Allocating physical capacity rights on an electricity transmission line

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Abstract

The inter-island HVDC line is a major transmission line in New Zealand, as it is the only link between its two islands. It enables the transfer of electricity between the South Island and the North Island. Because these transfers are generally beneficial to both the generators of the South Island and the consumers of the North Island, South Island generators are currently charged for the cost of the HVDC line based on a peak charge. We investigate an alternative scheme based on auctioning physical flow rights. Using a simplified supply-function equilibrium model we show that this is welfare optimizing in a perfectly competitive setting, but can result in inefficient dispatch and loss of rights revenue if generators bid for capacity strategically.

1 Introduction

The efficiency of electricity generation and transmission has become an important topic as regions around the world seek to reduce their energy costs. In pursuit of this goal many countries over the past 20 years have undertaken a restructuring of their electricity systems by removing the vertical integration of generation, transmission, distribution and retailing and attempting to replace it with separate horizontal layers, each made up of several competing organizations. Such competition has been introduced in the expectation that it will lead to an increase of efficiency and a decrease in electricity prices. The hope is also to provide signals for investments and new entry.

In many countries like New Zealand, Australia, Scandanavia, and most regions of North America the wholesale market for electricity is structured as a pool, where transmission is managed by a single entity often called the Independent System Operator (or ISO). Given demands of retail companies and wholesale purchasers, and offers of electricity at declared prices from generators, the ISO uses an optimization program to dispatch electricity so as to minimize the total revealed cost of meeting demand. The program also

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gives the prices of electricity for both generators and purchasers that may depend on their location. This structure has emerged as that preferred by the Federal Energy Regulation Commission (FERC) and instituted in many regions of the United States as the Standard Market Design (SMD) [1].

The transmission pricing regime in New Zealand has followed the basic principles laid out in the SMD[1]. In New Zealand, the transmission network is owned and operated by the ISO (a state-owned company called Transpower) whereas the generators are owned by a small number of private companies and state-owned enterprises. The economic dispatch of energy (and its price at each node) is determined by solving a linear programming model called SPD. Since the transmission of power is subject to congestion, and incurs energy losses, the price of energy is generally different from node to node, a setup known as *nodal pricing* or *locational marginal pricing*. A history of the development of these market arrangements is given in [19].

Figure 1: HVDC Line between the South Island and the North Island

A major component of the New Zealand transmission system is the High-Voltage Direct Current (HVDC) line joining the South Island to the North Island (see Figure 1). This was commissioned in 1965 (and upgraded in 1993) with the main purpose of conveying hydro-electric power from the South Island to the population centres in the North Island. Since the South Island generators tend to export electricity to the North Island, they decrease the North Island prices of electricity, but these generally remain higher than in the South. The benefits for South Island generators based on the higher

[1]Although its design and implementation predates the publication of the SMD by several years.

prices they can earn in the North due to the HVDC line has been estimated to be around NZ$240 million per year.

In its standard configuration the HVDC line consists of two parallel cables (or poles) although one of these is currently being repaired. The costs of maintaining and improving the HVDC line are borne by Transpower, who recover them by charging the users. These costs are estimated to be approximately NZ$88 million per year. Currently the charge for use is allocated entirely to South Island generators, in proportion to their last 12 peak generation periods over the last 4 years. Although it has been in use for some years, this charging scheme has understandably been unpopular with South Island generators, who view it as inequitable. Their claims are given some validity by observing that the HVDC link carries flows from North to South in winters when the hydro-electric lake levels are low (which has occurred in four of the last ten years).

In response to dissatisfaction with the current cost allocation scheme in New Zealand, a number of alternative proposals have been mooted. In this study we consider one recent candidate pricing scheme proposed by the NZIER [16]. This proposal which we shall call the *HVDC flow right model* is a variation on the flow-based transmission right proposals outlined in [8]. Under the NZIER scheme, generators bid for the use of capacity on the HVDC in a rights auction. Given an allocation of capacity rights, the power transfer of the generator from the South Island to the North Island is restricted by its allocation. Secondary trading of allocation rights is proposed to allow transfer of these to generators who value them the highest, and a spot market is also proposed in which small quantities of transmission rights can be traded at the margin at the same time as energy is dispatched by Transpower.

Allocating the cost of electricity transmission is a complicated problem that has attracted a lot of interest in the economics community, and (as mentioned above) has resulted in different solutions in different countries. Charging for the use of transmission must account for several sometimes conflicting objectives as follows:

1. It must result in an efficient utilization of capacity;

2. It must result in an income stream that is sufficient for expansion;

3. It must provide the correct economic signals for expansion.

In a perfectly competitive market, it is possible to devise transmission pricing schemes that give the most efficient utilization of existing capacity. Early descriptions of these are given by Hogan [12] and Chao and Peck [8]. In the FERC standard market design with full nodal pricing, the economic dispatch optimization (or at least a convex version of it) provides the most efficient capacity utilization and nodal prices that support this, assuming that generators do not act strategically. In this setting, point to point *financial transmission rights* (FTRs) provide a suitable mechanism for the allocation of transmission rentals to hedge location risk and support transmission capacity investment. As shown in [8] the same allocation can in principle be achieved with the assignment of physical *flowgate rights* to generators. In practice, the administration of markets for these instruments is very different, and seems to favour FTRs (see [13] and [9] for a discussion of these issues).

In real electricity markets, the returns from selling flowgate rights or FTRs are insufficient to cover the costs of providing transmission or expanding it. It has been estimated [18],[19] that the rental stream covers at most 30% of total cost. According to some economists[2] this missing money can be partly attributed to an unwillingness of regulators to let price differences between locations (and shortage probabilities) rise to levels that would support a transmisson line investment. In practice the extra cost of operating and expanding transmission assets must be recovered each year by the owner.

