On setting penalty parameters in electricity optimal dispatch software

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Abstract

We discuss the effects of setting penalty costs on artificial variables in electricity dispatch software. It is shown under a feasibility assumption that a choice of these can be made to give no shortfalls in grid security and energy, but a possible shortfall in spinning reserve.

1 Introduction

This note is concerned with the effects of choosing penalty costs on artificial variables in linear programs. It is well known in the folklore of linear programming that if there is a feasible solution to a linear program then one may set the costs of artificial variables using the "Big-M" method so as to ensure that no artificial variables are positive at the optimal solution. The question we wish to address in this paper is whether one can choose different M values for different artificial variables to guarantee that certain artificial variables are 0 when the linear program does not have a feasible solution.

The motivation for considering this question comes from the use of linear programming models to compute a security-constrained electricity dispatch in nodal electricity pools (see [1]). In these models there are different artificial variables representing energy shortfalls at the nodes, shortfalls in various grid-security constraints, and a spinning reserve shortfall. In situations in which these dispatch problems have no feasible solution, at least one of these artificial variables will be strictly positive at optimality. We show that as long as the system has sufficient energy to meet demand and grid security constraints then there is a choice of M values that ensures that only spinning reserve shortfall variables are positive at optimality.

2 Security-constrained dispatch

The dispatch model we consider is a linear program based on a DC-load flow model. This has the following form.

$$
\begin{array}{lll}
\text{P: minimize} & c^\top x + b^\top y + M_1^\top v + M_2 w + M_3^\top s & \\
& Ax + Bu + v = D, & \\
& \sum_{k=1}^{m_2} y_k + w = \beta z, & \\
& z \geq \sum_{j\in J(g)} x_j, & g = 1,2,\ldots,n, \\
& \sum_{j\in J(g)} x_j + \sum_{k\in K(g)} y_k \leq L_g, & g = 1,2,\ldots,n, \\
& Hu - s \leq f, & \\
& 0 \leq x_j \leq Q_j, & j = 1,2,\ldots,m_1, \\
& 0 \leq y_k \leq R_k, & k = 1,2,\ldots,m_2, \\
& u \in \mathcal{U},\ v \geq 0,\ w \geq 0, s \geq 0. &
\end{array}
$$

Here there are n generators who together make offers of tranches of energy Q_j at price c_j, $j = 1,2,\ldots,m_1$, and offers of reserve R_k at price b_k, $k = 1,2,\ldots,m_2$. Observe that a generator in this context is simply any entity that offers a stack, and does not necessarily reflect any particular ownership or physical constraints. For example a station of four large units that offers by unit is considered to be four generators in our model. Each offer j and k is associated with a generator g using the sets $J(g)$ and $K(g)$ that respectively give the offers of reserve and energy associated with each generator. Each generator g has a capacity L_g.

The amount of energy dispatched from offer j is represented by x_j and the amount of reserve dispatched from offer k is denoted y_k. The energy requirement is given by the vector D, and the first set of constraints define an energy flow balance at each node. Here u defines a vector of link flows that are constrained to lie in the convex set $\mathcal{U}$, representing, for example, capacity constraints and loop-flow conditions imposed by the DC-load flow assumption. The linear inequalities $Hu - s \leq f$ (where H is a fixed matrix and f is a given vector) represent grid security constraints that are added to ensure that the dispatch is robust in case of line outages. We represent any shortfalls in these constraints by s, which is penalized in the objective function by M_3.

The matrices A and B aggregate dispatch and link flows into a net flow into each node. We represent the shortfall of energy at each node by the vector v. The matrix B can be constructed to account for linear or piecewise linear line losses. In the latter case it is well known that the dispatch problem is not convex (see e.g. [2]). In most practical circumstances the linear program delivers a suitable dispatch, but should nodal prices become negative then the linear program can produce dispatch solutions that are physically impossible, and some form of enumeration is required to deliver the globally optimal solution to what is a non-convex optimization problem. Nevertheless we shall assume throughout this note that P does deliver the optimal dispatch, and leave the analysis of the special cases when it does not to a subsequent paper.

The spinning reserve requirement in this model is defined by the term βz, where β is a nonnegative adjustment factor that accounts for the availability of free reserve on the system, and z is the maximum risk, defined here to be the largest amount of energy dispatched to any generator. We represent the shortfall of reserve by the single variable w. (In practice there may be different types of spinning reserve but for simplicity we restrict attention to only one.)

If P has no feasible solution with $v = 0$ and $s = 0$ then there is no way of meeting the energy and security requirements without allowing some shortfall in energy or grid security, so we turn our attention to the situation in which P has a feasible solution in which $v = 0$ and $s = 0$.

