On Carbon Charges and Electricity Prices

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1 Introduction

This discussion paper[1] examines the effect on electricity prices of electricity generators increasing their fuel costs from a carbon tax or equivalent carbon charge. By looking at a very simple model of a single-node electricity pool market, we see (not surprisingly) that a carbon tax will increase electricity prices in such a model[2]. The degree of increase predicted by the model, however, depends on the type of equilibrium model that is used to represent participant behaviour. For a Cournot model with linear demand curves we see an increase in prices that is lower than the carbon tax, but with constant elasticity demand curves the increase in price is more than the tax with inelastic demand and less than the tax with elastic demand . In the equilibrium for linear demand curves, it is not guaranteed that all generators who emit CO_2 will reduce their emissions when required to pay a tax. In a simple symmetric supply-function duopoly model with capacities and a price cap under uniform demand, the equilibrium with a CO_2 charge increases average prices by less than the charge, but increases the likelihood of prices hitting the cap.

2 Cournot Model

We first look at prices in a Nash-Cournot equilibrium. In the basic Cournot model, we assume that each generator offers in a fixed quantity g_i, $i = 1, 2, \ldots, n$, which costs $C_i(g_i)$. Suppose we have a demand function whereby the price for demand $G = \sum_j g_j$ is $p(G)$. Each generator i assumes $g_{-i} = \sum_{j \neq i} g_j$ is fixed and optimizes its profit which is

$$P_i = g_i p(G) - C_i(g_i)$$

[1]This paper is a draft only and has not been submitted for peer review. The intention of posting the paper is to stimulate discussion in modelling the effects of carbon charges. The author welcomes any discussion or questions on this paper sent to him at the email address above.

[2]We make no such claim for a model with a transmission system and nodal pricing. Indeed it can be shown that in some circumstances prices in such a system may decrease with the addition of a carbon tax (see Downward, forthcoming).

$$\begin{aligned}
\frac{\partial P_i}{\partial g_i} &= p(G) + g_i \frac{\partial p}{\partial g_i} - C_i'(g_i) = 0 \\
p(G) &= C_i'(g_i) - g_i \frac{\partial p}{\partial g_i}
\end{aligned}$$

This gives a set of n simultaneous equations to be solved to yield the optimal offer x_i for each generator.

2.1 Linear Demand Curve

Suppose the price p is set by the inverse demand curve:

$$p(G) = A - BG$$

Now suppose generator i offers g_i. Suppose the cost of generating this amount is

$$C_i(g) = R_i g$$

Each generator i assumes $g_{-i} = \sum_{j \neq i} g_j$ is fixed and chooses generation g to optimize his profit:

$$\begin{aligned}
P_i(g) &= gp(G) - C_i(g) \\
&= g(A - BG) - R_i g \\
&= Ag - R_i g - gB(g + g_{-i})
\end{aligned}$$

The first order condition is

$$\frac{\partial P_i(g)}{\partial g} = A - R_i - 2Bg - Bg_{-i} = 0$$

which gives (on setting $g = g_i$)

$$\begin{aligned}
2Bg_i + Bg_{-i} &= A - R_i, \quad i = 1, \ldots, n, \\
2Bg_i + B(G - g) &= A - R_i, \quad i = 1, \ldots, n, \\
Bg_i + BG &= A - R_i, \quad i = 1, \ldots, n.
\end{aligned}$$

Summing the equations gives

$$BG + nBG = nA - \sum_{i=1}^{n} R_i$$

$$G = \frac{nA - \sum_{i=1}^{n} R_i}{B(n+1)}.$$

Now

$$\begin{aligned}
Bg_i &= A - BG - R_i \\
&= \frac{A + n\bar{R}}{n+1} - R_i
\end{aligned}$$

so

$$g_i = \frac{A + n\bar{R}}{B(n+1)} - \frac{R_i}{B}.$$

Also

$$\begin{aligned} p(G) &= A - BG = A - \frac{nA - \sum_{i=1}^{n} R_i}{n+1} \\ &= \frac{A + \sum_{i=1}^{n} R_i}{n+1} \\ &= \frac{A}{n+1} + \frac{n\bar{R}}{n+1} \end{aligned}$$

where $\bar{R}$ is the average marginal cost. As $n \to \infty$, we have

$$p(G) \to \bar{R}.$$

Also note that

$$p(G) = \bar{R} + \frac{A - \bar{R}}{n+1},$$

implying that the price markup over average marginal cost is additive and gets smaller as $\bar{R}$ increases. This indicates that increasing some R_i by adding a tax will not give commensurate price increases. We investigate this in the next subsection.