In New Zealand this cost recovery takes two forms. The additional costs of the high voltage AC transmission network in each island are recovered using a connection charge based on the transmission assets at the grid exit point of the consumer, and a coincident peak charge, in which each consumer pays a charge for every MW of power that was consumed in the 100 highest regional demand periods of the previous year[3]. The additional costs of the HVDC link are assigned to South Island generators as described above.

In a perfectly competitive market, the assignment of HVDC costs according to historical peak usage does not distort a generator's incentive to offer at marginal cost in any trading period[4]. This means that such charges will not have an impact on the efficient utilization of existing capacity. On the other hand, they might affect a generator's willingness to participate in the market. For example a small local generator in a remote part of the South Island might find it economically beneficial to become embedded with its local demand and avoid the charges, even though the participation of this generator in the national transmission system provides some benefits.

Cooperative game theory (see Nash [17], Shapley [20], and Aumann [6]) provides cost allocation schemes that encourage participation when it yields overall welfare benefits. These methods have not received much attention in the electricity market literature perhaps because of their complexity. Since the earliest discussion of their application to transmission planning [11] there have been a number of similar papers e.g. [22], [23], [21], [7], [14] with application in different jurisdictions. The application of these methodologies all assume that agents do not behave strategically in the energy market in response to the allocation of costs. Even if they do not, their exercise of market power in the energy market makes it difficult to compute the total

[2]The papers of Hogan generally take this position (see http://www.hks.harvard.edu/hepg). The argument is similar for investment in generation capacity, where price caps and probabilities of rationing are chosen at levels below which new investment in generation capacity would optimally occur.

[3]The country is divided into four regions. The coincident peak demand for each region is monitored and the 100 trading periods with highest demand are recorded for the year September 1 to August 31. Each customer's share of these peak demands is also recorded and determines its share of the total interconnection charge for the following year (from April 1-March 31).

[4]Since, the capacity choice of a rational generator is not likely to exceed its peak generation, an historical peak charge can be viewed as a capacity charge.

costs of different coalitions that are needed to analyze the game (e.g. by computing a Shapley value).

In this report we examine the *HVDC flow right model* from several perspectives. In the next section we describe the scheme in more detail. We show by example that a fixed allocation of transmission rights can result in an inefficient energy dispatch, demonstrating the need for a real-time balancing market in these instruments. Assuming perfect competition, we show that such a balancing market can result in a welfare-optimizing allocation of rights and dispatch. We then study the impact of strategic bidding in a stylized auction mechanism for allocating transmission rights by constructing a supply-function equilibrium for a system with two generators. The equilibrium model assumes that agents are paid a fixed price per MWh in the energy market, but strategically offer demand functions for HVDC capacity that limits their output. The computational results from this model show that the exercise of market power in the capacity auction leads to inefficient dispatch.

2 The HVDC flow right model

In this section, we present the HVDC flow right model in more detail and study its properties under an assumption of perfect competition. We shall also assume for the purposes of this section that there are no line losses. The HVDC flow right model allocates fractions of the HVDC line to particular generators. In its most simple form this allocation is fixed and constrains the dispatch of the ISO. In a pool dispatch of electricity the power flow on the HVDC cannot be easily split up and attributed to different generators. The electricity flow on the line does not belong to any particular generator, and in an optimal pool dispatch generators are indifferent between transporting it on the line and selling it locally. To determine how much the capacity of the HVDC line is contributing to each generator's dispatch the HVDC flow right model compares the optimal dispatch with one in which there is no HVDC line (which we call the *no-line solution*). The difference in dispatch levels is (if one discounts line losses) the capacity of the HVDC line that the optimal dispatch is deemed to allocate to each generator.

To establish some notation, let d be the demand in the South Island and g the demand in the North Island, and q_i and r_j the dispatch of generators i and j in South and North Islands respectively. The marginal cost of generator i is $C_i(q)$ and the marginal cost of generator j is $D_j(r)$. The capacity of the HVDC line is denoted K, and its flow is f. The optimal dispatch problem with the line is then defined to be

$$\begin{array}{lll} \text{DP}(K)\text{: min} & \sum_i \int_0^{q_i} C_i(q)dq + \sum_j \int_0^{r_j} D_j(r)dr & \\ & \sum_i q_i - f = d, & [\pi] \\ & \sum_j r_j + f = g, & [\mu] \\ & -K \le f \le K, & \\ & 0 \le q_i \le Q_i, \quad 0 \le r_i \le R_i. & \end{array}$$

Quantities in brackets denote Lagrange multipliers for DP(K). The Karush-Kuhn-Tucker conditions for a feasible solution (q, r, f) to be optimal for DP(K) are then

$$q_i.(C_i(q_i) - \pi) \leq 0, \quad (Q_i - q_i).(C_i(q_i) - \pi) \geq 0 \tag{1}$$

$$r_j.(D_j(r_j) - \mu) \leq 0, \quad (R_j - r_j).(D_j(r_j) - \mu) \geq 0 \tag{2}$$

$$(K + f).(\pi - \mu) \leq 0, \quad (K - f).(\pi - \mu) \geq 0 \tag{3}$$

The simplest version of a HVDC flow right model would solve DP(K) to give an optimal solution q_i^* and DP(0) to give a solution $\bar{q}_i$, and then charge generator i a cost proportional to $q_i^* - \bar{q}_i$. The proportionality constant could be fixed for all trading periods, or varying (to give an incentive to shift out of peaks). In the latter case it would need to be chosen carefully to enable the recovery of the full cost of the line.

