Submitted to *Operations Research*
manuscript OPRE-2014-10-589

Supply function equilibrium with taxed benefits

A. Downward*, A.B. Philpott and K. Ruddell
Electric Power Optimization Centre, University of Auckland, New Zealand, *a.downward@auckland.ac.nz

Supply function equilibrium models are used to study electricity market auctions with uncertain demand. We study the effects on supply function equilibrium of a system tax on the observed benefits of suppliers. Such a tax provides an incentive for agents to alter their offers to avoid the tax. We show how this surprisingly can lead to lower prices in equilibrium. The model is extended to a setting in which the agents are taxed on the benefits accruing to them from a transmission line expansion (in order to help fund the line). In these circumstances we study how incentives for agents to alter their bids varies with the relative size of the capacity expansion.

Key words: Supply Function Equilibrium, Electricity Markets

History: This paper was first submitted on 7 October 2014

1. Introduction

In electricity market auctions, producers typically submit amounts of generation that they are willing to supply at different prices. These offer curves are then cleared by a system operator in a pool to yield a system marginal price. All generation offered at a price equal or below this market price is dispatched. Each generator is then paid the system marginal price for all the energy they are dispatched. This leads to market rents accruing on infra-marginal offers (those with offer price below the system marginal price).

The offers of each generator may be modeled by a supply curve. In the face of uncertain demand, each agent seeks a curve to maximize expected profit, leading to the concept of supply-function equilibrium (SFE). SFE models have been applied to the study of electricity market auctions by a number of authors (Green and Newbery 1992, Holmberg and Newbery 2010). Although SFE models are not straightforward to work with, and there is a shortage of effective computational procedures to compute asymmetric SFE, these models deal with demand uncertainty in a natural way, a feature that makes them increasingly useful as intermittent renewable generation grows. For computational simplicity, it is customary in SFE models to assume symmetric players with identical costs and capacities, and a market with a price cap. The shock in demand is chosen so that demand exceeds the total supply capacity with some small probability. In these events the market clears at the price cap, and load is shed. Details can be found in the recent survey by Holmberg and Newbery (2010).

In this paper we study the effects on agent behaviour of a system tax levied on the surplus earned by inframarginal rents. Since the true marginal cost functions of the agents are not public knowledge, the rents are computed assuming that the supply function offered represents the agent's marginal cost of supply. The imposition of the tax alters the incentives of the agents in choosing what supply functions to offer to the auction. Their offer curves will adjust in such a way to minimize the tax paid, while not sacrificing too much profit. When electricity demand is deterministic, the agents can anticipate the market clearing price and their dispatch quantity. Given this dispatch point, each agent has an incentive to increase the prices of their inframarginal offers so as to reduce the apparent benefit (while maintaining their real benefit). One might then expect all offers to become perfectly elastic at the clearing price up to the anticipated dispatch quantity.

Uncertainty in demand alters this outcome. Agents offering horizontal bids might find if demand is lower than anticipated that they make no money at all. In such circumstances agents do better by offering an increasing supply curve that trades off the amount of tax paid against the need to earn some profit. One might expect this curve to mark up offers to recover the tax through higher prices, however we show that, in equilibrium, this strategy is only applied to the lower end of the supply curve, and at high prices agents may discount their offers.

Our study of such a tax is motivated by a proposal mooted by the New Zealand Electricity Authority to charge electricity market participants for transmission based on the benefits that accrue to them from these upgrades (see NZEA 2014). Although the details of this 'beneficiary-pays' scheme are still being negotiated, some effort has been devoted to estimating these benefits using the software used for dispatching the wholesale market and computing locational marginal prices.

The simplest version of this estimation process works as follows. After the market is dispatched with current transmission assets in place, the benefits of each agent are computed from their bid and offer curves. For a generator this benefit is measured by the rentals earned from inframarginal bids. As we have already remarked, this need not be the true benefit if these bids are marked up above the generators marginal cost. The dispatch software is then run again using the same bids and offers, but with the transmission assets de-rated to their pre-upgrade levels. The benefits for each agent are then computed under this counterfactual and subtracted from the previous estimates. If these are positive then the agents with positive net benefits contribute to the upgrade cost of the transmission system in proportion to these net benefits. A fuller description is provided by NZEA (2012).

The paper is laid out as follows. In section 2, we show how a tax on producer surplus gives rise to a best-response problem whose objective a convex combination of the best-response objective functions under uniform and pay-as-bid pricing. We then derive a symmetric equilibrium when such a tax is imposed on two agents at the upstream end of a constrained transmission line. Section 3 compares the response to the tax under SFE with a competitive model where all generators act as price-takers. Section 4 deals with the setting when the tax is calculated based on the difference between the actual and counter-factual dispatch from the expansion of the transmission line.

2. Supply function equilibrium

In this paper we confine attention to a duopoly in which each player chooses a monotone piecewise smooth curve $(q,p):[0,T]\to\mathbb{R}^2$ to maximize a best-response functional of the form (1). This is more general than Klemperer and Meyer's (1989) definition of the SFE problem. The first-order optimality condition for a profit-maximizing curve, given competitors' offers, gives a system of ordinary differential equations which, assuming symmetry of agents, yields a single ordinary differential equation that we can solve to find the equilibrium supply functions.

The market operates on locational marginal pricing; the system operator clears the market by choosing prices at each node to minimize the cost of meeting demand while satisfying the transmission capacity constraints. Suppliers are then paid according to some function (the pricing rule) of their offer curve and the local price.