2.2 Linear Demand Curve with Carbon Tax

Suppose for the moment that all thermal generators, $i = 1, 2, \ldots, k$, must pay the same carbon tax on their generation of $\alpha\bar{R}$, and the remaining $n - k$ generators are renewable and have zero carbon tax. Without a tax we have from the previous subsection:

$$p(G) = \frac{A}{n+1} + \frac{n\bar{R}}{n+1}$$

With a tax of $\alpha\bar{R}$ we get

$$\begin{aligned} B(G + g_i) &= A - R_i - \alpha\bar{R}, \quad i = 1, 2, \ldots, k \\ B(G + g_i) &= A - R_i, \quad i = k+1, 2, \ldots, n \end{aligned}$$

$$BG + nBG = nA - \sum_{i=1}^{n} R_i - k\alpha\bar{R}$$

$$G = \frac{nA - \sum_{i=1}^{n} R_i - k\alpha\bar{R}}{B(n+1)}$$

$$\begin{aligned} p(G) &= A - BG = A - \frac{nA - \sum R_i - k\alpha\bar{R}}{(n+1)} \\ &= \frac{A + \sum R_i + k\alpha\bar{R}}{n+1} \\ &= \frac{A}{n+1} + \frac{n\bar{R} + k\alpha\bar{R}}{n+1} \end{aligned}$$

Observe that the increase in price is $\frac{k\alpha\bar{R}}{n+1}$, which is less than $\alpha\bar{R}$, the fixed carbon tax (per MWh) for each thermal agent.

This result is much the same if we extend the model so that different generators incur different carbon taxes. In this case the thermal generators, $i = 1, 2, \ldots, k$, must pay a tax on their generation of $\alpha_i\bar{R}$, and the remaining $n-k$ generators are renewable and have zero tax. Without a tax we get

$$p(G) = \frac{A}{n+1} + \frac{n\bar{R}}{n+1}$$

With a tax of $\alpha_i\bar{R}$ we get

$$\begin{aligned} B(G+g_i) &= A - R_i - \alpha_i\bar{R}, \quad i = 1, 2, \ldots, k \\ B(G+g_i) &= A - R_i, \quad i = k+1, 2, \ldots, n \end{aligned}$$

$$BG + nBG = nA - \sum_{i=1}^{n} R_i - \sum_{i=1}^{k} \alpha_i\bar{R}$$

$$G = \frac{nA - \sum_{i=1}^{n} R_i - \sum_{i=1}^{k} \alpha_i\bar{R}}{B(n+1)}$$

$$\begin{aligned} p(G) &= A - BG = A - \frac{nA - \sum_{i=1}^{n} R_i - \sum_{i=1}^{k} \alpha_i\bar{R}}{(n+1)} \\ &= \frac{A + \sum_{i=1}^{n} R_i + \sum_{i=1}^{k} \alpha_i\bar{R}}{n+1} \\ &= \frac{A}{n+1} + \frac{n\bar{R} + \sum_{i=1}^{k} \alpha_i\bar{R}}{n+1} \end{aligned}$$

Observe that the change in price is $\frac{\sum_{i=1}^{k} \alpha_i\bar{R}}{n+1}$. Thus the price has gone up by less than $\frac{\sum_{i=1}^{k} \alpha_i\bar{R}}{k}$, which is the average carbon tax (per MWh) for each thermal agent. For example is $n = 2$ and $k = 1$, we have an increase in price of $\frac{\alpha\bar{R}}{3}$, or one third of the carbon charge incurred by the emitting generator.

Recall that the equilibrium generation with no tax is

$$g_i = \frac{A + n\bar{R}}{B(n+1)} - \frac{R_i}{B}.$$

With a carbon tax we get the generation level of carbon emitters to be

$$Bg_i = A - BG - R_i - \alpha_i\bar{R}$$

so

$$g_i = \frac{A}{B(n+1)} + \frac{n\bar{R} + \sum_{i=1}^{k} \alpha_i\bar{R}}{B(n+1)} - \frac{R_i}{B} - \frac{\alpha_i\bar{R}}{B}.$$