A more sophisticated flow right model allocates HVDC flow rights to generators through an auction. The optimal dispatch of electricity for a generator cannot then exceed its dispatch in the no-line solution by more than its allocated HVDC flow right. Formally let ρ_i be the HVDC flow right held by generator i, where

$$\sum_i \rho_i \leq K. \tag{4}$$

The optimal dispatch model for the whole system is then solved with the HVDC line being present but with constraints

$$q_i \leq \bar{q}_i + \rho_i \tag{5}$$

being imposed on the dispatch q_i of each generator. This gives

$$\begin{array}{lll} \text{DPR}(\rho)\text{: min} & \sum_i \int_0^{q_i} C_i(q)dq + \sum_j \int_0^{r_j} D_j(r)dr & \\ & \sum_i q_i - f = d, & [\pi(\rho] \\ & \sum_j r_j + f = g, & [\mu(\rho)] \\ & -K \leq f \leq K, & \\ & 0 \leq q_i \leq \bar{q}_i + \rho_i, & [\lambda(\rho)] \\ & 0 \leq q_i \leq Q_i, \quad 0 \leq r_i \leq R_i. & \end{array}$$

Quantities in brackets denote Lagrange multipliers for DPR(ρ). These all vary with ρ. It is clear that choosing ρ_i poorly for a given trading period can result in a suboptimal dispatch and considerable inefficiency. It is conceivable also that some agent might acquire all K units of the HVDC flow right in some trading period in which it can use its flow rights to restrict the dispatch of competitors.

Observe that the flow-right model we are discussing here involves *physical* rights, and as such the ability to strategically withhold transmission capacity from other players is not ruled out. This is not the case in most discussions of flowgate rights in the literature (see [9]) where withholding is not allowed and the flowgate right can only be exercised when the optimal dispatch (ignoring the rights ownership) constrains the flow in the line.

Given that such a fixed allocation might lead to inefficient dispatch, it is possible to construct a rebalancing of rights at the same time as dispatch to ensure that these correspond to the optimal dispatch with the line. The HVDC flow rights balancing auction requires each generator to submit a nonincreasing bid curve for extra HVDC flow rights in each trading period. (Generators with excess rights can offer to supply excess flow rights as well by offering a supply curve, but we suppose for simplicity that excess flow rights are only supplied inelastically by Transpower.) Given a collection of bids for rights, the auction allocates them to generators with highest bids until the market clears. In a uniform-priced auction all generators would pay the market clearing price for sold permits. A discriminatory or pay-as-bid format collects payment according to what is bid. There has been much discussion in the literature about which format is better, especially when agents may bid strategically (see [2]), but we will not discuss this aspect here. In what follows, we shall assume a uniform price format.

A simultaneous auction of flow rights and energy would operate as follows. An optimal solution (q^*, r^*, f^*) to DP(K) is computed and the South Island price π recorded and supplied to generators. Then an optimal solution $(\bar{q}, \bar{r}, \bar{f})$ is computed for DP(0). If $q^* \leq \bar{q}$, then there is no need to allocate flow rights from South to North[5]. We assume henceforth that $q^* \geq \bar{q}$ and $q^* \neq \bar{q}$. Each South Island generator i then makes an offer for extra dispatch capacity using a bid function $B_i(s)$, and the ISO then solves

$$
\begin{array}{lll}
\text{DPR: min} & \sum_i \int_0^{q_i} C_i(q)dq + \sum_j \int_0^{r_j} D_j(r)dr - \sum_i \int_0^{\rho_i} B_i(s)ds & \\
& \sum_i q_i - f = d, & [\pi_a] \\
& \sum_j r_j + f = g, & [\mu_a] \\
& -K \leq f \leq K, & \\
& 0 \leq q_i \leq \bar{q}_i + \rho_i, & [\lambda_i] \\
& \sum_i \rho_i \leq K & [\sigma] \\
& 0 \leq q_i \leq Q_i, \quad 0 \leq r_i \leq R_i, \quad 0 \leq \rho_i \leq Q_i - \bar{q}_i. &
\end{array}
$$

The solution to this problem allocates HVDC flow rights ρ_i to generator i, for which the ISO collects a payment. We have the following results.

Lemma 1 *If $\rho^* = q^* - \bar{q}$ then $\sum_i \rho_i^* \leq K$.*

Proof. We have $\sum_i \bar{q}_i - d = 0$ and $\sum_i q_i^* - d = f^*$, so $\sum_i \rho_i^* = \sum_i (q_i^* - \bar{q}_i) = f^* \leq K$. But $q_i \leq \bar{q}_i + \rho_i$ implies $\sum_i \rho_i \geq \sum_i (q_i - \bar{q}_i) = f$. ■

Lemma 2 *If (q, r, f, ρ) is feasible for DPR then $f \leq \sum_i \rho_i$.*

[5]If $\bar{q} \geq q^*$ and $\bar{q} \neq q^*$ then one might allocate HVDC flow rights to North Island generators for power flowing from North to South using the same methodology that we describe here. In the interests of brevity we choose to omit this case.

Proof. We have $\sum_i \bar{q}_i - d = 0$ and $\sum_i q_i - d = f$, so $\sum_i (q_i - \bar{q}_i) = f$. But $q_i \leq \bar{q}_i + \rho_i$ implies $\sum_i \rho_i \geq \sum_i (q_i - \bar{q}_i) = f$. ■

It follows from Lemma 1 that if we choose $\rho_i = q_i^* - \bar{q}_i$, then these rights satisfy (4) and so it is a feasible allocation for DPR. Furthermore, even with these rights imposing constraints (5) on dispatch, it is still the case that the optimal economic dispatch q^* is feasible for DPR. For short-term efficiency it would be useful to encourage the rights to be allocated in this way. This can be ensured by each generator bidding for rights at their marginal value, i.e the extra value that additional rights contribute to the generator's profit. This is shown by the following proposition.

