Price discovery can be inefficient

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Abstract

New Zealand has an energy-only wholesale electricity market that operates in real time. Before real time, the system operator publishes provisional prices from a sequence of auctions that use forecast demand. Electricity generators respond to these provisional prices by updating their offers. We show how this process can lead to inefficient dispatch and electricity prices.

1 Introduction

New Zealand has an energy-only wholesale electricity market that operates in real time. Before real time, the system operator publishes provisional prices from a sequence of auctions that use forecast demand. Electricity generators respond to these provisional prices by updating their offers. We show how this process can lead to inefficient dispatch and electricity prices.

We begin by presenting a stylized two-period model with two competing generators to show how an inflated forecast of demand might lead to an inefficient dispatch even if this forecast is corrected. We then discuss a setting in which a sequence of forecasts are made that converge to the final demand. This gives a sequence of provisional dispatches and prices, called a *pre-dispatch*, that models what happens in the New Zealand wholesale market. We prove a theorem that shows under mild assumptions that this process can lead to inefficient dispatch and prices. The final section of the paper looks at an example of a particular day, May 5, 2020.

2 A two-period model

Consider a simple example in which a competitive generator (1) with strictly increasing marginal cost function $c_1(u_1)$ competes with a generator (2) with capacity b and marginal cost $c_2 < c_1(0)$. If demand is $d > b$ then the dispatch for generator 1 and 2 respectively is $u_1 = d - b$, $u_2 = b$, the dispatch cost is

$$C = c_2 b + \int_0^{d-b} c_1(u)du,$$

and the clearing price is $\pi(d) = c_1(d - b)$. If generator 2 increases their offer price to any price $p < \pi(d)$ then the optimal dispatch and clearing price are the same. This could be interpreted as price-taking behaviour, i.e., generator 2 is not seeking to influence the price by their offer.

Now suppose that the actual load d is replaced by a forecast $\hat{d}$, and a provisional auction run with this forecast. If $\hat{d} > d$ then the provisional clearing price is $\pi(\hat{d}) = c_1(\hat{d} - b) > \pi(d)$. Now if generator 2 increases their offer price to $p = \pi(\hat{d})$ then the optimal dispatch and clearing price are the same as long as demand is the same as forecast. However if we dispatch plant to meet the actual load d then we obtain

$$\begin{aligned} \hat{u}_1 &= c_1^{-1}(\pi(\hat{d})) = \hat{d} - b > d - b, \\ \hat{u}_2 &= d - \hat{u}_1 < b, \end{aligned}$$

and the clearing price is $\pi(\hat{d})$. This is an inefficient dispatch since $c_2 < c_1(0) \leq c_1(u)$, $u \geq 0$ implies

$$c_2(\hat{d} - d) = \int_{d-b}^{\hat{d}-b} c_2 du < \int_{d-b}^{\hat{d}-b} c_1(u)du,$$

so the cost of $(\hat{u}_1, \hat{u}_2)$ is

$$c_2(b + d - \hat{d}) + \int_0^{\hat{d}-b} c_1(u)du > c_2 b + \int_0^{d-b} c_1(u)du = C.$$

Observe that the price markup $\pi(\hat{d}) - \pi(d)$ persists, even if the forecast $\hat{d} > d$ is corrected in a later period to a correct forecast. Moreover the dispatch is less efficient than the perfectly competitive dispatch. This simple example shows that inaccurate demand forecasts can lead to price markups when the uncertainty in demand is resolved.

3 Predispatch auctions

In this section we are interested in more general circumstances in which inaccurate predispatch demand forecasts lead to inefficient outcomes. Consider a setting where the same two generators as in the previous section compete, but a sequence of auctions is run for a given trading period for increasingly accurate demand forecasts $\hat{d}$. Suppose the ISO solves a sequence of dispatch problems $P(k)$, $k = 1, 2, \ldots$, with forecast demand d_k yielding clearing price $\pi(d_k)$. For convenience, we make the following assumption about forecasts.

Assumption 1 *Demand forecasts satisfy $d_k > b$ and are accurate enough so that $|d_k - d_l| < b$ for every $k, l > 0$.*

We require the following result.

Lemma 2 *Suppose generator 2 offers b at price p and demand is $d > b$. Then the clearing price $\pi(d, p)$ is continuous and nondecreasing in d and p.*

Proof. Suppose d is fixed. Then

$$\pi(d,p) = \begin{cases} c_1(d-b), & p < c_1(d-b), \\ p, & c_1(d-b) \le p < c_1(d), \\ c_1(d), & p \ge c_1(d), \end{cases}$$

which is continuous and nondecreasing in p.

Suppose p is fixed. Then

$$\pi(d,p) = \begin{cases} c_1(d), & d \le c_1^{-1}(p), \\ p, & c_1^{-1}(p) < d \le c_1^{-1}(p) + b, \\ c_1(d-b), & d > c_1^{-1}(p) + b, \end{cases}$$

which is continuous and nondecreasing in d. ■

We now prove the main result of this paper.