The reduction in generation level of generator i is

$$\begin{aligned} & \frac{A + n\bar{R}}{B(n+1)} - \frac{R_i}{B} - \left(\frac{A}{B(n+1)} + \frac{n\bar{R} + \sum_{i=1}^{k} \alpha_i\bar{R}}{B(n+1)} - \frac{R_i}{B} - \frac{\alpha_i\bar{R}}{B}\right) \\ = & \frac{\alpha_i\bar{R}}{B} - \frac{\sum_{i=1}^{k} \alpha_i\bar{R}}{B(n+1)} \\ = & \frac{(n+1)\alpha_i\bar{R} - \sum_{i=1}^{k} \alpha_i\bar{R}}{B(n+1)} \end{aligned}$$

and the total level of reduction of taxed generation is

$$\sum_{i=1}^{k} \frac{(n+1)\alpha_i \bar{R} - \sum_{i=1}^{k} \alpha_i \bar{R}}{B(n+1)} = \frac{(n+1)\sum_{i=1}^{k} \alpha_i \bar{R} - k\sum_{i=1}^{k} \alpha_i \bar{R}}{B(n+1)}$$
$$= \frac{(n+1-k)\sum_{i=1}^{k} \alpha_i \bar{R}}{B(n+1)}.$$

Observe that this is linear in α_i so that under this model we might expect to achieve any level of reduction in CO_2 by applying a large enough α_i to some generators. If this is chosen to be too high we will have $g_i < 0$, corresponding to a generator becoming a net buyer of electricity. These purchases do not give any direct reduction in CO_2, and so the model is not valid in this range.

A second remark worth making is that for small α_i we may get

$$\frac{(n+1)\alpha_i \bar{R} - \sum_{i=1}^{k} \alpha_i \bar{R}}{B(n+1)} < 0,$$

implying that in equilibrium some electricity generators might *increase* their emissions under a carbon tax.

Finally, it is important to realize that these results apply to simple one-node electricity systems. When a transmission network is present it is well known that Nash-Cournot equilibria might fail to exist under an assumption of full rationality of the players. Moreover, it is possible that in a transmission system, we might find paradoxical examples whereby a tax on CO_2 has the effect of decreasing electricity prices and *increasing* overall emissions. These paradoxes are investigated by Downward in a forthcoming paper.

2.3 Constant elasticity demand curve

For a linear demand curve we observe that in equilibrium, electricity prices decrease by less than the average carbon tax imposed on each thermal agent. We now revisit this result for nonlinear demand curves. Consider the inverse demand curve where γ measures the (constant) price elasticity of demand:

$$p(G) = p_0 \left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}}, \quad \gamma > 0.$$

Inverse demand curves for decreasing elasticity are plotted in Figure 1.

Suppose

$$C_i(g_i) = R_i g_i.$$

This gives

$$p(G) = C_i'(g_i) - g_i \frac{\partial p}{\partial g_i}.$$

Since

$$\frac{\partial p}{\partial g_i} = -p_0 G^{-\frac{1+\gamma}{\gamma}} \frac{G_0^{\frac{1}{\gamma}}}{\gamma}$$

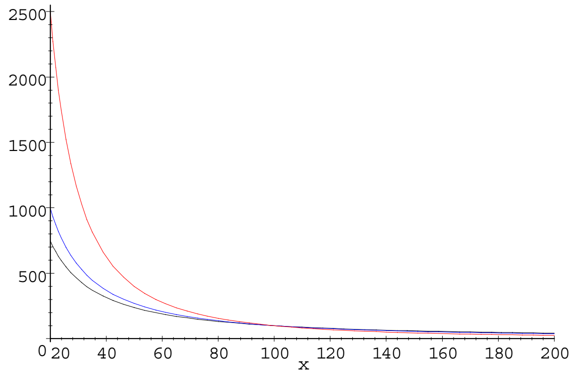

Figure 1: Inverse demand for varying elasticity

we get

$$p_0 \left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}} = R_i + g_i p_0 G^{-\frac{1+\gamma}{\gamma}} \frac{G_0^{\frac{1}{\gamma}}}{\gamma}, \quad i = 1, 2, \ldots, n.$$

a set of simultaneous equations to be solved.

Adding these equations gives

$$\begin{aligned}
np_0 \left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}} &= n\bar{R} + Gp_0 G^{-\frac{1+\gamma}{\gamma}} \frac{G_0^{\frac{1}{\gamma}}}{\gamma} \\
np_0 \left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}} &= n\bar{R} + p_0 G^{-\frac{1}{\gamma}} \frac{G_0^{\frac{1}{\gamma}}}{\gamma} \\
p_0 \left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}} &= \bar{R} + \frac{p_0}{n\gamma} \left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}} \\
p_0 (1 - \frac{1}{n\gamma}) \left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}} &= \bar{R} \\
\left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}} &= \frac{\bar{R}}{p_0(1 - \frac{1}{n\gamma})}
\end{aligned}$$

Thus the clearing price p satisfies

$$\begin{aligned}
p(G) &= p_0 \left(\frac{G}{G_0}\right)^{-\frac{1}{\gamma}} \\
&= \frac{\bar{R}}{(1 - \frac{1}{n\gamma})}
\end{aligned}$$

