Financial transmission rights and SPD

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[1]EPOC Winter Workshop, September 2, 2011.

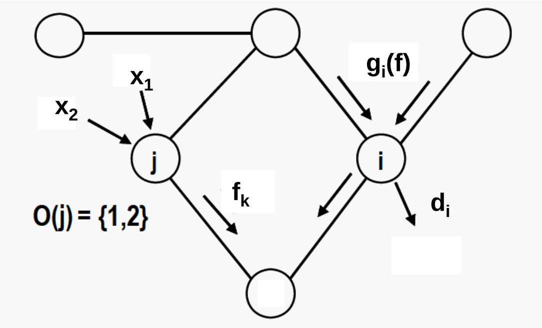
Motivation

- FTR design = economics+politics+mathematics
- How can mathematics help this process?
- What is known mathematically for FTRs in NZ setting?
- Are there any traps?
- Disclaimer: This is a talk about mathematics, not a recommendation for a specific FTR design.

Overview of talk

- An abstract formulation of SPD
- Revenue adequacy of FTR products
- Losses
- Reserve

Classical dispatch model (with losses)



$$
\begin{array}{llcll}
\min & \sum_i \sum_{j \in O(i)} c_j x_j & & & \\
\text{s.t.} & g_i(f) + \sum_{j \in O(i)} x_j - z_i & = & d_i, \quad i = 1, 2, \ldots, n & [\pi_i] \\
& f & \in & F, & \\
& x & \in & X, & \\
& z_i & \geq & 0, \quad i = 1, 2, \ldots, n. &
\end{array}
$$

g(f) = flow from the network into each node (demand)

Lossless flow

$$g_i(f) = \sum_{k \in \mathcal{F}(i)} -f_k + \sum_{k \in \mathcal{T}(i)} f_k$$

Quadratic losses

$$g_i(f) = \sum_{k \in \mathcal{F}(i)} \left(-f_k - \frac{r_k}{2} f_k^2 \right) + \sum_{k \in \mathcal{T}(i)} \left(f_k - \frac{r_k}{2} f_k^2 \right) \qquad (1)$$

Piecewise linear losses (SPD)

$$g_i(f) = \sum_{k \in \mathcal{F}(i)} -f_k + \sum_{k \in \mathcal{T}(i)} (f_k - T(f_k))$$

What is the set F?

In the classical case

$$f \in F$$

amounts to

$$-K \leq f \leq K, \quad \text{thermal limits}$$

$$Uf = 0, \quad \text{loop flow constraints (or similar)}$$

We might add

$$Wf \leq a, \quad \text{branch security constraints}$$

But we do not allow constraints mixing f with other variables e.g.

$$\sum_i \sum_{j \in O(i)} x_j \geq s^\top f$$

Rentals

Suppose that the optimal dispatch is x^*. Generators of energy earn a *generator rental*

$$R_g = \sum_i \pi_i \sum_{j \in O(i)} x_j^* - \sum_i \sum_{j \in O(i)} c_j x_j^* .$$

The difference between what is paid to the system clearing mananger by loads and what they pay generators is the *transmission rental*

$$R_t = \sum_i \pi_i d_i - \sum_i \pi_i \sum_{j \in O(i)} x_j^* .$$

Transmission rental formula

For an optimal feasible dispatch

$$g_i(f^*) + \sum_{j \in O(i)} x_j^* - z_i^* = d_i$$

$$\begin{aligned} R_t &= \sum_i \pi_i d_i - \sum_i \pi_i \sum_{j \in O(i)} x_j^* \\ &= \sum_i \pi_i (g_i(f^*) - z_i^*) \\ &= \sum_i \pi_i g_i(f^*) \end{aligned}$$

Revenue inequality theorem

For any feasible set of flows ($f \in F$)

$$\sum_i \pi_i g_i(f^*) \geq \sum_i \pi_i g_i(f).$$

The transmission rental R_t collected from the optimal dispatch equals or exceeds $\sum_i \pi_i g_i(f)$ for any set of branch flows in F.

