# Complementarity models for investment under risk

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# Summary

# Motivation: competition and investment

- Ongoing debate in New Zealand about investment and competition.
- (Current) New Zealand market: Workable competition with tolerated exercise of market power provides the missing risk premium for investing.
- (New) New Zealand market: Remove market power from the wholesale market to reduce prices and transfer wealth to consumers.
- "...a centralised power-buyer proposal would have a chilling effect on investment, which is likely to be large, widespread and long-lived." (Brent Layton, June 5, 2013)
- Our motivation is to study this effect in a wholesale market that is perfectly competitive.

# Market completeness

- Peaking plant cannot recover capital cost if dispatched at marginal cost: missing money removes the incentive to invest.
- A problem of market incompleteness among other things.
- Different remedies and debates over how to complete the market: capacity payments versus energy only markets.
- Stoft (2002) showed how missing money can be provided by VOLL pricing when agents maximize expected profit (see Philpott, EPOC 2012)
- Risk averse investors complicate this market solution.
- Q: Do perfectly competitive markets provide the right investment incentives when agents are risk averse?
- Q: What effect does vertical integration and contracting have on investment incentives?

# Our theoretical framework

(Heath and Ku 2004, Ralph and Smeers, 2011, Philpott, Ferris, Wets, 2014)

| Model assumptions | Perfect competition (complete risk market) | Perfect competition (incomplete risk market) | Imperfect competition |
|---|---|---|---|
| Risk neutral agents | Competitive equilibrium gives socially efficient solution. | Competitive equilibrium gives socially efficient solution. | Nash equilibrium can result in social inefficiency. |
| Risk averse coherent agents | Competitive equilibrium gives socially efficient solution with social risk measure. | Competitive equilibrium is not always socially efficient. | The most realistic model. |

If the market for hedging instruments is sufficiently rich, and agents use coherent risk measures, then there is a social risk measure that is optimized by a competitive equilibrium.

# Summary

# Dual representation of coherent risk measures
(Artzner et al, 1999, Shapiro & Ruszczynski, 2006)

A coherent risk measure $\rho$ of a random disbenefit $Z$ can be expressed as

$$\rho(Z) = \max_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z]$$

where $D$ is a convex set of probability measures called the risk set.

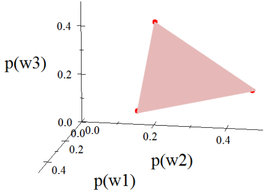
# Example: three outcomes

Consider possible disbenefit outcomes $Z(\omega_1) < Z(\omega_2) < Z(\omega_3)$ with equal probability. The coherent risk measure

$$\rho(Z) = \frac{3}{4}\mathbb{E}[Z] + \frac{1}{4}\max[Z]$$

has risk set

$$\mathcal{D}=\text{conv}\{(\frac{1}{2},\frac{1}{4},\frac{1}{4}),(\frac{1}{4},\frac{1}{2},\frac{1}{4}),(\frac{1}{4},\frac{1}{4},\frac{1}{2})\}.$$




$$\rho(Z) = \max_{\mu\in\mathcal{D}} \mathbb{E}_\mu[Z] = \frac{1}{4}Z(\omega_1) + \frac{1}{4}Z(\omega_2) + \frac{1}{2}Z(\omega_3).$$

# Example: ten outcomes

Suppose we have ten outcomes with equal probability. Our examples use a coherent risk measure $\rho$ that averages expected disbenefit and 20% conditional value at risk. This has risk set

$$
\begin{aligned}
\mathcal{D} \quad = \quad & \mathrm{conv}\{(\frac{6}{20},\frac{6}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20}), \\
& (\frac{6}{20},\frac{1}{20},\frac{6}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20}), \\
& (\frac{6}{20},\frac{1}{20},\frac{1}{20},\frac{6}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20}), \\
& \ldots, \\
& (\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{1}{20},\frac{6}{20},\frac{6}{20})\}.
\end{aligned}
$$