Different pricing rules give rise to different profit functionals that bidders seek to maximize. For example, uniform pricing, discriminatory (pay-as-bid) pricing, and taxed producer surplus models all have different objective functionals. When these integral functionals have the property that the integrand is linear in the derivative of the curve, then Theorem 1 gives a uniform set of optimality conditions.

THEOREM 1 **(Optimality conditions for best response)**. *Let $f,g:\mathbb{R}^2\rightarrow\mathbb{R}$ be piecewise continuously differentiable functions and $(q,p):[0,T]\rightarrow\mathbb{R}^2$ a continuously differentiable curve. A necessary condition for the curve (q,p) to maximize the functional*

$$\Pi(q,p)=\int_0^T\left(f(q,p)\frac{dq}{dt}+g(q,p)\frac{dp}{dt}\right)dt \tag{1}$$

is that $Z(q,p)=\frac{\partial g}{\partial q}-\frac{\partial f}{\partial p}=0$ at every point along the curve. Furthermore, if both components of (q,p) are nondecreasing in t then a sufficient condition for optimality is that $Z=0$ along (q,p) and $\frac{\partial}{\partial q}Z\leq 0$ everywhere.

Proof The functional Π is a line integral in the (q,p) plane. As this functional is linear in $\frac{dq}{dt}$ and $\frac{dp}{dt}$, we can apply Green's theorem, as in Anderson and Philpott (2002), to obtain the optimal curve. By Green's theorem, the integral of $\left(f(q,p)\frac{dq}{dt}+g(q,p)\frac{dp}{dt}\right)$ around any simple closed curve in the anticlockwise direction in the (q,p) plane is equal to the integral of Z over the area enclosed by the curve. Thus if $Z>0$ along part of the trajectory (q,p) then (by continuity of Z) there is an improving deviation to the right of the curve as it is traversed. Similarly if $Z<0$ along part of the trajectory (q,p) then there is an improving deviation to the left of the curve as it is traversed. Thus a maximal curve must have $Z(q,p)=0$.

Now suppose $Z=0$ along (q,p) and $\frac{\partial}{\partial q}Z\leq 0$ everywhere. If a candidate curve is nondecreasing and $\frac{\partial}{\partial q}Z\leq 0$ everywhere, then there can be no region to the right of the curve on which Z has a positive integral and so no improving deviations exist to the right of the curve as it is traversed. Similarly on the left there are no regions for which Z has a negative integral, so no improving deviations exist to the left. It follows that (q,p) is a global maximum. ■

Note that the condition $Z=0$ is equivalent to the Euler-Lagrange equation $\frac{d}{dp}F_{\dot{q}} - F_q = 0$ from the calculus of variations when the offer curve is modeled as a supply function $q(p)$. In this case, replacing t by p gives

$$F = f(q,p)\frac{dq}{dt} + g(q,p)\frac{dp}{dt} = f(q,p)\frac{dq}{dp} + g(q,p).$$

Then

$$\begin{aligned}\frac{d}{dp}F_{\dot{q}} - F_q &= \frac{d}{dp}f(q,p) - \frac{\partial f}{\partial q}\frac{dq}{dp} - \frac{\partial g}{\partial q} \\ &= \frac{\partial f}{\partial p} + \frac{\partial f}{\partial q}\frac{dq}{dp} - \frac{\partial f}{\partial q}\frac{dq}{dp} - \frac{\partial g}{\partial q} \\ &= -Z.\end{aligned}$$

We apply this theorem to both uniform- and discriminatory-price auctions, as well as auctions where the market operator taxes a portion of producer surplus.

2.0.1. Uniform-price auction In a uniform-price auction, the expected payoff to a firm offering a curve $(q(t), p(t))$ is

$$\begin{aligned}\Pi^U &= \int_0^T (qp - C(q))\, d\psi(q,p) \\ &= \int_0^T (qp - C(q))\left(\frac{dq}{dt}\psi_q + \frac{dp}{dt}\psi_p\right) dt,\end{aligned} \tag{2}$$

where $C(q)$ is the firm's cost to produce quantity q and $\psi(q,p)$ is the market distribution function (see Anderson and Philpott 2002), which gives the probability that a supplier is not fully dispatched if they offer the quantity q at price p. It can be interpreted as the measure of residual demand curves that pass below and to the left of the point (q,p). The integrand in Π^U is clearly linear in $\frac{dq}{dt}$ and $\frac{dp}{dt}$, so we can compute the Z function as in Anderson and Philpott (2002):

$$\begin{aligned}Z^U(q,p) &= \frac{\partial (qp - C(q))\psi_p}{\partial q} - \frac{\partial (qp - C(q))\psi_q}{\partial p} \\ &= (p - C'(q))\psi_p - q\psi_q,\end{aligned}$$

as the cross terms containing ψ_{qp} cancel.