Proposition 1 *Suppose that P has a feasible solution with $v = 0$ and $s = 0$. Then there are values of M_1, M_2 and M_3 that will give $v = 0$ and $s = 0$ in an optimal solution.*

Proof. Consider the finite set of all basic feasible solutions to P for which $v \neq 0$. Over all such solutions let

$$\delta_v = \min\{v_i | v_i > 0\}.$$

Similarly define

$$\delta_s = \min\{s_i | s_i > 0\},$$

where the minimum is taken over all basic feasible solutions to P for which $s \neq 0$. (If either of these sets of solutions are empty then any choice of M_1, M_2 and M_3 will give $v = 0$ or $s = 0$ respectively.)

Now given any value of M_2, define the vector M_1 so that each component $e_i^\top M_1$ satisfies

$$e_i^\top M_1 > \frac{(M_2\beta + m_1 \max\{c_j\} + m_2 \max\{b_k\}) \max\{L_g\}}{\delta_v}$$

and let the vector M_3 be defined so that each component $e_i^\top M_3$ satisfies

$$e_i^\top M_3 > \frac{(M_2\beta + m_1 \max\{c_j\} + m_2 \max\{b_k\}) \max\{L_g\}}{\delta_s}$$

Suppose with this choice of M_1, M_2 and M_3 that P has an optimal solution (x, y, z, v, w, s) with $v_i > 0$ for some i. Then $v_i \geq \delta_v$. We know that P has a feasible solution $(\bar{x}, \bar{y}, \bar{z}, 0, \bar{w}, 0)$. The objective value of this is

$$\begin{aligned} c^\top \bar{x} + b^\top \bar{y} + M_2\bar{w} &\leq \max\{L_g\}(m_1 \max\{c_j\} + m_2 \max\{b_k\}) + M_2\bar{w} \\ &\leq \max\{L_g\}(m_1 \max\{c_j\} + m_2 \max\{b_k\}) + M_2\beta\bar{z} \\ &\leq \max\{L_g\}(M_2\beta + m_1 \max\{c_j\} + m_2 \max\{b_k\}) \\ &< e_i^\top M_1 \delta_v \\ &\leq e_i^\top M_1 v_i \\ &\leq c^\top x + b^\top y + M_1^\top v + M_2 w + M_3 s, \end{aligned}$$

which contradicts the optimality of (x, y, z, v, w, s). A similar argument shows that assuming that P has an optimal solution (x, y, z, v, w, s) with $s_i > 0$ for some i yields a contradiction. So the above choices of M_1, M_2 and M_3 guarantee that $v = 0$ and $s = 0$ in an optimal solution. ■

The proposition above demonstrates the existence of M_1, M_2 and M_3 that guarantee that $v = 0$ and $s = 0$ in an optimal solution to P. To use these penalties in practice we require values for δ_v and δ_s, which are difficult to compute. In fact it is easy to see that the values M_1, M_2 and M_3 in the proposition are larger than actually required. In cases where degeneracy is not an issue it is possible to compute the minimum values required to ensure that $v = 0$ and $s = 0$ in an optimal solution to any given instance by fixing M_2 and solving P with sufficiently large values of M_1 and M_3 to give $v = 0$ and $s = 0$ in an optimal solution. Since these zero variables are non basic at optimality (assuming non degeneracy) then the dual variables (say, π and ρ respectively) on the energy and security constraints give infimal values for M_1 and M_3 that should be chosen to ensure $v = 0$ and $s = 0$ in an optimal solution. (This is because their reduced costs $M_1 - \pi$ and $M_3 - \rho$ will remain positive.) Thus as long as each component of M_1 is larger than corresponding components of π and each component of M_3 is larger than corresponding components of ρ, we will have $v = 0$ and $s = 0$ in an optimal solution to P.

3 The single-node case

In this section we examine a simpler linear programming model for which an explicit expression for M_1 and M_2 can be derived. We adopt the same notation, but now assume that all offers and load are located at a single node. This removes the need for the variables u and the grid security constraints.

$$
\begin{array}{lll}
\text{Q: minimize} & c^\top x + b^\top y + M_1 v + M_2 w & \\
& \sum_g \sum_{j\in J(g)} x_j + v = D, & \\
& \sum_{k=1}^{m_2} y_k + w = \beta z, & \\
& z \ge \sum_{j\in J(g)} x_j, & g = 1,2,\ldots,n, \\
& \sum_{j\in J(g)} x_j + \sum_{k\in K(g)} y_k \le L_g, & g = 1,2,\ldots,n, \\
& 0 \le x_j \le Q_j, & j = 1,2,\ldots,m_1, \\
& 0 \le y_k \le R_k, & k = 1,2,\ldots,m_2, \\
& v \ge 0,\ w \ge 0. &
\end{array}
$$

We assume throughout this section that $c \ge 0$ and $b \ge 0$.

First observe that when $\sum_g L_g < D$ every feasible solution to P will have

$$\sum_g \sum_{j\in J(g)} x_j \le \sum_g L_g < D$$

and so $v > 0$ in every feasible solution to P, no matter how large we make M_1. So $\sum_g L_g \ge D$ is a necessary condition for P to have a feasible solution with $v = 0$. It is easy to see (by choosing $y = 0$) that it is also sufficient. So we shall henceforth assume that $\sum_g L_g \ge D$, i.e. the total capacity of the market is big enough to meet the demand for energy.

