Single and Multi-settlement Approaches to Market Clearing Mechanisms under Demand Uncertainty

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Abstract

Electricity markets face a substantial amount of uncertainty. Traditionally this uncertainty has been due to varying demand. With the integration of larger proportions of volatile renewable energy, this added uncertainty from generation must also be faced. Conventional electricity market designs cope with uncertainty by running two markets: a day ahead or pre-dispatch market that is cleared ahead of time, followed by a realtime balancing market to reconcile actual realizations of demand and available generation. In such markets, the day ahead market clearing process does not take into account the distribution of outcomes in the balancing market. Recently an alternative so-called stochastic settlement market has been proposed (see e.g. Pritchard et al. [5] and Bouffard et al. [2]). In such a market, the ISO co-optimizes pre-dispatch and spot in one single settlement market. By considering all possible demand realizations ahead of time, pre and spot dispatch is deemed to be scheduled more efficiently. In this paper we consider simplified models for both market clearing mechanisms. Our models are targeted towards analyzing imperfectly competitive markets. We will demonstrate that this stochastic programming market clearing mechanism is indeed more efficient under the assumption of symmetry, however somewhat contrary to intuition, this result fails in an asymmetric example that we provide.

1 Introduction

Electricity markets face two key features that set them apart from other markets. The first is that electricity cannot be stored, so demand must equal supply at all times. This is particularly problematic given that demand for electricity is usually uncertain. Second, electricity is transported from suppliers to load over a transmission network with possible constraints. The combination of these two features means that in almost all electricity markets today an Independent System Operator (ISO) sets dispatch centrally and clears the market. Generators and demand-side users can make offers and bids, and the ISO will choose which are accepted according to a pre-determined settlement system.

The classic settlement system used in almost all existing electricity markets is one where the ISO sets dispatch to maximize social welfare. The ISO will ask for generators' cost functions, and buyers' willingness to pay functions, and choose dispatch to maximize welfare. This assumes that bidders have been truthful. A large part of demand however, is inelastic, created by residential and commercial consumers turning on and off electrical appliances without regard to price. The ISO's job then is to match supply to meet this demand at every moment in time. This becomes particularly difficult in the short-run (up to 24 hours before actual market clearing) as some types of generator (e.g. steam turbines and to some extent gas turbines) need to ramp up their generation slowly, and it is costly to change their output rapidly. Different markets have approached this problem in different ways.

One common approach used is to run two settlements. In the first settlement, usually run about 24 hours before clearing, generators make offers, and the ISO chooses a pre-dispatch. This gives generators an idea of what they might expect to be producing, so they can warm up their plant, etc to prepare to produce most efficiently at this level. This first market is run based on an estimate of what demand is expected to be, so a second 'balancing market' is run soon before the market actually clears. In this second market, new sets of offers are submitted and upon market clearing the dispatches can deviate from pre-dispatch levels. Both settlement markets are based on that described above - maximizing social welfare, but they are run separately, and the result of one is not tied to the other.

Another option is used in New Zealand. Here generators can place offers for a given half hour period up to two hours beforehand. During the actual half hour, the ISO will then run a settlement every five minutes, using the same bids each time, to figure out dispatch. Any generator may then be asked to deviate at 5 minutes notice. Note that in this case, the same offer curves are used in the pre-dispatch phase as well as the actual half hour in question. In the two settlement markets, expected demand is used to clear the pre-dispatch quantities.

These market mechanisms are all workarounds designed to compensate for a settlement system that is basically devised to maximize welfare assuming demand is deterministic. In a deterministic auction, one should know how much generation capacity is available and what the demand level is while this is not always the case in real electricity markets. Intermittent generation (like wind and solar energy) and demand level are usually hard to predict. This complicates the problem of dispatching generators in reality.

An alternative to deterministic settlement systems is to use a stochastic settlement process. In a stochastic settlement, the ISO can choose both pre-dispatch and short-run deviations for each generator to maximize expected social welfare in one step. By co-optimizing both together, we might expect a stochastic settlement system to do better (on average) than two deterministic settlements. The idea of a stochastic settlement can be attributed to Bouffard et al., Wong and Fuller, and Pritchard et al. [2, 5, 6]. In these two-stage, single settlement models, the pre-dispatch clears with information about the future distribution of uncertainties in the system (e.g. demand and volatile renewable generation,) and information about deviation costs for each generator. These models assume that each firms' offers and deviation costs are truthful. In an imperfectly competitive market, this assumption is not valid. The question then remains: can the stochastic settlement auction give better expected social welfare when firms are behaving strategically? That is the question explored by this paper.

We start by introducing a simplified version of the two settlement market that operates in New Zealand. We will then introduce a simplified version of the stochastic programming mechanism for clearing electricity markets. As we will look at this mechanism as a game, and we try to analyse results under a steady state, we need to find the equilibria. We therefore need simplified and tractable versions of these mechanism. Thus we modify the offered supply function by firms from step functions to linear functions. We will present results pertaining to the existence of equilibria for each market clearing mechanism. Finally we construct these equilibria and compare social welfare attached to them under the different clearing mechanisms.

In section 2, we discuss some of the related literature, then we introduce the assumptions underpinning the market environment. We introduce and prove properties of the two settlement and stochastic settlement auctions respectively. Finally we present proofs of how the two auctions compare. Section 6 concludes the paper.

2 The Market Environment

In this paper, we aim to compare different market designs for electricity. We begin by presenting assumptions that are common to all markets we consider, features of what we call the *market environment*. These include such considerations as the number of firms, the costs firms face, the structure of demand and so forth.

Assumption 2.1 *The market environment may be defined by the following features.*

- *Electricity is traded over a network with no transmission constraints and no line losses, thus we may consider all trading as taking place at a single node.* [1]
- *Demand for electricity is uncertain, and may realize in one of $s \in \{1, \cdots, S\}$ possible outcomes (scenarios), each with probability θ_s. Demand in state s is assumed to be linear, and defined by the inverse demand function $p_s = Y_s - ZQ$, where Q is the quantity of electricity and p_s is the market price in scenario s. Without loss of generality, assume $Y_1 < Y_2 < ... < Y_S$. We will denote the expected value of Y_s by $Y = \sum_s \theta_s Y_s$.*
- *There are n symmetric firms wishing to sell electricity.*
- *For a given firm i in scenario s, we will denote the pre-dispatch quantity by q_i, and any short-run* change *in production by $x_{i,s}$. Thus a generator's actual production in scenario s is equal to $q_i + x_{i,s}$, which we denote by $y_{i,s}$.*
- *Each firm i's* long-run *cost function is $\alpha q_i + \frac{\beta}{2} q_i^2$, where q_i is the quantity produced by firm i, and $\beta > 0$.*
- *Each firm's* short-run *cost function is $\alpha\left(q_i + x_{i,s}\right) + \frac{\beta}{2}\left(q_i + x_{i,s}\right)^2 + \frac{\delta}{2} x_{i,s}^2$, where q_i is the long-run expected dispatch of firm i, and $q_i + x_{i,s}$ is the actual short-run dispatch and $\delta > 0$.*

[1] This assumption may also shed light on any scenario where line capacities do not bind, even if in other scenarios they do bind.

- *As minimum marginal cost of generation should not be more than maximum price of electricity, we assume*

$$\alpha \leq Y_s \qquad \forall s \in \{1, \ldots, S\}.$$

- *There is an Independent System Operator (ISO) who takes bids and determines dispatch and prices according to the given market design.*
- *All the above assumptions are common knowledge to all participants in the market.*

Our assumptions on generators' cost functions are particularly critical to the analysis that follows, and deserve further explanation. Generators face two distinct costs when generating electricity. If given sufficient advance notice of the quantity they are to dispatch, the generator can plan the allocation of turbines to produce that quantity most efficiently. This is what we mean by a *long-run* cost function. The interpretation of this is the lowest possible cost at which a generator can produce quantity q. In electricity markets, however, demand fluctuates at short notice, and the ISO may ask a generator to change its dispatch at short notice. In this case, generators may not have enough time to efficiently reallocate its turbines. For example, many thermal turbines take hours to ramp-up. Most likely, the generator will have to adopt a less efficient production method, such as running some turbines above their rated capacity which also increases the wear on the turbines. Thus there is some inherent cost in deviating from an expected pre-dispatch in the short-run. This cost can be incurred even if the requested deviation is negative. We assume that the generator will be unable to revert to the most efficient mode of producing this quantity $q_{i,s} + x_{i,s}$ in the short-run, so pays a penalty cost. Note that this imposes a positive penalty cost upon the generator for making the short-run change, even if the change is negative. This penalty cost is additively imposed on top of the 'efficient' cost of producing at the new level. We call this cost the *deviation cost*. Note that we assume the symmetric case in which cost of generation and deviation is determined through the same constant parameters (α, β, δ).

