The MIDAS touch:

solving hydro bidding problems using mixed integer programming

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Acknowledgements

- Anes Dallagi , EDF R&D France
- Andrew Kerr, Meridian Energy New Zealand
- PGMO
- OSIRIS EDF R&D
- FMJH

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  - Meridian participate in the NZ Spot market using the Waitaki hydro-scheme

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  3. Solve for next period based on observed prices & dispatch

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- Respect physical constraints of the hydro-scheme (i.e. storage bounds and water flow balance)
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- Meet ancillary requirements:
  - Frequency containment reserve
  - Frequency restoration reserve

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  - Partition overall price distribution into segments
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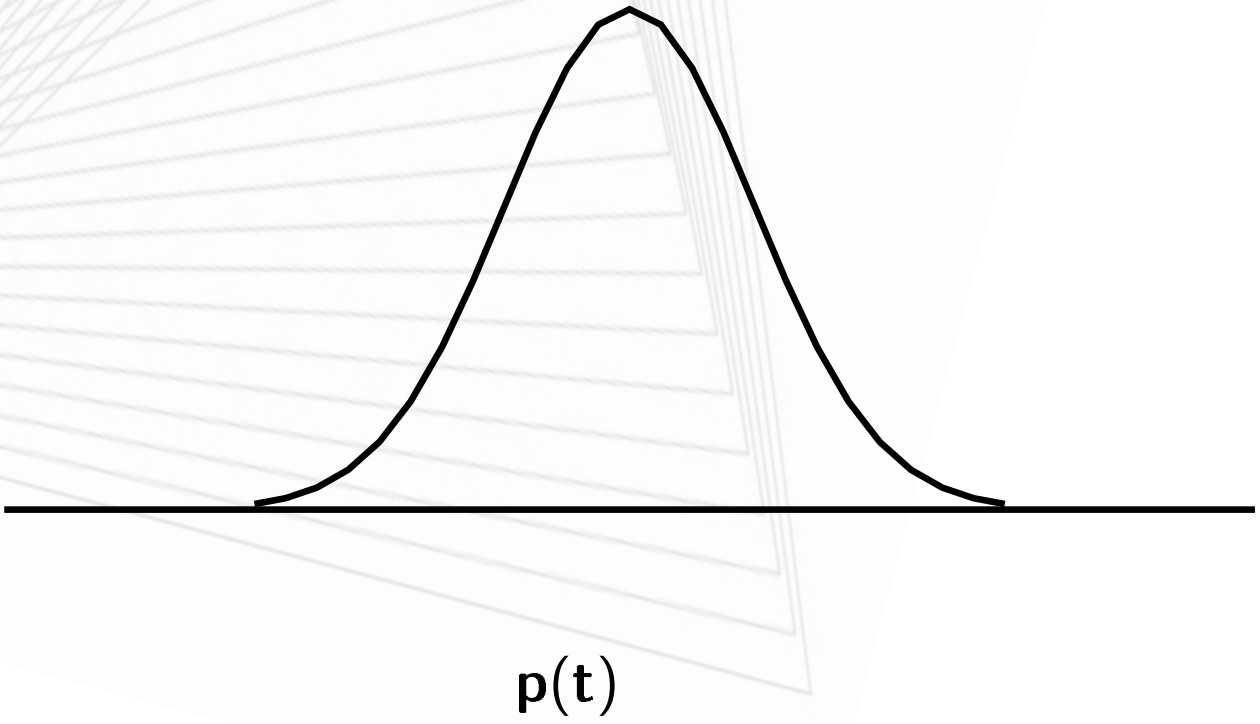
- Assumptions:
  - Price taking hydro power producers
  - Deterministic inflow
  - Discrete finite offers (i.e. an offer stack)

Formulation I: Modelling price

- Market clearing price $p(t)$ is random variable



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- Market clearing price $p(t)$ is random variable
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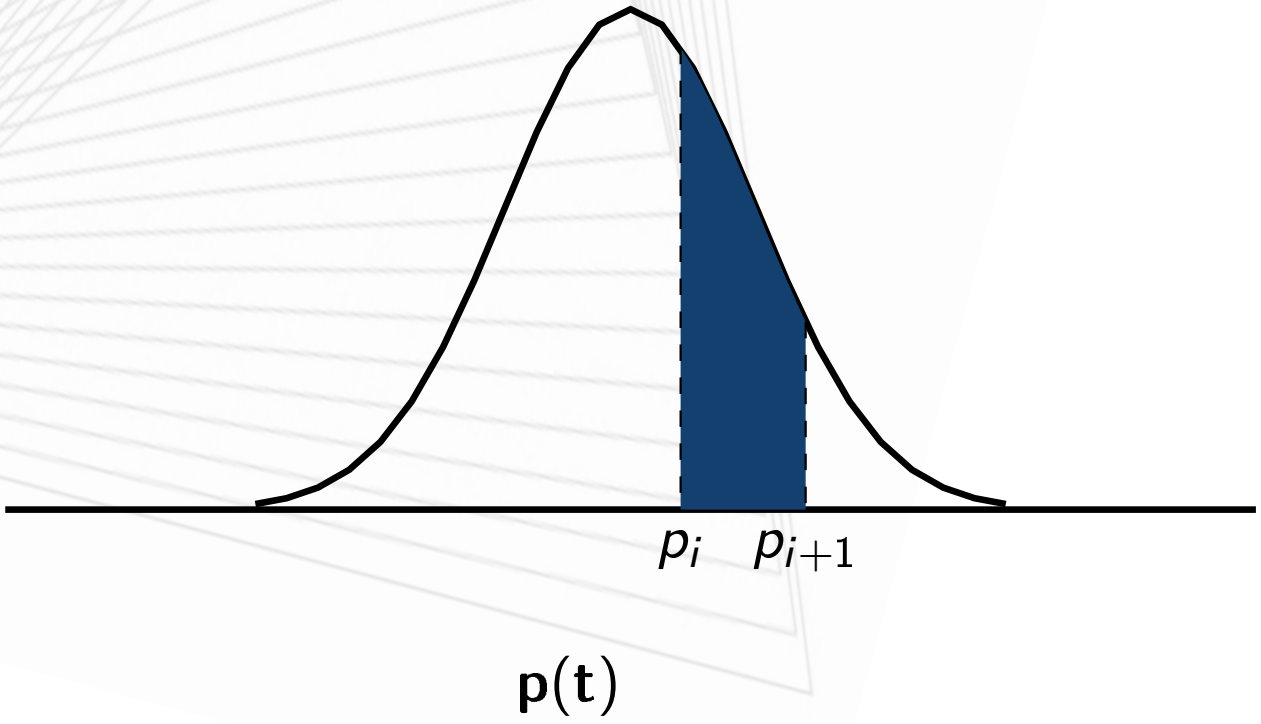

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- Transition probability of moving from segment i to segment j:

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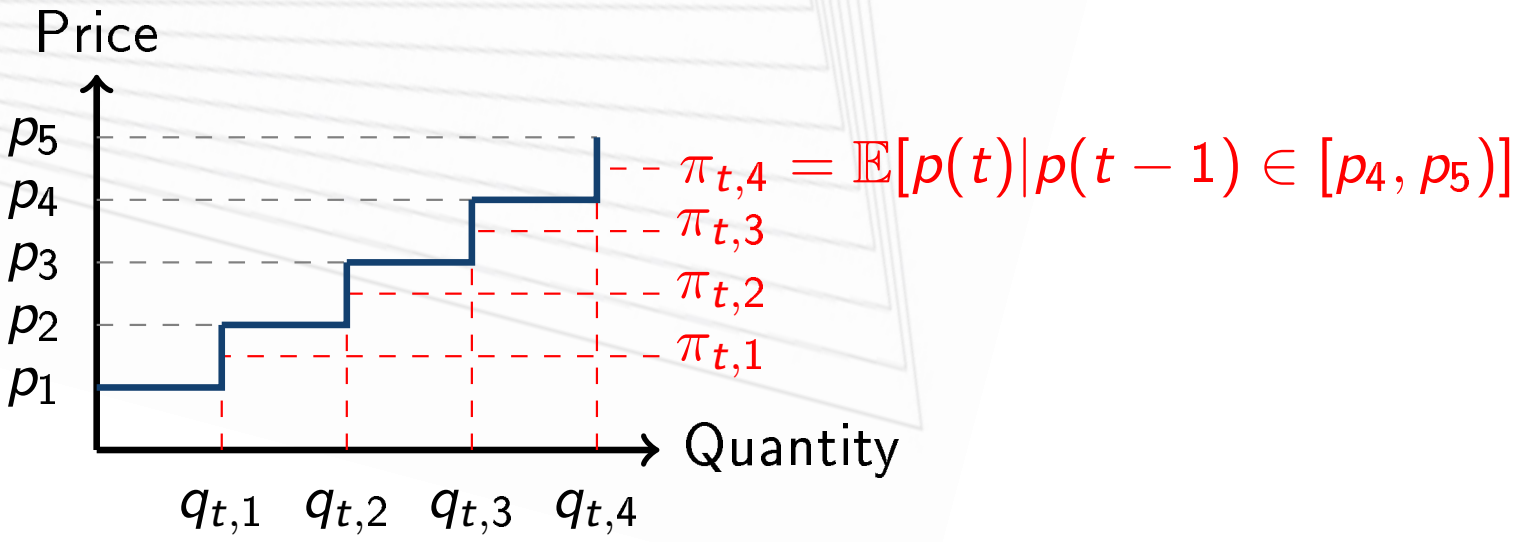

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$$\begin{aligned} V_{t,i}(x_{t,i}) = \quad & \max \mathbb{E}[\text{Profit}(q_t)] \quad + \mathbb{E}[\text{FutureProfit}(x_{t+1,j})] \\ & \sum_{j=1}^{M} P_{i,j}(t)\pi_{t,j}q_{t,j} \quad + \sum_{j=1}^{M} P_{i,j}(t)V_{t+1,j}(x_{t+1,j}) \end{aligned}$$

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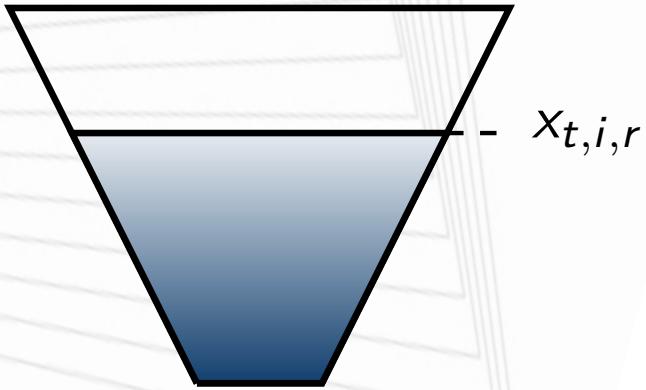

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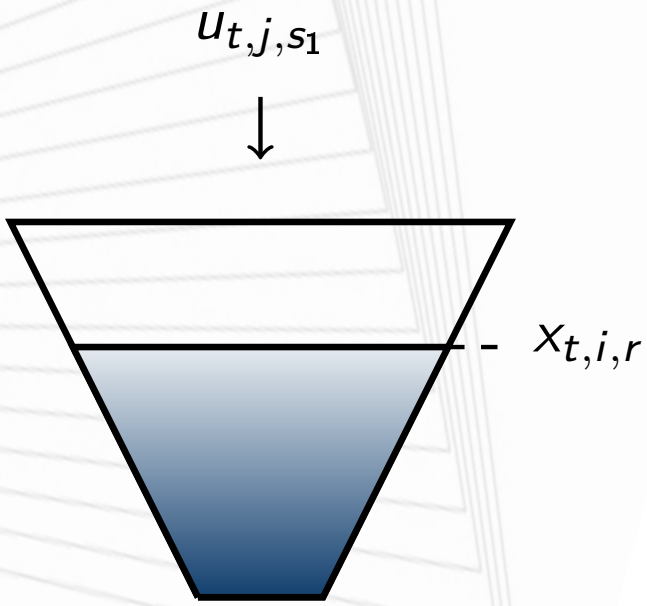

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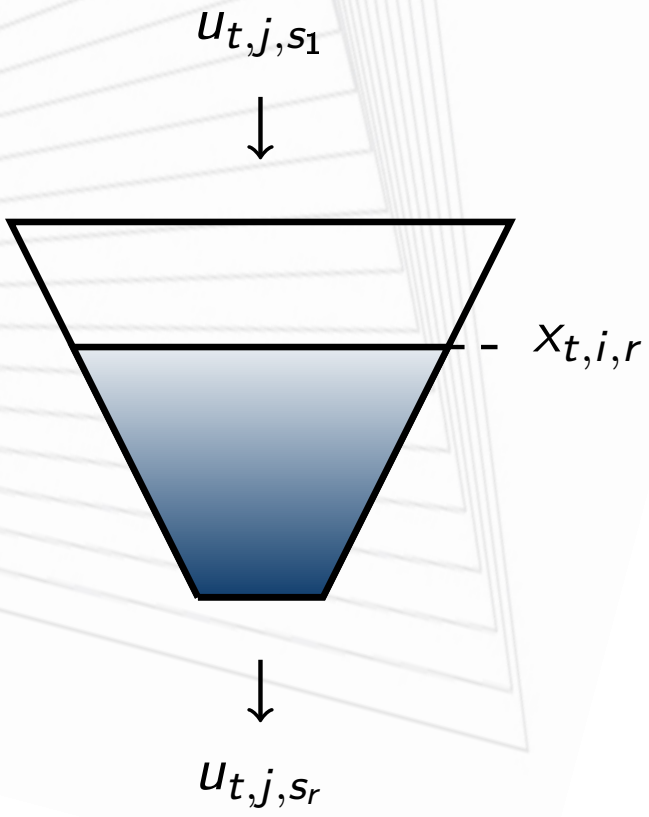

