Electricity offer optimization over consecutive trading periods

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Motivation

Much attention paid in the literature to single period optimization and equilibrium models, but...

- constraints and dynamics important over a trading day
- what are the key challenges in optimization in pool markets in this framework?
- put this in a revenue-optimization framework
- focus on a unit commitment problem for simplicity

Single-unit commitment

Assume generator offers power from a single unit with

operating range $q \in Q$,

operating cost $C(q)$,

start-up cost U,

shut-down cost D,

For each trading period $k = 1, \ldots, K$, first determine whether to offer, and then what curve to offer.

Dynamic programming recursion for unit commitment

Let $V_k(1)$ ($V_k(0)$) be the optimal expected profit the generator can make from the end of trading period k to the end of period K, if the unit is running (not running) at the end of period k. We assume that

$$V_K(0) = V_K(1) = 0.$$

Let

$$R_k = \max_{q \in Q}\{E[p](q - q_c) - C(q) + q_c p_c\}$$

and

$$S_k = q_c(p_c - E[p]).$$

This gives the recursion:

$$V_{k-1}(0) = \max\{S_k + V_k(0), R_k - U + V_k(1)\},$$

$$V_{k-1}(1) = \max\{S_k - D + V_k(0), R_k + V_k(1)\}.$$

Extension to a Markov price process

Let price follow a Markov process with states $\pi \in \Pi_k$. Then let

$$R_k(\pi) = \max_{q \in Q} \{E[p|\pi](q - q_c) - C(q) + q_c p_c\}$$

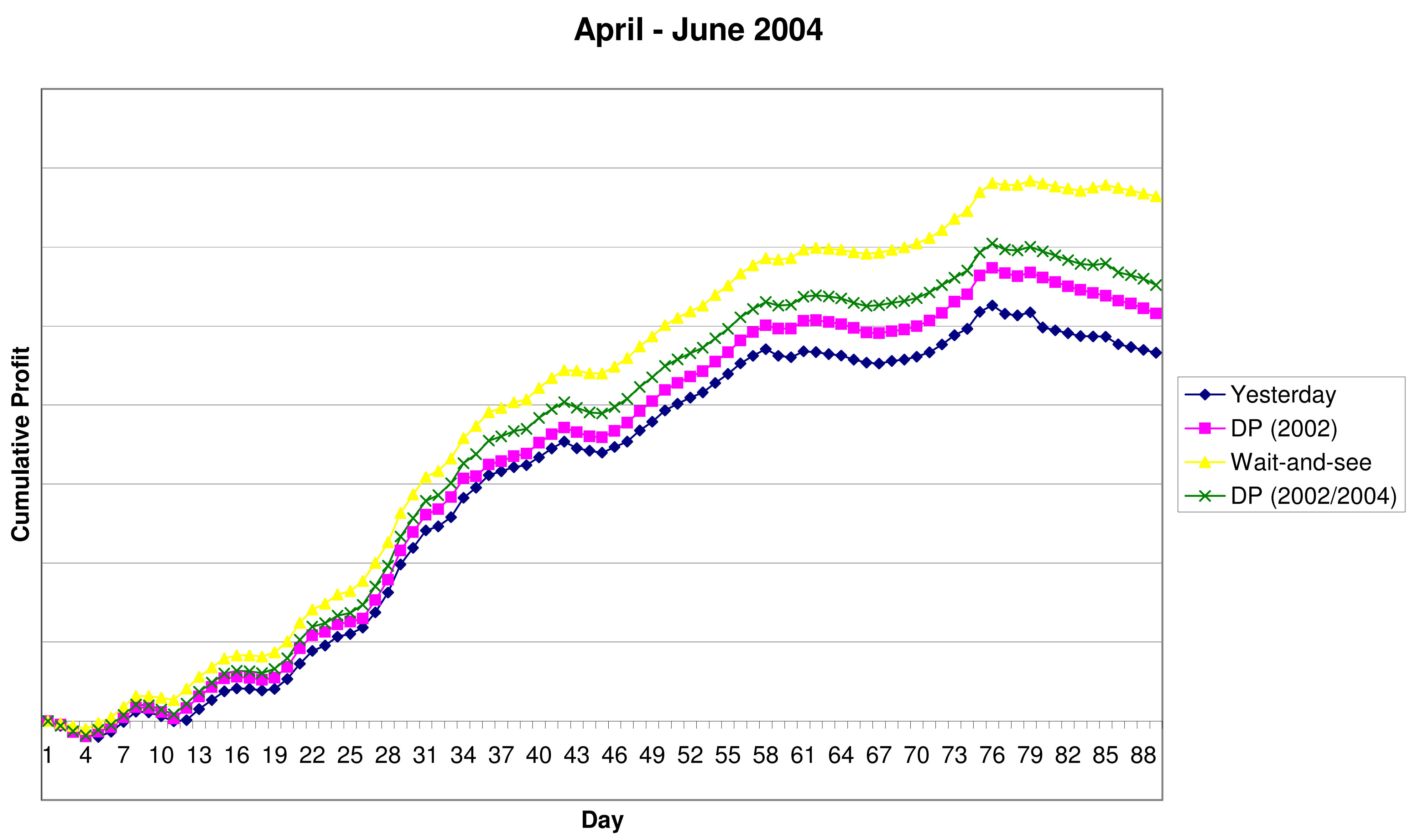
and

$$S_k = q_c(p_c - E[p|\pi]).$$

This gives the recursion:

$$V_{k-1}(0, \pi) = \max\{S_k + E[V_k(0, p)|\pi], R_k - U + E[V_k(1, p)|\pi]\},$$

$$V_{k-1}(1, \pi) = \max\{S_k - D + E[V_k(0, p)|\pi], R_k + E[V_k(1, p)|\pi]\}.$$