Proposition 3 *Suppose the ISO solves a sequence of dispatch problems $P(k)$, $k = 1, 2, \ldots$, with forecast demand $d_k \to d$, each yielding clearing price $\pi(d_k)$. Assume the forecasts satisfy Assumption 1. If for $k = 0, 1, 2, \ldots$, generator 2 offers b at price $p = \pi(d_k)$ to problem $P(k+1)$, where $\pi(d_0) = c_2$, then $\pi(d_k) \to \pi(d)$ if and only if $d_k \le d$ for every k.*

Proof. We show that $\pi(d_k) = \max_{l \le k}\{c_1(d_l - b)\}$, so $\pi(d_k)$ is a nondecreasing sequence. This shows that $\pi(d_k)$ does not converge to $\pi(d)$ if any demand forecasts $d_k > d$. Conversely $\max_{l \le k}\{c_1(d_l - b)\}$ is easily shown to converge to $c_1(d - b)$ if $d_k \uparrow d$.

Suppose as an induction hypothesis that for some k, $\pi(d_k) = \max_{l \le k}\{c_1(d_l - b)\}$. This is true for $k = 1$, since

$$\pi(d_1) = c_1(d_1 - b) > c_2 = \pi(d_0).$$

By Lemma 1, we have that the clearing price resulting from problem $P(k+1)$ is

$$\begin{aligned} \pi(d_{k+1}) &= \begin{cases} c_1(d_{k+1} - b), & \pi(d_k) < c_1(d_{k+1} - b), \\ \pi(d_k), & c_1(d_{k+1} - b) \le \pi(d_k) < c_1(d_{k+1}), \\ c_1(d_{k+1}), & \pi(d_k) \ge c_1(d_{k+1}), \end{cases} \\ &= \begin{cases} c_1(d_{k+1} - b), & \max_{l \le k}\{c_1(d_l - b)\} < c_1(d_{k+1} - b), \\ \max_{l \le k}\{c_1(d_l - b)\}, & c_1(d_{k+1} - b) \le \max_{l \le k}\{c_1(d_l - b)\} < c_1(d_{k+1}), \\ c_1(d_{k+1}), & \max_{l \le k}\{c_1(d_l - b)\} \ge c_1(d_{k+1}). \end{cases} \end{aligned}$$

Now $\max_{l \leq k}\{c_1(d_l - b)\} \geq c_1(d_{k+1})$ implies that $c_1(d_l - b) \geq c_1(d_{k+1})$ for some $l \leq k$, so $d_l - b \geq d_{k+1}$ contradicting Assumption 2. Thus the third range for $\pi(d_{k+1})$ cannot occur and

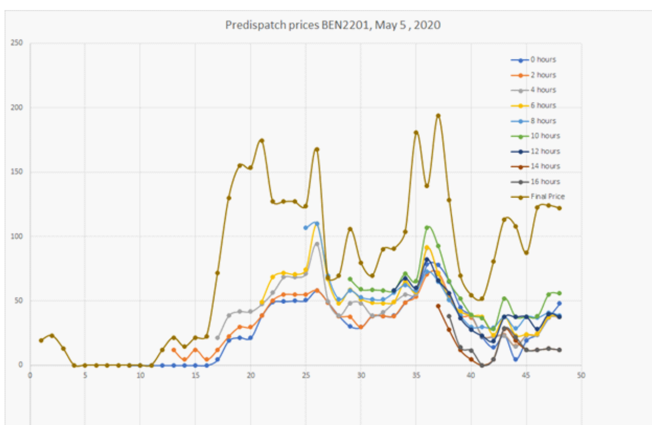
$$\begin{aligned} \pi(d_{k+1}) &= \begin{cases} c_1(d_{k+1} - b), & \max_{l \leq k}\{c_1(d_l - b)\} < c_1(d_{k+1} - b), \\ \max_{l \leq k}\{c_1(d_l - b)\}, & c_1(d_{k+1} - b) \leq \max_{l \leq k}\{c_1(d_l - b)\}, \end{cases} \\ &= \max_{l \leq k+1}\{c_1(d_l - b)\}, \end{aligned}$$

which yields the result. ∎

This result shows that a sequence of solutions to dispatch auctions in which agents seek to discover the price might lead to an inefficient outcome. The setting of the theorem with two generators having specific cost structures does not represent all the nuances of the New Zealand market, but simply highlights a deleterious feature of a repeated auction mechanism. Precluding generators from updating offers in light of pre-dispatch information would converge on a perfectly competitive solution if their original offers were at marginal cost.

4 Example

How often does the phenomenon descibed above occur in actual pre-dispatch schedules? It would be interesting to examine forecast demands for predispatch and corresponding offers and clearing prices, and see whether the behaviour descibed above occurs. Pre-dispatch prices are published on the EMI site of the New Zealand Electricity Authority. We extracted these for May 5, 2020, and plot them below.



Predispatch prices and final prices for Benmore on May 5, 2020. Each coloured line shows a predispatch price schedule (PRSL) computed for the next 72 trading periods, but displayed up to the end of the day.

Observe that predispatch prices are generally increasing, and converge to prices that are below the final price. It seems unlikely that demand forecasts were also increasing towards their true final values as the forecast interval gets smaller, as this would imply a systematic bias (although a cold front getting colder faster than predicted might lead to this outcome). A systematic examination of historical demand forecasts should be able to resolve this.

5 Conclusion

It is difficult to say if the imperfection in predispatch described above warrants a redesign of the New Zealand auction mechanism. It has always been taken as an article of faith in the New Zealand wholesale market that more frequent auctions lead to better outcomes, as they enable price discovery. This is certainly the case in auction settings where purchasers' valuations are private information and the seller wishes to extract (and capture) as much of this value as she can. In an electricity market that is regulated to maximize the long-term benefit of consumers, it would seem to be strange to move towards more frequent predispatch auctions that might decrease this benefit.