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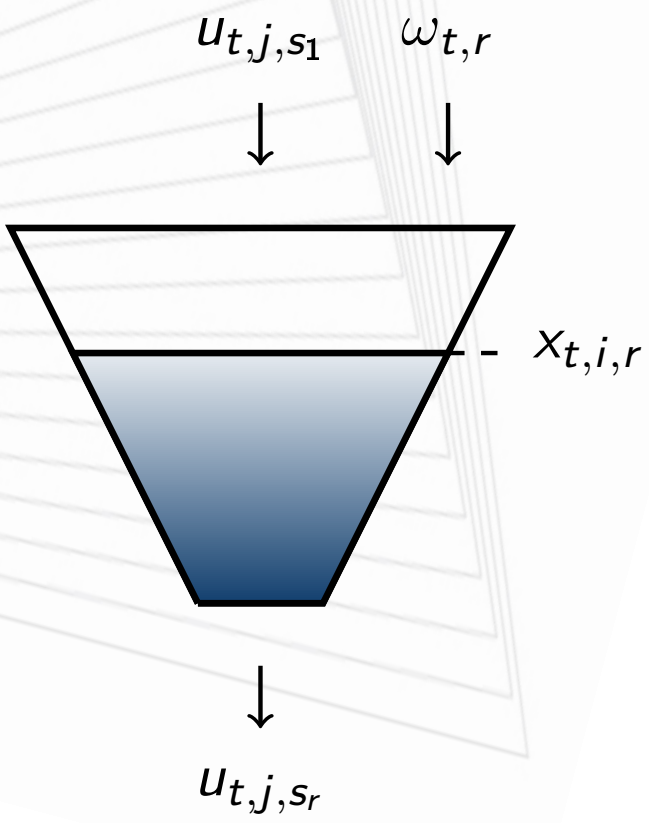

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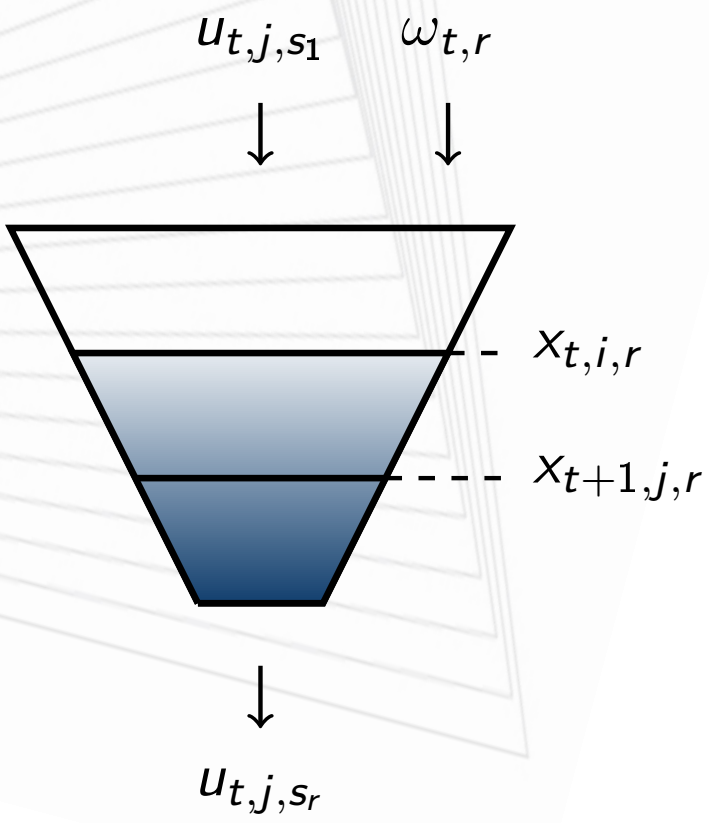

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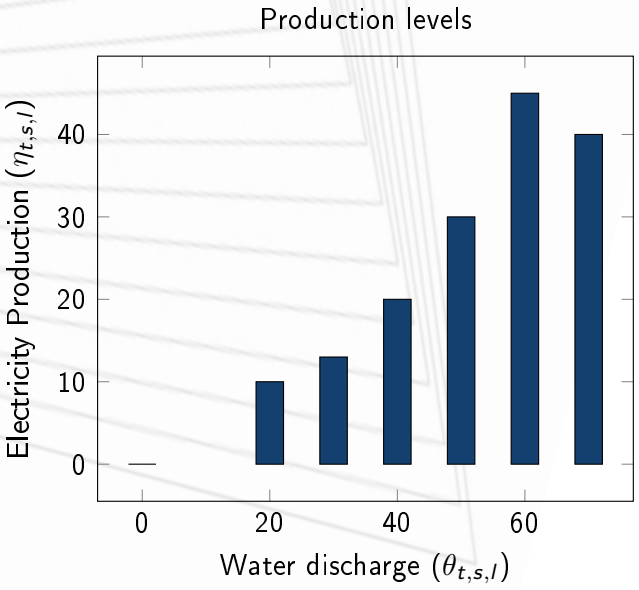

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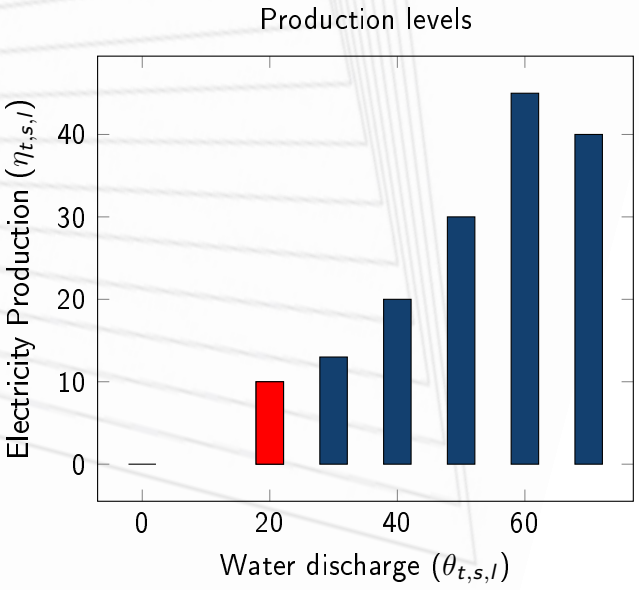

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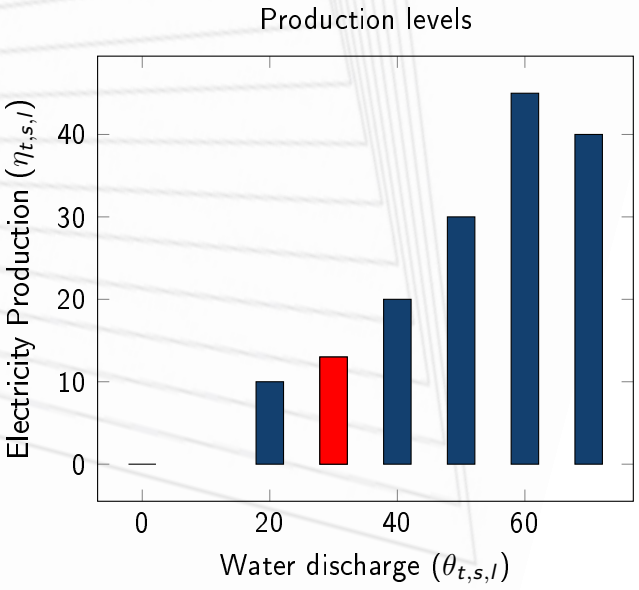

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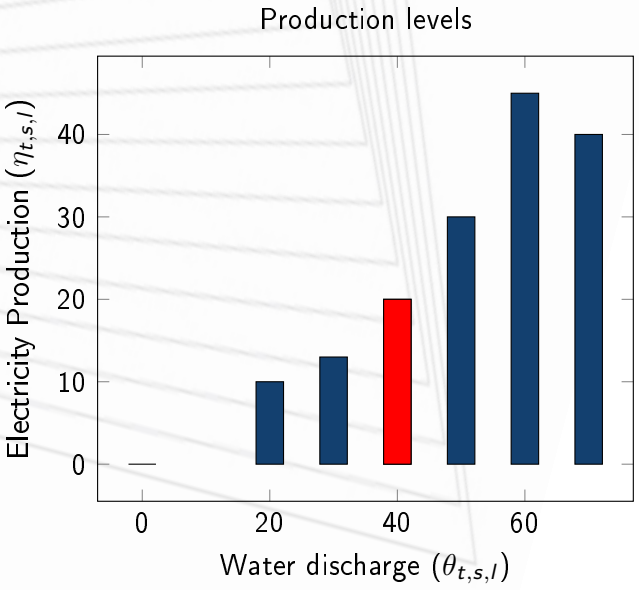

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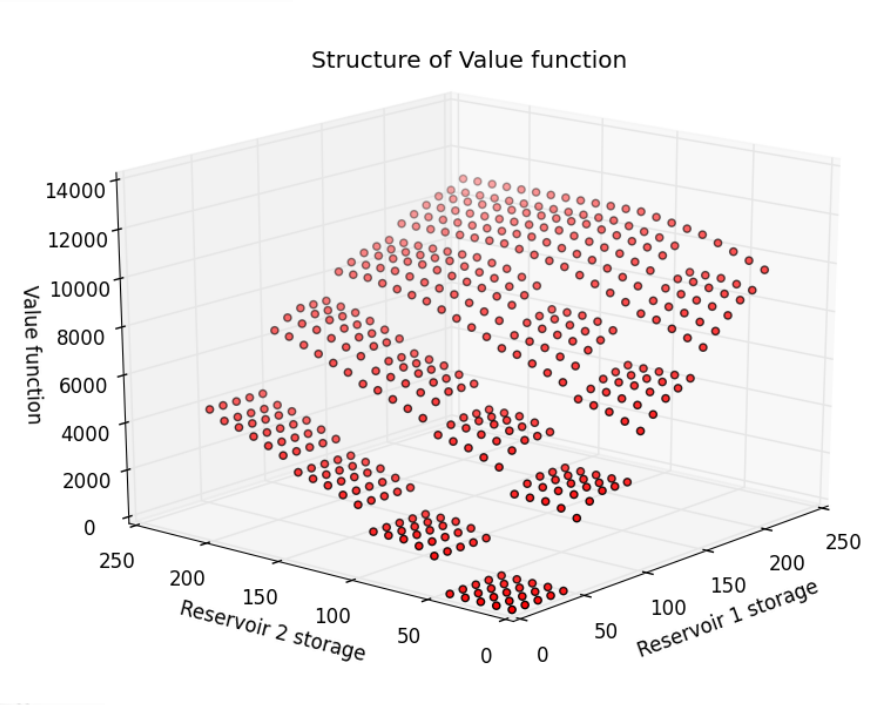

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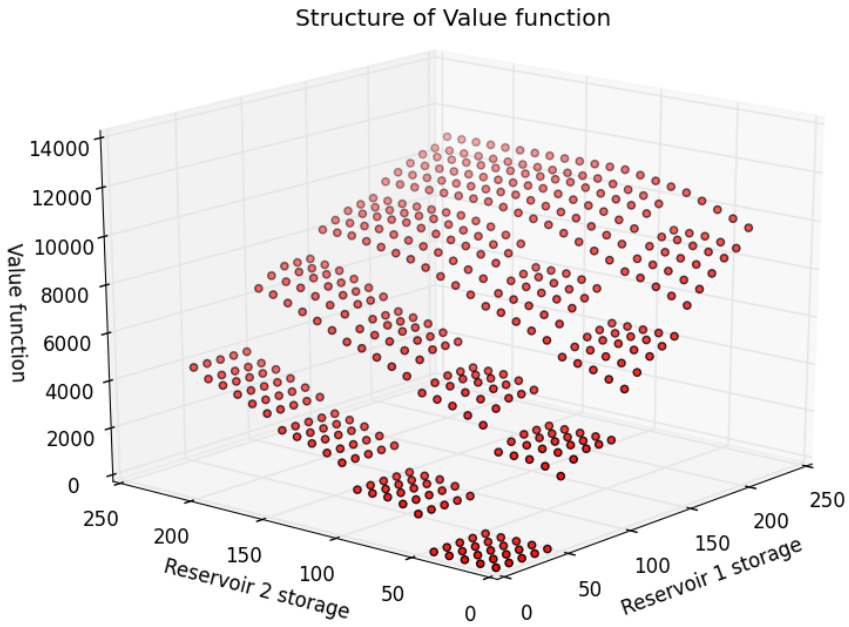