Proposition 3 *If $B_i(s) = \pi - C_i(\bar{q}_i + s)$, and $\rho^* = q^* - \bar{q}$, then (q^*, r^*, f^*, ρ^*) solves DPR.*

Proof. First observe that (q^*, r^*, f^*, ρ^*) satisfies the constraints of DPR, where $\sum_i \rho_i^* \leq K$ follows from Lemma 1. We show that if $B_i(s) = \pi - C_i(\bar{q}_i + s)$ then there exist Lagrange multipiers so that (q^*, r^*, f^*, ρ^*) satisfies the Karush-Kuhn-Tucker conditions for DPR. The objective function of DPR can be rewritten

$$
\begin{aligned}
& \sum_i \int_0^{q_i} C_i(q)dq + \sum_j \int_0^{r_j} D_j(r)dr - \sum_i \int_0^{\rho_i} (\pi - C_i(\bar{q}_i + s))ds \\
= & \sum_i \int_0^{q_i} C_i(q)dq + \sum_i \int_{\bar{q}_i}^{\bar{q}_i + \rho_i} C_i(s)ds + \sum_j \int_0^{r_j} D_j(r)dr - \pi \sum_i \rho_i
\end{aligned}
$$

Let $\mu_a = \mu$, $\pi_a = \pi$, $\lambda_i = \pi - C_i(q_i^*)$. With these choices the Lagrangian for DPR (ignoring simple bounds) is

$$
\begin{aligned}
\mathcal{L} = & \sum_i \int_0^{q_i} C_i(q)dq + \sum_i \int_{\bar{q}_i}^{\bar{q}_i + \rho_i} C_i(s)ds + \sum_j \int_0^{r_j} D_j(r)dr - \pi \sum_i \rho_i \\
& + \pi(d + f - \sum_i q_i) + \mu(g - f - \sum_i r_i) + \sum_i (\pi - C_i(q_i^*))(q_i - \bar{q}_i - \rho_i) \\
& + \sigma(\sum_i \rho_i - K)
\end{aligned}
$$

This has derivatives

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q_i} &= C_i(q_i) - \pi + \pi - C_i(q_i^*) \\
\frac{\partial \mathcal{L}}{\partial r_j} &= D_j(r_j) - \mu \\
\frac{\partial \mathcal{L}}{\partial f} &= \pi - \mu \\
\frac{\partial \mathcal{L}}{\partial \rho_i} &= C_i(\bar{q}_i + \rho_i) - \pi - (\pi - C_i(q_i^*)) + \sigma
\end{aligned}
$$

Now choosing $q = q^*$ gives $\frac{\partial \mathcal{L}}{\partial q_i} = 0$ so

$$
q_i . \frac{\partial \mathcal{L}}{\partial q_i} \leq 0, \quad (Q_i - q_i) . \frac{\partial \mathcal{L}}{\partial q_i} \geq 0 \tag{6}
$$

and choosing $r = r^*$ gives

$$r_j.\frac{\partial \mathcal{L}}{\partial r_j} \leq 0, \quad (R_j - r_j).\frac{\partial \mathcal{L}}{\partial r_j} \geq 0 \tag{7}$$

because $D_j(r_j^*) - \mu$ satisfies (2). Similarly choosing $f = f^* \in [-K, K]$ gives

$$(K + f).\frac{\partial \mathcal{L}}{\partial f} \leq 0, \quad (K - f).\frac{\partial \mathcal{L}}{\partial f} \geq 0 \tag{8}$$

Finally choosing $\rho^* = q^* - \bar{q}$ (which is nonnegative by assumption) gives

$$\frac{\partial \mathcal{L}}{\partial \rho_i} = 2C_i(q_i^*) - 2\pi + \sigma.$$

If $\sum_i (q_i^* - \bar{q}_i) < K$, then choosing $\sigma = 0$ gives $\frac{\partial \mathcal{L}}{\partial \rho_i} = 2C_i(q_i^*) - 2\pi$. Since $\rho^* = q^* - \bar{q} \geq 0$, the optimality conditions (1) give

$$(\rho_i^* + \bar{q}_i).\frac{\partial \mathcal{L}}{\partial \rho_i} \leq 0, \quad (Q_i - \bar{q}_i - \rho_i^*).\frac{\partial \mathcal{L}}{\partial \rho_i} \geq 0.$$

Note that these conditions imply that if $\frac{\partial \mathcal{L}}{\partial \rho_i} > 0$ then $\rho_i^* + \bar{q}_i = 0$. This implies that $\rho_i^* = \bar{q}_i = 0$. Thus we have $\rho_i^*.\frac{\partial \mathcal{L}}{\partial \rho_i} \leq 0$ which gives

$$\rho_i^*.\frac{\partial \mathcal{L}}{\partial \rho_i} \leq 0, \quad (Q_i - \bar{q}_i - \rho_i^*).\frac{\partial \mathcal{L}}{\partial \rho_i} \geq 0. \tag{9}$$

If $\sum_i (q_i^* - \bar{q}_i) = K$, then we have $\sum_i \rho_i^* = K$. By Lemma 2, $f^* \leq \sum_i \rho_i^*$, so increasing $\sum_i \rho_i^*$ beyond K will not yield any extra benefits since $f^* \leq K$. So we choose Lagrange multiplier $\sigma = 0$ and apply the same argument as above to yield (9).

Thus $\mu_a = \mu$, $\pi_a = \pi$, $\lambda_i = \pi - C_i(q_i^*)$, $\sigma = 0$ and (q^*, r^*, f^*, ρ^*) satisfy the Karush-Kuhn-Tucker conditions (6)-(9) for DPR, and so (q^*, r^*, f^*, ρ^*) is an optimal solution for DPR as these conditions are necessary and sufficient for a convex program. ■

Proposition 3 shows that a bid for flow rights s of $B_i(s) = \pi - C_i(\bar{q}_i + s)$ gives the welfare maximizing dispatch if the market is operated as described above. Since such a bid function represents the marginal profit that each generator will earn from an extra flow right, this result is not surprising. The next section discusses the situation in which generators can bid strategically for flow rights. This is shown to lead to an inefficient dispatch of energy.