We first show that as long as M_1 and M_2 are chosen large enough then we can assume that the solutions we will work with have $\sum_{j\in J(g)} x_j + \sum_{k\in K(g)} y_k = L_g$ for every g.

Proposition 2 *There exists some constant M such that if $M_1, M_2 > M$ then any optimal solution to Q with $\sum_{j\in J(g)} x_j + \sum_{k\in K(g)} y_k < L_g$ has $v = 0$, $w = 0$.*

Proof. Suppose (x, y, z, v, w) is an optimal solution to Q with $v > 0$, and $\sum_{j\in J(g)} x_j + \sum_{k\in K(g)} y_k < L_g$ for some g. Choose

$$\delta = \frac{L_g - \sum_{j\in J(g)} x_j - \sum_{k\in K(g)} y_k}{1+\beta}$$

and let j be any element of $J(g)$ and k be any element of $K(g)$. Then there exists a feasible solution

$$(x + \delta e_j, y + \beta\delta e_k, z + \delta, v - \delta, w)$$

for Q with objective function value

$$c^\top x + c_j\delta + b^\top y + \beta\delta b_k + M_1 v - M_1\delta + M_2 w.$$

The increase in objective is $c_j\delta + \beta\delta b_k - M_1\delta$ which is strictly negative as long as $M_1 > \max_j\{c_j\} + \beta\max\{b_k\}$.

Similarly if $w > 0$, and $\sum_{j\in J(g)} x_j + \sum_{k\in K(g)} y_k < L_g$ for some g, then choose

$$\delta = L_g - \sum_{j\in J(g)} x_j - \sum_{k\in K(g)} y_k$$

and let k be any element of $K(g)$. Then there exists a feasible solution

$$(x, y + \delta e_k, z, v, w - \delta)$$

for Q with objective function value

$$c^\top x + b^\top y + b_k\delta + M_1 v + M_2 w - M_2\delta.$$

The increase in objective is $b_k\delta - M_2\delta$ which is strictly negative as long as $M_2 > \max_k\{b_k\}$.

Thus if we choose $M = \max_j\{c_j\} + (1+\beta)\max\{b_k\}$ then either $v > 0$ or $w > 0$ yields a contradiction. ■

We shall assume from now on that M_1 and M_2 are chosen sufficiently large so that any optimal solution to Q with positive artificial variables has $\sum_{j\in J(g)} x_j + \sum_{k\in K(g)} y_k = L_g$ for every generator g. This makes sense from the interpretation of M_1 and M_2 as penalty costs on constraint violation.

Proposition 3 *Any optimal solution to Q with $y = 0$ has $v = 0$.*

Proof. If an optimal solution has $y = 0$, then $\sum_{j\in J(g)} x_j = L_g$ for every g. Thus $\sum_g \sum_{j\in J(g)} x_j = \sum_g L_g \geq D$, and so $v = 0$. ■

Proposition 4 *There exists some constant M such that if $M_1-(1+\beta)M_2 > M$ then any optimal solution to Q with $y \neq 0$ has $v = 0$.*

Proof. Suppose (x, y, z, v, w) is an optimal solution to Q. Suppose for some k that $y_k > 0$ and $v > 0$, and choose $\delta = \min\{y_k, v\}$. Now choose some j in $J(g)$ where g is such that $k \in K(g)$. Then there exists a feasible solution

$$(x + \delta e_j, y - \delta e_k, z + \delta, v - \delta, w + (1+\beta)\delta)$$

for Q with objective function value

$$c^\top x + c_j\delta + b^\top y - b_k\delta + M_1 v - M_1\delta + M_2 w + M_2(1+\beta)\delta.$$

The increase in objective is

$$\begin{aligned} & c_j\delta - b_k\delta - M_1\delta + M_2(1+\beta)\delta \\ = \quad & -(b_k - c_j + M_1 - M_2(1+\beta))\delta \end{aligned}$$

Now let $M > \max_j\{c_i\}$. Thus

$$\begin{aligned} M_1 - M_2(1+\beta) &> M \\ &\geq c_j - b_k \end{aligned}$$

and so the increase in objective is negative, contradicting the optimality of (x, y, z, v, w). ■

It follows that if we choose $M_2 = \max_j\{c_j\} + 1 + (1+\beta)\max\{b_k\} + 1$ and $M_1 = (1+\beta)M_2 + \max_j\{c_j\} + 1$ as penalty parameters on w and v then any optimal solution to Q will have $v = 0$, as long as there is enough offered energy to meet demand.

References

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[2] A.B. Philpott and G. Pritchard, Financial transmission rights in convex pool markets, *Oper. Res. Letters*, 32, 2, (2004) 109-113.