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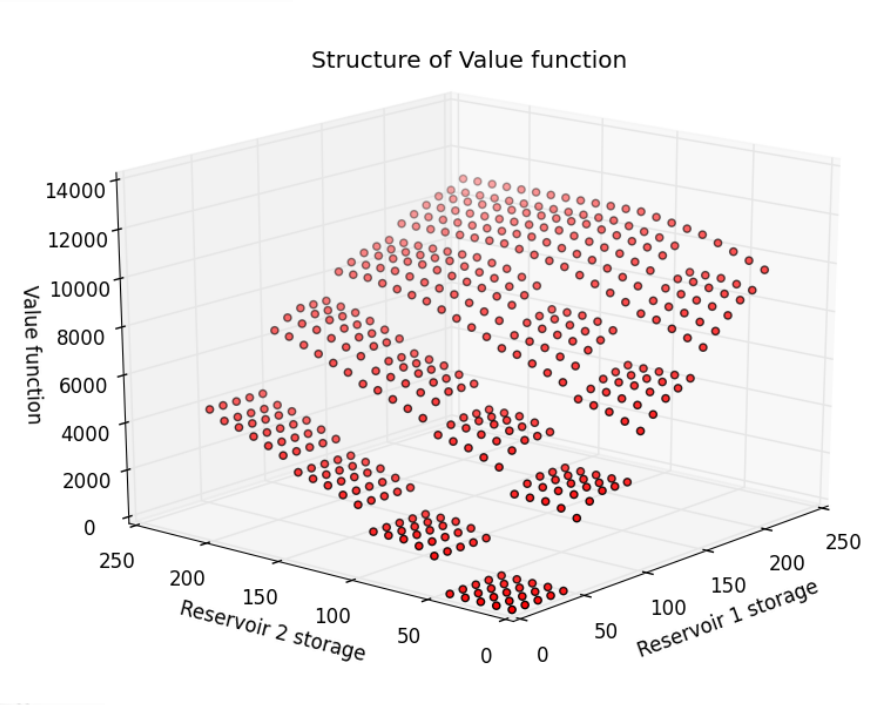

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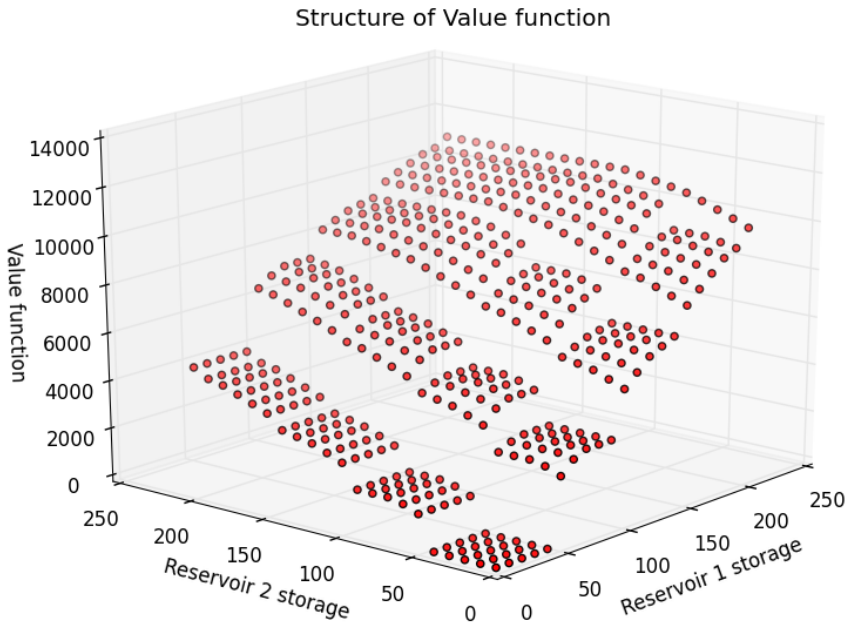
- Makes HERBS hard to solve



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- **Stochastic dual dynamic program (SDDP)** (Pereira & Pinto [3])
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- **Linear decision rules** (Braathen & Eriksrud [2])
  - Optimal policy not guaranteed, due to restriction of the decision space

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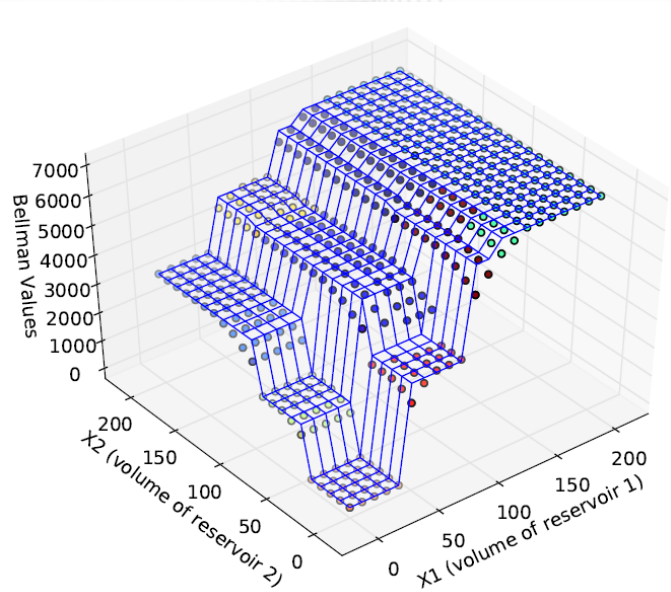
- Stochastic optimization algorithm designed to solve **sequential decision problems under uncertainty**
- Creates an outer approximation of $V_{t+1,j}(x_{t+1,j})$
- Uses piece-wise constant functions to represent the plateaus of $V_{t+1,j}(x_{t+1,j})$



Formulation I

Formulation I

- $\hat{v}_{t+1,j}$: Variable that approximates the value function $V_{t+1,j}(x_{t+1,j})$

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- Use series of binary variables & Big M constraints to couple $x_{t+1,j}$ to $\hat{v}_{t+1,j}$ using $a_{j,h,r}$ & $b_{j,h}$

Formulation II

- For period $t+1$ & price segment j:

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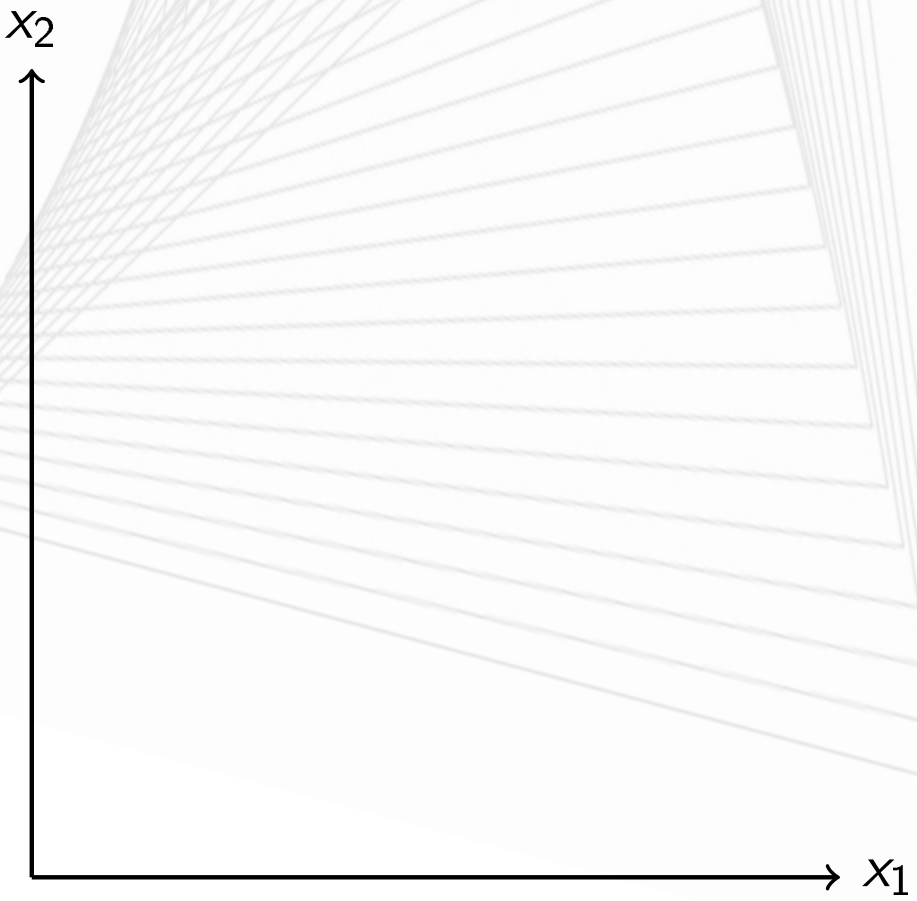

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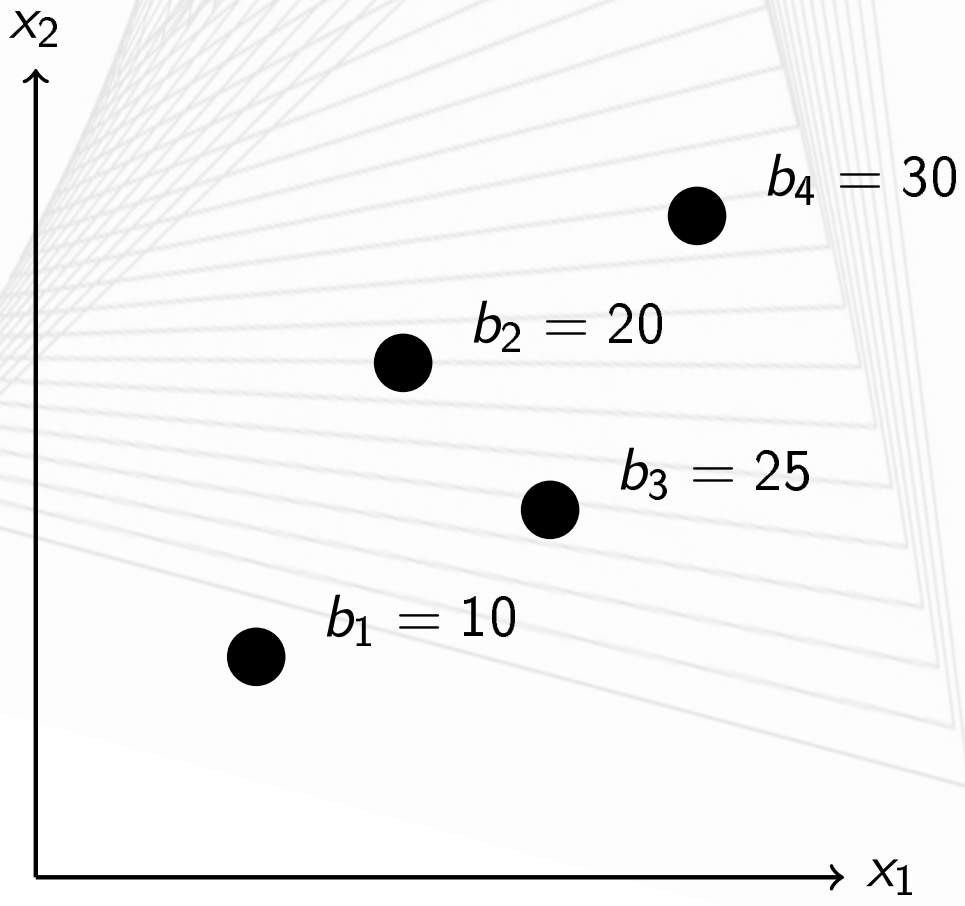
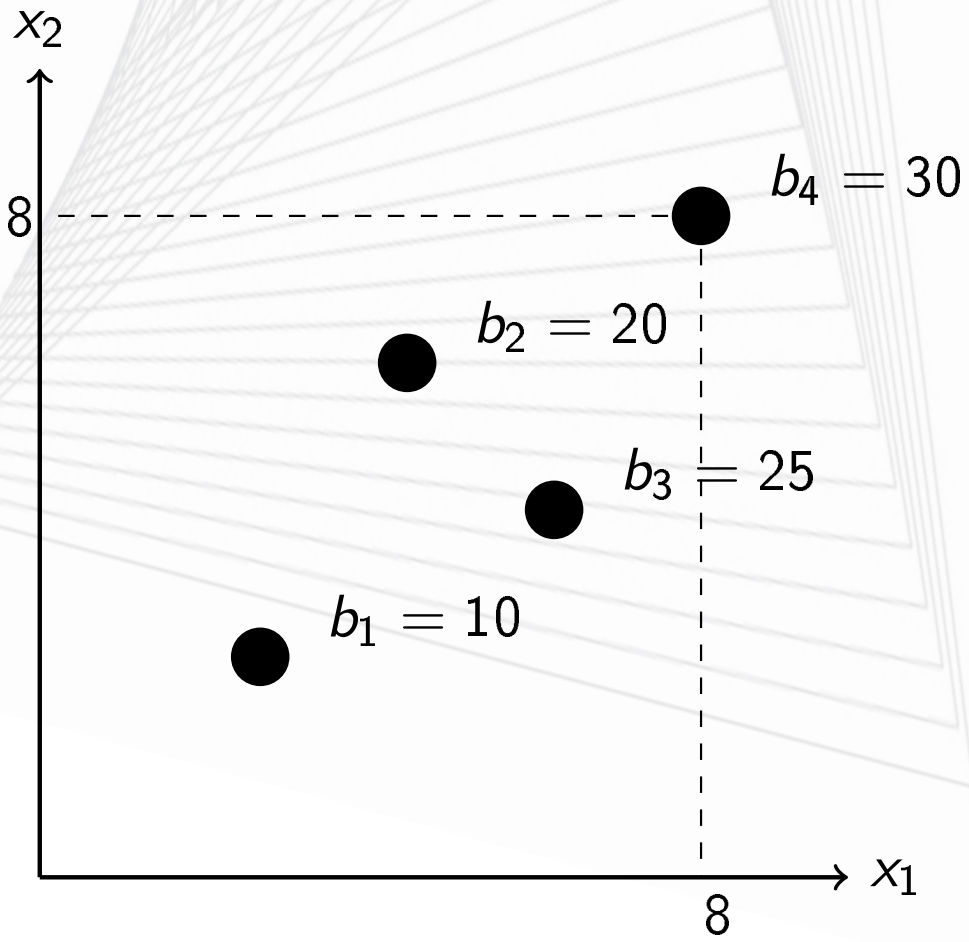