3 Cost allocation with imperfect competition

We now turn our attention to the effect of strategic behaviour of generators on the outcomes of an auction for flow rights. We first review the equilibrium conditions for 2 players in a single-node energy market when players offer supply functions (see [15],[5]).

3.1 Supply-function equilibrium in energy

Suppose each player has linear marginal costs $c_i(q) = c_i q$ and demand H is inelastic and random with probability distribution F. Each company's profit is then

$$R_i(q, p) = pq - \frac{c_i q^2}{2}.$$

Consider generator 1. Suppose that the other generators' offers aggregate to form the supply function $S(p)$. Then the market distribution function (see [4]) faced by generator 1 is

$$\psi(q, p) = F(q + S(p))$$

and an optimal response is defined by

$$\begin{aligned} Z_1(q,p) &= \frac{\partial R_1}{\partial q}\frac{\partial \psi}{\partial p} - \frac{\partial R_1}{\partial p}\frac{\partial \psi}{\partial q} \\ &= ((p - c_1 q)S'(p) - q)F'(q + S(p)). \end{aligned}$$

By [4, Lemma 4.2] $Z = 0$ defines an optimal response to $S(p)$. If $F'(q + S(p)) > 0$ then this defines the response $q(p)$ as solving

$$(p - c_1 q(p))S'(p) - q(p) = 0$$

giving

$$S'(p) = \frac{q(p)}{p - c_1 q(p)}.$$

In a symmetric supply-function equilibrium (SFE) with $n+1$ generators having identical marginal costs cq, each generator has n rivals giving total supply defined by $S(p) = nq(p)$, which leads to

$$nq'(p) = \frac{q(p)}{p - cq(p)}.$$

To solve this ordinary differential equation, consider the inverse supply function $p(q)$. Then

$$p'(q) = \frac{1}{q'(p)} = n\frac{p(q) - cq}{q}$$

so

$$p'(q) - n\frac{p(q)}{q} = -nc.$$

Multiplying by q^{-n} yields

$$q^{-n}p'(q) - nq^{-n}\frac{p(q)}{q} = -ncq^{-n}$$

or

$$\big(q^{-n}p(q)\big)' = -ncq^{-n}.$$

If $n > 1$ then

$$\big(q^{-n}p(q)\big) = nc\frac{q^{1-n}}{n-1} + \alpha$$

so

$$p(q) = \frac{nc}{n-1}q + \alpha q^n$$

which corresponds to the result in [5].

In the special case where $n = 1$ (i.e. a duopoly) we have

$$q'(p) = \frac{q(p)}{p - cq(p)}$$

which is the first-order linear ordinary differential equation

$$p'(q) - \frac{p(q)}{q} = -c.$$

Multiplying by q^{-1} yields

$$\big(q^{-1}p(q)\big)' = -cq^{-1}$$

which has general solution

$$p(q) = -cq\ln q + \alpha q. \tag{10}$$

For the curve defined by (10) to produce a symmetric Nash equilibrium, it must be non-decreasing and non-negative. This requires α to be large enough so that the curves remain increasing for all values of demand. Observe that there is no upper bound here for α, and we can define a family of supply-function equilibrium for any α sufficiently large.

Proposition 4 *Suppose the support of F' is $(0, 1)$. If $\alpha \geq c - c\ln 2$ then the offer curve*

$$p_\alpha(q) = \begin{cases} -cq\ln q + \alpha q, & q \in [0, \frac{1}{2}], \\ \frac{1}{2}c\ln 2 + \frac{1}{2}\alpha, & q \in (\frac{1}{2}, 1] \end{cases}$$

defines a symmetric SFE.

Proof. Note that $p_\alpha(q) \geq 0$ and for $q \in [0, \frac{1}{2}]$, $p'_\alpha(q) = -c - c\ln q + \alpha \geq 0$ as long $\alpha \geq c + c\ln q$, which is guaranteed by $\alpha \geq c - c\log 2$. If the other generator offers $p_\alpha(q)$ then

$$Z(q, p) = ((p - cq)S'(p) - q)F'(q + S(p))$$

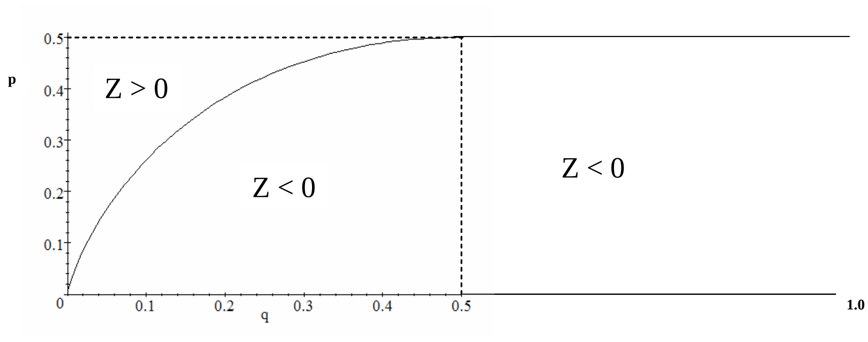

Figure 2: Plot of $Z(q,p)$ for example $p_\alpha(q)$ where $c=1$ and $\alpha = 1-\ln 2$.