Our goal is to compare the outcomes of different markets imposed upon this environment. To be able to draw comparisons in different paradigms, we need to examine the steady state behaviour of participants under the different market clearing mechanisms. To this end, we need to compute equilibria arising under the different market clearing mechanisms. In order to make the computations tractable, we will restrict the firms to offer linear supply functions in the following sections of this paper.

3 Two Settlement Model

In this section we will introduce a two-settlement market which is inspired by the market clearing mechanism as it operates in New Zealand. In the New Zealand market, firms bid a step supply function for a given half hour period. The bid is made at least two hours in advance.

The market will then be cleared six times, every five minutes during the given half hour period.[2] We simplify the situation by assuming the market clears only twice; once after the bids are submitted, but before demand is realised. This we call the 'pre-dispatch settlement' which tells the generators approximately how much they should produce. Once demand is realized, the same bids will be used to determine actual dispatch in what we call the 'spot settlement'. The difference between pre-dispatch and spot dispatch is a generator's short-run deviation, which is subject to potentially higher costs as we described earlier.

3.1 Mathematical Model

Our simplified mathematical model for the two settlement market has two distinct stages; pre-dispatch and spot. Each generator i bids a supply function $a_i + b_i q_i$ before the pre-dispatch market to represent their quadratic costs. At this point, demand is uncertain. The ISO will then use the generator's bid twice: once to clear the pre-dispatch market, and once again after demand is realized to clear the spot market. The pre-dispatch market determines the pre-dispatch quantities each generator is asked to dispatch, and the spot market determines the final quantities the generators are asked to dispatch. As in reality, in both the pre-dispatch and spot markets, the ISO aims to maximize social welfare, assuming generators are bidding their true cost functions. Since demand is unknown in pre-dispatch, the ISO will nominate (and use) an expected demand (and will not consider the distribution of demand).

$$\begin{array}{rcll} \min\ z & = & \sum_{i=1}^{n} \left(a_i q_i + \frac{b_i}{2} q_i^2\right) - \left(YQ - \frac{Z}{2}Q^2\right) & \\ s.t. & & \sum_i q_i - Q = 0 & [f] \end{array} \tag{1}$$

From this first settlement, the ISO can extract a forward price f equal to the shadow price on the (expected demand balance) constraint. Recall that the pre-dispatch quantity for generator i is denoted by q_i. After pre-dispatch is determined, true demand is realized, and the ISO then clears the spot market (using the specific demand scenario that has been realized) to maximize welfare by solving (2).

$$\begin{array}{rcll} \min\ z & = & \sum_{i=1}^{n} \left(a_i y_{i,s} + \frac{b_i}{2} y_{i,s}^2\right) - \left(Y_s C_s - \frac{Z}{2}C_s^2\right) & \\ s.t. & & \sum_i y_{i,s} - C_s = 0 & [p_s] \end{array} \tag{2}$$

Here again the ISO computes a spot price p_s as the shadow price on the constraint. (Note that we can eliminate the constraint and substitute C_s in the objective, however imposing this constraint enables the easy introduction of the price as the shadow price attached to the constraint.) The generator is not permitted to change its bid after pre-dispatch, but does face the usual additional deviation cost δ for its short-run deviation.

Note that in both ISO optimization problems (1, 2) we have dispensed with non-negativity constraints on the pre-dispatch and dispatch quantity respectively. We will demonstrate that

[2] Within this five minute period a frequency keeping generator will match any small changes in demand. We ignore this aspect of the market, as frequency-keeping is purchased via fixed annual contracts and does not impact the market.

the resulting equilibria of our two settlement market model will always have associated non-negative pre-dispatch and dispatch quantities. We have eliminated the non-negativity constraints following the convention of supply function equilibrium models (see e.g. [4, 1]) in order to enable the analytic computation of equilibrium supply offers.

In the last section of this paper, we return to this point, enforce non-negativity constraints, and present numerical experiments where the equilibrium offers (and associated dispatch quantities,) are computed using global optimization techniques.

Firm i's profit in scenario s in this market is then given by

$$u_{i,s}^{TS}\left(q_i, x_{i,s}\right) = fq_i + p_s\left(y_{i,s} - q_i\right) - \left(\alpha y_{i,s} + \frac{\beta}{2}y_{i,s}^2 + \frac{\delta}{2}\left(y_{i,s} - q_i\right)^2\right). \quad (3)$$

3.2 Equilibrium Analysis of the Two Settlement Market

In this section we will present equilibria of the two settlement market model. We will first compute the optimal dispatch quantities from the ISO's optimal dispatch problems (1) and (2) for any number of players. We will then embed these quantities in the generator's expected profit function and allow the generators to simultaneously optimize over their (linear) supply function parameters to obtain equilibrium offers.

Proposition 3.1 *Problem (1) is a convex program with a strictly convex objective. Its unique optimal solution and the corresponding optimal dual f are given by*

$$\begin{aligned} f &= \frac{Y + ZA}{ZB + 1} \\ q_i &= fB_i - A_i \end{aligned}$$

where $A_i = \frac{a_i}{b_i}$, $B_i = \frac{1}{b_i}$, $A = \sum_i A_i$ and $B = \sum_i B_i$.

Proof Note that problem (1) has a single linear constraint and that its objective is a strictly convex quadratic as we have assumed that $b_i > 0$ and $Z > 0$. The problem therefore has a unique optimal solution delivered by the first order conditions provided below.

$$\begin{aligned} Q - \sum_i q_i &= 0 &(4)\\ f - Y + ZQ &= 0 &(5)\\ -f + a_i + b_i q_i &= 0 \quad \forall i &(6) \end{aligned}$$

Using equation (5) we can rewrite equation (6) as

$$Y - ZQ = a_i + b_i q_i \quad \forall i \quad (7)$$

Now summing over all i we obtain

$$\sum_i q_i = (\sum_i \frac{1}{b_i})(Y - ZQ) - (\sum_i \frac{a_i}{b_i}) \quad (8)$$

Note that $B = \sum_i \frac{1}{b_i}$ and $A = \sum_i A_i$. This together with equations (8) and (4) yields

$$Q = \frac{BY - A}{ZB + 1}.$$

Now substituting Q from the above into equation (5), we obtain

$$f = \frac{Y + ZA}{ZB + 1}.$$

Similarly substituting Q into (7) yields

$$q_i = B_i(Y - Z\frac{BY - A}{ZB + 1}) - A_i.$$

This equation simplifies to

$$q_i = fB_i - A_i,$$

and we obtain the expressions in the statement of the proposition.

Proposition 3.2 *For each scenario s, problem (2) is a convex program with a strictly convex objective. Its unique optimal solution and the corresponding optimal dual p_s, are given by*

$$\begin{aligned} p_s &= \frac{Y_s + ZA}{ZB + 1} \\ y_{i,s} &= p_s B_i - A_i \end{aligned}$$

where A_i, B_i, A and B are defined above in proposition (3.1).

Proof Note that problems (2) and (1) are structurally identical, therefore the simple proof of proposition (3.1) applies again here.

Remark Note from the above that the pre-dispatch price (and quantity) are equal to the expected spot market prices (and quantities respectively). That is

$$f = \sum_{s=1}^{S} \theta_s p_s. \tag{9}$$

We will now compute the linear supply functions resulting from the equilibrium of the two settlement market game laid out in (2.1). Before we begin with the firm computations, we will establish a technical lemma that we utilize in establishing the equilibrium results.