Formulation II

- For period $t+1$ & price segment j:

$$a_4 = (8, 8)$$



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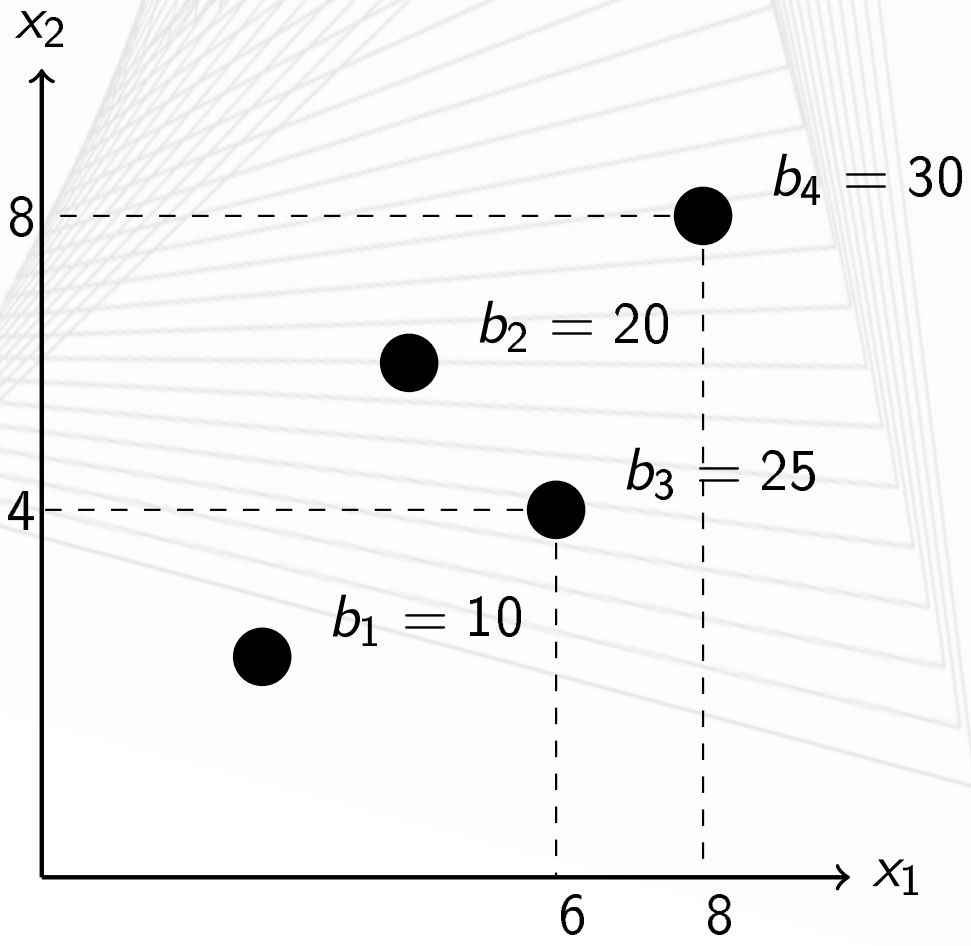
$$a_4 = (8, 8)$$
$$a_3 = (6, 4)$$



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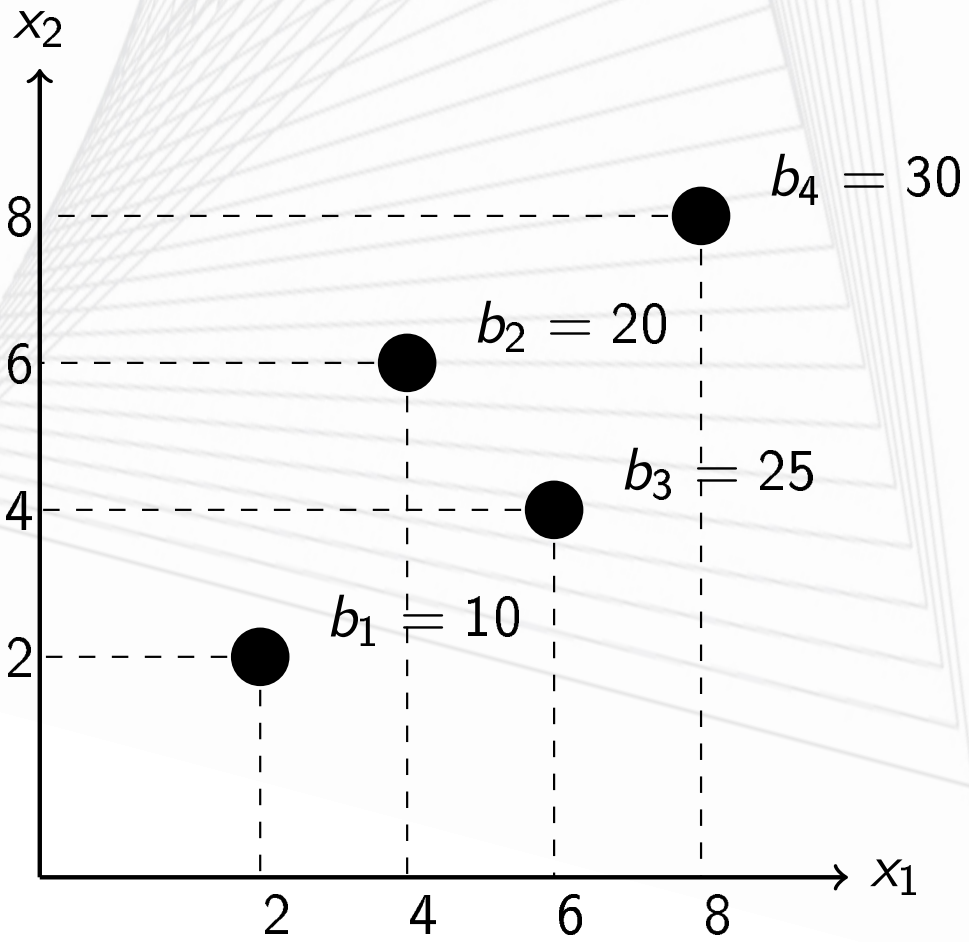

$$a_4 = (8, 8)$$
$$a_3 = (6, 4)$$
$$a_2 = (4, 6)$$
$$a_1 = (2, 2)$$

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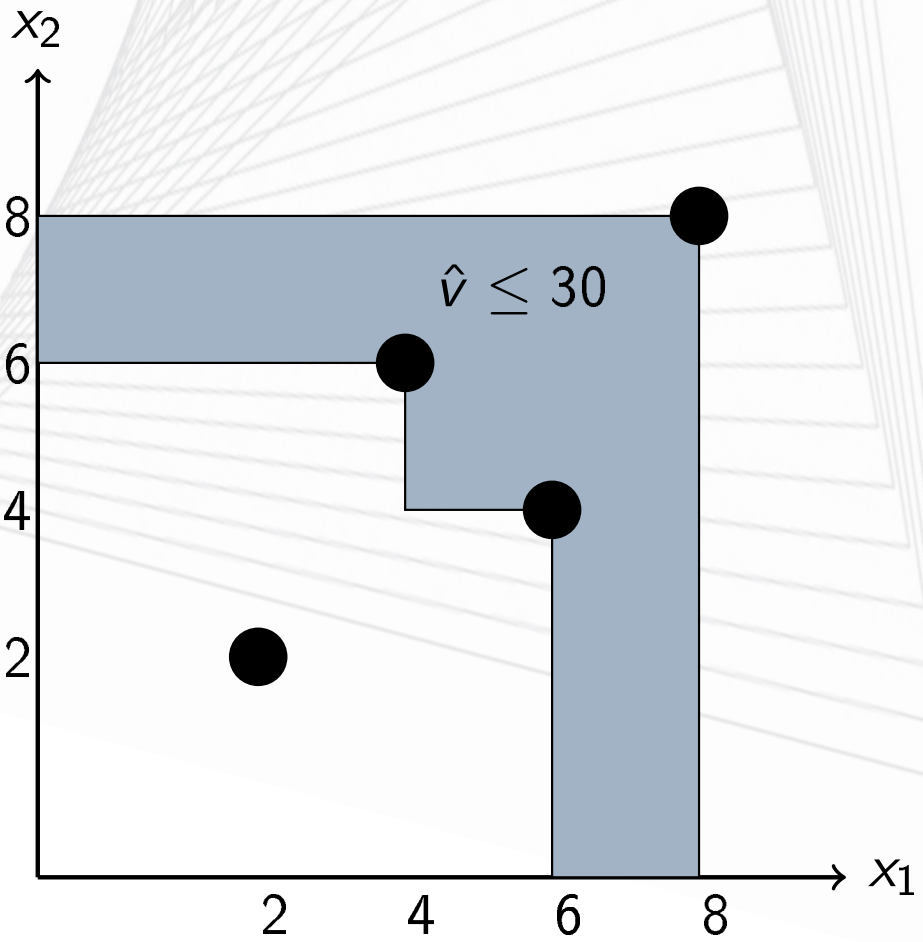

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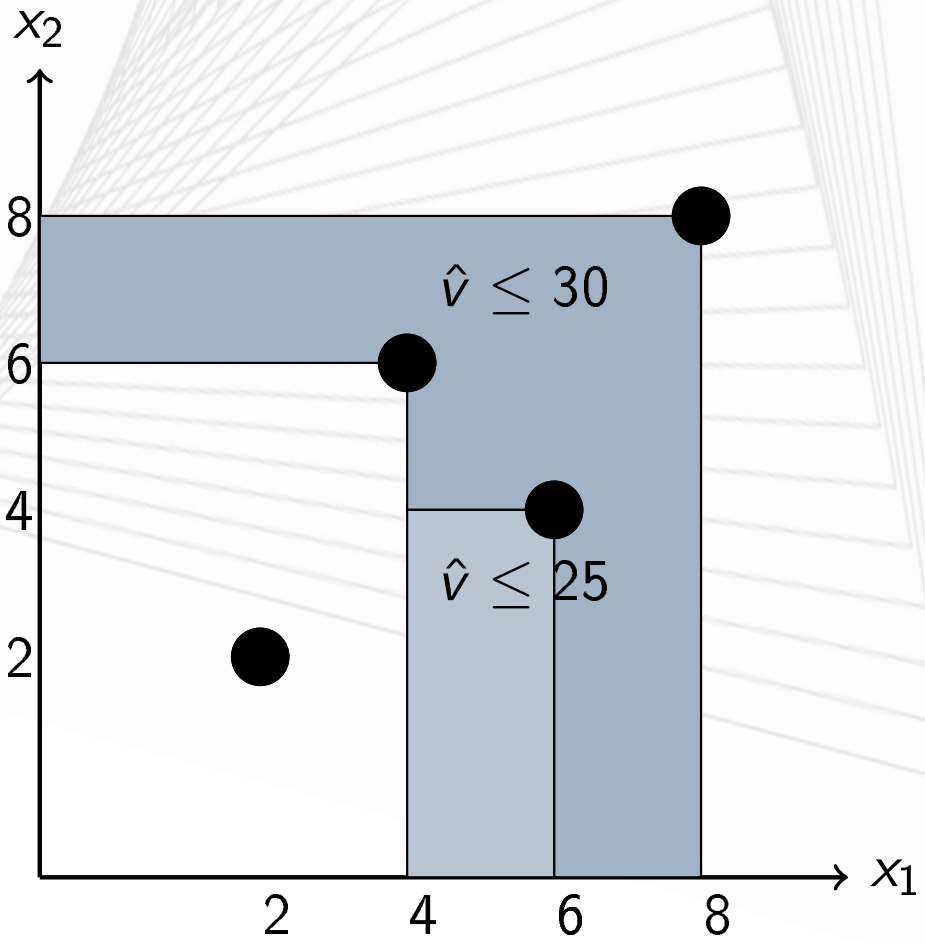

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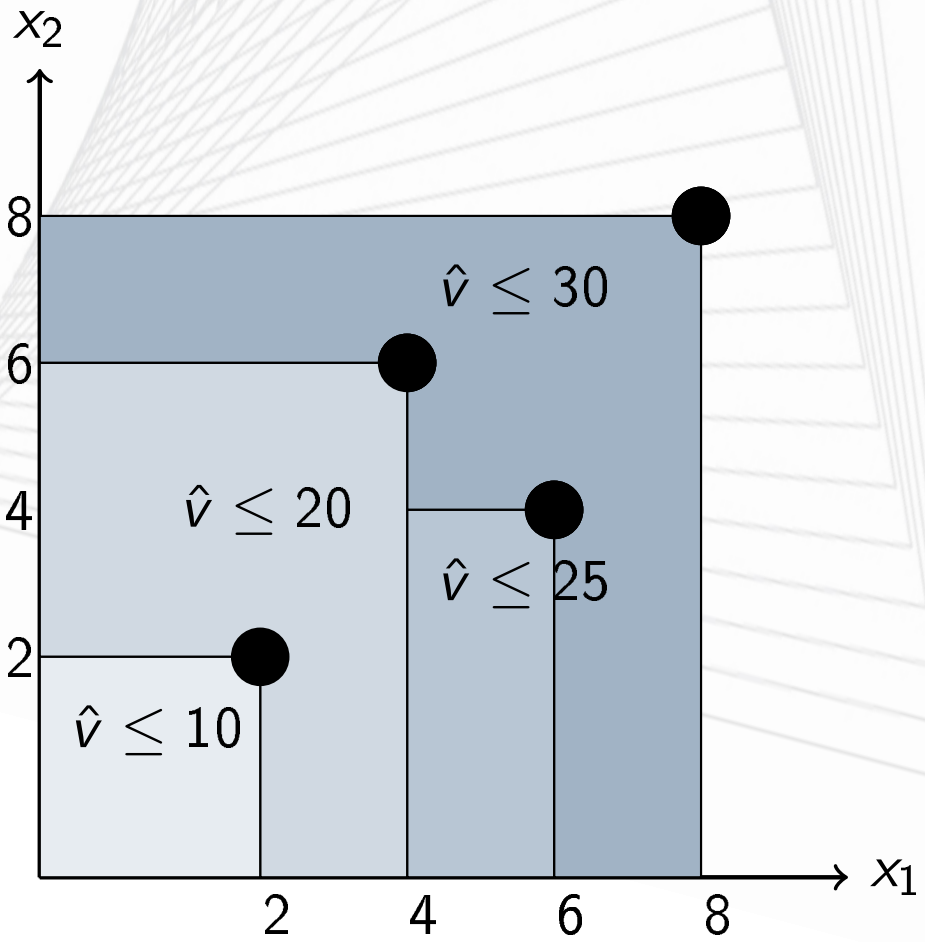

