet.	Complete	388	349	-	142
Averse	Compet.	1 product	388	348	-	144
Averse	Compet.	0 products	376	339	-	176
Averse	Cournot	Complete	345.4	234	838	131
Averse	Cournot	1 product	345	234	838	133
Averse	Cournot	0 products	342	230	838	155

$$\begin{aligned}\text{Investment Margin} \;&=\; \text{Risk Margin (when risk averse)}\\ &\quad+ \text{Strategic Margin (when Cournot)}\end{aligned}$$

$= \text{net profit of investing 1 unit capacity} = (\tau(P_\omega - C)Y_\omega - Ix)/x$

Notes on Investment, Strategic & Risk Margins

We breakdown the investment margin in two part: the strategic margin and the risk margin.

The strategic margin is due to the strategic behaviour of the duopoly and is equal to $(\frac{B}{2}Q_\omega) \times (\frac{\tau Q_\omega}{x}) = \tau B Q_\omega^2/(2x)$.

The risk margin is the other source of profit for an investment i.e. $(P_\omega - \frac{B}{2}Q_\omega) \times (\frac{\tau Q_\omega}{x}) - I$. The margin due to risk is similar to the risk neutral competitive case

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The risk margin is the other source of profit for an investment i.e. $(P_\omega - \frac{B}{2}Q_\omega) \times (\frac{\tau Q_\omega}{x}) - I$. The margin due to risk is similar to the risk neutral competitive case

Outline

Concluding thoughts

- We use risk trading to explore equilibria under risk aversion
  - Risk aversion is not an imperfection...
  - But incompleteness of financial markets is.
  - Risky capacity equilibria span competitive & strategic (Cournot) spot markets, complete & incomplete risk markets
- Risky capacity equilibria are, to some extent, tractable
  - Risky competitive capacity equilibrium with complete market $\Leftrightarrow$ risk averse optimization. **Valid in multistage case** [Philpott-etal-16] [deMaere-etal-17]
  - **Risky Cournot capacity equilibrium with complete market and linear demand** $\Leftrightarrow$ **Nash game with 2 agents** [deMaere-etal-17]
  - **Existence of risky equilibria in complete and incomplete cases comparable to RN cases** [deMaere-etal-17]
- Can risk trading be away to discuss non-financial hedging? What about issues like capacity markets?

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