2.0.2. Discriminatory-price auction In a discriminatory-price (pay-as-bid) auction (Anderson et al. (2013)), the expected payoff is

$$\Pi^D := \int_0^T [p - C'(q)] [1 - \psi(q,p)] \frac{dq}{dt} dt.$$

Again this is linear in $\frac{dp}{dt}$ and $\frac{dq}{dt}$ and theorem 1 holds with

$$Z^D = (p - C'(q)) \psi_p - (1 - \psi(q,p)).$$

2.0.3. Tax on producer surplus Suppose that some fraction $\alpha \in (0,1)$ of the observed producer surplus earned by a generator is paid as tax. Such a tax is unlikely to be applied by a real regulator, but we will later see that it is equivalent, over part of price-quantity space, to the proposed beneficiaries-pay scheme that will be analyzed in section 4. If the market clears at quantity q for a generator at price π then the generator receives

$$\begin{aligned} R(q,\pi) &= q\pi - C(q) - \alpha \int_0^q (\pi - p(t)) dt \\ &= q\pi - C(q) - \alpha q\pi + \alpha \int_0^q p(t) dt = (1-\alpha)(q\pi - C(q)) + \alpha \left(\int_0^q p(t) dt - C(q) \right). \end{aligned}$$

This is a convex combination of uniform and pay-as-bid pricing with multiplier α. Thus the total payoff will be

$$\Pi^A = (1-\alpha) \Pi^U + \alpha \Pi^D. \tag{3}$$

We can write down the optimality conditions for the problem faced by a generator maximizing Π^A. These use the scalar field defined by $Z^A(q,p) = (1-\alpha) Z^U + \alpha Z^D$. Thus

$$\begin{aligned} Z^A(q,p) &= (1-\alpha)((p - C'(q)) \psi_p - q\psi_q) + \alpha((p - C'(q)) \psi_p - (1 - \psi(q,p))) \\ &= (p - C'(q)) \psi_p - (1-\alpha) q\psi_q - \alpha (1 - \psi(q,p)). \end{aligned} \tag{4}$$

2.1. Example

We illustrate the above analysis with a simple example with two symmetric agents, located at node 1 of the two-node network shown in figure 1. Here there is a price-taking, random and price-inelastic demand ε at node 2. The line connecting the two nodes has capacity K, which is less than the

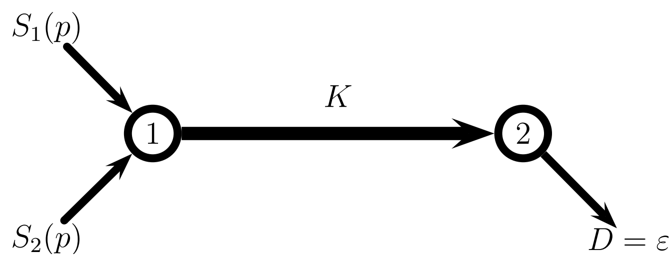

Figure 1 Symmetric equilibrium example. Line has capacity K.

maximum demand at node 2. There is no demand at node 1. This is the simplest possible model of a transmission-constrained network.

Suppose there is a proportional tax α imposed on the observed surplus of each agent. We apply the optimality conditions of the previous section to look for an equilibrium in symmetric duopoly. Suppose the other player offers a piecewise smooth supply function $S(p)$ and demand has cumulative probability distribution function F. Then

$$\begin{aligned}\psi(q,p) &= \Pr[\varepsilon < q + S(p)] \\ &= F(q + S(p))\end{aligned}$$

and, substituting into (4)

$$Z^A(q,p) = (p - C'(q))\, S'(p)\, f(q + S(p)) - (1-\alpha)\, q f(q + S(p)) - \alpha[1 - F(q + S(p))]. \quad (5)$$

Here $f = F'$ is the probability density of the demand shock. When this is strictly positive we can divide (5) through by $f(q + S(p))$ to obtain

$$\hat{Z}^A(q,p) := (p - C'(q))\, S'(p) - (1-\alpha)\, q - \alpha \frac{1 - F(q + S(p))}{f(q + S(p))}. \quad (6)$$

By theorem 1, the curve defined implicitly by $\hat{Z}^A(q,p) = 0$ is a profit-maximizing response if it is monotone and $\frac{\partial}{\partial q}\hat{Z}^A(q,p) \le 0$ for all p and q, i.e.

$$-C''(q)\, S'(p) - (1-\alpha) - \alpha \frac{\partial}{\partial q}\left[\frac{1 - F(q + S(p))}{f(q + S(p))}\right] \le 0. \quad (7)$$

The term $G(x) = \frac{1-F(q+S(p))}{f(q+S(p))}$ is the inverse hazard rate of the distribution. In Holmberg's (2009) model, for pure pay-as-bid pricing, $\alpha = 1$ and marginal costs are constant, so it is necessary that $G' \ge 0$ for (7) to hold. This restricts the analysis to probability distributions that decay faster than the exponential distribution, which has $G' = 0$. If the tax rate α is less than 1 then we are less restricted in our choice of probability distribution for the demand shock. For instance, as shown in the example below, if $\alpha < \frac{1}{2}$, then (7) holds for a uniform distribution.