Now $Z=0$ along the curve $p_\alpha(q)$ and is positive above the curve and negative below it as shown in Figure 2.Since for fixed p, $(p-cq)S'(p)-q$ is decreasing in q, we also have that $Z<0$ in the rectangle $[\frac{1}{2},1]\times[0,p_\alpha(\frac{1}{2})]$. Since $S(p_\alpha(\frac{1}{2})) = \frac{1}{2}$, $F'(q+S(p_\alpha(\frac{1}{2}))=0$ for $q>\frac{1}{2}$, and so $Z=0$ along the horizontal line $(q,\frac{1}{2}c\ln 2+\frac{1}{2}\alpha)$, $q\in(\frac{1}{2},1]$. Indeed $Z=0$ everywhere above this line segment as $F'(q+S(p_\alpha(\frac{1}{2}))=0$ in this region as well. It follows that $p_\alpha(q)$ is an optimal response to $p_\alpha(q)$, and so forms a symmetric supply-function equilibrium. ■

Observe that when the demand is h, generators always get dispatched $h/(n+1)$. Then, the expected profit made by each generator (assuming $n>2$) is

$$\begin{aligned}
\mathbb{E}[R(p_a(\frac{H}{n+1}),\frac{H}{n+1})] &= \mathbb{E}[\left(\frac{nc}{n-1}\frac{H}{n+1}+\alpha(\frac{H}{n+1})^n\right)\frac{H}{n+1}-\frac{1}{2}c\left(\frac{H}{n+1}\right)^2] \\
&= \mathbb{E}[\frac{nc}{n-1}\left(\frac{H}{n+1}\right)^2+\alpha(\frac{H}{n+1})^n\frac{H}{n+1}-\frac{1}{2}c\left(\frac{H}{n+1}\right)^2] \\
&= \frac{1}{2}\frac{n+1}{n-1}c\mathbb{E}[\left(\frac{H}{n+1}\right)^2]+\alpha\mathbb{E}[(\frac{H}{n+1})^{n+1}]
\end{aligned}$$

so for fixed n the expected profit is linear in α, and so can be arbitrarily high. One way of refining the equilibrium concept so that there is a unique supply-function equilibrium, is to set a price cap $\bar{p}$. Companies would now bid supply functions with the highest value of α that will give a dispatch where their maximum dispatched output corresponds to the price cap. This condition is $p_\alpha(\frac{h_{max}}{n+1})=\bar{p}$, or

$$\frac{nc}{n-1}\frac{h_{max}}{n+1}+\alpha(\frac{h_{max}}{n+1})^n=\bar{p}$$

$$\alpha=\left(\bar{p}-\frac{nc}{n-1}\frac{h_{max}}{n+1}\right)(\frac{h_{max}}{n+1})^{-n}$$

Since $p_\alpha(h)$ is an increasing function of α, this gives us an upper bound for α. Note that the price cap needs to be high enough so that this upper bound remains higher than the marginal cost of generation at $\frac{h_{max}}{n+1}$. This condition is $\bar{p} \geq c\frac{h_{max}}{n+1}$. Observe that α can be negative as long as it gives a non-decreasing offer curve at $q = \frac{h_{max}}{n+1}$.

3.2 Supply-function equilibrium in physical flow rights

The supply-function equilibrium decribed above can be used to derive a supply-function equilibrium for generators bidding for HVDC flow rights. In order to derive this, we simplify the energy auction and assume that all energy is traded at a fixed price π. Generators then bid strategically for flow rights. We assume for simplicity that all generators are located in the South Island and sell only in the North Island. Thus each generator's dispatch of energy must not exceed their flow right allocation

We denote the profit for generator i when dispatched quantity q in the energy market by

$$\Pi_i(q) = \pi q - c_i \frac{q^2}{2}.$$

If the clearing price for flow rights is p then the generator must pay qp for the rights to dispatch q. The profit of generator i when dispatched quantity q becomes

$$R_i(q, p) = \pi q - c_i \frac{q^2}{2} - pq.$$

In the auction we suppose that each generator i bids a non-increasing curve $q = B_i(p)$ for flow rights. For each level of demand h, the solution to $\sum_i B_i(p) = h$ determines a clearing price p, and associated profit $R_i(q, p)$.

To compute a supply-function equilibrium, we let $r = \pi - p$, and appeal to the results of the previous section. Given any non-increasing curve $B_i(p)$ there is a non-decreasing curve $S_i(r) = B_i(\pi - r)$. If the rest of the generators offer a curve that aggegates to $S(r)$ then an optimal response for generator 1 is defined by

$$\begin{aligned} Z_1(q, r) &= \frac{\partial R_1}{\partial q}\frac{\partial \psi}{\partial r} - \frac{\partial R_1}{\partial r}\frac{\partial \psi}{\partial q} \\ &= ((r - c_1 q)S'(r) - q)F'(q + S(r)). \end{aligned}$$

When $F'(q + S(r)) \neq 0$, we have

$$q(r) = \frac{rS'(r)}{1 + c_1 S'(r)}$$

We can use this to derive some equilibria in different settings.

3.2.1 Symmetric duopoly

Suppose there are two identical generators with $c_1 = c_2 = c$. Then by symmetry $q(r) = S(r)$, and we have

$$q(r) = \frac{rq'(r)}{1 + cq'(r)}$$

By (10) this has solution

$$r(q) = -cq \ln q + \alpha q$$

so

$$\begin{aligned} p(q) &= \pi - r(q) \\ &= \pi - \alpha q + cq \ln q \end{aligned}$$

Note that $p_\alpha(q) = \pi + cq \log q - \alpha q$ is a decreasing curve, as long as $q \leq e^{-1+\frac{\alpha}{c}}$.

Proposition 5 *Suppose the support of F' is $(0,1)$. If $\alpha \in [c - c\ln 2, 2\pi - c\ln 2]$, then the bidding curve*

$$p_\alpha(q) = \begin{cases} \pi - \alpha q + cq \ln q, & q \in [0, \frac{1}{2}], \\ \pi - \frac{1}{2}\alpha - \frac{1}{2}c\ln 2, & q \in (\frac{1}{2}, 1] \end{cases}$$

defines a symmetric SFE.