$$
\rho(Z) = \max_{\mu \in \mathcal{D}} \sum_{i=1}^{10} \mu_i Z(\omega_i).
$$

# Example: welfare calculations for an investment plan.

| Welfare ($M) | Low retail demand | | | | | High retail demand | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Inflows | 40% | 55% | 70% | 85% | 100% | 40% | 55% | 70% | 85% | 100% | Expected | Risk Adj |
| Thermal | 48.61 | -33.47 | -35.60 | -35.82 | -35.82 | 204.19 | 47.51 | -30.56 | -34.10 | -34.38 | 6.06 | -14.88 |
| Hydro | 235.05 | 78.50 | 73.54 | 71.69 | 71.69 | 478.92 | 282.10 | 154.31 | 147.12 | 145.98 | 173.89 | 122.79 |
| Retail 1 | 331.46 | 407.77 | 411.01 | 411.73 | 411.73 | 324.41 | 501.54 | 588.38 | 593.24 | 593.78 | 457.50 | 392.72 |
| Retail 2 | 331.46 | 407.77 | 411.01 | 411.73 | 411.73 | 324.41 | 501.54 | 588.38 | 593.24 | 593.78 | 457.50 | 392.72 |
| Hydro gentailer | 380.07 | 374.30 | 375.41 | 375.91 | 375.91 | 528.60 | 549.06 | 557.83 | 559.15 | 559.40 | 463.56 | 419.21 |
| Thermal gentailer | 566.51 | 486.26 | 484.54 | 483.42 | 483.42 | 803.33 | 783.64 | 742.69 | 740.36 | 739.76 | 631.39 | 557.40 |
| Industry | 969.05 | 1080.13 | 1083.37 | 1084.09 | 1084.09 | 899.60 | 978.32 | 1044.43 | 1047.67 | 1048.03 | 1031.88 | 983.10 |
| Total | 1915.63 | 1940.69 | 1943.32 | 1943.41 | 1943.41 | 2231.52 | 2311.02 | 2344.95 | 2347.18 | 2347.20 | 2126.83 | 2,027.50 |
| Price | $83.44 | $51.74 | $50.82 | $50.61 | $50.61 | $129.84 | $80.80 | $61.93 | $61.01 | $60.90 | | |

The yellow column shows the result of evaluating the policy of each agent using its risk set. Each agent's risk adjusted welfare sums to 1876. With vertical integration this is 1960. The orange figure 2027.5 is risk-adjusted total social welfare.

# Questions

- Each agent has a risk set. What is the risk set of society?
- If we could derive such a set then we can improve the total risk-adjusted social welfare by a social plan (better than the risk neutral one above)?
- Can such an optimal social plan arise as a competitive equilibrium of risk averse agents with different risk sets?
- What are the best market mechanisms to use to give equilibria that are close to socially optimal?

# Summary

# Types of capacity planning models

- Deterministic social planning model (GEM - Bishop and Bull)
- Stochastic risk-neutral social planning model (GEMSTONE-Girardeau et al)
- Stochastic risk-neutral equilibrium model is equivalent to a social planning problem.
- Stochastic risk-averse equilibrium model (CRAGE- Kok et al)
- Stochastic risk-averse equilibrium model with hedging (CRAGE- Kok et al)

# CRAGE=Competitive Risk-Averse Generation Expansion
(Kok, Philpott, Zakeri, 2014)

CRAGE has risk averse agents: generators, retailers, gentailers, industrial load
All have known coherent risk measures.
Ownership (e.g. vertical integration) is exogeneous.
An auctioneer announces contract price and wholesale spot prices in each scenario.
Each agent then solves a two-stage risk-averse stochastic programming problem:

- Stage 0: choose increase in generation capacity and purchase contract positions at contract price.
- Stage 1: Offer all generation capacity at short-run marginal cost in the spot market and earn market rents minus contract payments.

If contracts and generation quantities clear their markets then we have a competitive equilibrium.

# Agent problem (generator)

Given contract price $f$ and electricity wholesale price $p(\omega)$, choose capacity expansion $x$, purchase contract $Q_g$ and sell generation $u(\omega)$ to solve

$$
\begin{aligned}
\text{GP:} \quad \min \quad & K(x) && + \quad \rho_g(Z_g) \\
\text{s.t.} \quad & u(\omega) && \leq \quad x\phi(\omega), \\
& Z_g(\omega) && = \quad -p(\omega)u(\omega) + C(u(\omega)) + (f - p(\omega))Q_g, \\
& x, u(\omega) && \geq \quad 0.
\end{aligned}
$$

# Agent problem (retailer)

Given retail demand $d(\omega)$, retail price $\pi$, contract price $f$ and electricity wholesale price $p(\omega)$, purchase contract $Q_r$ and purchase $d(\omega) - s(\omega)$ to solve

$$
\begin{aligned}
\text{RP:} \quad \min \quad & \rho_r(Z_r) \\
\text{s.t.} \quad Z_r(\omega) &= (p(\omega) - \pi)(d(\omega) - s(\omega)) \\
& \quad + (f - p(\omega))Q_r + (\text{VOLL} - \pi)s(\omega), \\
s(\omega) &\leq d(\omega), \\
s(\omega) &\geq 0.
\end{aligned}
$$

# Agent problem (industrial)

Given industrial demand $e(\omega)$, value of electricity $v <$VOLL, contract price $f$ and electricity wholesale price $p(\omega)$, purchase contract $Q_i$ and purchase $e(\omega) - r(\omega)$ to solve

$$
\begin{array}{llcl}
\text{IP:} & \min & \rho_i(Z_i) & \\
 & \text{s.t.} & Z_i(\omega) = & (p(\omega) - v)(e(\omega) - r(\omega)) \\
 & & & +(f - p(\omega))Q_i, \\
 & & r(\omega) \leq & e(\omega), \\
 & & r(\omega) \geq & 0.
\end{array}
$$