Lemma 3.3 *Assume that function $f(x, y) : R^2 \to R$ is defined on a $D_x \times D_y$ with $D_x, D_y \subseteq R$. Furthermore assume that $x^*(y) \in D_x$, maximizes $f(x, y)$ for any arbitrary but fixed y. Also assume $g(y) = f(x^*(y), y)$ is maximized at $y^* \in D_y$. Then, $f(x, y)$ is maximized at $(x^*(y^*), y^*)$.*

Proof Note that for any $(x, y) \in D_x \times D_y$,

$$f(x, y) \le f(x^*(y), y)$$

by the assumption on $x^*(y) \in D_x$. Furthermore $f(x^*(y), y) \le f(x^*(y^*), y^*)$. Clearly then

$$f(x, y) \le f(x^*(y^*), y^*) \quad \text{for any } (x, y) \in D_x \times D_y.$$

3.2.1 Firm i's computations

In this section we will focus on firm i's expected profit function. Note that using equation (9) we obtain

$$u_i^{TS} = E_s[u_{i,s}^{TS}] \;=\; \sum_{s=1}^{S} \theta_s \left(p_s y_{i,s} - \left(\alpha y_{i,s} + \frac{\beta}{2} y_{i,s}^2 + \frac{\delta}{2} \left(y_{i,s} - q_i\right)^2 \right) \right).$$

Using propositions (3.1) and (3.2), we can re-write u_i^{TS} as a function of a_i and b_i. In order to find a maximum of u_i^{TS} (for a fixed set of competitor offers) we appeal to a transformation that will yield concavity results for u_i^{TS}. We consider u_i^{TS} to be a function of A_i and B_i (instead of a_i and b_i). Note that the transformation ($A_i = \frac{a_i}{b_i}, \quad B_i = \frac{1}{b_i}$) is a one-to-one transformation.

Proposition 3.4 *Let all competitor (linear) supply function offers be fixed. The following maximizes u_i^{TS} (and is therefore firm i's best response).*

$$\begin{aligned} B_i &= \frac{1 + ZB_{-i}}{Z + \beta + \delta + Z(\beta + \delta)B_{-i}} \\ A_i &= \frac{\alpha + B_i \left(Z\alpha - \delta \left(Y + ZA_{-i}\right)\right) + Z\alpha B_{-i}}{2Z + \beta + Z\beta B_{-i}} \end{aligned}$$

Proof We can show that u_i^{TS} is a concave function of A_i, assuming B_i is a fixed parameter. Here we have dispensed with the expression for u_i^{TS} as a function of A_i and B_i as it is long and rather complicated. This expression can be found in the online technical companion [3].

We note that u_i^{TS} is a smooth function of A_i and B_i. Let $A_{-i} = \sum_{j \neq i} A_j$ and $B_{-i} = \sum_{j \neq i} B_j$. Then

$$\frac{\partial^2 u_i^{TS}}{\partial {A_i}^2} = -\frac{\left(1 + ZB_{-i}\right)\left(2Z + \beta + Z\beta B_{-i}\right)}{\left(1 + ZB\right)^2} \leq 0$$

Let B_i be arbitrary but fixed. As u_i^{TS} is a concave function of A_i the first order condition yields an expression for $A_i^*(B_i)$, the value of A_i that maximizes u_i^{TS} (for the fixed B_i).

$$A_i^*(B_i) = \frac{(1+ZB_{-i})(-Y+\alpha-ZA_{-i}+Z\alpha B_{-i}) + B_i(Z(Y+ZA_{-i}) + (Z\alpha+\beta Y+Z\beta A_{-i})(ZB_{-i}+1))}{(1+ZB_{-i})(2Z+\beta+Z\beta B_{-i})}.$$

We can embed $A_i^*(B_i)$ into u_i^{TS} and find the maximizer in terms of B_i. Lemma (3.3) then can be applied to demonstrate that the end result delivers the maximum of u_i^{TS}.

After embedding this value of A_i^* into the profit function, the derivative with respect to B_i of u_i^{TS} is.

$$\frac{du_i}{dB_i} = \frac{\left(Y^2 - \sum_s \theta_s Y_s^2\right)\left(-1 + (Z + \beta + \delta)B_i + Z\left(-1 + (\beta + \delta)B_i\right)B_{-i}\right)}{\left(1 + ZB\right)^3}$$

$B_i^* = \frac{1+ZB_{-i}}{Z+\beta+\delta+Z(\beta+\delta)B_{-i}}$, is the zero of this derivative. Recall that $Y = \sum_s \theta_s Y_s$, therefore Jensen's inequality implies $Y^2 - \sum_s \theta_s Y_s^2 \leq 0$. Thus, $\frac{du_i}{dB_i} \geq 0$, when $B_i < {B_i}^*$, and $\frac{du_i}{dB_i} \leq 0$, when $B_i > B_i^*$. In other words, u_i is a quasi-concave function of B_i and is maximized at $B_i = {B_i}^*$.

Note that evaluating A_i^* at B_i^* yields

$$A_i^* = \frac{\alpha + B_i\left(Z\alpha - \delta\left(Y + ZA_{-i}\right)\right) + Z\alpha B_{-i}}{2Z + \beta + Z\beta B_{-i}}.$$

From the above, we can obtain the equilibrium of the two settlement model by solving all best responses simultaneously. This gives the unique and symmetric solution

$$\text{2S-EQM:} B_i = \frac{2}{-(n-2)Z + \beta + \delta + \sqrt{(n-2)^2 Z^2 + 2nZ(\beta+\delta) + (\beta+\delta)^2}} \tag{10}$$

$$A_i = \frac{\alpha + (nZ\alpha - Y\delta)B_i}{2Z + \beta + (n-1)Z(\beta+\delta)B_i} \tag{11}$$

As we discussed earlier, these equilibrium offers yield non-negative pre-dispatch and dispatch quantities. The computations to show the non-negativity of these quantities can be found in the technical companion [3].

Proposition 3.5 *The equilibrium pre-dispatch and spot production quantities of the firms in the two settlement market are non-negative, i.e.*

$$\begin{aligned} q_i &\geq 0 & \forall i, \\ y_{i,s} &\geq 0 & \forall i, s. \end{aligned}$$

4 Stochastic Settlement Market

4.1 SFSP Model

We now introduce the market model we will use to analyze a stochastic settlement market. As discussed in the introduction, the stochastic settlement market contains only a single stage of bidding, but the market clearing procedure takes into account all possible realizations of demand when determining dispatch. The market works as follows. When the market opens, demand is uncertain. Firms are allowed to bid their 'normal' cost functions (the cost of producing a given output most efficiently) and a 'penalty' cost function that they would need to be paid to deviate in the short-run. Since firms have quadratic cost functions, they can bid their actual costs by submitting a linear supply function. Each firm i chooses a_i and b_i to bid the linear supply function $a_i + b_i q$, and d_i to bid the (marginal) penalty cost $d_i q$. Note that as with the two-settlement model, these bids (a_i, b_i, d_i) need not be their true values (α, β, δ). The offered b_i should have a positive quantity and d_i should be non-negative.

After generators have placed their bids, the ISO computes the market dispatch according to the stochastic settlement model (outlined below). At this point demand is still uncertain. The ISO chooses two key variables. The first is the *pre-dispatch* quantity for every firm. This is the quantity the ISO asks each firm to prepare to produce before demand is realized. This is the pre-dispatch quantities q_i for firm i as defined in Section 2. The second is the *short-run deviation* that each generator will be asked to make in any possible scenario. This deviation is the variable $x_{i,s}$ defined in Section 2, representing the deviation of firm i in scenario s. The ISO can choose both pre-dispatch and short-run deviations simultaneously, while aiming to maximize expected social welfare. The ISO assumes that generators have bid their true costs.