Financial Transmission Rights (FTRs)

What is a *financial transmission right* (mathematically)?
It is a vector $h(\alpha)$ of "offtakes" at the nodes of the network, where $h_i(\alpha)$ is the offtake at node i. The FTR $h(\alpha)$ generates a payment to its holder of

$$\sum_i \pi_i h_i(\alpha)$$

For simplicity, we will restrict attention to FTRs of the form

$$h(\alpha) = \begin{bmatrix} -r_1 & 0 & \ldots & 0 & r_n \end{bmatrix}^\top$$

A *balanced* FTR $h(\alpha)$ has $\sum_i h_i(\alpha) = 0$.

Simultaneous feasibility

Suppose we allocate some FTRs as vectors $h(\alpha)$, $\alpha \in A$. Then these are *simultaneously feasible* if

$$\begin{array}{lrcll} \text{SF:} & g_i(v) - z_i & = & \sum_\alpha h_i(\alpha), & i = 1, 2, \ldots, n, \\ & z_i & \geq & 0, & i = 1, 2, \ldots, n, \\ & v & \in & F, & \end{array}$$

has a feasible solution v, z.

Observations

- A set of FTRs can be defined by a flow vector v
- In SPD framework, v is by convention the flow *leaving* each node.
- The vector v can be anything in F.
- In SPD we have

$$g_i(v) = \sum_{k \in \mathcal{F}(i)} -v_k + \sum_{k \in \mathcal{T}(i)} (v_k - T(v_k))$$

where $T(v)$ are piecewise linear transmission losses.

- The total payout for the set of FTRs is $\sum_i \pi_i g_i(v)$.

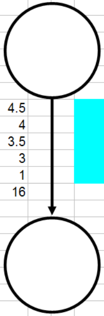
Simultaneous feasibility implies revenue adequacy

- Consider FTRs between two nodes B and O with prices π_B and π_O.
- Suppose a set of FTRs is now defined by $\{(-h_B(\alpha), h_O(\alpha)), \alpha \in A\}$
- Aggregate to get $(-\sum_\alpha h_B(\alpha), \sum_\alpha h_O(\alpha))$
- Find a flow $v \in F$ that satisfies SF.
- This implies

$$\pi_O \sum_\alpha h_O(\alpha) - \pi_B \sum_\alpha h_B(\alpha) = \sum_i \pi_i g_i(v) \leq \sum_i \pi_i g_i(f^*)$$

Example

						OfferQ	Offer P	
Price	5					50	5	
						22	110	
	Tranche Flow	Tranche Capacity	Loss Slope			22		
	5	5	0.1	4.5	4	Line	Rentals	50
	5	5	0.2	4	3			
	5	5	0.3	3.5	2		Received	160
	5	5	0.4	3	1		Paid	110
	2	5	0.5	1	0		Check	50
Sent	22		Received	16				
						Loss =	6	
		Demand	Supply					
Price	10	16	16					



An FTR of (-20, 15) is simultaneously feasible. It earns a payment of \$10*15 - \$5*20=\$50.

An FTR of (-20, 14) is also simultaneously feasible. It earns a payment of \$10*14 - \$5*20=\$40.

Simultaneous feasibility with losses

Any set of balanced FTRs will not be simultaneously feasible.

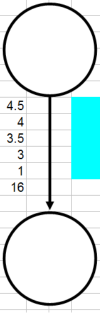
$$
\begin{array}{lrcll}
\text{SF:} & g_i(v) - z_i & = & -r, & i = 1, \\
& g_i(v) - z_i & = & 0, & i = 2, \ldots, n-1, \\
& g_i(v) - z_i & = & r, & i = n, \\
& z_i & \geq & 0, & i = 1, 2, \ldots, n, \\
& v & \in & F &
\end{array}
$$

is feasible only if

$$\sum_{i=1}^{n} g_i(v) \geq 0$$

which cannot happen if there are any nonzero losses.