We now choose some specific problem data to illustrate the equilibrium. Suppose that the demand shock is uniformly distributed on $[0, \bar{\varepsilon}]$ and $\alpha < \frac{1}{2}$. Assume that the line capacity K is infinitesimally smaller than $\bar{\varepsilon}$. Suppose that marginal costs for each agent are the same and are constant ($C' = c$). Then

$$\begin{aligned}\frac{\partial}{\partial q}\hat{Z}^A(q,p) &= -(1-\alpha) - \alpha\frac{\partial}{\partial q}\left[\frac{1-(q+S(p))/\bar{\varepsilon}}{1/\bar{\varepsilon}}\right] \\ &= 2\alpha - 1 \\ &< 0\end{aligned}$$

and so solving $Z^A = 0$ gives a symmetric equilibrium. If we set $q(p) = S(p) = Q(p)$, then the condition $Z^A = 0$ gives

$$Q'(p) = \frac{(1-3\alpha)Q}{p-c} + \frac{\alpha\bar{\varepsilon}}{p-c}.$$

This is a first order linear ODE which can be solved using an integrating factor to give

$$Q(p) = k(p-c)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha},$$

where k is a constant of integration that can be chosen to satisfy an endpoint condition. As the line has a capacity that binds at the highest levels of demand, there exists a unique endpoint condition $q(\bar{p}) = \frac{K}{2} \approx \frac{\bar{\varepsilon}}{2}$ for which no profitable deviation is possible (see Holmberg 2008).

We can compute the changes in welfare of each agent from the change in equilibrium. Suppose $K = 1$ and $\bar{\varepsilon} = 1$, and consider first the case where $\alpha = 0$, there is a price cap at $\bar{p} = 1$ and constant marginal costs of $c = 0$. In perfect competition each generator would offer at price equal to marginal cost and earn no profit. However, in our supply function game the equilibrium curve S has equation $q(p) = \frac{p}{2}$. As there are two firms, the total supply is $2S(p) = p$, and as the market clears when supply equals demand $2S(p) = D(p,\varepsilon) = \varepsilon$, we can write the market price as a function of demand as $p(\varepsilon) = \varepsilon$. The expected consumer surplus (assuming all consumers value electricity at $\bar{p}$) is

$$\begin{aligned}CS &= \int_0^{\bar{\varepsilon}} \varepsilon(\bar{p} - p(\varepsilon))\, f(\varepsilon)\, d\varepsilon \\ &= \frac{1}{6}.\end{aligned}$$

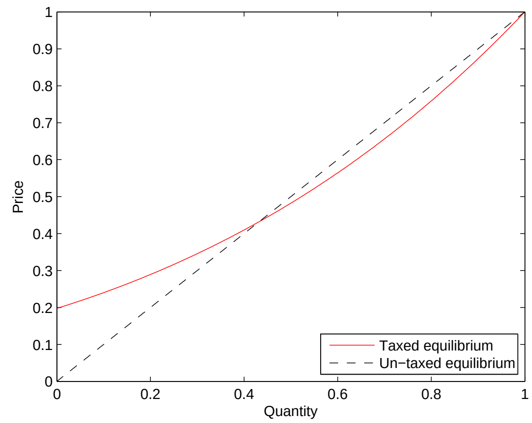

Figure 2 Equilibrium supply curves for no tax (dashed) and a 25% tax on perceived surplus (solid).

The expected producer revenue is the firm's objective function $\Pi^U = \frac{1}{6}$. Their expected observed surplus however, is $\frac{1}{12}$. If a tax is applied to this curve, the firms each pay α of their perceived surplus, so if $\alpha = \frac{1}{4}$, then each firm pays $\frac{1}{48}$ in tax, leaving net profit of $\frac{7}{48}$. As shown in figure 2, the tax gives an incentive for firms to change the shape of their offer curve. Our firms will settle on a new equilibrium curve S^A which has equation

$$q(p) = \frac{3}{2}p^{\frac{1}{4}} - 1$$

with inverse

$$p(q) = \frac{16}{81}(q+1)^4.$$

It is simple to check that this is monotone and solves $Z^A = 0$.

The expected consumer surplus is 0.1737; the expected producer profit, before tax, is 0.1632; and the producer surplus perceived by the market operator is 0.066. Each producer pays taxes of 0.0165 and so earns 0.1467 net profit.

Table 1 summarizes the changes in producer and consumer surplus between the untaxed and taxed scenarios. The overall effect of the tax is a small transfer of welfare from producers to consumers. Though higher prices are charged at times of low demand, this is offset by lower prices higher up the offer curves. Note that social surplus (the sum of consumer and producer surpluses $CS + 2\Pi^U$) does not change with the introduction of the tax; this is because demand is inelastic. Also note that expected consumer surplus actually rises once firms adjust to the tax, as the new equilibrium SFE is more competitive for the higher demand realizations.

Curve	α	CS	Π^U	Π^A	Tax per firm	Social Surplus
S	0.25	0.1666	0.1666	0.1458	0.0208	0.5
S^A	0.25	0.1737	0.1632	0.1467	0.0165	0.5

Table 1 Benefits and taxes under a producer-surplus tax.

In this model where all supply and demand in node 2 is inelastic, there are no congestion rents accruing to the system operator because there is never a price differential between the nodes. All load that cannot be satisfied through the transmission line is lost, and when this happens the market power of the suppliers in node 1 lets them charge the price cap. If there were elastic demand or supply in the downsteam node, then there would be positive congestion rent.

2.2. Dependence on demand shock distribution

The first-order condition for symmetric equilibrium under the producer surplus tax (6) has a term that is proportional to the inverse hazard rate of the demand-shock distribution. This indicates that the higher the inverse hazard rate, the flatter the supply function equilibrium bid will be, and the higher markups will be for small output levels. In figure 3 we plot the solutions to the ODE for the density distributions shown in figure 4. Their inverse hazard rates are plotted in figure 5.