Now consider a a carbon charge of $\alpha_i \bar{R}$ on agents $i = 1, 2, \ldots, k$. This increases $\bar{R}$ by the fixed constant $\frac{\sum_{i=1}^k \alpha_i \bar{R}}{n}$. The price then increases by

$$\Delta p(G) = \frac{\sum_{i=1}^k \alpha_i \bar{R}}{n(1 - \frac{1}{n\gamma})}$$

$$= \frac{\sum_{i=1}^{k} \alpha_i \bar{R}}{(n - \frac{1}{\gamma})}.$$

The average carbon charge paid by the emitting plant is $\frac{\sum_{i=1}^{k} \alpha_i \bar{R}}{k}$, and so the change in price exceeds this if $(n - \frac{1}{\gamma}) < k$ or

$$\gamma < \frac{1}{n-k}.$$

This means that the markup in price depends on the price elasticity of demand. For inelastic demand (small γ) this model predicts high markups from carbon charges.

3 Supply-Function Equilibrium Model

The results in the previous section show that the effect of a carbon tax on electricity prices depends on the elasticity of demand. It is generally accepted, at least in the short term, that electricity demand is very inelastic to price. This means that high markups might result from carbon taxes under a Cournot model. An alternative approach seeks an equilibrium in supply functions. We illustrate this with a simple duopoly model.

Consider two agents $i = 1, 2$, who offer electricity to a pool market. Suppose that their marginal costs are $R_1 = R_2$. Suppose that each agent has capacity K, and that the market has a price cap P. Let demand have a uniform distribution on $[0, 2K]$. Then a symmetric supply-function equilibrium is given by the offer curves

$$P_i(q) = R_i + \frac{P - R_i}{K} q.$$

A plot of the supply function equilibrium is shown in Figure 2 for $K = 5$ and $P = 4$. The price is distributed uniformly between 1 and 4, with average 2.5.

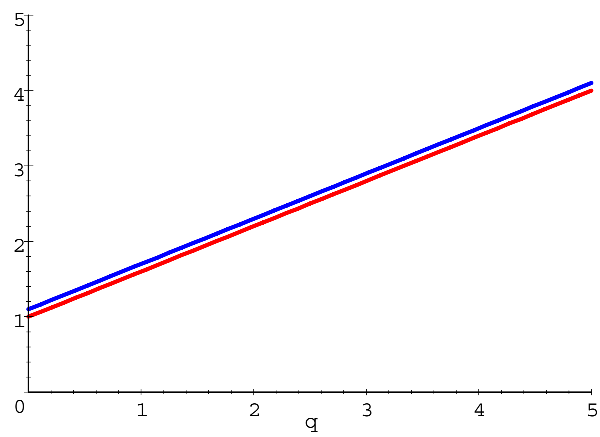

Figure 2: Plot of supply function equilibrium for $R_1 = R_2 = 1$.

Now consider the case where $R_1 < R_2$. This would occur if both generators had marginal cost R_1, but now generator 2 incurs a carbon charge of $R_2 - R_1$. The supply function equilibrium has the following form:

$$P_1(q) = \begin{cases} R_2 & 0 \leq q \leq \frac{R_2-R_1}{P-R_1}K \\ R_1 + \frac{P-R_1}{K}q & \frac{R_2-R_1}{P-R_1}K \leq q \leq K \end{cases}$$

$$P_2(q) = \begin{cases} R_2 + \frac{P-R_1}{K}q & 0 \leq q \leq \frac{P-R_2}{P-R_1}K \\ P & \frac{P-R_2}{P-R_1}K \leq q \leq K \end{cases}$$

This equilibrium is shown in Figure 3 for $K = 5$ and $P = 4$. Here the blue curve is player 1 and the red curve is player 2.

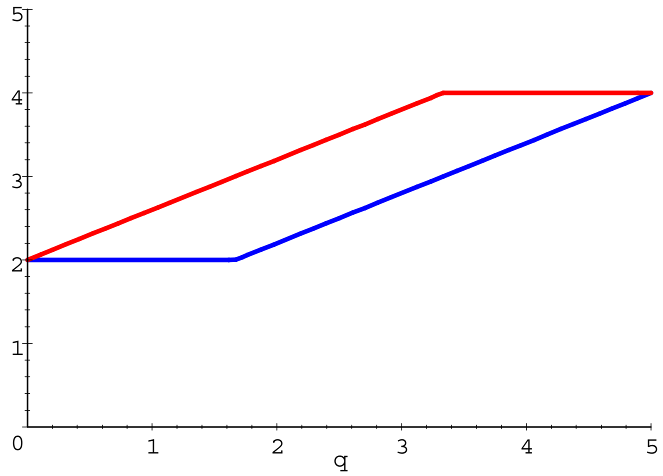

Figure 3: Plot of supply function equilibrium for $R_1 = 1$ and $R_2 = 2$.