In the final stage, demand is realized, and the ISO will ask generators to modify their pre-dispatch quantity according to the short-run deviation for the particular scenario. Each generator ends up producing $q_i + x_{i,s}$. Two prices are calculated during the course of optimizing welfare. The first is the shadow price on the pre-dispatch quantities. We will denote this by f. The second are the shadow prices on each of the deviations, for each of the scenarios. We will denote these by p_s for scenario s. Each generator is paid f per unit for its pre-dispatch quantity q_i, and p_s for its deviations $x_{i,s}$. Thus in realization s, generator i makes profit equal to

$$u_{i,s}^{SS}\left(q_i, x_{i,s}\right) = f q_i + p_s x_{i,s} - \alpha\left(q_{i,s} + x_{i,s}\right) + \frac{\beta}{2}\left(q_{i,s} + x_{i,s}\right)^2 + \frac{\delta}{2} x_{i,s}^2. \tag{12}$$

Mathematically, the stochastic optimization problem solved by the ISO can be represented as follows. [3]

ISOSP:

$$\begin{array}{rcll}
\min z & = & \sum_{s=1}^{S} \theta_s \left(\sum_{i=1}^{n} \left[a_i(q_i + x_{i,s}) + \frac{b_i}{2}(q_i + x_{i,s})^2 + \frac{d_i}{2} x_{i,s}^2\right] - (Y_s C_s - \frac{Z}{2} C_s^2)\right) & \\
s.t. & & \sum_i q_i - Q = 0 & [f] \\
 & & Q + \sum_i x_{i,s} - C_s = 0 \quad \forall s \in \{1, \dots, S\} & [p_s]
\end{array}$$

Q and C_s stand for the total contracted (or pre-dispatched) quantity and total consumption in scenario s respectively. Note that we could have eliminated the two equality constraints. However, their dual variables are the market prices f and p_s respectively, so for clarity we have left them in.

4.2 Characteristics of the Stochastic Optimization Problem

We begin by presenting a series of results that simplify the set of solutions to the ISOSP problem. These results drastically simplify the subsequent analysis of firms' behaviour in equilibrium under stochastic settlement.

Lemma 4.1 *In the stochastic settlement market clearing, the expected deviation of firm i from pre-dispatch quantity q_i^* is zero, that is, the optimal solution to ISOSP will always satisfy*

[3] This is a modified version of Pritchard et al.'s problem. There is only one node and thus no transmission constraints, and demand is elastic.

$$\sum_s \theta_s x^*_{i,s} = 0.$$

Proof Let us assume Q^*_i and $x^*_{i,s}$ form ISOSP's optimal solution. Let us define for each i and s the quantity $k_{i,s} := q^*_i + x^*_{i,s}$, the total production of firm i in scenario s. Note that $C_s = \sum_i q^*_i + \sum_i x^*_{i,s}$. Assume, on the contrary, that there exists at least one firm j such that $\sum_s \theta_s x^*_{j,s} \neq 0$. The optimal objective value of ISOSP is then given by

$$\sum_i \sum_s \theta_s (a_i k_{i,s} + \frac{b_i}{2}(k_{i,s})^2) + \sum_i \sum_s \theta_s \frac{d_i}{2}(x^*_{i,s})^2 + Y_s \sum_i k_{i,s} - \frac{Z}{2}(\sum_i k_{i,s})^2. \tag{13}$$

Note that as $\sum_s \theta_s x^*_{j,s} \neq 0$, the term $\sum_i \sum_s \theta_s \frac{d_i}{2}(x^*_{i,s})^2$ is positive. Now, for a fixed i and $k_{i,s}$ given from above, consider the problem

$$\begin{aligned} \min_{q_i, x_{i,s}} w &= \frac{d_i}{2} \sum_{s=1}^{S} \theta_s x^2_{i,s} \\ \forall s : q_i + x_{i,s} &= k_{i,s}. \end{aligned} \tag{14}$$

This problem clearly reduces to the univariate problem

$$\min_{q_i} w = \sum_{s=1}^{S} \theta_s (k_{i,s} - q_i)^2,$$

which is optimized at

$$q_i = \sum_{s=1}^{S} \theta_s k_{i,s}.$$

Define $\hat{q_i}$ and $\hat{x_{i,s}}$ by

$$\hat{q_i} = \begin{cases} q^*_i, & i \neq j \\ \sum_{s=1}^{S} \theta_s k_{j,s} & \text{otherwise,} \end{cases}$$

and

$$\hat{x_{i,s}} = \begin{cases} x^*_{i,s}, & i \neq j \\ k_{j,s} - \hat{q_j} & \text{otherwise.} \end{cases}$$

By definition, $\hat{q_i} + \hat{x_{i,s}} = q^*_i + x^*_{i,s}$ for all i and s. It is easy to see that the quantities $\hat{q_i}$ and $\hat{x_{i,s}}$ yield a feasible solution to ISOSP. Furthermore, the objective function evaluated at $\hat{q_i}$ and $\hat{x_{i,s}}$ is given by

$$\sum_i \sum_s \theta_s (a_i k_{i,s} + \frac{b_i}{2}(k_{i,s})^2) + Y_s \sum_i k_{i,s} - \frac{Z}{2}(\sum_i k_{i,s})^2.$$

This value is strictly less than the objective evaluated at q^*_i and $x^*_{i,s}$ (given by 13), as we have already established that $\sum_i \sum_s \theta_s \frac{d_i}{2}(x^*_{i,s})^2 > 0$. This yields the contradiction that proves the result.

Corollary 4.2 *In the stochastic problem ISOSP, if $q_i^* + x_{i,s}^* \geq 0$ is satisfied $\forall s \in \{1, \ldots, S\}$ then $q_i^* \geq 0$ will hold.*

Proof In lemma 4.1 we established that $\sum_s \theta_s x_{i,s}^* = 0$. Therefore there exists a scenario s' such that $x_{i,s'}^* \leq 0$. clearly then $q_i^* + x_{i,s'}^* \geq 0$ implies $q_i^* \geq 0$.

Lemma 4.1 is the crucial result that drives the rest of our characterizations. We now use this to prove that the ISO's optimization problem can be split into two separate problems: one to clear the pre-dispatch market, and the other to clear the spot market.

Lemma 4.3 *Problem ISOSP is equivalent to the following optimization problem which is separable in the pre-dispatch and the spot market variables*

$$\begin{aligned} z &= \sum_{i=1}^{n}\left(a_i q_i + \frac{b_i}{2}q_i^2\right) - Y\sum_{i=1}^{n} q_i + \frac{Z}{2}\left(\sum_{i=1}^{n} q_i\right)^2 \\ &\quad + \sum_{i=1}^{n}\left(\frac{b_i + d_i}{2}\sum_{s=1}^{S}\theta_s x_{i,s}^2\right) - \sum_{i=1}^{n}\sum_{s=1}^{S}\theta_s Y_s x_{i,s} + \frac{Z}{2}\sum_{s=1}^{S}\theta_s\left(\sum_{i=1}^{n} x_{i,s}\right)^2 . \end{aligned}$$

Proof Substituting for C_s from constraints into the objective function of ISOSP yield

$$\begin{aligned} z &= \sum_{s=1}^{S}\theta_s\Bigg(\sum_{i=1}^{n}\left(a_i\left(q_i + x_{i,s}\right) + \frac{b_i}{2}\left(q_i + x_{i,s}\right)^2 + \frac{d_i}{2}x_{i,s}^2\right) \\ &\quad - Y_s\sum_{i=1}^{n}\left(q_i + x_{i,s}\right) + \frac{Z}{2}\left(\sum_{i=1}^{n}\left(q_i + x_{i,s}\right)\right)^2\Bigg) \end{aligned}$$

Rearranging the above we obtain:

$$\begin{aligned} z &= \sum_{i=1}^{n}\left(a_i q_i + \frac{b_i}{2}q_i^2\right) - Y\sum_{i=1}^{n} q_i + \frac{Z}{2}\left(\sum_{i=1}^{n} q_i\right)^2 \\ &\quad + \sum_{i=1}^{n}\left(a_i\sum_{s=1}^{S}\theta_s x_{i,s}\right) + \sum_{i=1}^{n}\left(\frac{b_i + d_i}{2}\sum_{s=1}^{S}\theta_s x_{i,s}^2\right) - \sum_{i=1}^{n}\sum_{s=1}^{S}\theta_s Y_s x_{i,s} + \frac{Z}{2}\sum_{s=1}^{S}\theta_s\left(\sum_{i=1}^{n} x_{i,s}\right)^2 \\ &\quad + \sum_{i=1}^{n}\left(q_i b_i\sum_{s=1}^{S}\theta_s x_{i,s}\right) + \sum_{s=1}^{S}\theta_s Z\sum_{i=1}^{n}\sum_{j=1}^{n} q_i x_{j,s} \end{aligned}$$

Note that the first part of the objective above is a function of pre-dispatch quantities q_i and the second only a function of the spot dispatches $x_{i,s}$. Furthermore, observe that in both of the terms in the third part, the factor $\sum_{s=1}^{S}\theta_s x_{j,s}$ appears. We can therefore appeal to lemma (4.1) and eliminate this last part. This completes the proof.