April - June 2004
Cumulative Profit
Day
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43
46
49
52
55
58
61
64
67
70
73
76
79
82
85
88
Yesterday
DP (2002)
Wait-and-see
DP (2002/2004)

Price making using market distribution functions

[Anderson and Philpott, 2002]

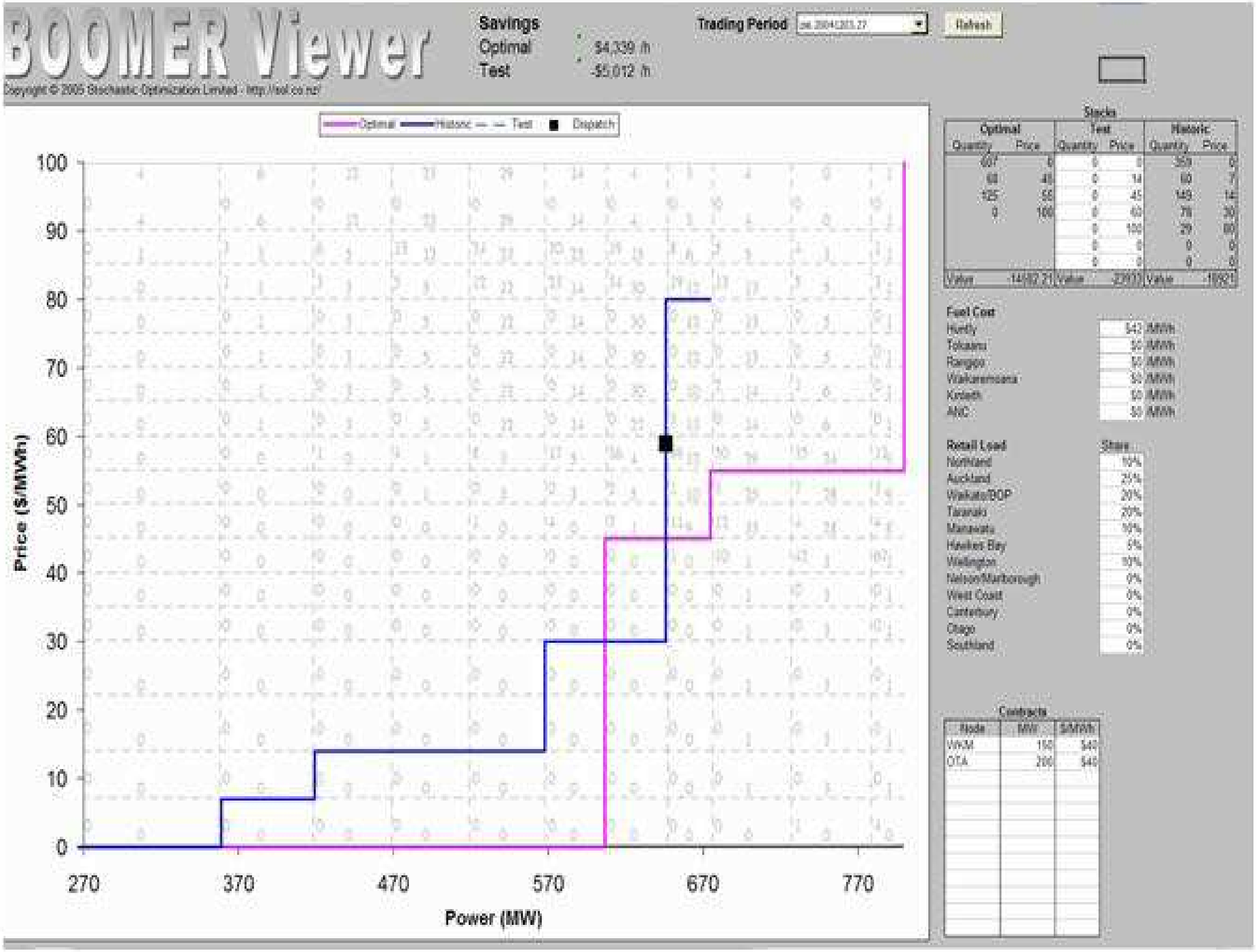
$\psi(q,p)$ gives a probability measure on possible residual demand curves at the location at which a generator offers quantity q at price p.

$$\psi(q,p) = \text{Pr(an offer of } (q,p) \text{ is not fully dispatched)}$$

Payoff from offering supply curve s is

$$P = \int_s R(q,p) d\psi(q,p).$$

Can extend this to a transmission grid model using market simulation (e.g. Boomer).


BOOMER Viewer
Copyright © 2005 Stochastic Optimization Limited - http://sol.co.nz/
Savings
Optimal
$4,339 /h
Test
-$5,012 /h
Trading Period
Refresh
Optimal
Historic
Test
Dispatch
Price ($/MWh)
Power (MW)
Stacks
Optimal
Test
Historic
Quantity
Price
Value
Fuel Cost
Huntly
Tokaanu
Rangipo
Waikaremoana
Kinleith
ANC
Retail Load
Share
Northland
Auckland
Waikato/BOP
Taranaki
Manawatu
Hawkes Bay
Wellington
Nelson/Marlborough
West Coast
Canterbury
Otago
Southland
Contracts
Node
MW
$/MWh
WKM
OTA