# Competitive equilibrium conditions

$$(Q_g, x, u) \in \arg\min \mathrm{GP},$$

$$(Q_r, s) \in \arg\min \mathrm{RP},$$

$$(Q_i, r) \in \arg\min \mathrm{IP},$$

$$0 \leq Q_g + Q_r + Q_i \perp f \geq 0,$$

$$0 \leq u + s + r - d - e \perp p \geq 0.$$

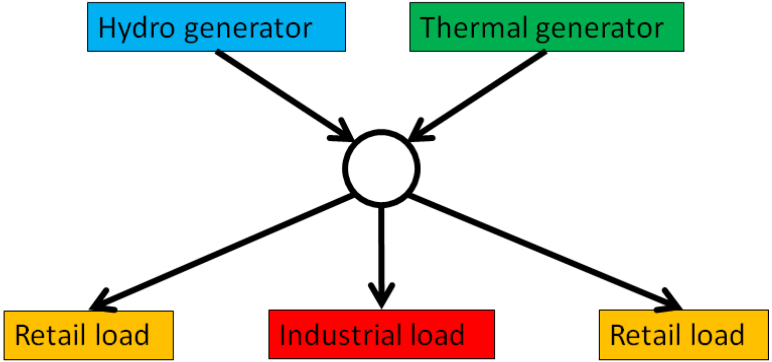
# CRAGE in GAMS using EMP

```
file info / '%emp.info%' /;
put info;
put / 'equilibrium';
loop(a,
put / 'min ' ttlDB(a);                * The objective value for agent a
loop ( pl$(map_pl_a(pl,a)),
put / x(pl);
);
put / t(a), CFD_secs(a);               *Decision variables
loop(omega,
put / DB(a,omega), DB_p(a,omega);
);
```

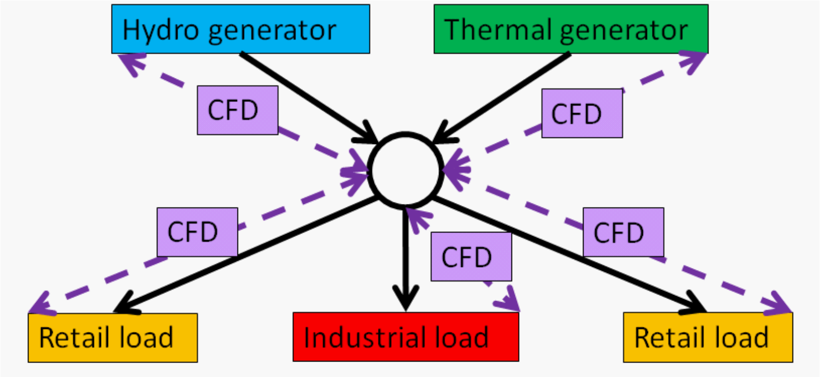
```
put / FirstStageObj_agent(a);                    *First stage objective
loop(omega,
put / SecondStageObj_agent(a,omega), DefCVaR_agent(a,omega);
*Second stage objective
loop(lb,
loop( pl$(gen(pl) and map_pl_a(pl,a)),
put / CapacityCon_gen(pl,lb,omega);
);
loop(pl$((not gen(pl)) and map_pl_a(pl,a)),
put / CapacityCon_dem(pl,lb,omega)  ) ) );
****************************************
put / 'vifunc MarketCon_MCP p';
put / 'vifunc CFD_SecCon_MCP CFD_p';
putclose info /;
```

# Summary

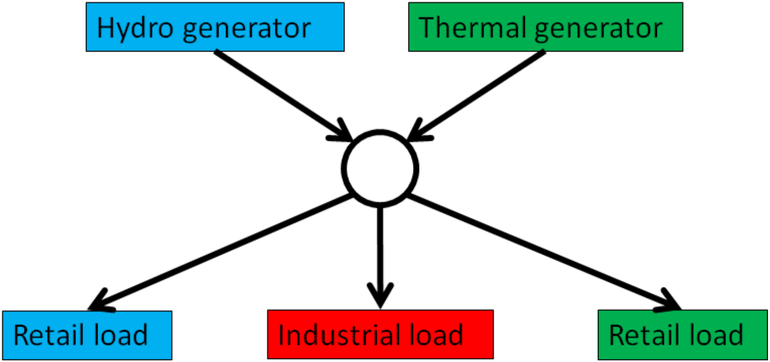
# Toy example


Hydro generator
Thermal generator
Retail load
Industrial load
Retail load

# With contracts

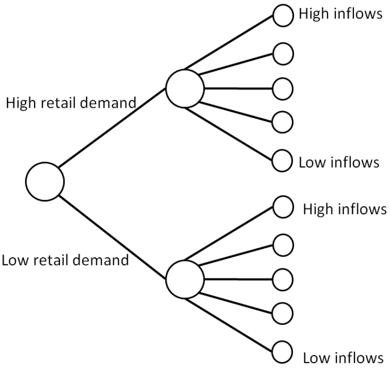


# With vertical integration




With vertical integration there are three agents.