The rest of this section is devoted to deriving explicit expressions for the solution of ISOSP. In the next section we will use these expressions to arrive at best response functions for the firms and subsequently in constructing an equilibrium for the stochastic market settlement. In order to simplify the equations and arrive at explicit solutions, we will transform the space of the parameters of ISOSP (firm decision variables). We will use the following transformation $H : \begin{pmatrix} \mathbb{R} \\ \mathbb{R}^+ - \{0\} \\ \mathbb{R}^+ \end{pmatrix} \to \begin{pmatrix} \mathbb{R} \\ \mathbb{R}^+ - \{0\} \\ \mathbb{R}^+ - \{0\} \end{pmatrix}$ that is one-to-one and onto.

$$\begin{pmatrix} A_i \\ B_i \\ R_i \end{pmatrix} = H \begin{pmatrix} a_i \\ b_i \\ d_i \end{pmatrix} := \begin{pmatrix} a_i/b_i \\ 1/b_i \\ 1/(b_i + d_i) \end{pmatrix}$$

If we further define

$$A = \sum_i A_i, \quad B = \sum_i B_i \quad \text{and } R = \sum_i R_i,$$

ISOSP reduces to minimizing the following:

$$\begin{aligned} z \;&=\; \sum_{i=1}^{n} \left(\frac{A_i}{B_i} q_i + \frac{1}{2B_i} q_i^2 \right) - Y \sum_{i=1}^{n} q_i + \frac{Z}{2} \left(\sum_{i=1}^{n} q_i \right)^2 \\ &\quad + \sum_{s=1}^{S} \theta_s \left[\sum_{i=1}^{n} \frac{1}{2R_i} x_{i,s}^2 - (Y_s - Y) \sum_{i=1}^{n} x_{i,s} + \frac{Z}{2} \left(\sum_{i=1}^{n} x_{i,s} \right)^2 \right]. \end{aligned}$$

Note as before (lemma (4.3)) that the above is separable in q_i's and $x_{i,s}$'s, we can therefore solve the two stages separately. Note also that the two problems are convex optimization problems therefore KKT conditions will readily produce the optimal solution (for derivation please refer to the technical companion [3]).

Proposition 4.4 *If (q, x, f, p) represents the solution of ISOSP, then we have*

$$q_i = \frac{(Y + ZA)B_i}{1 + ZB} - A_i \tag{15}$$

$$x_{i,s} = \frac{(Y_s - Y)R_i}{1 + ZR} \tag{16}$$

$$f = \frac{Y + ZA}{1 + ZB}$$

$$p_s = \frac{Y + ZA}{1 + ZB} + \frac{Y_s - Y}{1 + ZR}$$

Observe from the expression for f that this forward price (paid on pre-dispatch quantities) is independent of any deviation costs in the spot market.

As we have observed, ISOSP can be separated into two different (sets of) market clearing problems, one for the pre-dispatch market and the other over the spot market (in each scenario). Therefore the upshot of market clearing through our stochastic program is to find the intersection of the expected demand curve with the aggregate supply offer curve (to find pre-dispatch quantities) and then to find the intersection of the demand curve (for each scenario) with the aggregate deviation offer curves as depicted in Figures 1 and 2 below.

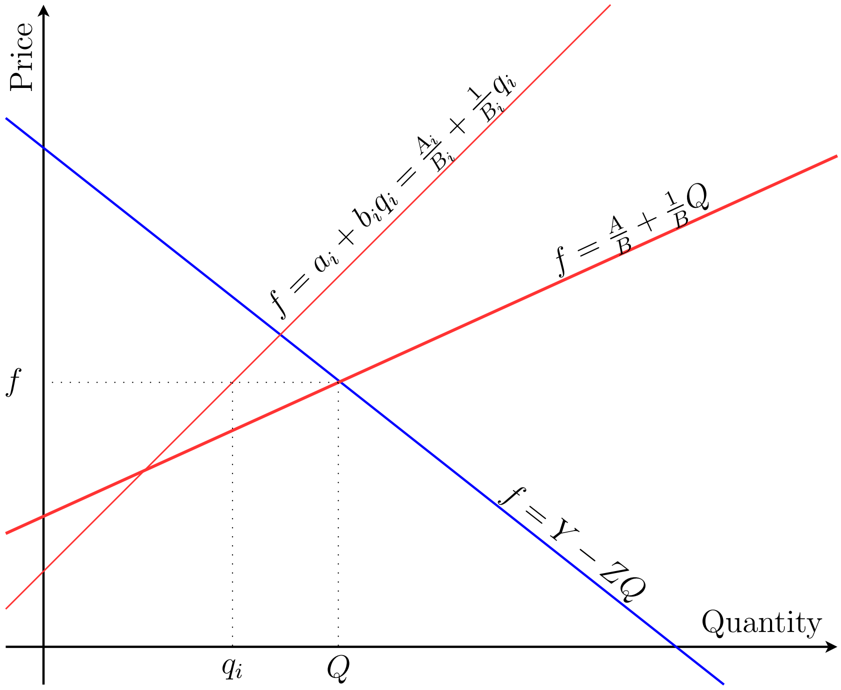

Figure 1: Market clearing of the forward market using firms' supply functions as an equivalent representation of ISOSP problem

Corollary 4.5 *In the solution of ISOSP, forward price is equal to the expected spot market price.*

Proof This is simply observed from proposition 4.4.

The fact that contract price is equal to the expected spot market price, implies that there is no systematic arbitrage.

4.3 Equilibrium Analysis of the Stochastic Settlement Market

In section (4.1) we presented firm i's profit under scenario s in equation (12). In our market model, we assume that all firms are risk neutral and therefor interested only in maximizing their expected profit. Firm i's expected profit is given by

$$u_i = fq_i + \sum_{s=1}^{S} \theta_s \left(p_s x_{i,s} - \left(\alpha(q_i + x_{i,s}) + \frac{\beta}{2}\ (q_i + x_{i,s})^2 + \frac{\delta}{2} x_{i,s}^2 \right) \right), \qquad (17)$$

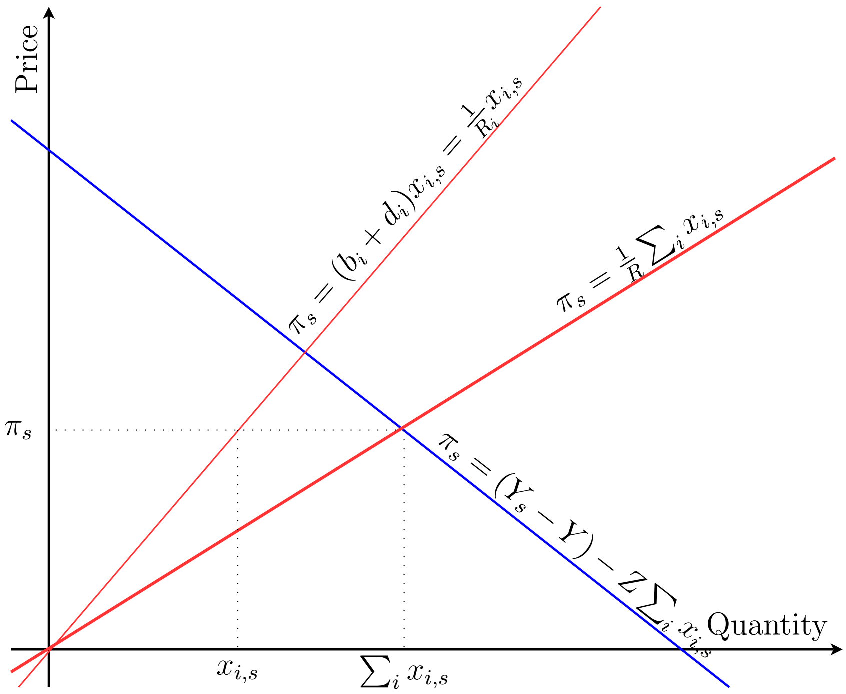

Figure 2: Market clearing of the spot market using firms' supply functions as an equivalent representation of ISOSP problem