Unit commitment for MDF

Suppose we have a known MDF $\psi_k(q,p)$ for each trading period $k = 1, \ldots, K$. Let

$$R_k = \max_s \int_s (pq - C(q) + q_c(f-p)) d\psi_k(q,p),$$

and

$$S_k = \int_0^\infty q_c(f-p) d\psi_k(0,p).$$

This gives the recursion:

$$V_{k-1}(0) = \max\{S_k + V_k(0), R_k - U + V_k(1)\},$$

$$V_{k-1}(1) = \max\{S_k - D + V_k(0), R_k + V_k(1)\}.$$

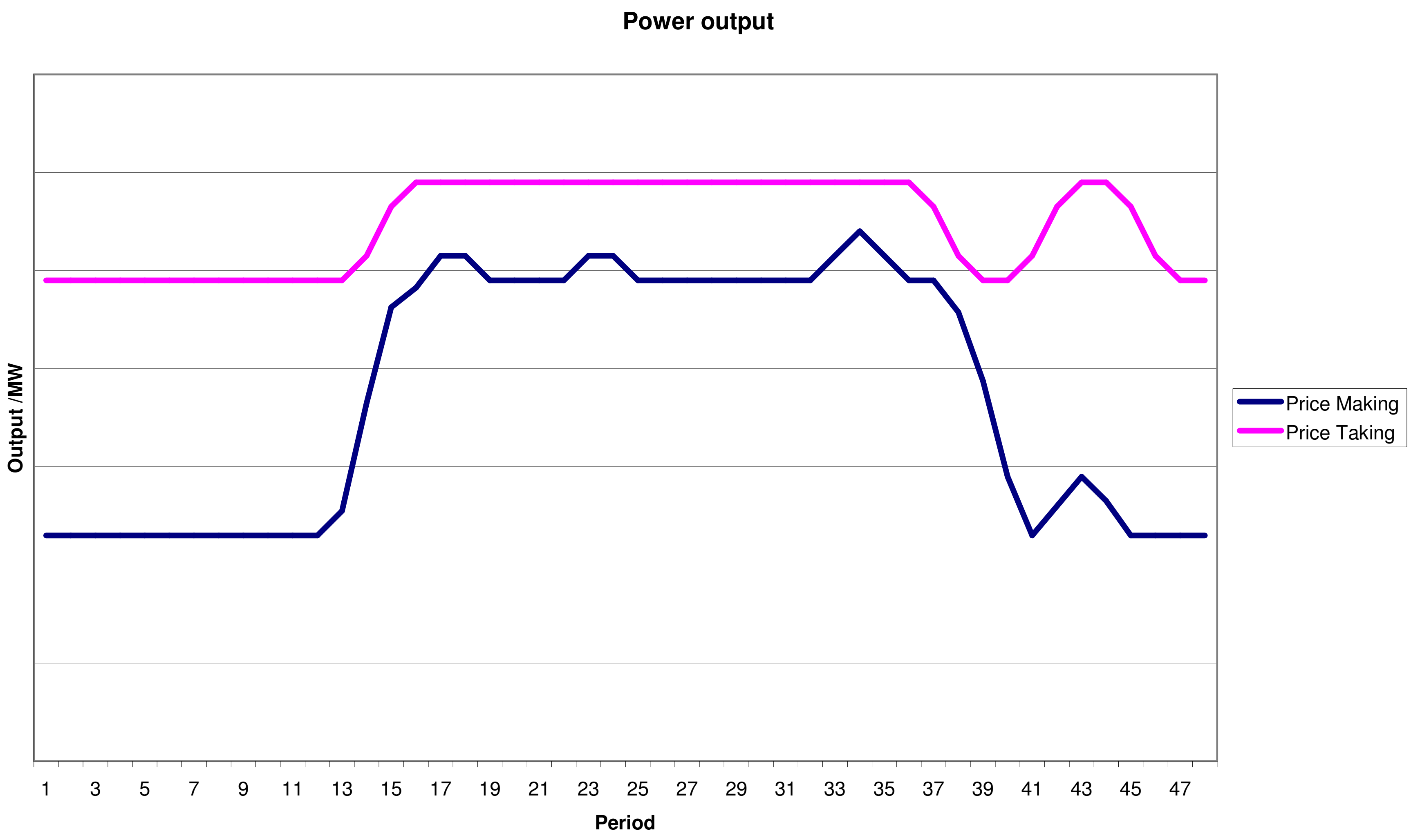
The solution

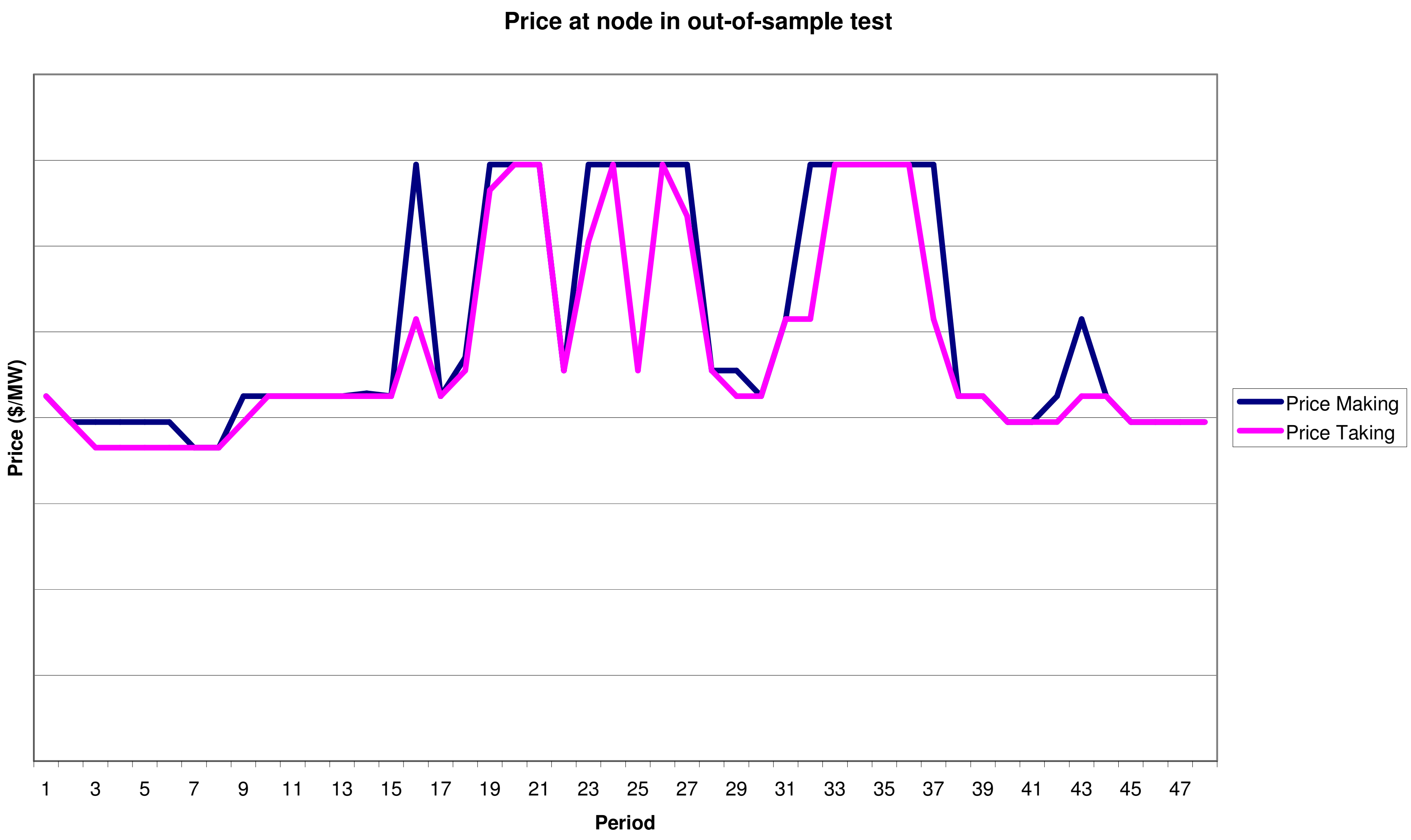
Recursion gives a single sequence of on/off decisions for a trading day

This is a "*here-and-now*" commitment - no recourse to change plan in mid-stream like the Markov model

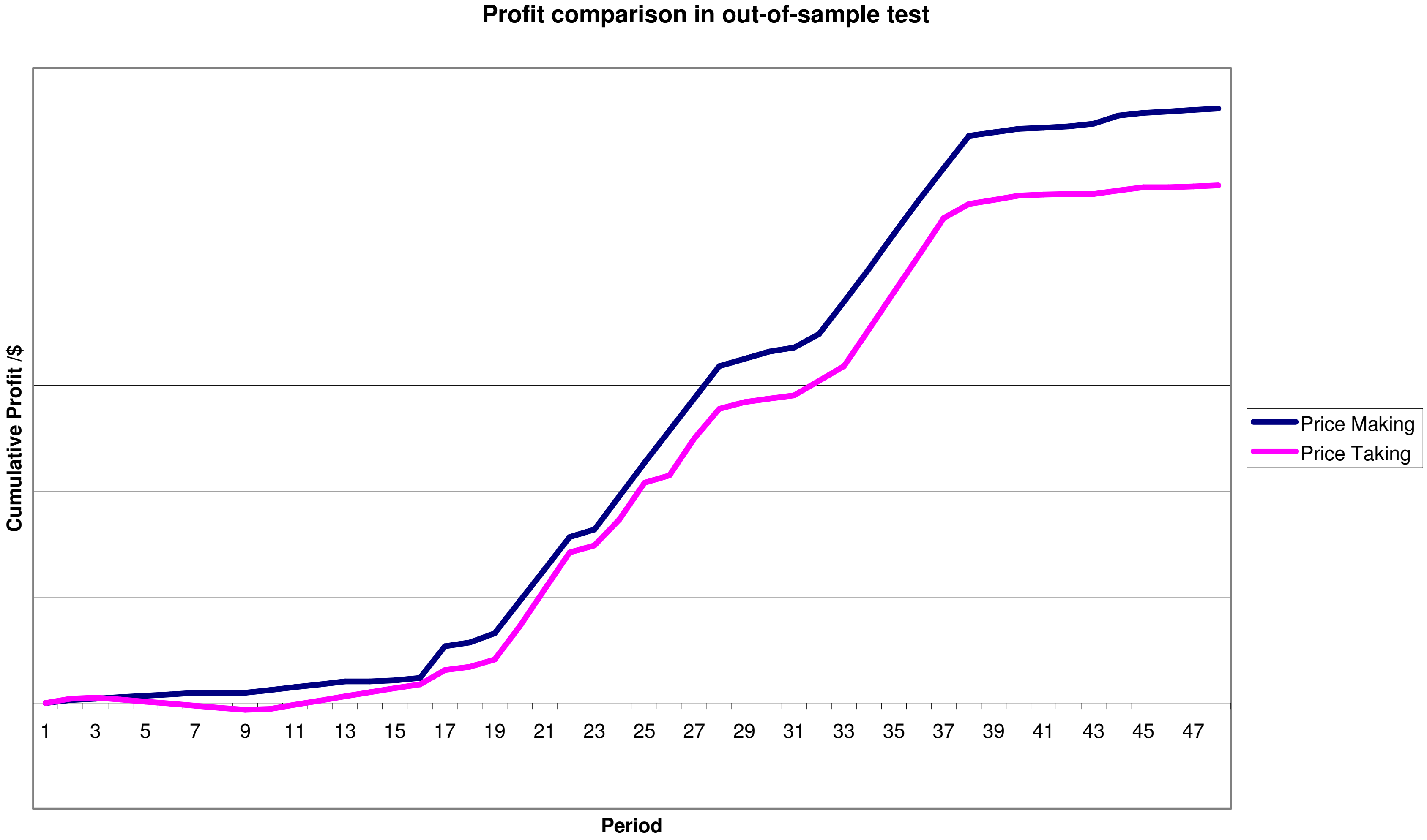
We might seek to test this on some out-of-sample trading days