From figures 3 and 5, we can see that when the inverse hazard rate becomes large it dominates the ODE. So when it is almost certain that dispatch will be above the current point on the curve, the curve becomes very elastic. In the limit, if we are certain to be dispatched above the current point on the curve, then minimizing the perceived surplus is the only incentive acting and the curve should thus be perfectly elastic.

3. Modelling all firms as price-takers

It is instructive to model the effects of a producer-surplus tax under conditions of perfect competition, i.e. where all generators act as price-takers. This can serve as a benchmark to measure market power in the 'strategic' oligopolistic market.

When players are price-takers, the only market information to which they can respond is the price. It is as though they face residual demand curves at fixed prices (that are perfectly inelastic). Hence the market distribution function to which the firms react depends only on price.

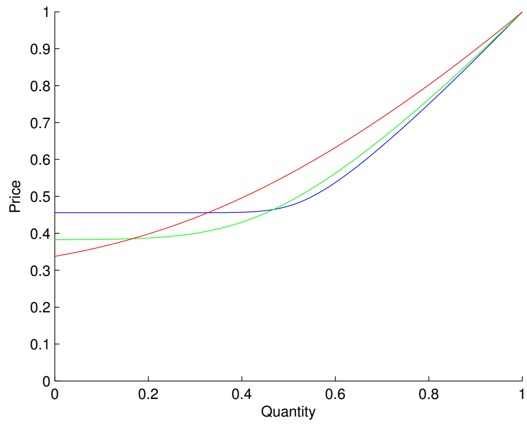

Figure 3 Equilibria with normally distributed shock with mean 0.7 and different standard deviations 0.1, 0.2 and 0.5

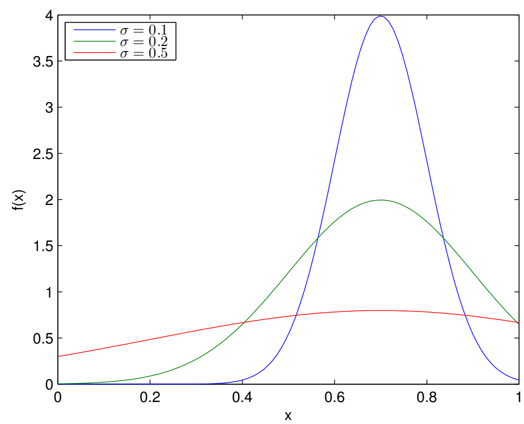

Figure 4 Densities of demand shocks used in figure 3

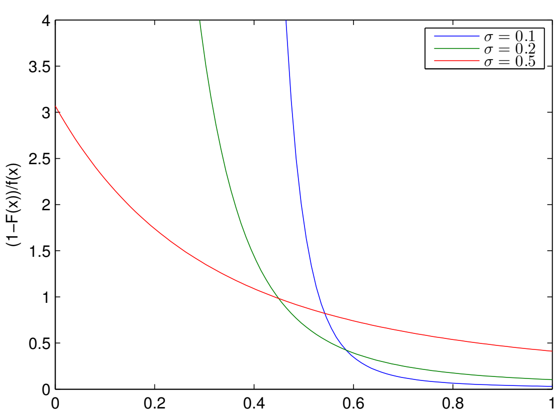

Figure 5 Inverse hazard rates of demand shocks in figure 4

If we write $h(p) = \frac{\partial \psi}{\partial p}$ for the density of the distribution of prices, then, since the revenue function is the same as for Π^A above, theorem 1 gives

$$(p - c'(q))\, h(p) - \alpha\, (1 - \psi(p)) = 0 \tag{8}$$

as the first order condition for equilibrium. The parameter α takes us from a uniform-price auction to pay-as-bid as it varies from 0 to 1.

3.1. ODE for equilibrium

Suppose that the market distribution function arises from a one-dimensional shock ε with density f and cumulative distribution function F.

The system operator dispatches from the aggregate supply curve by setting $Q(p) = \varepsilon$. This induces the market distribution function as a distribution on prices

$$\Psi(p) = F(\varepsilon) = F\left(Q(p)\right). \tag{9}$$

The density of this is then obtained by the chain rule,

$$\psi(p) = \Psi'(p) = f\left(Q(p)\right) Q'(p). \tag{10}$$

We substitute (9) and (10) into (8) to obtain the ODE

$$(p - c'(q))\, f(Q)\, Q' - \alpha\, (1 - F\left(Q\right)) = 0. \tag{11}$$

This represents a fixed point in that the producers are maximizing their profit given the price distribution f; simultaneously the system operator dispatches by choosing prices so that supply and demand intersect.

By the same argument as in Federico and Rahman (2003), the price bid for the last unit offered is equal to its marginal cost. This is because there is classical Bertrand competition for the units sold under congestion conditions. This competition pushes the top of the price distribution down toward marginal cost, where it finds equilibrium.

3.2. Example

To compare with the strategic SFE (section 2.1), we introduce rising marginal costs so that the price-taking equilibrium is non-trivial

$$c'(q) = \gamma q. \tag{12}$$

The first-order condition (8) becomes

$$(p - \gamma q)\frac{1}{\bar{\varepsilon}}q' - \alpha\left(1 - \frac{q}{\bar{\varepsilon}}\right) = 0$$
$$Q'(p) = \alpha\frac{\bar{\varepsilon} - Q}{p - \gamma Q} \tag{13}$$

This is a non-linear ODE, but it can be solved without great difficulty. If we take the same parameters as in the SFE example above ($\alpha = 0.25$, $\bar{\varepsilon} = 1$), but add a linear marginal cost with coefficient $\gamma = 0.5$, we can compare outcomes. Figure 6 shows the solution to (13) alongside the solution to the equivalent ODE for supply function equilibrium,

$$Q'(p) = \frac{(1 - 3\alpha)Q}{p - \gamma Q} + \frac{\alpha\bar{\varepsilon}}{p - \gamma Q}. \tag{14}$$

Note that markups increase uniformly when all firms are price takers. Note also that the SFE is determined by the price-cap, but this has no influence on the price-taking equilibrium.