The above expression for u_i can be expanded and we can observe that

$$\begin{aligned} u_i &= fq_i - (\alpha q_i + \frac{\beta}{2}q_i^2) \\ &\quad + \sum_{s=1}^{S} \theta_s \left(p_s x_{i,s} - \frac{\beta+\delta}{2} x_{i,s}^2 \right) \\ &\quad - \alpha \sum_{s=1}^{S} \theta_s x_{i,s} - \beta q_i \sum_{s=1}^{S} \theta_s x_{i,s} \end{aligned}$$

Note that from lemma (4.1), the generator would know that for any admissible bid, the corresponding expected deviation from pre-dispatch quantities $\sum_{s=1}^{S} \theta_s x_{i,s} = 0$. Therefore the expected profit for the generator becomes

$$\begin{aligned} u_i &= fq_i - (\alpha q_i + \frac{\beta}{2}q_i^2) \\ &\quad + \sum_{s=1}^{S} \theta_s \left(p_s x_{i,s} - \frac{\beta+\delta}{2} x_{i,s}^2 \right). \end{aligned}$$

We can use the expressions obtained from proposition (4.4) to write u_i as follows.

$$\begin{aligned} u_i &= -\frac{1}{2}\beta A_i^2 + \frac{A_i\left(-ZA+\alpha+ZB\alpha+ZA\beta B_i+Y\left(-1+\beta B_i\right)\right)}{1+ZB} \\ &+\frac{1}{2(1+ZB)^2(1+ZR)^2}\Bigg(\\ &2(1+ZR)^2(ZA+Y)(ZA+Y-(1+ZB)\alpha)B_i-(1+ZR)^2(ZA+Y)^2\beta B_i^2 \\ &+(1+ZB)^2R_i\left(-2+(\beta+\delta)R_i\right)\left(Y^2-\sum_s \theta_s Y_s^2\right)\Bigg) \end{aligned} \tag{18}$$

Although this expression of the expected profit for the generator is rather ugly, it does have the advantage that upon differentiating with respect to R_i, all dependence on A_i and B_i drops and we are left with

$$\frac{du_i}{dR_i} = \frac{\left(Y^2-\sum_s \theta_s Y_s^2\right)\left(-1+(Z+\beta+\delta)R_i+ZR_{-i}\left(-1+(\beta+\delta)R_i\right)\right)}{(1+ZR)^3}. \tag{19}$$

Recall that $R_{-i}=\sum_{j\neq i}R_j$. (For verification of this derivative term see the technical companion [3].) The fact that this derivative is free of A_i and B_i indicates that u_i is separable in R_i and (A_i,B_i), that is

$$u_i(A_i,B_i,R_i)=g_i(A_i,B_i)+h_i(R_i) \tag{20}$$

Equation (20) enables us to maximize u_i by maximizing g_i and h_i over (A_i,B_i) and R_i respectively. This is helpful as we can establish quasi-concavity results for g_i and h_i separately.

We start our investigations by examining g_i. The full expression for g_i can be found in the technical companion [3]. Holding B_i fixed, note that

$$\frac{d^2 g_i}{dA_i^2} = -\frac{\left(1+ZB_{-i}\right)\left(2Z+\beta+Z\beta B_{-i}\right)}{(1+ZB)^2}.$$

This demonstrates that g_i is concave in A_i for any fixed B_i. Furthermore, for any fixed B_i, we can use the first order conditions to find $A_i^*(B_i)$, i.e. the value of A_i that maximizes $g_i(A_i,B_i)$ for the fixed B_i.

$$A_i^*(B_i)=\frac{\left(1+ZB_{-i}\right)\left(\alpha-ZA_{-i}+Z\alpha B-Y\right)+\left(Y+ZA_{-i}\right)\left(Z+\beta+Z\beta B_{-i}\right)B_i}{\left(1+ZB_{-i}\right)\left(2Z+\beta+Z\beta B_{-i}\right)} \tag{21}$$

To find the optimal value for g_i, we can now appeal to lemma (3.3) and substitute the expression for $A_i^*(B_i)$ in $g_i(A_i^*(B_i),B_i)$. Surprisingly, upon undertaking this substitution, it can be observed that $g_i(A_i^*(B_i),B_i)$ is a constant value. Figure 3 depicts g_i.

To uncover the intuition behind this feature of g_i, we can offer the following mathematical explanation. We observe that

$$\begin{aligned} \frac{dg_i}{dA_i} &= \frac{-\left(1+ZB_{-i}\right)\left(Y-\alpha+ZA_{-i}+(2Z+\beta)A_i+ZB_{-i}\left(-\alpha+\beta A_i\right)\right)}{(1+ZB)^2} \\ &+\frac{\left(Z(Y+\alpha)+Y\beta+Y(Z\alpha+Y\beta)B_{-i}+ZA_{-i}\left(Z+\beta+Z\beta B_{-i}\right)\right)B_i}{(1+ZB)^2} \end{aligned}$$

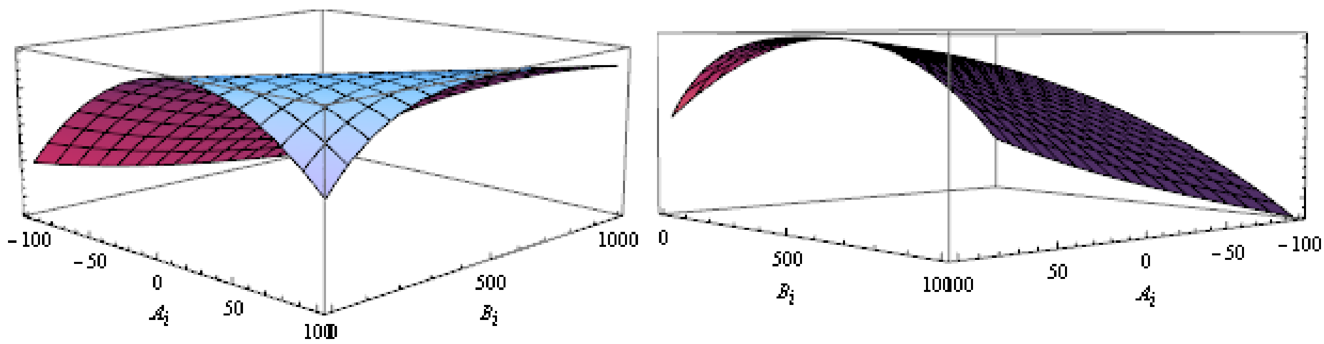

Figure 3: Two views of the function g_i. Note that the optimal value of g_i is obtained along a continuum, for any value of B_i.

and that

$$\frac{dg_i}{dB_i} = -\frac{Y + ZA}{1 + ZB} \cdot \frac{dg_i}{dA_i}.$$

Therefore, stationary conditions enforced in A_i will also imply stationarity in B_i.

As $g_i(A_i^*(B_i), B_i)$ is constant for any $B_i > 0$, for any value of $B_i > 0$, the tuple $(A_i^*(B_i), B_i)$ is an argmax of g_i for any positive B_i. The following analysis on h_i will explain how optimal R_i is constrained by the value of B_i.

Proposition 4.6 *Suppose that R_{-i} is fixed. Then h_i is optimized at*

$$R_i^* = \min\{B_i, \frac{1 + ZR_{-i}}{Z + \beta + \alpha + Z(\beta + \delta)R_{-i}}\}$$

Proof Note that at

$$\hat{R}_i = \frac{1 + ZR_{-i}}{Z + \beta + \alpha + Z(\beta + \delta)R_{-i}} \tag{22}$$

The derivative $\frac{dh_i}{dR_i} = \frac{du_i}{dR_i}$ vanishes. Also recall from Jensen's inequality that $Y^2 \leq \sum_s \theta_s Y_s^2$. It can therefor be seen from (19) that this derivative is positive for $R_i < \hat{R}_i$ and negative for $R_i > \hat{R}_i$. Recall further that the definition of B_i and R_i require $R_i \leq B_i$. Therefore, in optimizing h_i, we need to enforce this constraint and we obtain

$$R_i^* = \min\{B_i, \frac{1 + ZR_{-i}}{Z + \beta + \alpha + Z(\beta + \delta)R_{-i}}\}.$$

We now return to u_i, the expected profit function for firm i. As $u_i(A_i, B_i, R_i) = g_i(A_i, B_i) + h_i(R_i)$, we can start by obtaining the maximum value of g_i attained at a point $(A_i^*(B_i), B_i)$ for any positive B_i. Subsequently, we proceed to optimize $h_i(R_i)$. Proposition (4.6) readily delivers the optimal R_i. We have therefore proved the following theorem.