- cannot test this strategy on the observed price sequence
- simulate it with historical offers and loads using Boomer
- this assumes that all other agents are also "here-and-now" optimizers


Power output
Output /MW
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
43
45
47
Period
Price Making
Price Taking



Price at node in out-of-sample test
Price ($/MW)
Period
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
43
45
47
Price Making
Price Taking



Profit comparison in out-of-sample test
Cumulative Profit /$
Period
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
43
45
47
Price Making
Price Taking

Some questions

We know that other agents are not all “here-and-now” optimizers - they change offers regularly in response to market conditions.

So assumption is likely to be violated in practice...

What about a Markov model?

> Suppose $\psi_k(q, p, \pi)$ is a Markov chain dependent on market “*state*” π
>
> Our offer would be a *policy* that would be dependent on observed states π and state of the unit.
>
> What would π represent? Observed spot market prices or some function thereof.
>
> How would we model state transitions of $\psi_k(q, p, \pi)$?

Is there an equilibrium in policies and what does it look like?

Example of optimizing in policies

Symmetric Cournot duopoly with unlimited capacity (ignore unit effects)

Assume constant production cost $c = 2$.

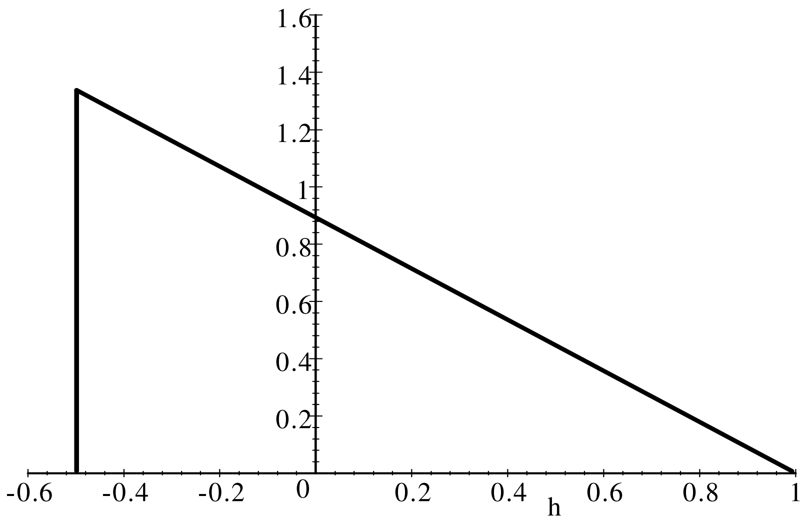
Assume a linear demand function $D(p) = 16 - p$ with a (random) demand "shock" H.

Assume that H is i.i.d. and $E[H] = 0$.

$$D = 16 - p + H$$

Example: triangular density function for H

$$f(h) = \begin{cases} \frac{8}{9}(1-h) & -\frac{1}{2} < h \leq 1 \\ 0 & \text{otherwise} \end{cases}$$



Symmetric duopoly under Nash-Cournot equilibrium

A chooses offer q_A to maximize its expected profit assuming B has fixed offer q_B.

$$\begin{aligned} E\left[R_A(q_A, q_B)\right] &= E\left[q_A(p-c)\right] \\ &= E\left[q_A(16 - q_A - q_B + H - 2)\right] \\ &= 14q_A - q_A^2 - q_A q_B \end{aligned}$$

$$2q_A + q_B = 14$$

By symmetry

$$q_A^N = q_B^N = \frac{14}{3} = 4.667$$

and

$$E[p] = 16 - \frac{14}{3} - \frac{14}{3} = 6.667$$

$$\begin{aligned} E\left[R_A(q^N, q^N)\right] &= 14\left(\frac{14}{3}\right) - \left(\frac{14}{3}\right)\left(\frac{14}{3}\right) - \left(\frac{14}{3}\right)\left(\frac{14}{3}\right) \\ &= 21.7777. \end{aligned}$$

Infinitely repeated game

Maximize present value of expected profit discounted at rate r.

When $r = 0.1$, expected discounted profit for each agent playing Nash-Cournot strategy is

$$E\left[R_A(q^N)\right](1 + \left(\frac{1}{1.1}\right) + \left(\frac{1}{1.1}\right)^2 + \ldots) = \frac{21.7777}{1 - \left(\frac{1}{1.1}\right)} = 239.55.$$

This strategy is an equilibrium in the infinitely repeated game.

Can we do better than 239.55?

Markov-Cournot game in trigger strategies

Suppose a trigger price $\bar{p}$ is common knowledge to both players.

The game is in two states depending on observed price p_{k-1} from previous period:

$\pi = 1$	$p_{k-1} < \bar{p}$	action	$q = q_1$	response	$q = x_1$
$\pi = 2$	$p_{k-1} \geq \bar{p}$	action	$q = q_2$	response	$q = x_2$

Actions in current stage depend on the state of the game, and are chosen to maximize expected present value of profit.