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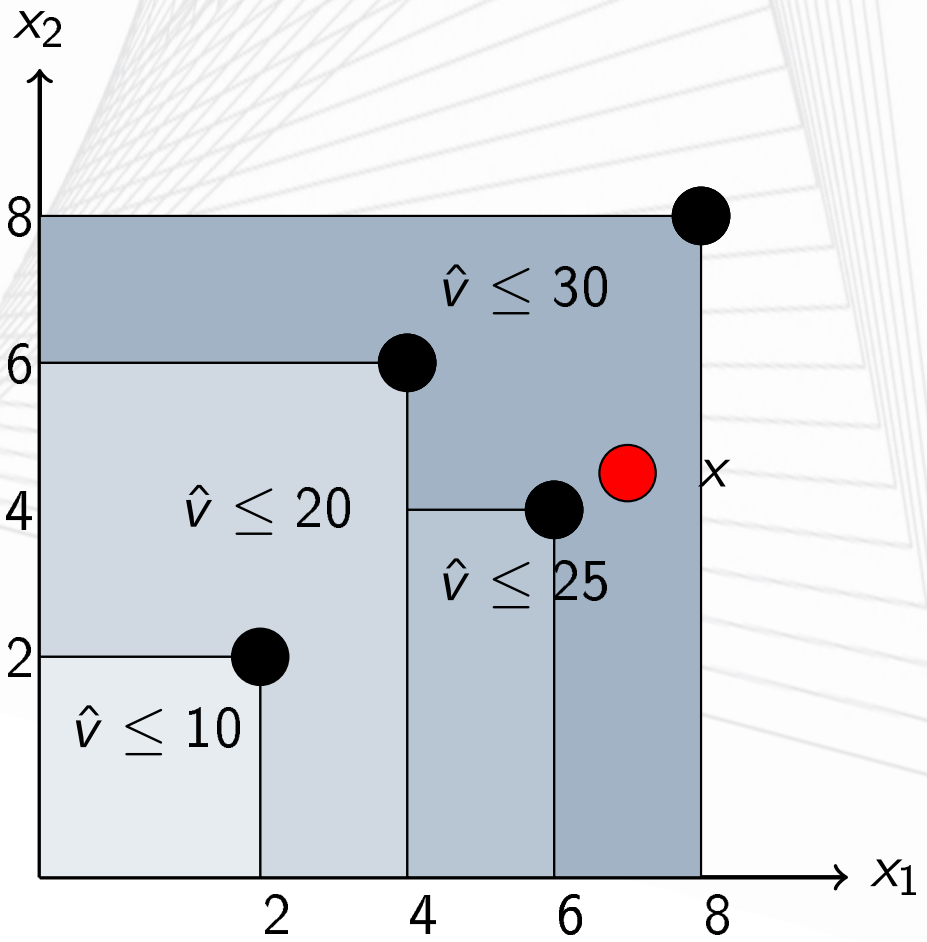

$\hat{v} \leq 30$

$x_1 > 6;$
Or $x_1 > 4$
Or $x_1 > 0$

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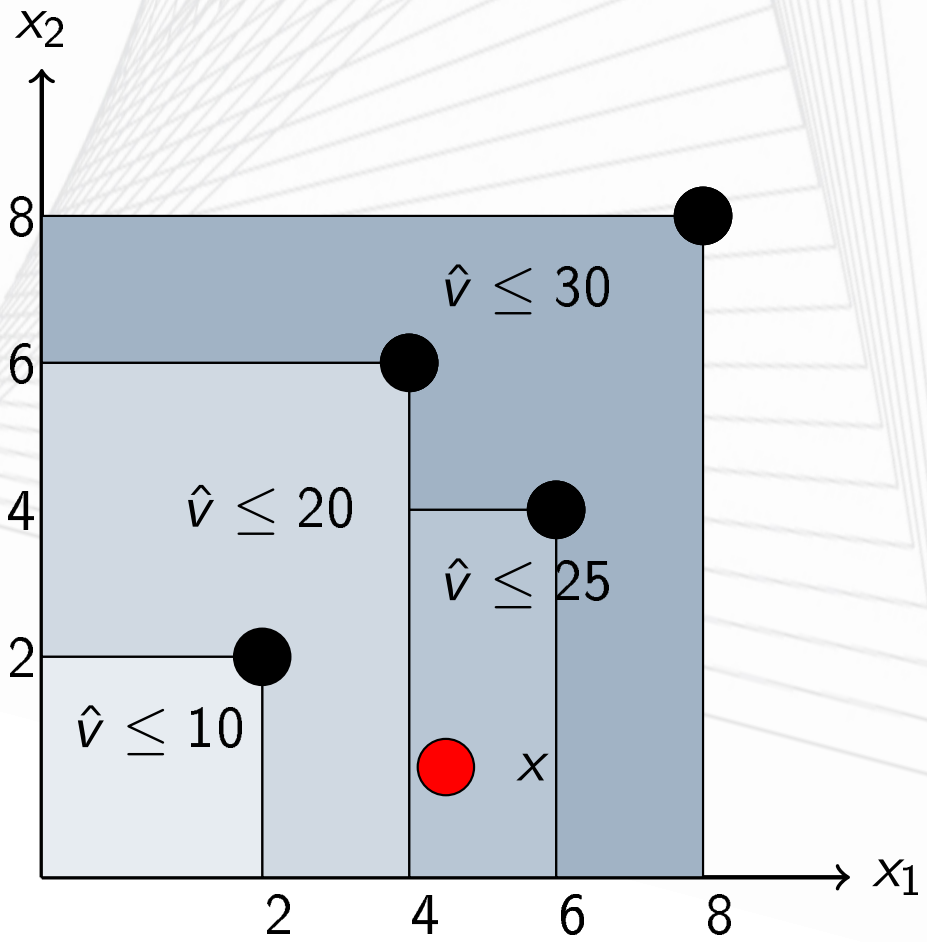

$\hat{v} \leq 30$;
And $\hat{v} \leq 25$

$x_1 > 4$

$x_2 > 0$

Description of the MIDAS algorithm

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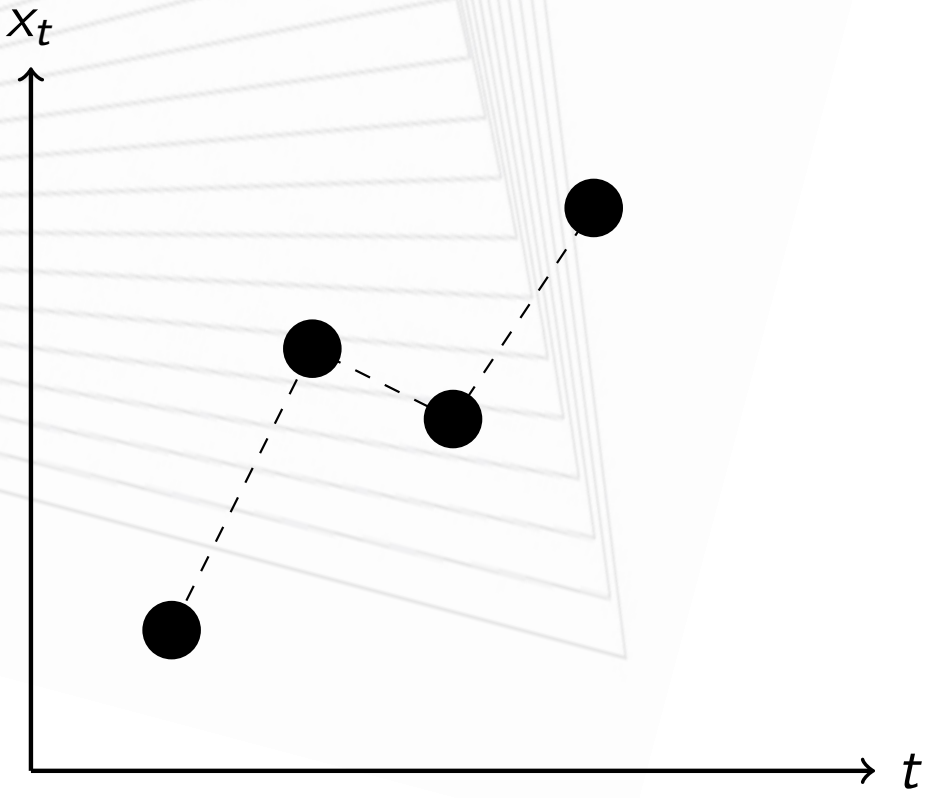

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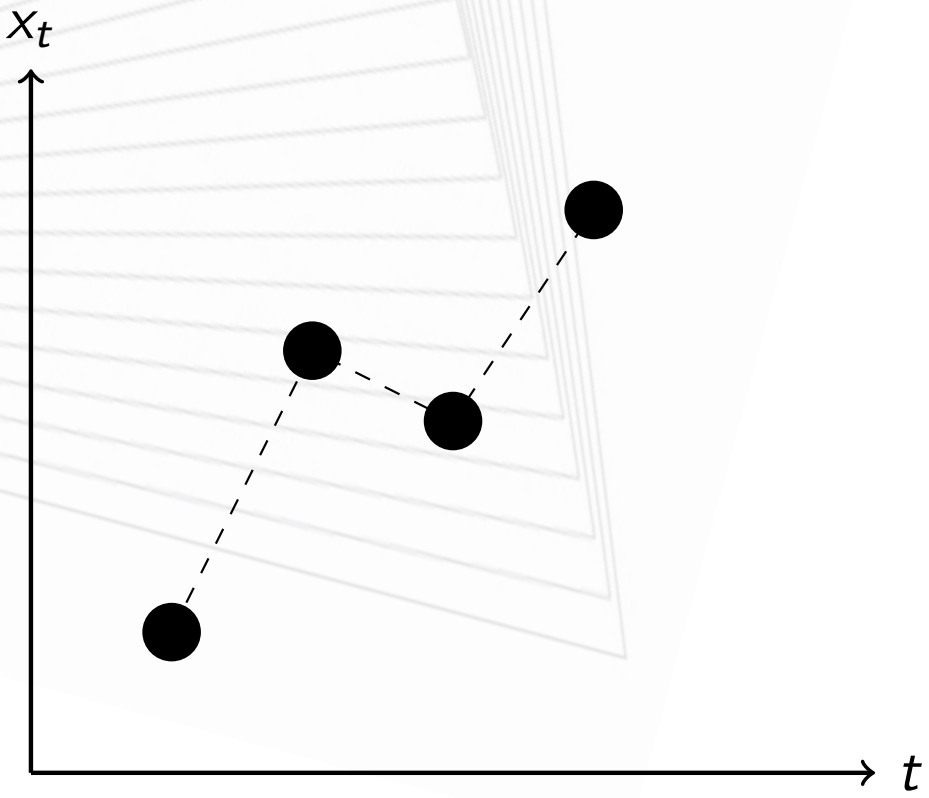

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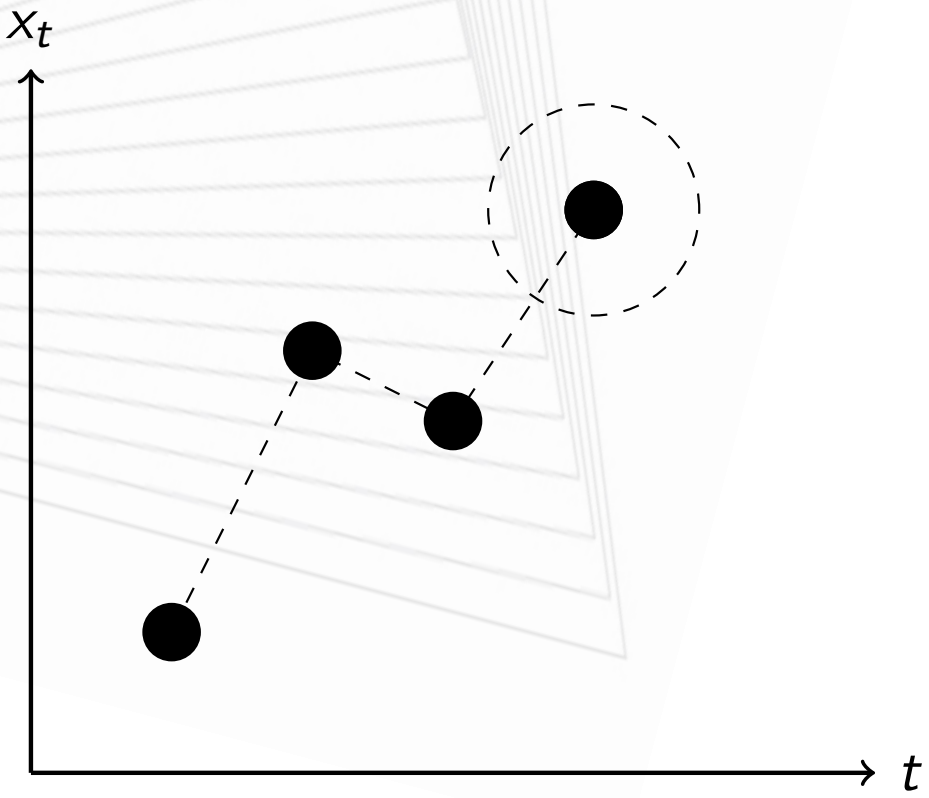