An interesting feature of this equilibrium is that player 1 bids their price up to the lowest marginal cost of the competitor for low demand outcomes (in effect acting as if they were subject to the carbon tax as well). The player who is subject to the tax bids at the price cap at the top of their stack.

The industry stack is

$$P_1(q) = \begin{cases} R_2 & 0 \leq q \leq \frac{R_2-R_1}{P-R_1}K \\ R_2 + \frac{(P-R_2)}{(K+\frac{P-R_2}{P-R_1}K-\frac{R_2-R_1}{P-R_1}K)}(q - \frac{R_2-R_1}{P-R_1}K) & \frac{R_2-R_1}{P-R_1}K < q \leq K + \frac{P-R_2}{P-R_1}K \\ P & K + \frac{P-R_2}{P-R_1}K < 2K \end{cases}$$

as shown in Figure 4.

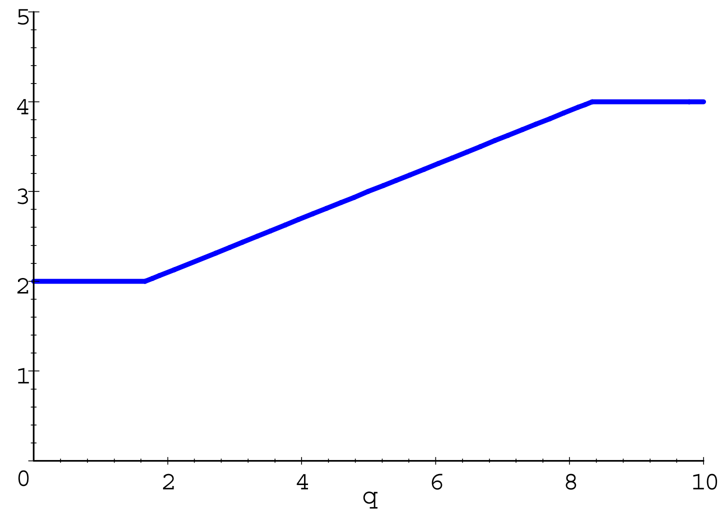

Figure 4: Industry stack for asymmetric supply function equilibrium

This gives the distribution of electricity prices shown in Figure 5.

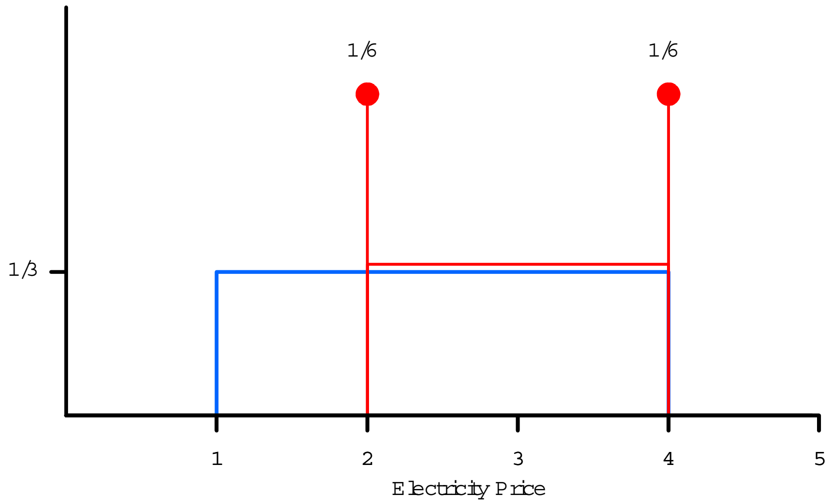

Figure 5: Distribution of electricity price before (blue) and after (red) carbon charge

It is easy to see from this that that the price is distributed between 2 and 4, with average price 3. The price has increased by 0.5 which is less than the carbon charge. There is now a probability of $\frac{1}{6}$ that the price hits the cap, and the price equals (its lowest value of) 2 with same probability.

4 Conclusions

It is not clear what conclusions can be drawn from these simple examples, except possibly that care should be taken in any game-theoretical analysis of this issue. The results obtained will depend critically on the assumptions made about the particular oligopoly model. It is well known that supply-function equilibrium models predict more competitive behaviour than Cournot models with the same demand curves, and so one might expect less price-impact from carbon charges in the former model. Even so, it is surprising that this impact is not more dramatic for the inelastic demand case that we have presented.