Markov-Cournot game with two states and identical players

$$
\begin{aligned}
V(1) &= \max_{x_1}\left\{E[R(x_1,q_1)] + \frac{1}{1+r}\left(P_{11}(x_1,q_1)V(1) + P_{12}(x_1,q_1)V(2)\right)\right\} \\
V(2) &= \max_{x_2}\left\{E[R(x_2,q_2)] + \frac{1}{1+r}\left(P_{21}(x_2,q_2)V(1) + P_{22}(x_2,q_2)V(2)\right)\right\}
\end{aligned}
$$

Transition probabilities for the example

$$p = 16 - (x + q) + H$$

$$p_t < \bar{p} \iff H < x + q + \bar{p} - 16$$

So

$$\begin{aligned} P_{12}(x_1, q_1) &= \Pr(p_t > \bar{p} \mid x_1 + q_1 \text{ offered}) \\ &= \Pr(H > x_1 + q_1 + \bar{p} - 16) \end{aligned}$$

$$\begin{aligned} P_{21}(x_2, q_2) &= \Pr(p_t < \bar{p} \mid x_2 + q_2 \text{ offered}) \\ &= \Pr(H < x_2 + q_2 + \bar{p} - 16) \end{aligned}$$

Markov perfect equilibrium (MPE) satisfies

$$
\begin{aligned}
V(1) &= \max_{x_1} \left\{ E[R(x_1, q_1] + \frac{1}{1+r} \begin{pmatrix} \Pr(H < x_1 + q_1 + \bar{p} - 16) V(1) \\ + \Pr(H > x_1 + q_1 + \bar{p} - 16) V(2) \end{pmatrix} \right\} \\
V(2) &= \max_{x_2} \left\{ E[R(x_2, q_2] + \frac{1}{1+r} \begin{pmatrix} \Pr(H < x_2 + q_2 + \bar{p} - 16) V(1) \\ + \Pr(H > x_2 + q_2 + \bar{p} - 16) V(2) \end{pmatrix} \right\}
\end{aligned}
$$

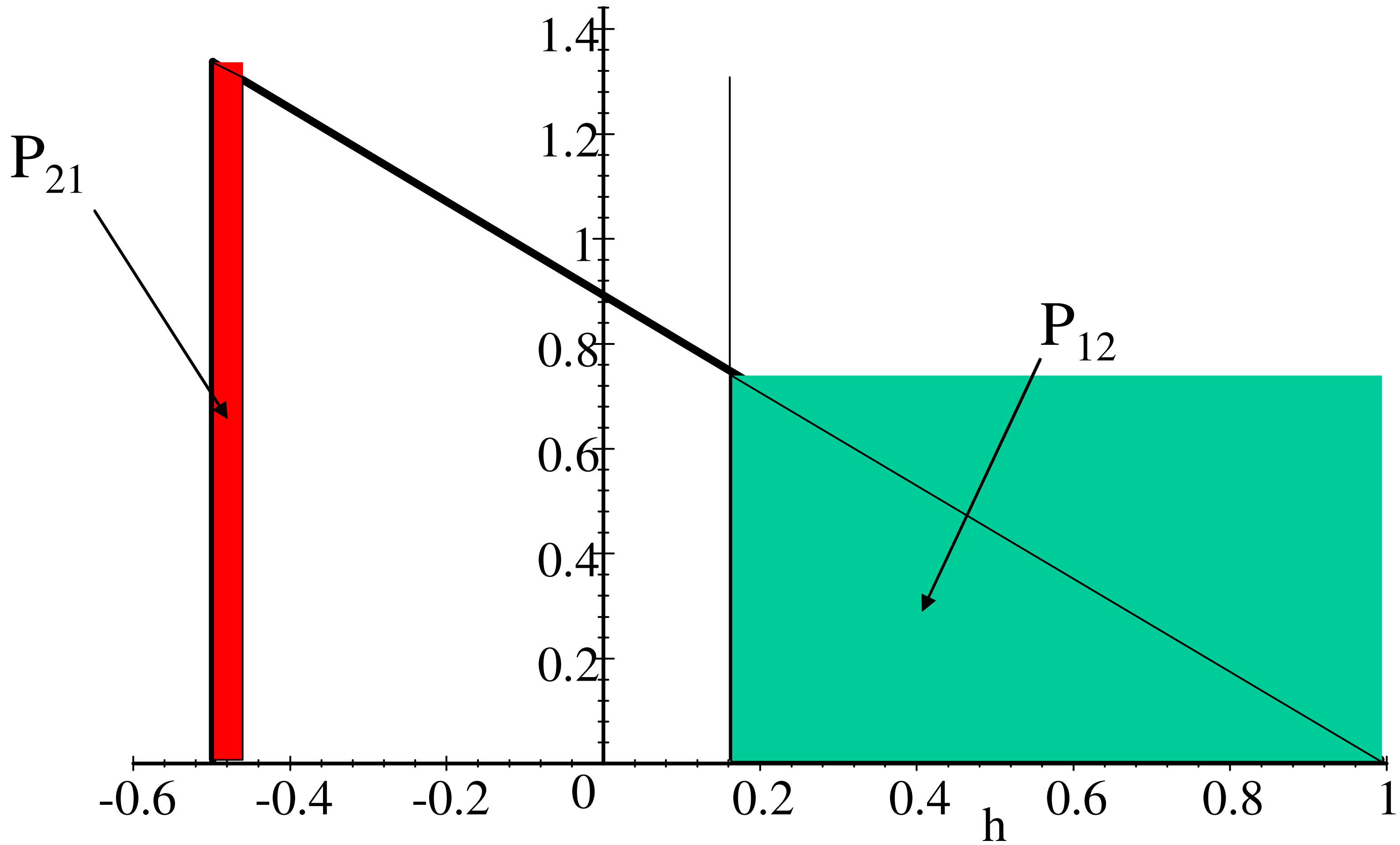
A Markov perfect equilibrium for example