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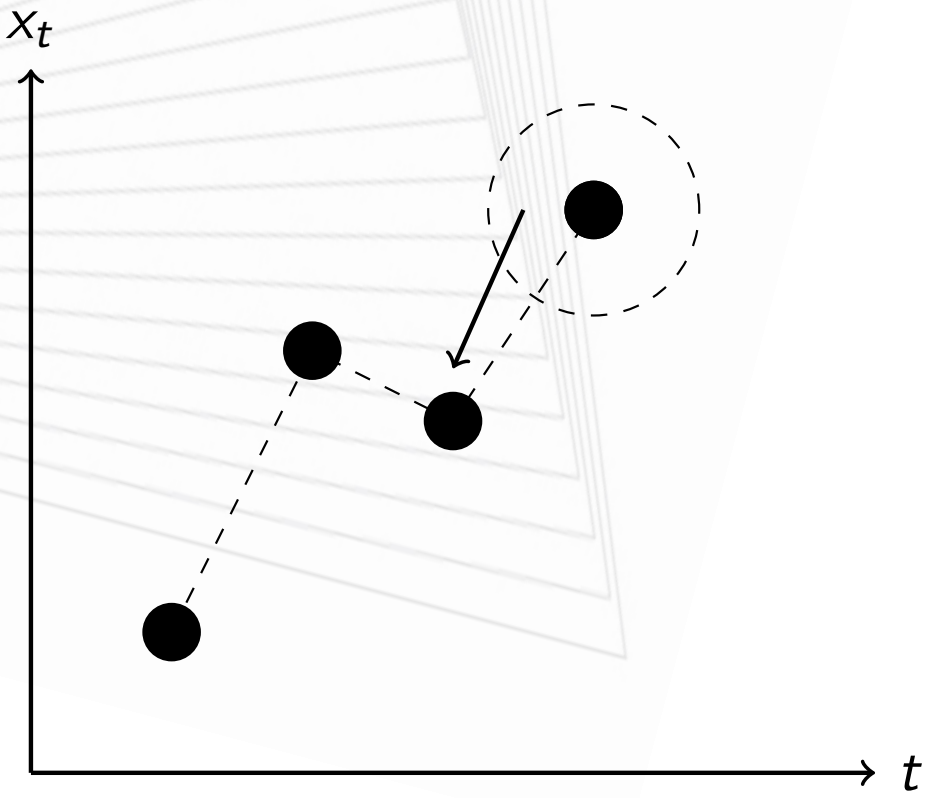

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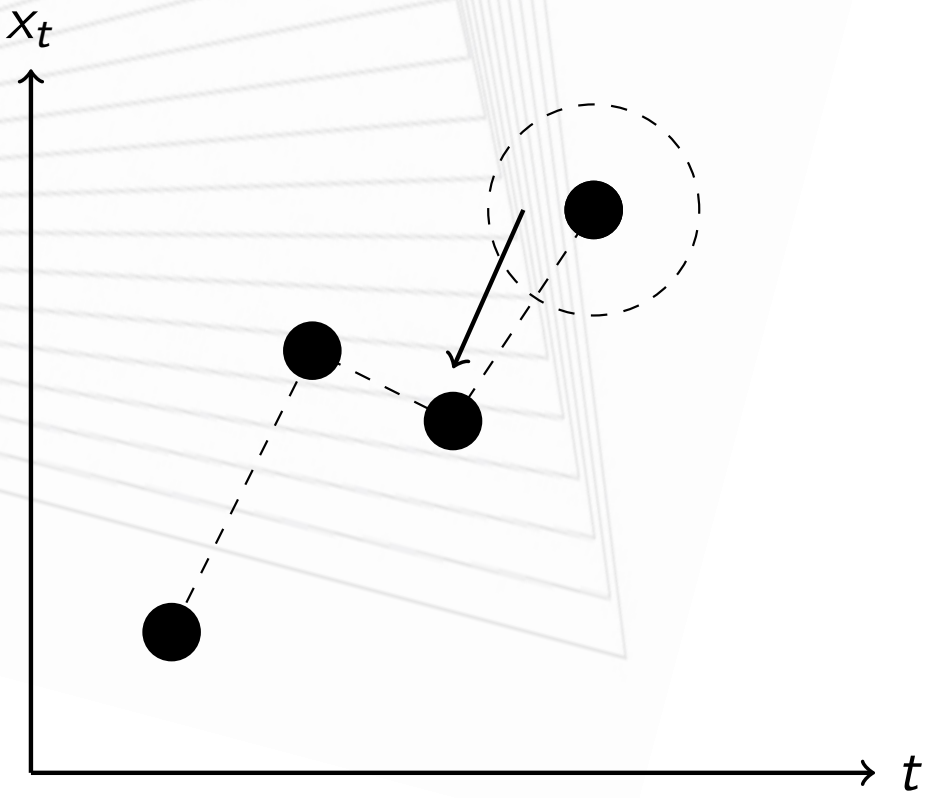

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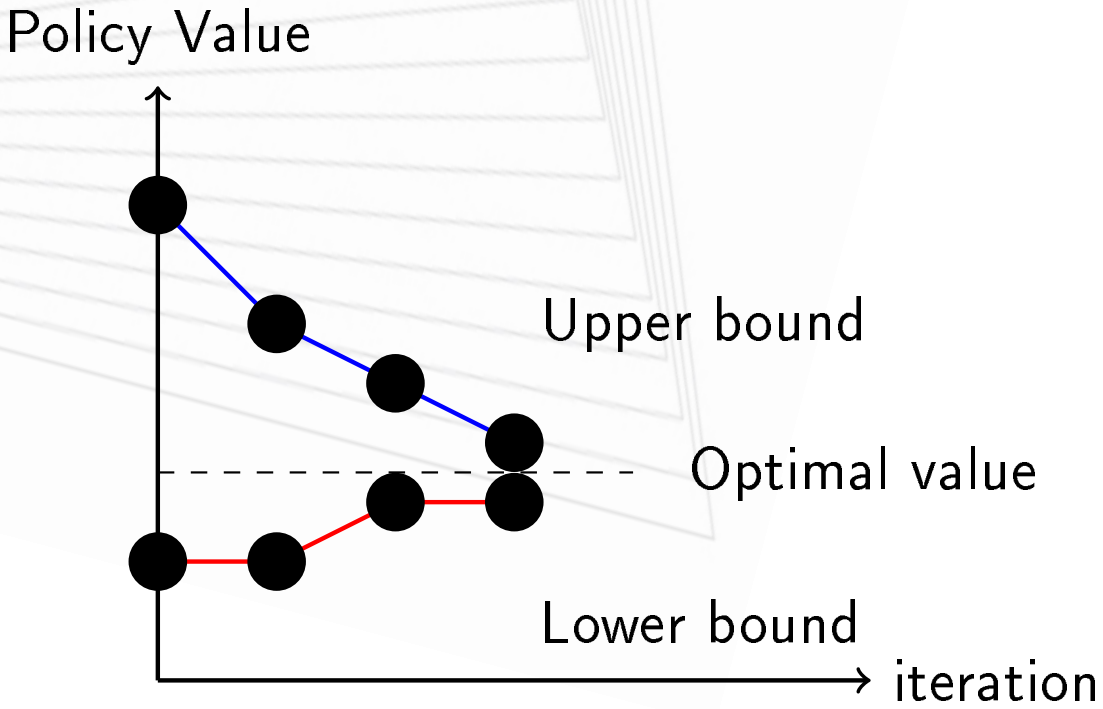

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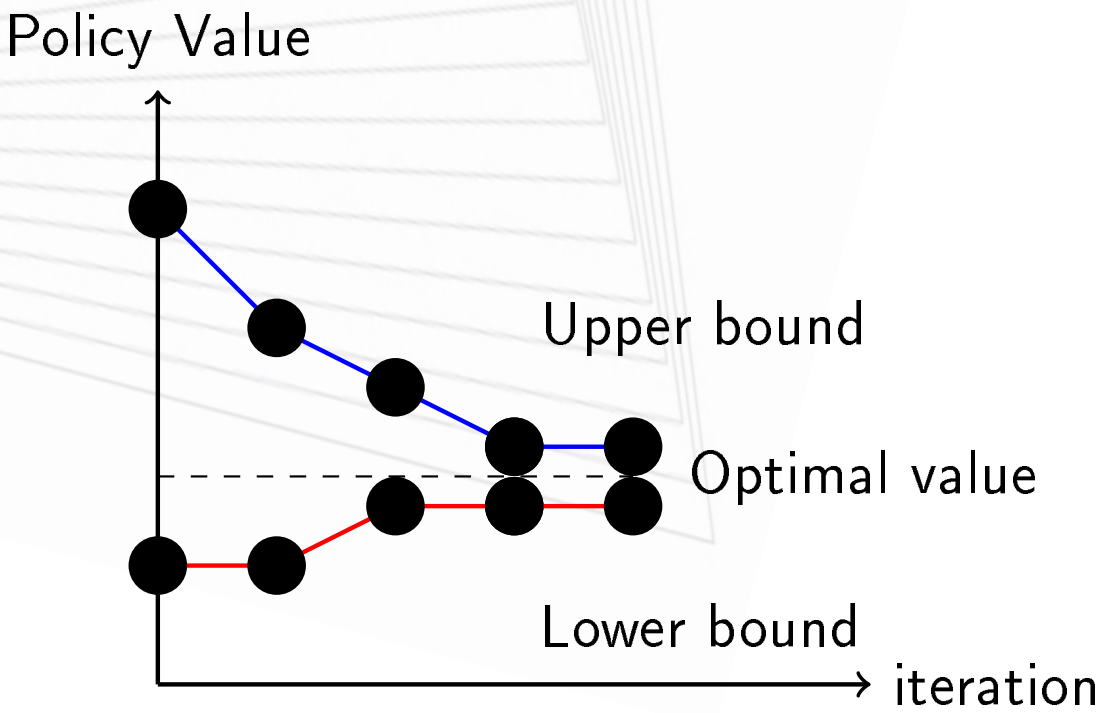

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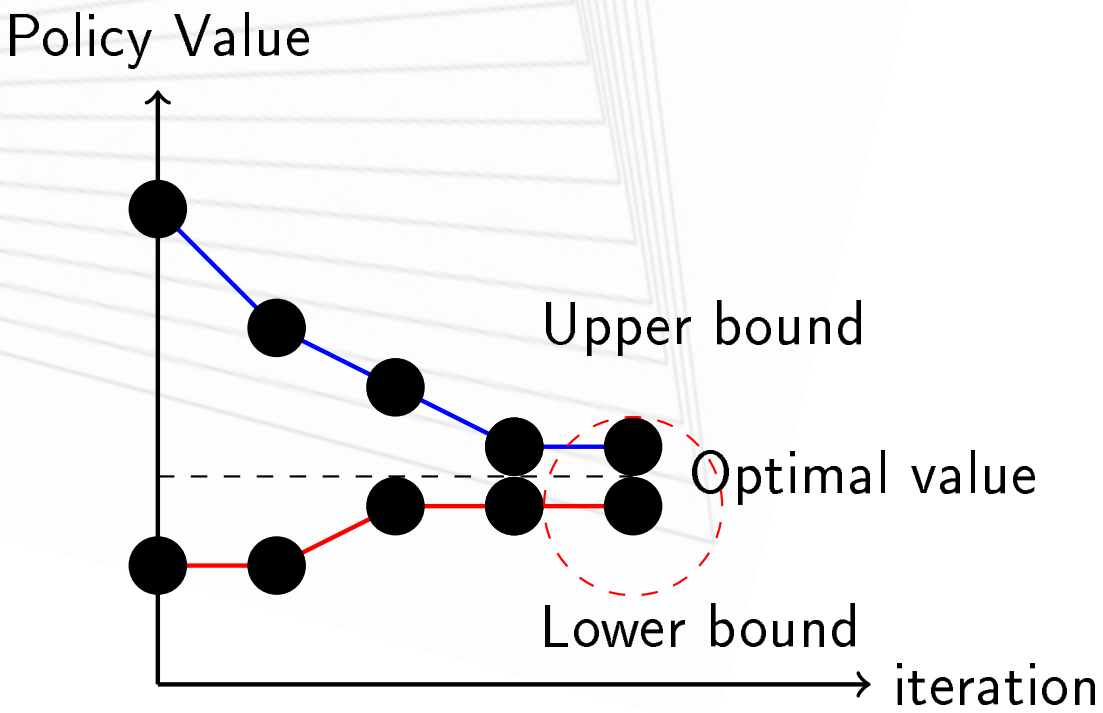