Theorem 4.7 *The best response of firm i, holding competitor offers fixed, is to offer in*

$$A_i = \frac{(1 + ZB_{-i})(\alpha - ZA_{-i} + Z\alpha B - Y) + (Y + ZA_{-i})(Z + \beta + Z\beta B_{-i}) B_i}{(1 + ZB_{-i})(2Z + \beta + Z\beta B_{-i})},$$

$$R_i = \frac{1 + ZR_{-i}}{Z + \beta + \alpha + Z(\beta + \delta)R_{-i}},$$

and any choice of B_i where

$$B_i \geq \frac{1 + ZR_{-i}}{Z + \beta + \alpha + Z(\beta + \delta)R_{-i}}.$$

Theorem 4.7 indicates that there exist a situation with multiple (infinite) equilibria in this game. To prevent the problem of unpredictability, caused by multiple equilibria, we assume that the ISO chooses B_i as a system parameter. This parameter is identical for and known to all participants. This also provides the ISO with the authority to choose B_i in a way to obtain better equilibrium (i.e. an equilibrium that yields higher social welfare).

Proposition 4.8 *The unique symmetric equilibrium quantities of the stochastic settlement market are as follows.*

$$a_i = \frac{\alpha - Y + B_i\left(-Z(Y(n-2) - (2n-1)\alpha) + Y\beta + Z(n-1)(Zn\alpha + Y\beta)B_i\right)}{B_i\left(Z(n+1) + \beta + Y(n-1)(Zn + \beta)B_i\right)} \tag{23}$$

$$d_i = \max\{0, \frac{-Z(n-2) + \beta + \delta + \sqrt{Z^2(n-2)^2 + 2Zn(\beta+\delta) + (\beta+\delta)^2}}{2} - \frac{1}{B_i}\} \tag{24}$$

One important feature of the equilibrium values are the non-negativity of the pre-dispatch and dispatch. This is important, because we neglected the non-negativity constraints in ISOSP in the first place.

Theorem 4.9 *If $(\mathbf{q}^*, \mathbf{x}^*)$ represents the equilibrium of the stochastic settlement market, following equations always hold.*

$$\forall i, s : q_i^* + x_{i,s}^* \geq 0$$

$$\forall i : q_i^* \geq 0$$

Though, the equilibrium pre-dispatch and dispatch are non-negative, one might raise an objection that a game without the non-negativity constraints embedded in the ISO's optimization problem is different with the original game. Therefore, there is no assurance the found equilibrium is also the equilibrium of the original game. The following theorem proves that the obtained equilibrium values are also the equilibrium of the original game with non-negativity constraints. The proof of this theorem is quite lengthy and consists of several lemmas, which can be found in the technical companion [3].

Theorem 4.10 *The equilibrium of the symmetric stochastic settlement game without the non-negativity constraints in ISOSP is also the equilibrium of the stochastic settlement game with the non-negativity constraints.*

5 Comparison of the Two Markets

The key question we asked in the introduction is which model performs better when firms bid strategically. Our criterion for comparing the two models is social welfare. In the environment of our markets, social welfare is defined as

$$\begin{aligned}\mathrm{SW} = &\sum_{s=1}^{S} \theta_s \left(Y_s \left(\sum_{i=1}^{n} y_{i,s} \right) - \frac{Z}{2} \left(\sum_{i=1}^{n} y_{i,s} \right)^2 \right) \\ &- \sum_{s=1}^{S} \theta_s \left(\sum_{i=1}^{n} \left(\alpha y_{i,s} + \frac{\beta}{2} {y_{i,s}}^2 + \frac{\delta}{2} \left(y_{i,s} - q_i \right)^2 \right) \right) \end{aligned} \tag{25}$$

The next proposition establishes that when firms are bidding strategically, the stochastic settlement market dominates the two settlement market provided the ISO chooses the slope of the supply function sufficiently low.

Proposition 5.1 *When the parameter b_i is chosen less than the threshold value of $\hat{b}$, where*

$$\hat{b} = \frac{-Z(n-2) + \beta + \delta + \sqrt{Z^2(n-2)^2 + 2Zn(\beta+\delta) + (\beta+\delta)^2}}{2},$$

social welfare in the stochastic settlement market is higher than that in the two-settlement market.

Proof To prove the proposition, we show when $b_i = \hat{b}$, we can conclude $\mathrm{SW}^{\mathrm{SS}} = \mathrm{SW}^{\mathrm{TS}}$. Then, we demonstrate $\mathrm{SW}^{\mathrm{SS}}$ is a decreasing function of b_i, and therefore, $\mathrm{SW}^{\mathrm{SS}} \geq \mathrm{SW}^{\mathrm{TS}}$, when $b_i \leq \hat{b}$.

When $b_i = \hat{b}$, it is easy to show that equilibrium quantities are identical in the stochastic settlement and two settlement markets. (equations (23), (11), (10))

$$\begin{aligned} B_i^{\mathrm{SS}} &= B_i^{\mathrm{TS}} \\ A_i^{\mathrm{SS}} &= A_i^{\mathrm{TS}} \\ R_i^{\mathrm{SS}} &= B_i^{\mathrm{SS}} \end{aligned}$$

Under this situation we can show that $y_{i,s}$ and q_i formulae (from propositions 3.1, 3.2, and 4.4) simplifies to

$$\begin{aligned} q_i^{\mathrm{SS}} &= q_i^{\mathrm{TS}} = \frac{YB_i - A_i}{1+ZB} \\ y_{i,s}^{\mathrm{SS}} &= y_{i,s}^{\mathrm{TS}} = \frac{Y_s B_i - A_i}{1+ZB} \end{aligned}$$

Therefore social welfare of these models (equation 25) are the same providing $b_i = \hat{b}$. Note that we can rewrite social welfare formula (25) as

$$SW = \sum_{s=1}^{S} \theta_s \left(Y_s \left(\sum_{i=1}^{n} y_{i,s} \right) - \frac{Z}{2} \left(\sum_{i=1}^{n} y_{i,s} \right)^2 - \sum_{i=1}^{n} \left(\alpha y_{i,s} + \frac{\beta}{2} {y_{i,s}}^2 + \frac{\delta}{2} {x_{i,s}}^2 \right) \right).$$

Note that $x_{i,s}$ is independent of b, and therefore,

$$\frac{dW}{db_i} = -\frac{1}{b_i^2} \sum_{i,s} \frac{dW}{dy_{i,s}} \frac{dy_{i,s}}{dB_i}.$$

On the other hand, we show (in the technical companion [3]) that

$$\frac{dy_{i,s}}{dB_i} = \frac{(Y-\alpha)(n-1)Z^2}{(Z + nZ + \beta + (n-1)Z(nZ+\beta)B_i)^{\,2}} \geq 0.$$

Note that according to our assumptions $\forall s$, $\alpha \leq Y_s$. Also, this derivative is a fixed number independent of i and s. Thus,

$$\frac{dW}{db_i} = -\frac{1}{b_i^2} \frac{dy_{i,s}}{dB_i} \sum_{i,s} \frac{dW}{dy_{i,s}}.$$

On the other hand,

$$\frac{dW}{dy_{i,s}} = \theta_s \left(Y_s - \alpha - (Zn + \beta) y_{i,s} \right).$$

Hence,

$$\begin{aligned} \sum_{s} \frac{dW}{dy_{i,s}} &= Y - \alpha - (Zn+\beta)\, q_i \\ &= \frac{bZ(Y-\alpha)}{Z(n-1)(nZ+\beta) + b((n+1)Z+\beta)} \geq 0. \end{aligned}$$

In sum, we can conclude that,

$$\frac{dW}{db_i} \leq 0.$$

Example Consider a market with two symmetric generators as defined in table 1.

Figure 4 shows how the social welfare of the stochastic settlement mechanism is affected by the choice of b. It also shows that for enough small bs, the stochastic settlement mechanism has a higher equilibrium social welfare in comparison with the two settlement mechanism. Thus, for the rest of this example, we assume the ISO chooses $b = 0.001$ to increase the equilibrium social welfare.