Revenue inadequacy with losses



						OfferQ	Offer P	
Price	5					50	5	
						22	110	
	Tranche	Tranche	Loss			22		
	Flow	Capacity	Slope					
	5	5	0.1	4.5	4	Line	Rentals	50
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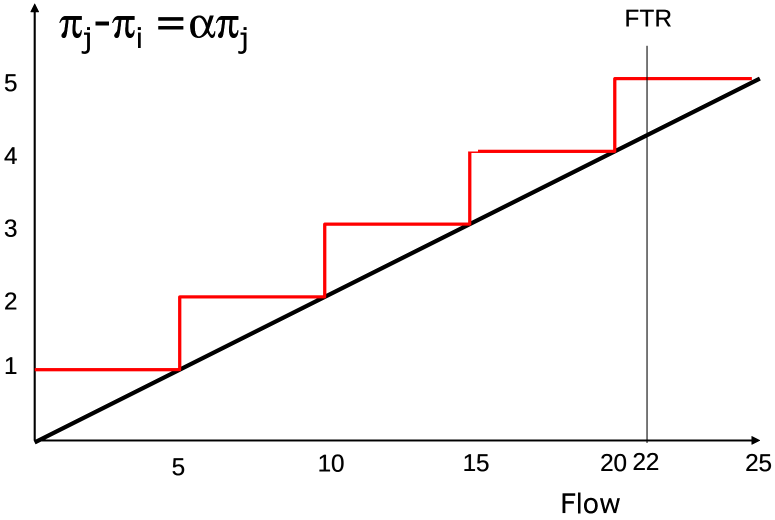
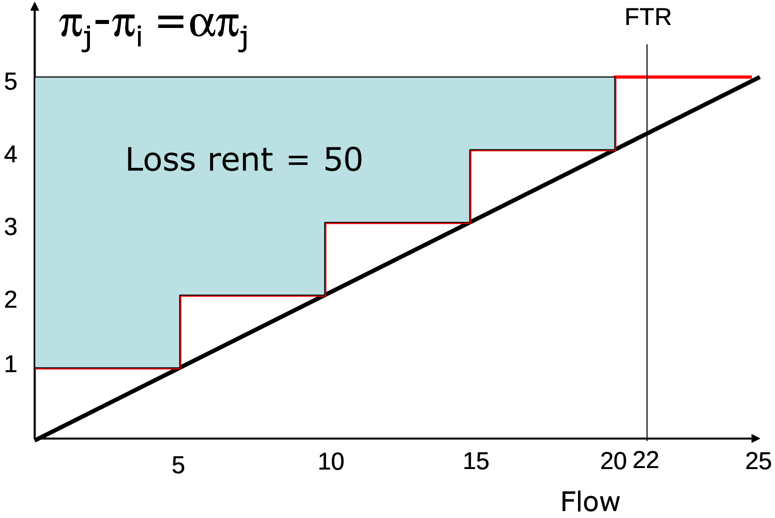
	5	5	0.3	3.5	2		Received	160
	5	5	0.4	3	1		Paid	110
	2	5	0.5	1	0		Check	50
Sent	22		Received	16				
						Loss =	6	
		Demand	Supply					
Price	10	16	16					

A balanced FTR (-22, 22) earns a payment of $(10-5)*22=$110.
But $R_t = \$50$. The $60 shortfall is the cost of the 6MWh loss.

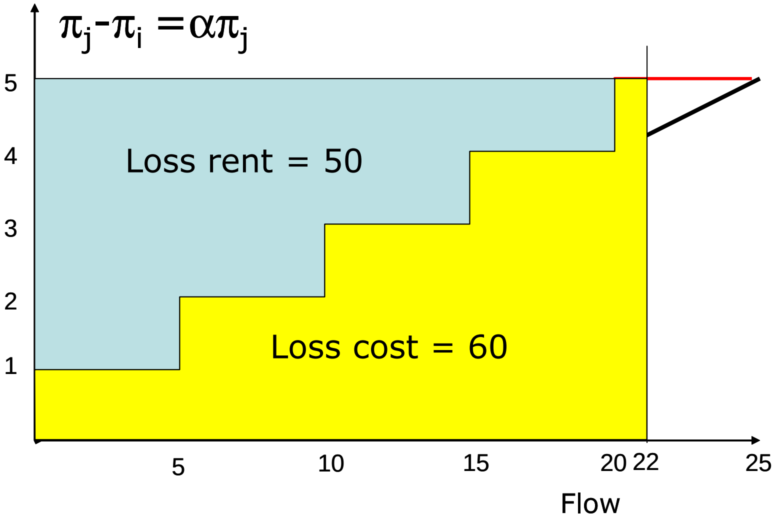
In graphical terms



In graphical terms



In graphical terms



Losses require modifications to classical design

Can do one of the following to ensure revenue adequacy:

- Restriction on volume of FTR allocation.
- Loss support contracts to accompany a balanced FTR.
- Unbalanced FTRs $(-1, 1-\alpha)$ for some α.
- Discounted balanced FTRs
- Some combination of the above.