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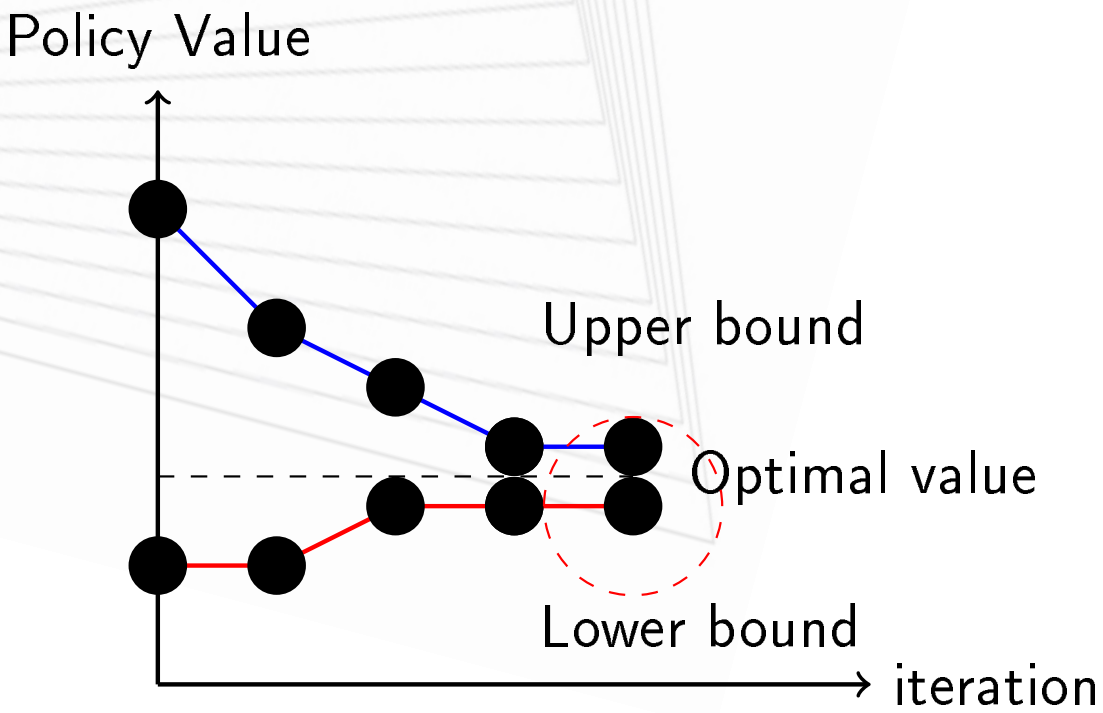

Hydro-scheme data

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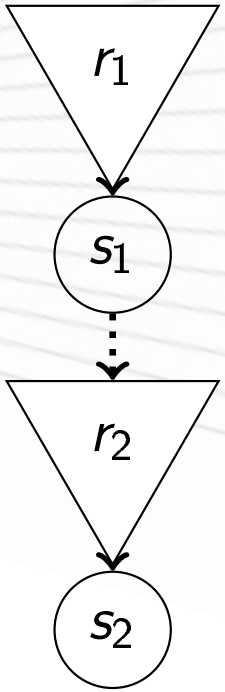
- Two identical, cascaded reservoirs



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- Two identical, cascaded reservoirs
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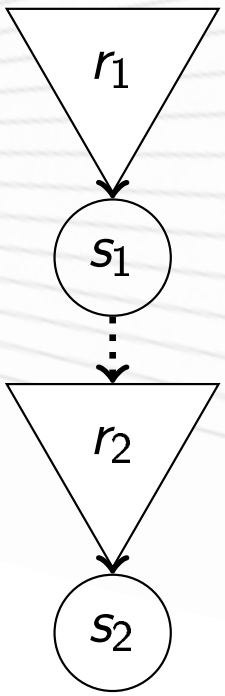

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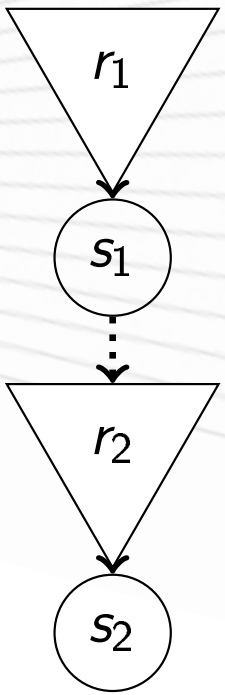

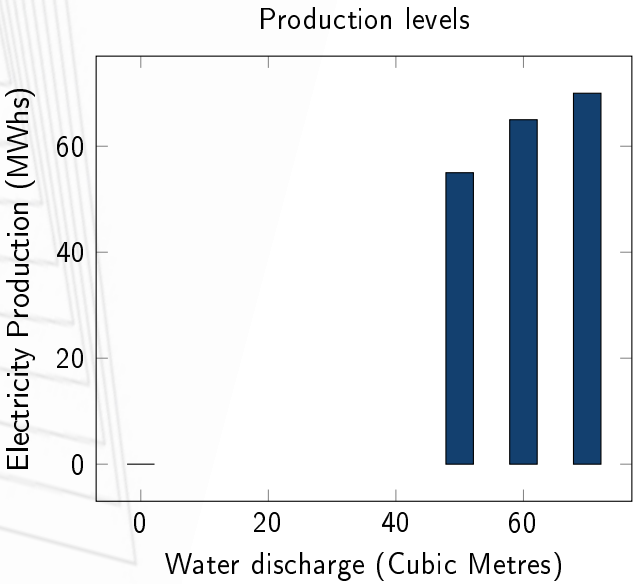

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  - **MIDAS policy**: On average 98% of optimal value
  - **SDDP policy**: On average 93% of optimal value

	Mean	**Median**	**Upper quartile**	**Lower quartile**
SDDP	92.92	88.22	81.18	96.35
MIDAS	97.93	99.06	97.18	99.67

Approximation of value function

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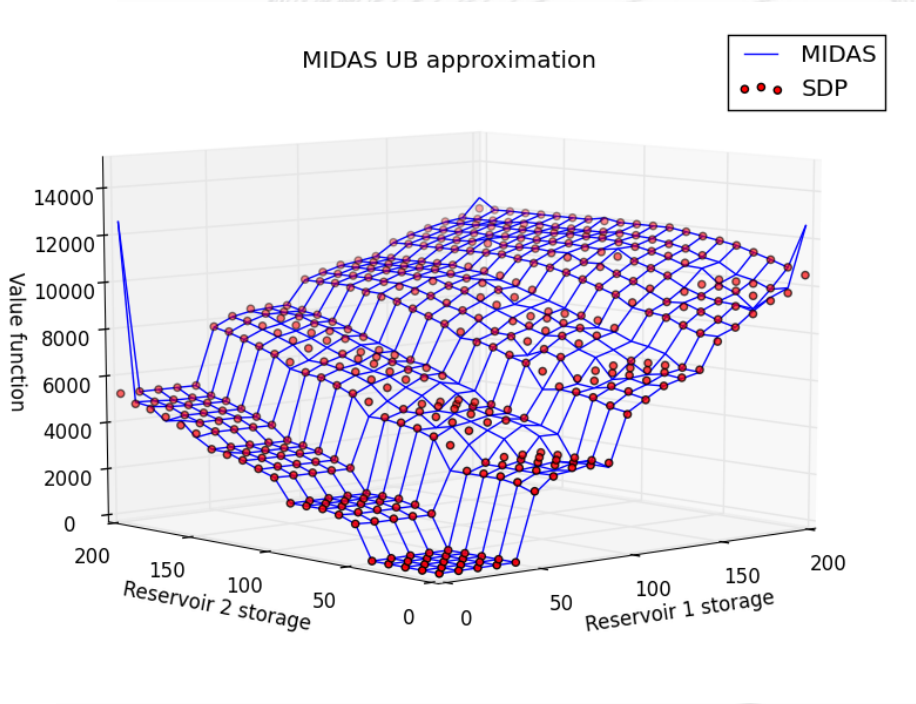

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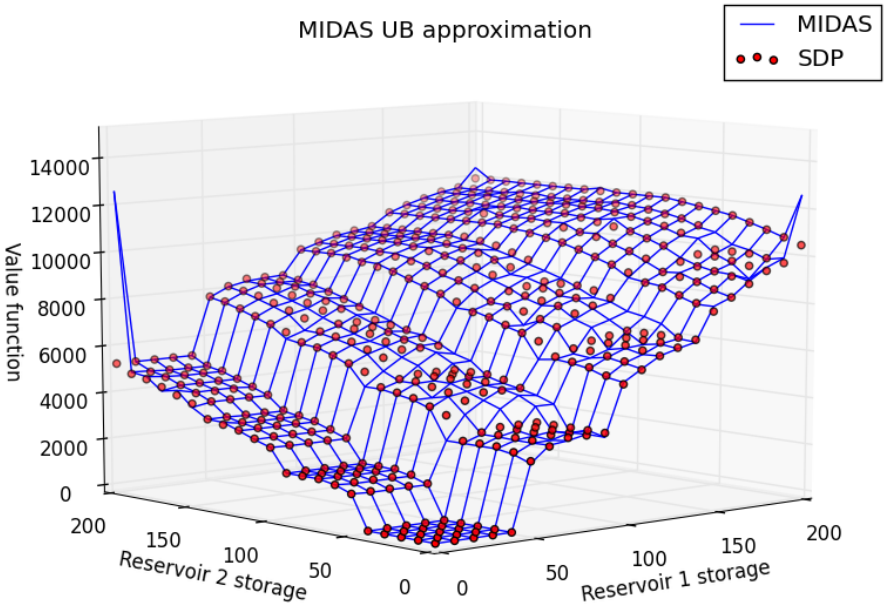

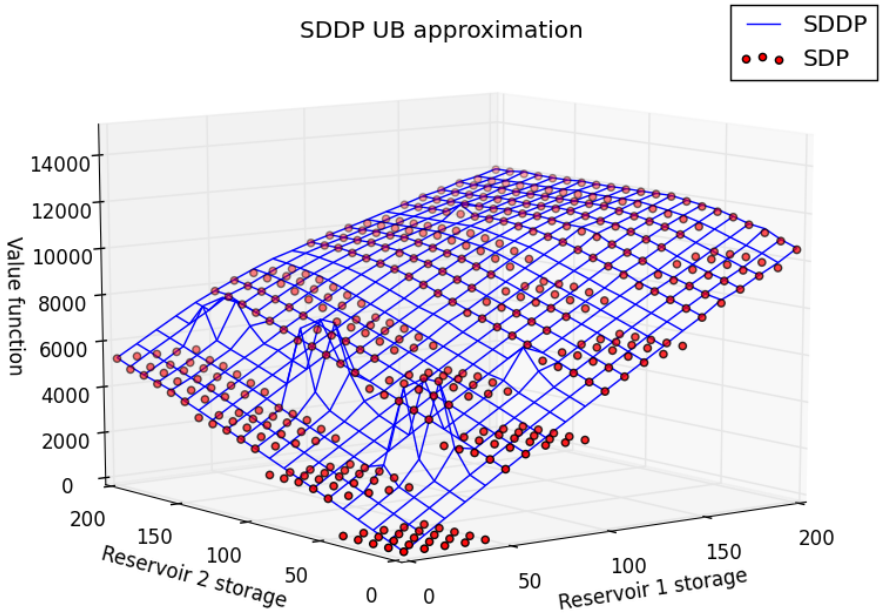

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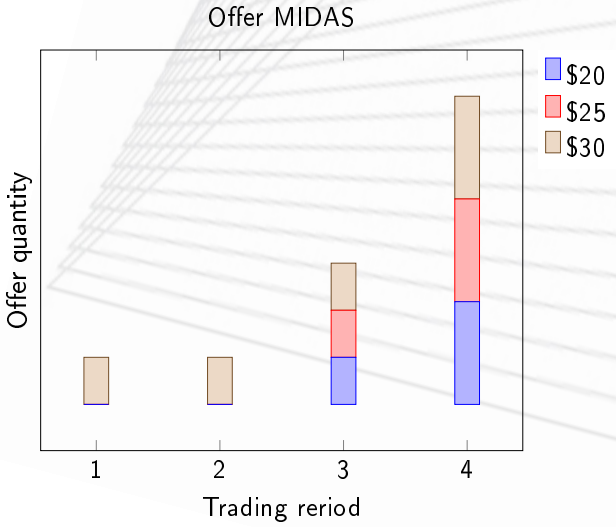

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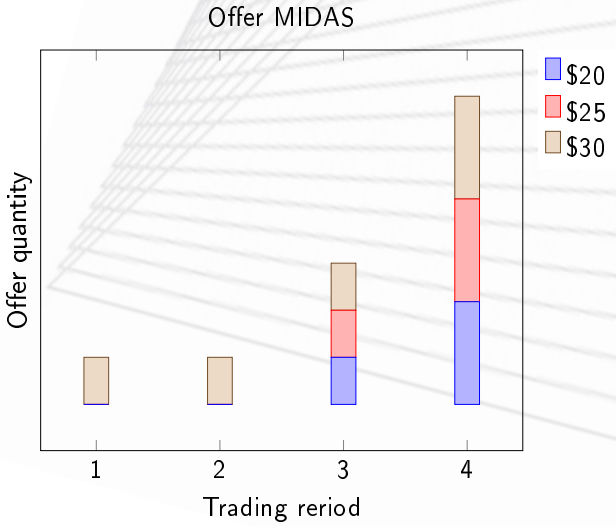