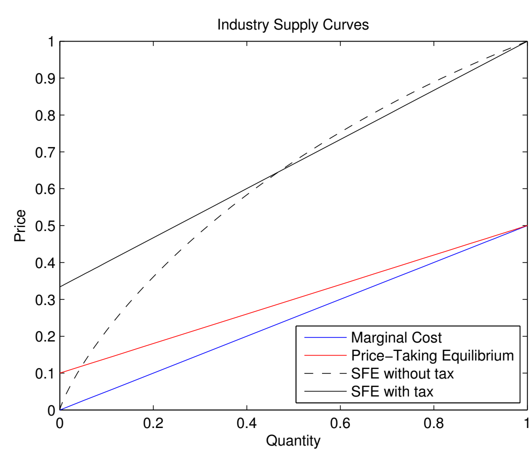

Figure 6 Equilibrium with price-taking producers, and strategic producers competing in supply functions

4. Line capacity expansion

We now consider a model in which the transmission line is expanded from capacity J to capacity K, and a proportional tax on observed benefits is levied to recover the costs of the line expansion. The model is again a simple two-node network as in Figure 1, with symmetric players at one node, and an inelastic demand shock ε at the downstream end of the line.

The motivation for the model is a proposal for a new transmission pricing scheme to cover large grid investments in New Zealand. The NZ wholesale electricity market is dispatched according to a combined energy and reserve co-optimization in real time with bids covering half-hour trading periods. A range of transmission pricing schemes are under consideration that include various combinations of

- Congestion rents
- Locational ‘postage-stamp’ charges (GIT, AoB)
- A tax on observed benefits, calculated at each trading period (SPD).

Congestion rents accrue to the system operator already in the NZ wholesale market. The way they are redistributed does not have an effect on the strategic behavior in the real-time market. Locational charges, once the rate has been announced, function as an additional cost of production and so their effect on market is just a uniform markup across all levels of output. The SPD scheme differs from other transmission pricing methods promoted as beneficiary-pays in that the benefits are calculated as part of the dispatch, based on actual bids to the market. In fact the only cost information used by the regulator comes from the submitted bid function. Beneficiaries-pay transmission cost recovery schemes in NY (see Hogan (2011)) and Argentina (see Pollitt (2008)) apply charges as locational ‘postage-stamp’ fees based on an ex-ante analysis of benefits arising from network expansion.

Presently, in the NZ wholesale market, suppliers submit bid stacks (that we model as curves). In the delivery period, demand and intermittent supply are realized. The system operator solves the dispatch problem to satisfy demand at least cost based on the bids submitted and subject to transmission capacity constraints. The solution gives price and production levels for all generators. The beneficiaries-pays scheme proposed as the ‘SPD method’ in NZEA (2014) makes an adjustment to payments at dispatch by solving a counter-factual dispatch problem. The dispatch is solved a second time, with transmission lines de-rated to their pre-expansion capacity. The difference in producer surplus between the two dispatches is calculated and generators are paid the price from the main dispatch minus a portion of this difference. The portion of perceived benefits to be charged, α is declared in advance. Demand may also be charged, but as its bids in the spot market are inelastic, it has no way of strategically responding to the tariff in the short term.

Suppose player 2 offers a supply function $S_2(p)$. If player 1 offers quantity q at price p then the market distribution function is just the probability that either the total quantity offered $q+S_2(p)$ exceeds the line capacity K or that the combined offers of the two firms $q+S_2(p)$ at price p exceeds the demand shock ε;

$$\psi(q,p) = \Pr\left[q + S_2(p) > \min(K,\varepsilon)\right].$$

If the demand shock is uniformly distributed on $[0,\bar{\varepsilon}]$, then we obtain the following piecewise definition for ψ:

$$\psi(q,p) = \begin{cases} \frac{q+S_2(p)}{\bar{\varepsilon}} & \text{if } q < K - S_2(p) \\ 1 & \text{if } q \geq K - S_2(p). \end{cases} \tag{15}$$

The partial derivatives are $\psi_q = \frac{1}{\bar{\varepsilon}}$ and $\psi_p = \frac{S_2'}{\bar{\varepsilon}} = S_2'\psi_q$ when $q \leq K - S_2(p)$ and both are zero otherwise. There is a jump in the value of $\psi(q,p)$ lying on the curve $q = K - S_2(p)$.