Parameter	Value
α, β, δ	$50, 1, 0.5$
Y_1, Y_2, Z	$100, 150, 1$
θ_1, θ_2	$0.5, 0.5$
n	2

Table 1: The market environment for the example

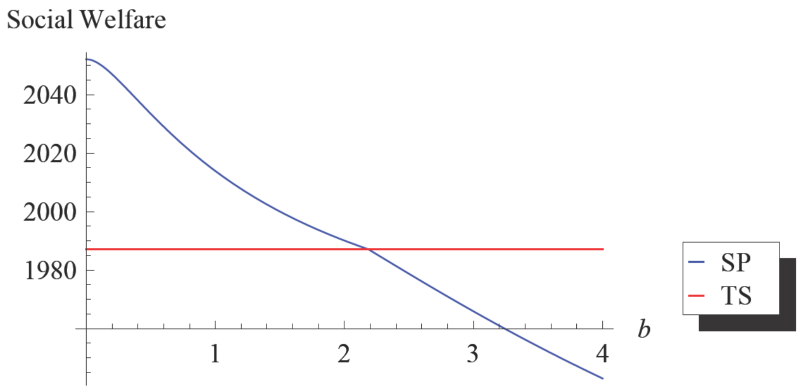

Figure 4: The effect of b on the social welfare of stochastic settlement model and an how it compares to the two settlement mechanism

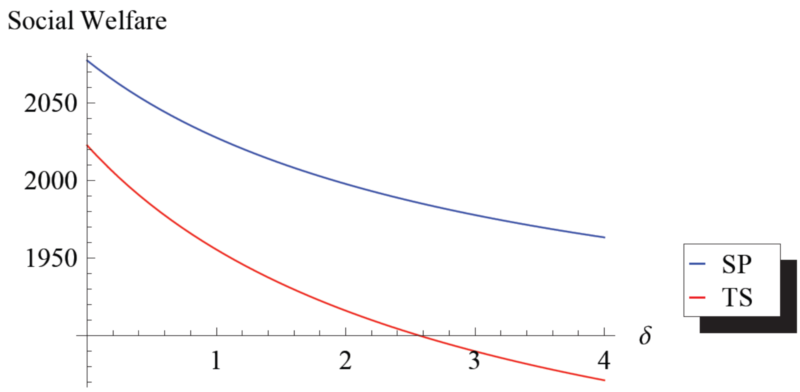

Figure 5: The SP mechanism yields higher social welfare for different δ values

Another interesting experiment is to investigate the effect of δ on these mechanisms.

Figures 5, 6, and 7 compare the stochastic settlement and the two settlement mechanisms for this market, however for different δ values. A first observation is the stochastic settlement mechanism increases social and consumer welfare and decreases producer welfare in comparison

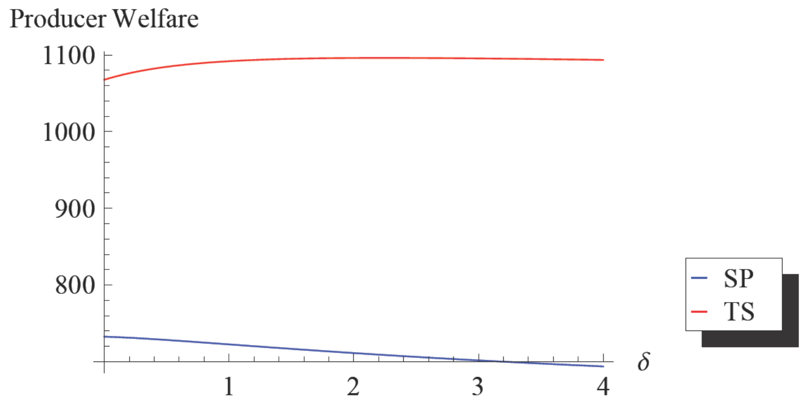

Figure 6: The SP mechanism yields lower producer welfare for a range of examples (i.e. different δ values)

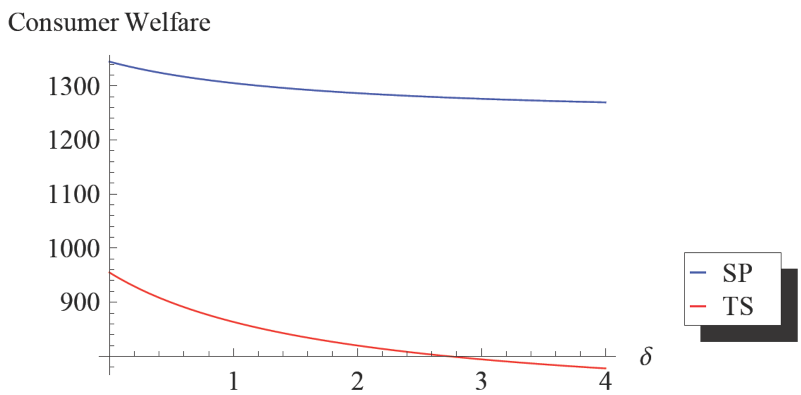

Figure 7: The SP mechanism yields higher consumer welfare for a range of examples (i.e. different δ values

with the two settlement mechanism. Also, these effects are enhanced by increasing δ. This is an expected result that the strength of the stochastic settlement is bolder when cost of deviation is higher.

It is also interesting to investigate the effect of competition on these mechanisms. To do so, we can test the effect of number of firms on these mechanism.

Figure 8 shows the difference in the social welfare of our two mechanism as a function of n. It shows that when the number of generators increase, the performance of the stochastic and two settlement mechanisms converges.

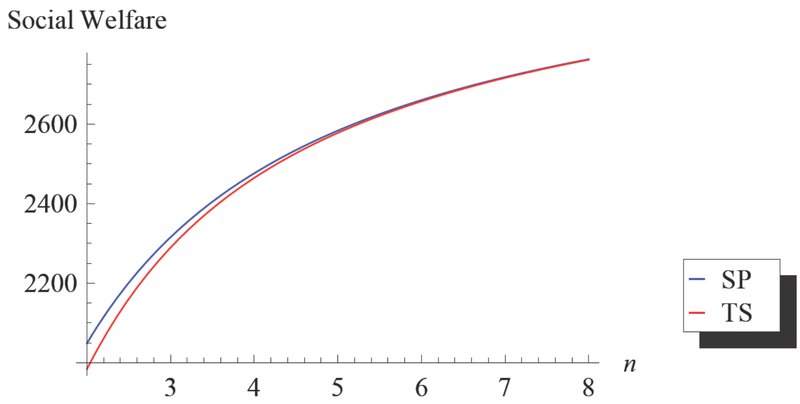

Figure 8: Social welfare of the two settlement mechanism converges that of the stochastic mechanism when n increases. Competition increases with a bigger market.

6 Conclusion

In this paper, we set up a modelling environment in which we were able to capture key elements of a stochastic settlement auction versus a two settlement auction. In particular, we were able to model firms' best responses in these markets, and so find equilibrium behaviour in each. We find that in our model, the stochastic settlement auction dominates the two settlement auction when measuring expected social welfare.

References

[1] F. Bolle. Supply function equilibria and the danger of tacit collusion. the case of spot markets for electricity. *Energy Economics*, 14(2):94–102, 1992.

[2] F. Bouffard, F. D. Galiana, and A. J. Conejo. Market-clearing with stochastic security-part i: formulation. *Power Systems, IEEE Transactions on*, 20(4):18181826, 2005.

[3] J. Khazaei, G. Zakeri, and S. Oren. Technical companion for the paper "single and multi-settlement approaches to market clearing mechanisms under demand uncertainty".

[4] P. Klemperer and M. Meyer. Supply function equilibria in oligopoly under uncertainty. *Econometrica*, 57(6):1243–1277, 1989.

[5] G. Pritchard, G. Zakeri, and A. Philpott. A Single-Settlement, Energy-Only electric power market for unpredictable and intermittent participants. *Operations Research*, Apr. 2010.

[6] S. Wong and J. Fuller. Pricing energy and reserves using stochastic optimization in an alternative electricity market. *Power Systems, IEEE Transactions on*, 22(2):631638, 2007.