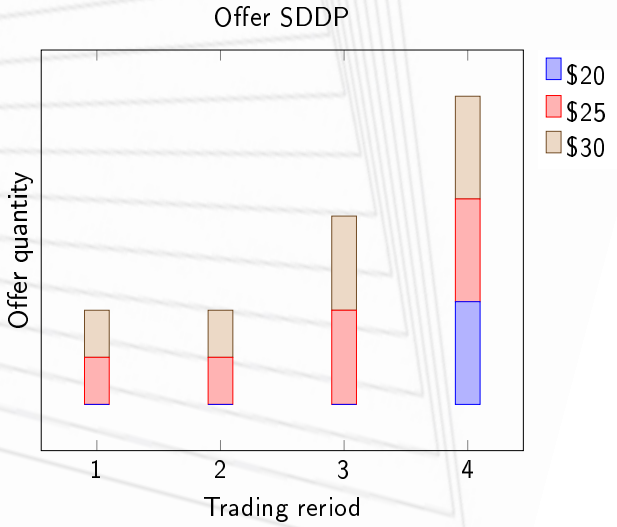

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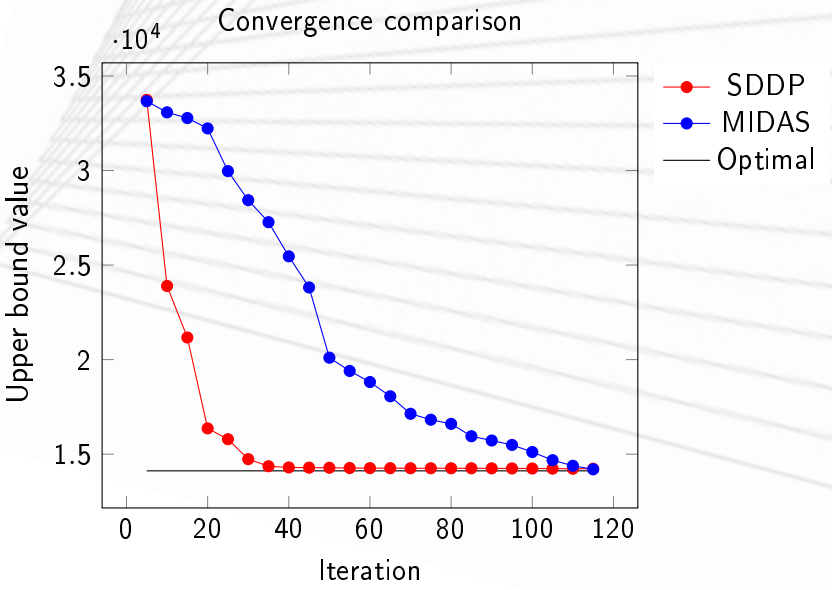

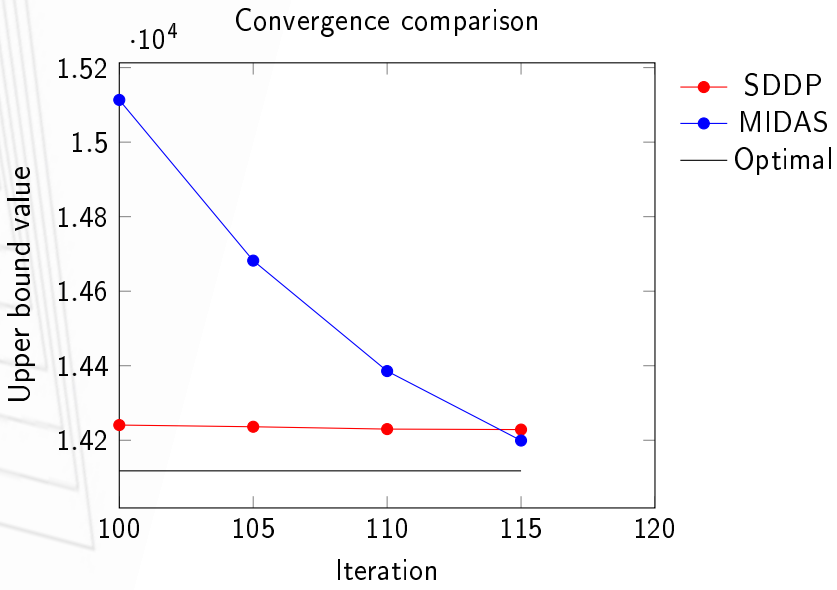

Comparison of computation II

- **Low storage:** $x_{t,i} = (100, 50)$

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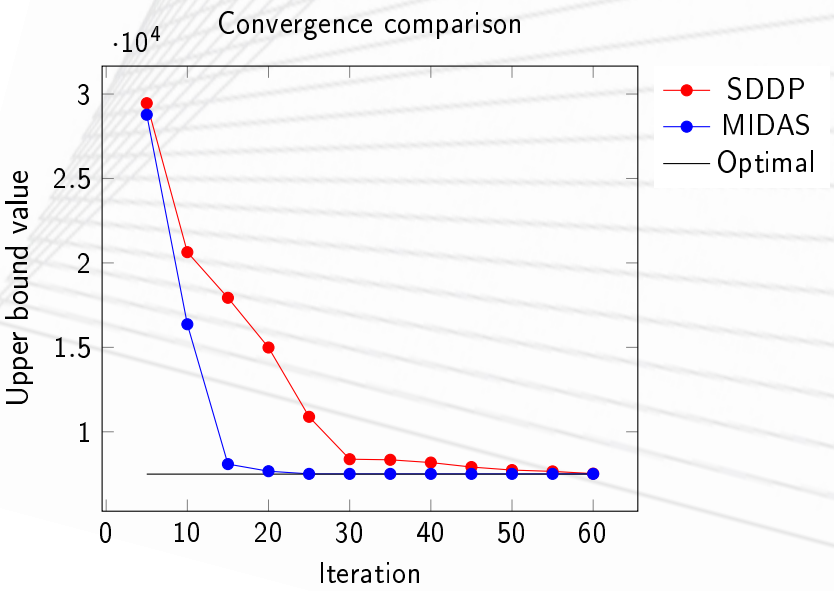

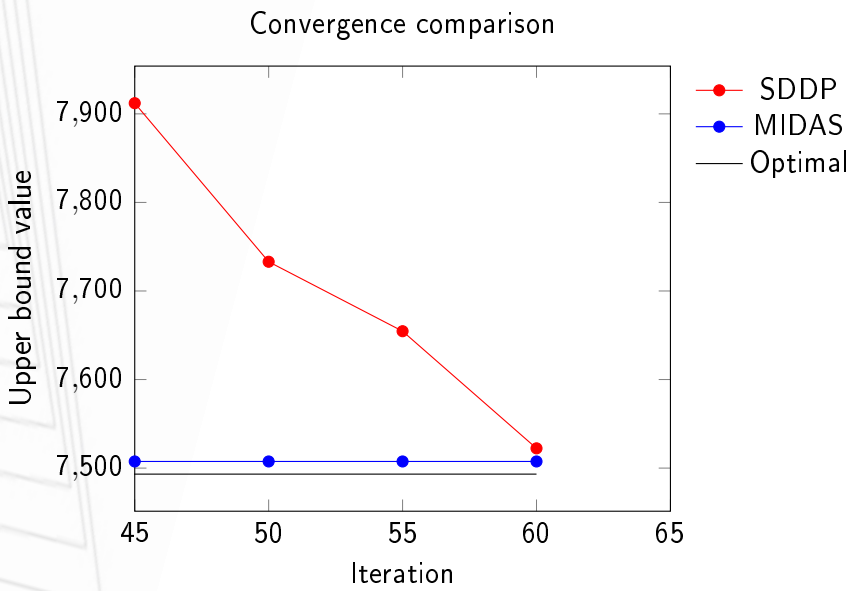

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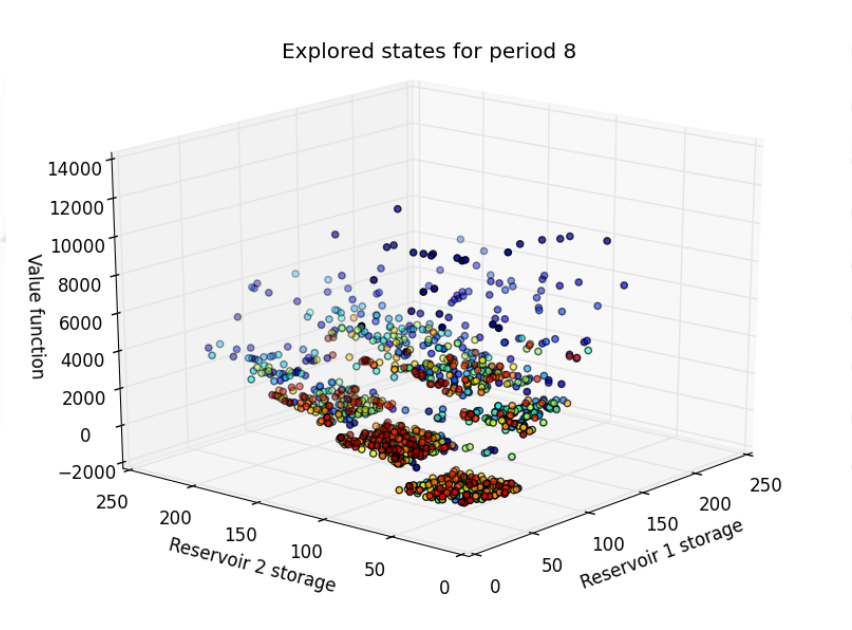

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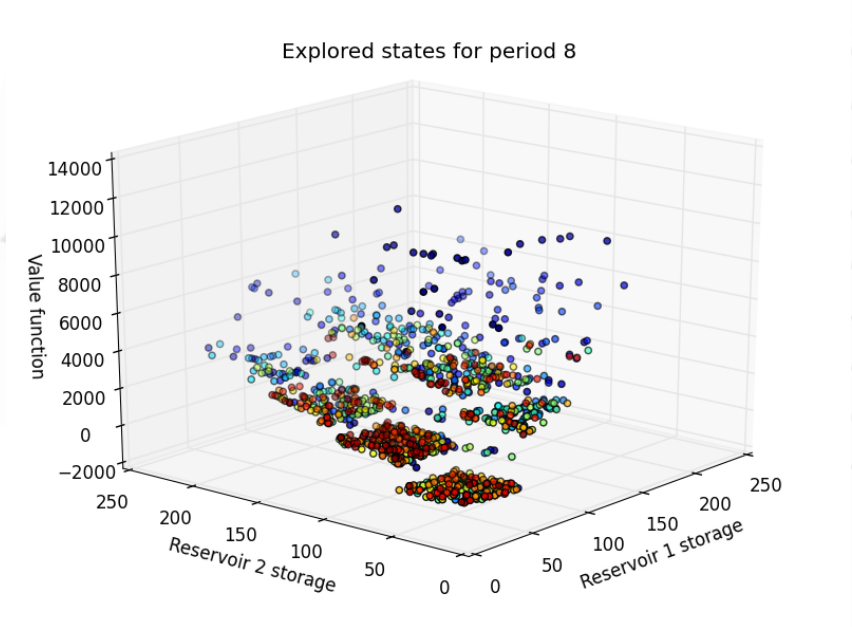

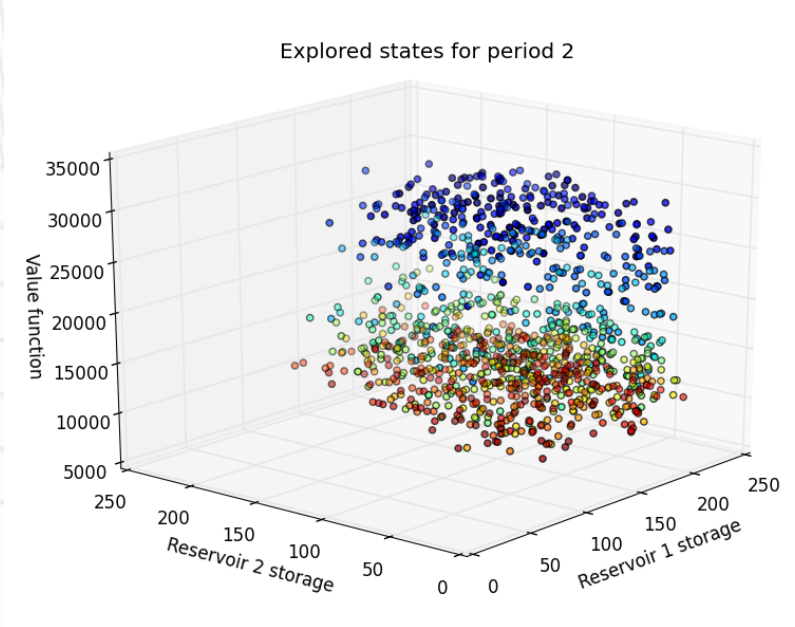

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  - Solving MIPs with many Big M constraints & tight integer feasibility tolerance

Questions

Thank you for listening!

Questions?

References I

[1] Pritchard, G. and Zakeri, G. *Market offering strategies for hydroelectric generators. Operations Research,* 51(4):602–612, 2003.

[2] Löhndorf, N., Wozabal, D. and Minner, S. *Optimizing Trading Decisions for Hydro Storage Systems Using Approximate Dual Dynamic Programming. INFORMS,* 61(4):801-823, 2013.

[3] Pereira M.V.F. and Pinto L.M.V.G. *Multi-stage stochastic optimization applied to energy planning. Mathematical Programming,* 52(1):359–375, 1991.

[4] Boomsma, T.K., Juul N. and Fleten S.E. *Bidding in sequential electricity markets: The Nordic cas European Journal of of Operation Research,* 238(3):797–809, 2014.

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[1] Fleten, S.E. and Kristoffersen, T.K. *Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer. European Journal of Operational Research,* 181(2):916–928, 2007.

[2] Braathen, J and Eriksrud, A.L. *Hydropower Bidding Using Linear Decision Rules. Institutt for industriell økonomi og teknologiledelse,* 2013.