State 1	$p_{t-1} < 7.667$	both players choose	$q_1 = 4.25$
State 2	$p_{t-1} \geq 7.667$	both players choose	$q_2 = 3.931$

$$\begin{aligned} P_{12}(q_1, q_1) &= \Pr(p_t > \bar{p} \mid 2q_1 \text{ offered}) \\ &= \Pr(H > 2q_1 + \bar{p} - 16) \\ &= \Pr(H > 0.1667) = 0.3086 \end{aligned}$$

$$\begin{aligned} P_{21}(q_2, q_2) &= \Pr(p_t < \bar{p} \mid 2q_2 \text{ offered}) \\ &= \Pr(H < 2q_2 + \bar{p} - 16) \\ &= \Pr(H < -0.4713) = 0.0379 \end{aligned}$$

Random transitions



Value iteration with optimal policy gives

$$V(1) = q_1(14 - 2q_1) + \frac{1}{(1+r)}((1 - 0.3086)V(1) + 0.3086V(2))$$
$$V(2) = q_2(14 - 2q_2) + \frac{1}{(1+r)}(0.0379V(1) + (1 - 0.0379)V(2))$$

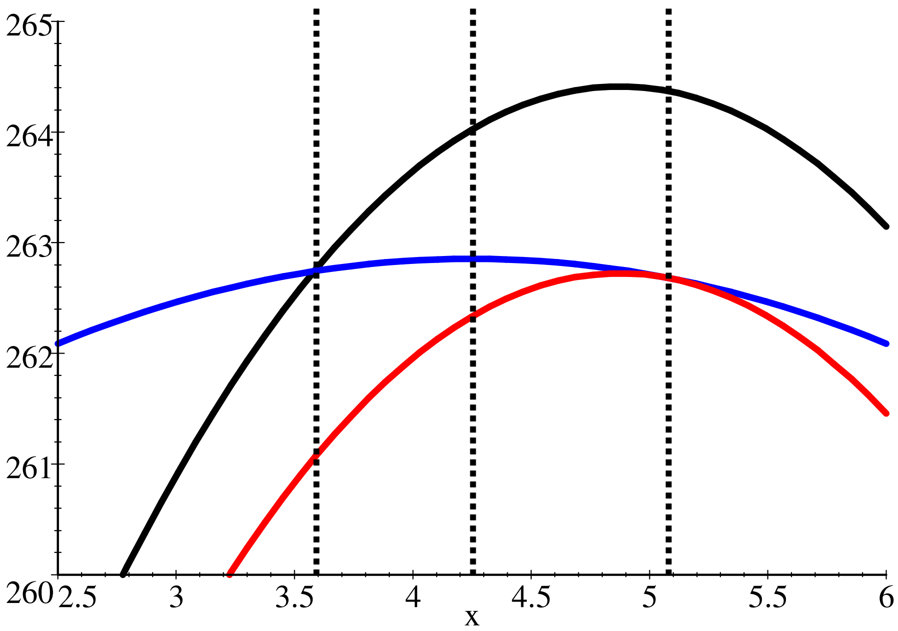
Solving gives

$$V(1) = 262.85$$
$$V(2) = 264.71$$

Why is this a Markov perfect equilibrium?

Optimal response x to $q_1 = 4.25$ satisfies Bellman equation

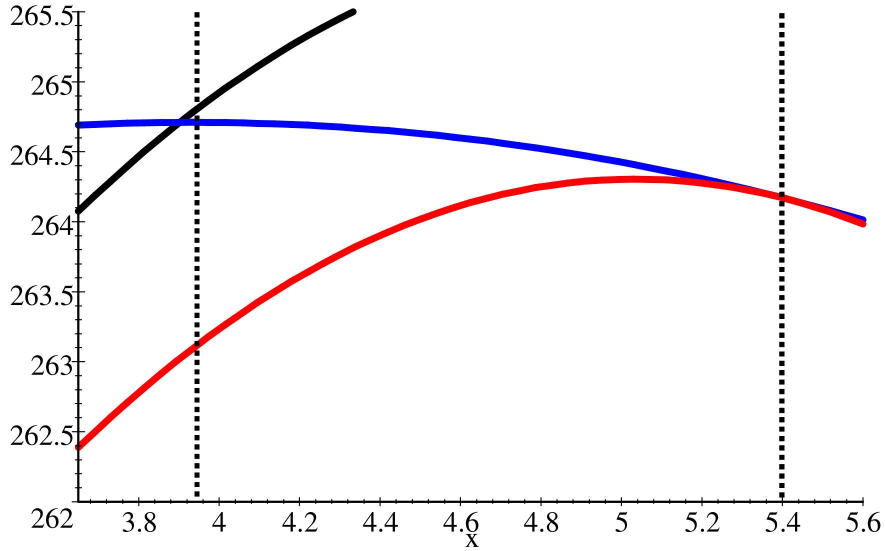
$$V(1) = \max_x \begin{cases} x(14-(x+q_1)) + \frac{1}{1.1}V(2), & x \le 3.5833 \\ x(14-(x+q_1))+ \\ \frac{1}{1.1}\left(P_{11}(x,q_1)V(1) + P_{12}(x,q_1)V(2)\right), & 3.5833 < x \le 5.0833 \\ x(14-(x+q_1)) + \frac{1}{1.1}V(1), & 5.0833 < x \end{cases}$$