4.1. Payoffs

We now look at the payoffs. The actual pre-tax producer profit if a generator is dispatched $\theta(\varepsilon)$ at price $\pi(\varepsilon)$ under demand realization ε is

$$P(\varepsilon) = \pi(\varepsilon)\,\theta(\varepsilon) - C(\theta(\varepsilon)).$$

The system operator assumes that the submitted curve is marginal cost and observes a different surplus. The observed surplus is

$$\sigma(\varepsilon) = \int_0^{t(\varepsilon)} (\pi(\varepsilon) - p(t))\,\frac{dq}{dt}dt,$$

where $t(\varepsilon)$ satisfies $(q(t), p(t)) = (\theta(\varepsilon), \pi(\varepsilon))$. Integrating by parts gives

$$\begin{aligned}\sigma(\varepsilon) &= [(\pi(\varepsilon) - p(t))q(t)]_{t=0}^{t(\varepsilon)} - \int_0^{t(\varepsilon)} -q(t)\frac{dp}{dt}dt \\ &= \int_0^{t(\varepsilon)} q(t)\frac{dp}{dt}dt.\end{aligned}$$

Taking the expectation of this surplus over all demand outcomes gives

$$\begin{aligned}\mathbb{E}[\sigma] &= \int_0^{\bar{\varepsilon}}\int_0^{t(\varepsilon)} q(t)\frac{dp}{dt}dt\, f(\varepsilon)\, d\varepsilon \\ &= \int_0^{T}\left(\int_{\varepsilon(t)}^{\bar{\varepsilon}} f(h)\, dh\right) q(t)\frac{dp}{dt}\, dt.\end{aligned}$$

But

$$\begin{aligned}\int_{\varepsilon(t)}^{\bar{\varepsilon}} f(h)\, dh &= \Pr(h > \varepsilon(t)) \\ &= 1 - \psi(q(t), p(t))\end{aligned}$$

so

$$\mathbb{E}[\sigma] = \int_0^{T}(1 - \psi(q(t), p(t)))\, q(t)\frac{dp}{dt}\, dt.$$

This fact can also be derived form the observation that the apparent producer surplus is precisely the difference between profits under uniform and discriminatory pricing.

The clearing price π depends on the demand realization ε and the network configuration, so the two network configurations give two price functions, the actual price of dispatch $\pi(\varepsilon)$ and the counter-factual price $\hat{\pi}(\varepsilon)$. These in turn give rise to two distinct realizations of producer surplus $\sigma(\varepsilon)$ and $\hat{\sigma}(\varepsilon)$.

For a given level of demand, our firm will be dispatched at one price and quantity in the actual and a different price and quantity in the counter-factual network. The market distribution functions give the distributions of these price-quantity pairs relative to the firm's offer. The firm pays a portion α of the difference between the perceived surplus in the actual network and counter-factual network cases; i.e. the tax is

$$\alpha(\sigma(\varepsilon) - \hat{\sigma}(\varepsilon)).$$

This gives profit net of tax of

$$R(\varepsilon) = P(\varepsilon) - \alpha(\sigma(\varepsilon) - \hat{\sigma}(\varepsilon)).$$

The generator then constructs an offer curve to maximize this tax-adjusted profit. Since $R(\varepsilon)$ is the linear combination of three terms, we can express the expectation as a linear combination of the individual expectations. Here $P(\varepsilon)$ and $\sigma(\varepsilon)$ are evaluated using the market distribution function ψ assuming a full line capacity, whereas $\hat{\sigma}(\varepsilon)$ is evaluated using the counter-factual market distribution function ϕ assuming the unexpanded capacity. The expected profit over the entire supply curve is

$$\begin{aligned}
\Pi^L &= \mathbb{E}[P] - \alpha\left(\mathbb{E}[\sigma] - \mathbb{E}[\hat{\sigma}]\right) \\
&= \int_0^T (pq - C(q))\left(\frac{dp}{dt}\psi_p + \frac{dq}{dt}\psi_q\right) dt - \alpha\left(\int_0^T q\left[1-\psi(q,p)\right]\frac{dp}{dt}\,dt - \int_0^T q\left[1-\phi(q,p)\right]\frac{dp}{dt}\,dt\right) \\
&= \int_0^T \left((pq - C(q))\left(\frac{dp}{dt}\psi_p + \frac{dq}{dt}\psi_q\right) - \alpha q\left(\phi(q,p) - \psi(q,p)\right)\frac{dp}{dt}\right) dt.
\end{aligned}$$

The resulting Z function is

$$Z^L = (p - C'(q))\,\psi_p - q\psi_q - \alpha\left(q\left(\phi_q - \psi_q\right) + \phi - \psi\right). \tag{16}$$

In our model with a one-dimensional shock in the downstream node, $[\phi - \psi]$ is non-zero only when

$$J < q + S_2(p) \leq K,$$

in which case $\phi = 1$. Hence our functional Π^L can be thought of as piecewise defined; equal to Π^U when $\phi - \psi = 0$ and Π^A otherwise.

As the integrand in Π^L is discontinuous along the line $J - q - S_2(p) = 0$, we can no longer rely on Theorem 1 directly. The following corollary gives sufficient conditions for an optimal supply function against the functional Π^L.

Suppose f and g are as in Theorem 1 except for a jump discontinuity along a curve that we define implicitly by $h(q,p) = 0$. Without loss of generality we can fix $T_1 \in (0,T)$ and require that $h(q(T_1), p(T_1)) = 0$; any curve that crosses $h = 0$ can be re-parametrized so this is true. Then we can give sufficient conditions on optimal curves as a corollary to Theorem 1.