Proof. The proof follows along the same lines as that of Proposition 4. ∎

Because we are looking for symmetric Nash equilibria, the generators will always be dispatched half of the total demand. Hence, their expected profits are equal and are

$$\begin{aligned} \mathbb{E}[R] &= \mathbb{E}[\pi\frac{h}{2} - c\frac{h^2}{8} - p_\alpha(\frac{h}{2})\frac{h}{2}] \\ &= \mathbb{E}[\pi\frac{h}{2} - c\frac{h^2}{8} - (\pi - \alpha\frac{h}{2} + c\frac{h}{2}\ln\frac{h}{2})\frac{h}{2}] \\ &= \mathbb{E}[\frac{1}{8}h^2\left(-c + 2\alpha + 2c\ln 2 - 2c\ln h\right)]. \end{aligned}$$

Thus $\mathbb{E}[R]$ is an increasing function of α. The least competitive equilibrium therefore has maximum $\alpha = 2\pi - c\ln 2$. This choice of α means that the bid for the purchase of half of the maximum flow rights is 0. With this choice of α the total expected revenue collected from the ISO for the HVDC charge is

$$\begin{aligned} \mathbb{E}[hp_\alpha(\frac{h}{2})] &= \mathbb{E}[h(\pi - (2\pi - c\ln 2)\frac{h}{2} + c\frac{h}{2}\ln\frac{h}{2})] \\ &= \mathbb{E}[\pi h - \pi h^2 + \frac{1}{2}ch^2\ln h]. \end{aligned}$$

We may compare this with the expected revenue collected from a non-strategic bid curve $B_i(s) = \pi - cs$. This is

$$\mathbb{E}[h(\pi - c\frac{h}{2})] = \mathbb{E}[\left(\pi h - c\frac{h^2}{2}\right)].$$

At any demand level $h < 1$ we get

$$\pi h - c\frac{h^2}{2} - (\pi h - \pi h^2 + \frac{1}{2}ch^2\ln h) = h^2(\pi - \frac{1}{2}c - \frac{1}{2}c\ln h) > 0$$

since the energy price $\pi > \frac{1}{2}c$, the marginal cost of generation at half the maximum demand. Thus the revenue earned by the ISO is decreased in every demand realization due to strategic bidding for HVDC flow rights. If we compute the expected rights revenue from both settings in the case where H has a uniform distribution then

$$\mathbb{E}[\pi h - \pi h^2 + \frac{1}{2}ch^2 \ln h] = \frac{1}{6}\pi - \frac{1}{18}c$$

and

$$\mathbb{E}[h(\pi - c\frac{h}{2})] = \frac{1}{2}\pi - \frac{1}{6}c$$

a threefold increase when strategic bidding is precluded.

3.2.2 Asymmetric duopoly

Let us now consider an asymmetric case, where the marginal costs are $c_1(q) = c_1 q$ and $c_2(q) = c_2 q$. We have the best response of player 1 is

$$S_1(r) = \frac{rS_2'(r)}{1 + c_1 S_2'(r)}$$

Since $S_i(r) = B_i(\pi - r)$, this gives

$$B_1(\pi - r)(1 - c_1 B_2'(\pi - r)) = -rB_2'(\pi - r)$$

and $p = \pi - r$ gives on rearranging

$$B_1(p) = B_1(p)c_1 B_2'(p) - (\pi - p)B_2'(p)$$

$$B_2'(p) = -\frac{B_1(p)}{(\pi - p) - c_1 B_1(p)} \tag{11}$$

Similarly

$$B_1'(p) = -\frac{B_2(p)}{(\pi - p) - c_2 B_2(p)} \tag{12}$$

This gives a system of simultaneous nonlinear differential equations in p. In order to solve this system we follow the approach of Anderson and Hu [3]. We discretize $[0, 1]$ into n equal intervals, and we define the $4(n+1)$ values of B_1, B_1', B_2 and B_2' at the endpoints of these intervals (called *grid points*). The relations (11) and (12) applied at each grid point give a set of equations. We also add conditions that ensure monotonicity and smoothness of the solution, as well as requiring that $B_1(0) = B_2(0) = \pi$. The resulting system of simultaneous nonlinear equations and inequalities can be formulated as a nonlinear programming problem[6] and solved by GAMS/CONOPT[10]. The numerical solution for this system where $\pi = 3$, $c_1 = 1$, and $c_2 = 5$ is plotted in Figure 3.

[6]The GAMS source code for this is provided in the Appendix to this paper.

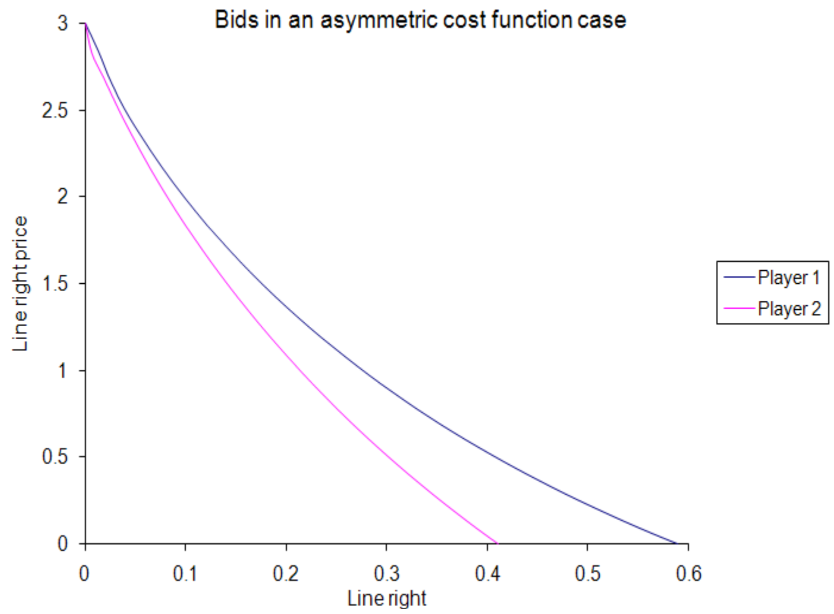

Figure 3: Supply-function equilibrium with $\pi = 3$, and asymmetric costs $c_1 = 1$, $c_2 = 5$.