# Uncertainty is modeled using scenarios




In example there are ten equally likely scenarios. Demand is either high or low, and varying inflow levels influence hydro generation capacity for that outcome.

# Stochastic risk-neutral social planning model

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Inflows | 40% | 55% | 70% | 85% | 100% | 40% | 55% | 70% | 85% | 100% | Expected | Risk Adj |
| Thermal | 48.61 | -33.47 | -35.60 | -35.82 | -35.82 | 204.19 | 47.51 | -30.56 | -34.10 | -34.38 | 6.06 | -14.88 |
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| Price | $83.44 | $51.74 | $50.82 | $50.61 | $50.61 | $129.84 | $80.80 | $61.93 | $61.01 | $60.90 | | |

Risk neutral solution from previous slide: hydro plant expands by 1500 and thermal plant expands by 600. Thermal plant makes losses in some high inflow years. Agents' risk-adjusted welfare is 1876.

# Competitive Risk-Averse Equilibrium

| | | RN | RA | VI | CFD | CFD_VI | AD | AD_VI |
|---|---|---|---|---|---|---|---|---|
| Expansion decisions | Thermal | 600.00 | 690.91 | 697.45 | 535.53 | 600.00 | 590.55 | 590.55 |
| | Hydro | 1500.00 | 1272.73 | 1256.36 | 1661.16 | 1500.00 | 1523.63 | 1523.63 |
| | | | | | | | | |
| Risk adjusted welfare ($M) | Thermal | -14.88 | 5.47 | | 3.33 | | 6.07 | |
| | Hydro | 122.79 | 159.81 | | 162.13 | | 154.38 | |
| | Retail 1 | 392.72 | 289.02 | | 402.30 | | 411.20 | |
| | Retail 2 | 392.72 | 289.02 | | 402.30 | | 411.20 | |
| | Hydro gentailer | 419.21 | 413.71 | 412.45 | 410.12 | 417.48 | 417.27 | 418.51 |
| | Thermal gentailer | 557.40 | 571.96 | 574.40 | 564.52 | 566.90 | 565.58 | 566.82 |
| | Industry | 983.10 | 954.44 | 953.63 | 1041.66 | 1043.11 | 1044.66 | 1042.18 |
| | Total | 2027.50 | 2018.55 | 2017.52 | 2027.00 | 2027.50 | 2027.52 | 2027.51 |

# Vertical integration

| | | RN | RA | VI | CFD | CFD_VI | AD | AD_VI |
|---|---|---|---|---|---|---|---|---|
| Expansion decisions | Thermal | 600.00 | 690.91 | 697.45 | 535.53 | 600.00 | 590.55 | 590.55 |
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# Contracts

| | | RN | RA | VI | CFD | CFD_VI | AD | AD_VI |
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| | | | | | | | | |
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# Competitive Risk-Averse Equilibrium

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# Contracts and vertical integration

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| Expansion decisions | Thermal | 600.00 | 690.91 | 697.45 | 535.53 | 600.00 | 590.55 | 590.55 |
| | Hydro | 1500.00 | 1272.73 | 1256.36 | 1661.16 | 1500.00 | 1523.63 | 1523.63 |
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| | Total | 2027.50 | 2018.55 | 2017.52 | 2027.00 | 2027.50 | 2027.52 | 2027.51 |

# Summary

# This is the end

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# References

R. Aid, G. Chemla, A. Porchet & N Touzi. Hedging and vertical integration in electricity markets. Management Science, 57, 2011.

P. Artzner, F. Delbaen, J-M. Eber & D. Heath, Coherent measures of risk, Mathematical Finance, 9, 1999.

D. Heath & H. Ku. Pareto equilibria with coherent measures of risk. Mathematical Finance, 14(2), 2004.

R. Meade & S. O'Connor. Comparison of long-term contracts and vertical integration in decentralised electricity markets. ISCR Report, 2008.

R.Meade. Vertical integration vs vertical separation in an imperfectly competitive industry such as electricity with retail, wholesale and forward markets. ISCR Report, 2012.

D. Ralph & Y. Smeers. Pricing risk under risk measures: an introduction to stochastic-endogenous equilibria, 2011.