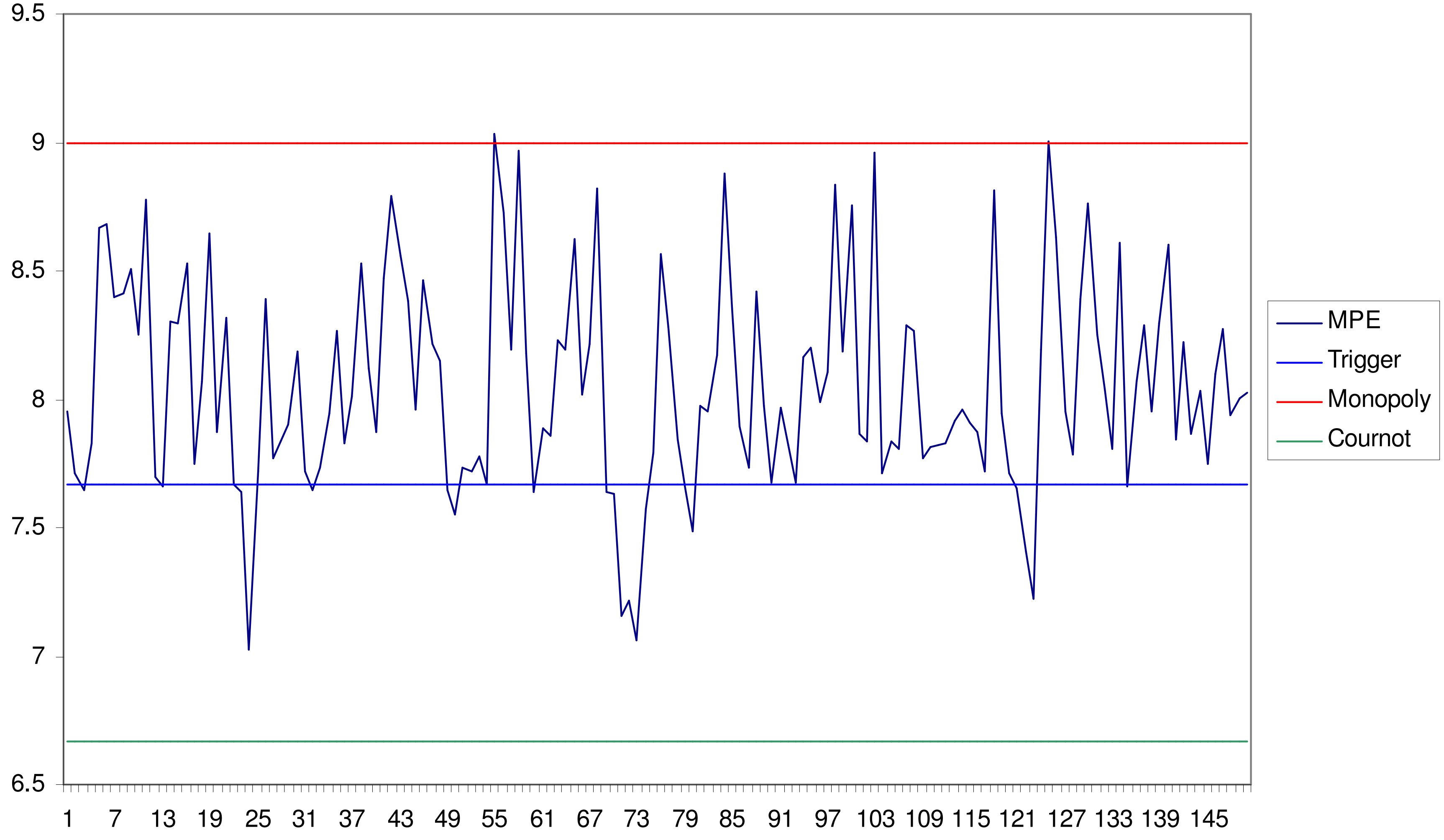
Optimal action in the high-price state

Optimal response x to $q_2 = 3.931$ satisfies Bellman equation

$$V(2) = \max_x \begin{cases} x(14 - (x + q_2)) + \frac{1}{1.1} V(2), & x \le 3.9023 \\ x(14 - (x + q_2)) + \\ \frac{1}{1.1} \left(P_{21}(x, q_2) V(1) + P_{22}(x, q_2) V(2) \right), & 3.9023 < x \le 5.4023 \\ x(14 - (x + q_2)) + \frac{1}{1.1} V(1), & 5.4023 < x \end{cases}$$



Simulation of prices over 150 plays of MPE



Maximizing profit

Recall expected discounted profit for each agent playing Nash-Cournot strategy is

$$E\left[R_A(q^N)\right](1+\left(\frac{1}{1.1}\right)+\left(\frac{1}{1.1}\right)^2+\ldots)=\frac{21.7777}{1-\left(\frac{1}{1.1}\right)}=239.55.$$

Compare

$$V(1)=262.85$$

$$V(2)=264.71$$

so we can expect to do better than repeated Nash-Cournot.

Conclusions

We can contruct (contrived) examples where both players capture half the monopoly profit.

Many equilibria exist in these models due to the *Folk Theorem* (e.g. Fudenberg and Maskin, *Econometrica*, 1986, Fudenberg, Levine and Maskin, *Econometrica*, 1994)

This lessens their predictive power - but models give insights into how one might attack multi-period optimization in price-setting context.

The end