It is interesting to look at the efficiency of such a dispatch. An efficient dispatch needs to satisfy $c_1q_1 = c_2q_2$, which means that $q_1/q_2 = c_2/c_1 = 5$. However, as one can see from Figure 3 the effect of the auction is to bring the dispatches of the two generators closer together, so that the value of q_1/q_2 is at most 1.5. This means that the dispatch is not efficient. When total demand is h we have a system optimum of $q_1 = \frac{5h}{6}$, $q_2 = \frac{h}{6}$, with total system cost of $\frac{1}{2}\left(\frac{5h}{6}\right)^2 + \frac{5}{2}\left(\frac{h}{6}\right)^2 = \frac{5}{12}h^2$. In contrast the solution $q_1 = \frac{3h}{5}$, $q_2 = \frac{2h}{5}$ has a total system cost when demand is h of at least $\frac{1}{2}\left(\frac{3h}{5}\right)^2 + \frac{5}{2}\left(\frac{2h}{5}\right)^2 = \frac{29}{50}h^2$ which is about a 40% increase in dispatch cost.

4 Conclusion

The simple examples in this paper illustrate that allocating physical transmission rights for the HVDC is possible as long as agents bid their true marginal value for those rights (given perfect knowledge of the South Island clearing price in the system optimal dispatch). This begs the question of why we would choose to allocate the rights by auction in any case. A simpler scheme would be to allocate the costs of the HVDC link directly to generators depending on the values of $q_i^* - \bar{q}_i$. This would give an efficient energy dispatch *ceteris paribus* but incentives would exist for generators to change their offers of energy to avoid some of the extra charges. These incentives are illustrated by the simple supply-function equilibrium results we discuss.

We have not attempted to quantify the extent of the inefficiency produced by strategic offering in a real market. Indeed the increase in energy prices from strategic offering will also create some allocative inefficiency (if demand is allowed to respond) that we have not discussed here. It may be argued that the effects discussed here are not material in real electricity markets in which the offering process would be considerably more complicated than the simple model we present. Nevertheless, illustrations of inefficiency in these simple models should serve as a warning to market designers not to expect too much from the efficiency of physical transmission right auctions, unless the market is closely monitored to prevent undue exercise of market power.

5 Appendix

```
* GAMS/CONOPT SFE model
  *
  * Written by Andy Philpott, based on Xinmin Hu's model
  * Assume a discretization of demand shock
  * Gives a supply-function response
  * Note:  Asymmetric SFE
  SETS
  i player / 1, 2 /
  k discretization /1*51/;
  ;
  scalar delta >0 to make constraints strict
  /0.005/ ;
  scalar r
  /3/ ;
  parameter c(i) cost coefficient /
  1 1
  2 5 / ;
  parameter d(k) demand shock /
  1 0.00
  2 0.02
  3 0.04
  4 0.06
  :
  :
  49 0.96
  50 0.98
  51 1.00
  / ;
  parameter q0(i,k) ;
  parameter p0(k);
  positive variable
  q(i,k) quantity offered by player i at demand k
  pi(k) clearing price at demand level k ;
  variable
  s(i,k) slope of other player
  t(i,k) slope of player i
```

```
pt(k)
u(k) error terms
error;
equation
objective
response(i,k)
demand(k)
stdef1(k)
stdef2(k)
decreasing1(i,k)
decreasing2(i,k)
decreasing3(k)
tilde(i,k)
monotonic1(k)
monotonic2(k)
monotonic3(i,k);
objective ..  error =e= 1000*sum(k,u(k)*u(k));
response(i,k) ..  s(i,k)*(r - pi(k) - c(i)*q(i,k)) + q(i,k)
=e= 0;
demand(k)..  sum(i, q(i,k)) + u(k) =e= d(k) ;
stdef1(k)..  t('1',k) =e= s('2',k) ;
stdef2(k)..  s('1',k) =e= t('2',k);
decreasing1(i,k)..  t(i,k) =l= 0;
decreasing2(i,k)$(ord(k) lt card(k))..  q(i,k+1)- q(i,k)
=g= 0;
decreasing3(k)$(ord(k) lt card(k))..  pi(k+1) - pi(k)
=l= 0;
tilde(i,k)$(ord(k) lt card(k))..  q(i,k+1) - q(i,k)
- t(i,k+1)*pi(k+1)+ t(i,k)*pi(k) + (t(i,k+1)-t(i,k))* pt(k)
 =e= 0;
monotonic1(k)..  pi(k) - delta =g= pt(k) ;
monotonic2(k)$(ord(k) lt card(k))..  pt(k) - delta
=g= pi(k+1) ;
monotonic3(i,k)$(ord(k) lt card(k))..  q(i,k) + delta
=l= q(i,k+1) ;
model sfe /all / ;
option nlp=conopt ;
option decimals = 8;
t.up(i,k)= -0.001 ;
s.up(i,k)= -0.001 ;
t.lo(i,k)= -5.0 ;
s.lo(i,k)= -5.0 ;
q.up(i,'1')= 0;
q.lo(i,'1')= 0;
pi.up('51')=0.000;
pi.lo('51')=0.000;
pi.up('1')=3.000;
```

```
pi.lo('1')=3.000;
solve sfe minimising error using nlp;
FILE RES /D:sfeLEN50.out/;
PUT RES;
RES.nd=5;
RES.nw=10;
RES.ap=0;
put ''Quantity1 Quantity2 Price PTilde '' / ;
loop (k,
put q.l('1',k);
put '' '';
put q.l('2',k);
put '' '';
put pi.l(k)
put '' '';
put pt.l(k)
/;
) put /;
```

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