Unbalanced and discounted FTRs

An *unbalanced* FTR $(-r, (1-\alpha)r)$ gives

$$
\begin{array}{rrcll}
\text{SF:} & g_i(v) - z_i & = & -r, & i = 1, \\
& g_i(v) - z_i & = & 0, & i = 2, \ldots, n-1, \\
& g_i(v) - z_i & = & (1-\alpha)r, & i = n, \\
& z_i & \geq & 0, & i = 1, 2, \ldots, n, \\
& v & \in & F, &
\end{array}
$$

which will be feasible for some v and z.

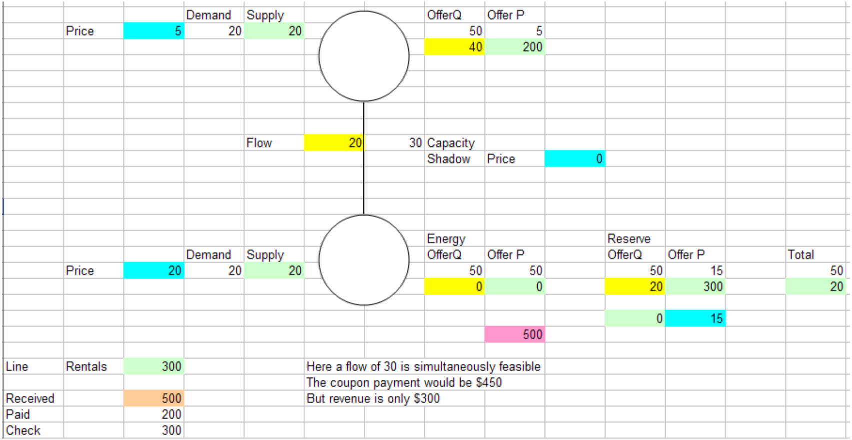
An $\alpha-$*discounted* balanced FTR from i to j pays $(1-\alpha)\pi_j - \pi_i$ per MW of balanced FTR. It is equivalent to an unbalanced FTR.

Reserve-constrained dispatch (SPD)

$$
\begin{array}{llcll}
\min & \sum_i \sum_{j\in O(i)} c_j x_j + \sum_i \sum_{j\in R(i)} b_j y_j & & & \\
\text{s.t.} & g_i(f) + \sum_{j\in O(i)} x_j - z_i & = & d_i, & i = 1,2,\ldots,n \\
& \sum_i \sum_{j\in R(i)} y_j - w_1 & = & q^\top x, & \\
& \sum_i \sum_{j\in R(i)} y_j - w_2 & = & s^\top f, & \\
& f & \in & F, & \\
& (x,y) & \in & X, & \\
& z_i & \geq & 0, & i = 1,2,\ldots,n \\
& w_i & \geq & 0, & i = 1,2.
\end{array}
$$

This contains constraints mixing f and y and so we cannot apply the SF test as it is.

Example of two node problem



The rentals add up to $300, but an FTR of 30 MW requires 30*($20-$5)=$450.

What is going on here?

- Assume that the funding for reserve is coming from another source (e.g. a levy). So the revenue inadequacy is not about paying for existing reserve from transmission rentals.
- An FTR of up to 20 MW is feasible without interfering with reserve constraint. The payouts are then less than $300.
- If the FTR exceeds 20MW then it requires more reserve at $15/MW. This is the shadow price on the reserve constraint. But this extra money does not come from the dispatch.
- Some reserve support is needed if we want to allocate more than 20MW of FTR
- Observe that it does not help if we consider the South generator as the reserve provider.

Conclusions

- Classical results ignore losses and ancillary constraints - mathematics is then "simple".
- Balanced FTRs are less complex for trading, so more liquid.
- Disadvantage is the complexity in ensuring that they can be funded from rentals.