Corollary 1 **(Sufficient condition for optimal bidding with discontinuous payoff)**. *The following conditions are sufficient for a curve $(q,p):[0,T] \to \mathbb{R}^2$ to maximize* (1)*:*

1. *$Z(q,p) = 0$ at all points along the curve except T_1.*

2. *$Z_q < 0$ wherever $h \neq 0$.*

3. *The curve is continuous.*

4. *The curve crosses $h = 0$ exactly once, at T_1.*

Proof The proof is by a simple decomposition argument. Consider the problem of choosing separate maximal curves for (1) over $[0,T_1]$ and $[T_1,T]$ with the boundary conditions (upper and lower respectively) $h(q(T_1),p(T_1)) = 0$. Because of condition 4, these two subproblems satisfy the assumptions of Theorem 1. Conditions 1 and 2 are, by Theorem 1, sufficient for optimality in these subproblems.

To pass from separately maximizing over the two subintervals to globally maximizing over all of $[0,T]$, we add the constraint that the curve be continuous at T_1. As our optimal solution satisfies the additional constraint, it is still optimal. □

4.2. Example

We now consider an example. The base levels of parameters are as follows:

$$\begin{aligned}
\bar{\varepsilon} &= 1 && \text{maximum shock} \\
c &= 0 && \text{marginal cost} \\
K &= 0.8 && \text{enlarged line capacity} \\
J &= 0.2 && \text{restricted line capacity} \\
\alpha &= \tfrac{1}{4} && \text{tax rate} \\
\bar{p} &= 1 && \text{price cap}
\end{aligned} \tag{17}$$

The first order condition for an SFE is

$$(p - C')\,\psi_p - q\psi_q = 0 \text{ for } q < J - S_2(p) \tag{18}$$

$$(p - C')\,\psi_p - (1-\alpha)\,q\psi_q - \alpha\,(1 - \psi(q,p)) = 0 \text{ for } q > J - S_2(p)\,. \tag{19}$$

Here, as in the previous example,

$$\psi(q,p) = \begin{cases} \frac{q + S_2(p)}{\bar{\varepsilon}} & \text{if } q \leq K - S_2(p) \\ 1 & \text{if } q > K - S_2(p)\,, \end{cases}$$

so

$$\psi_p = \frac{S_2'(p)}{\bar{\varepsilon}} \text{ and } \psi_q = \frac{1}{\bar{\varepsilon}}.$$

Replacing $S_2(p)$ and q by $Q(p)$ in (19) yields

$$(p-c)\,Q'\,(p) - (1-\alpha)\,Q\,(p) - \alpha\,(\bar{\varepsilon} - 2Q\,(p)) = 0,$$

which can be solved using an integrating factor, whereby

$$Q\,(p) = k\,(p-c)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha}, \tag{20}$$

with k a constant of integration. To pass through the price cap we require

$$k\,(\overline{p}-c)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha} = \frac{K}{2}$$
$$k = \left(\frac{K}{2} + \frac{\alpha\bar{\varepsilon}}{1-3\alpha}\right)(\bar{p}-c)^{3\alpha-1}.$$

The equation $Z = 0$ gives an ordinary differential equation for $q < J - S_2\,(p)$ with general solution

$$q\,(p) = k\,(p-c)\,. \tag{21}$$

Where k is a constant of integration. For continuity of the curve, we choose $k = \frac{J}{2(p^*-c)}$, where p^* solves $Q\,(p) = \frac{J}{2}$ in (20). Our equilibrium candidate is thus

$$Q(p) = \begin{cases} \left(\frac{K}{2} + \frac{\alpha\bar{\varepsilon}}{1-3\alpha}\right)\left(\frac{p-c}{\bar{p}-c}\right)^{1-3\alpha} - \frac{\alpha\bar{\varepsilon}}{1-3\alpha} & \text{if } p \geq p^* \\ \frac{J}{2}\frac{p-c}{p^*-c} & \text{if } p < p^*. \end{cases} \tag{22}$$

Observe that the exponent of $\left(\frac{p-c}{\bar{p}-c}\right)$ vanishes when $\alpha = \frac{1}{3}$. In that case the differential equation

$$(p-c)S'(p) + (3\alpha - 1)Q(p) = \alpha\bar{\varepsilon}$$

becomes

$$Q'(p) = \frac{\alpha\bar{\varepsilon}}{(p-c)},$$

so the symmetric equilibrium supply functions are

$$Q(p) = \alpha\bar{\varepsilon}\log\frac{p-c}{\bar{p}-c} + \frac{K}{2} \text{ for } Q > \frac{J}{2}.$$

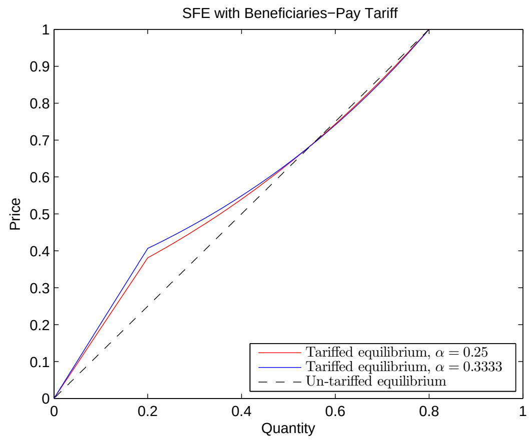

Figure 7 Plot of untaxed equilibrium offer (dashed) and taxed equilibrium offers (solid) when maximum demand is 1 and $\alpha = \frac{1}{4}$ (red) and $\alpha = \frac{1}{3}$ (blue).

The supply-function equilibria for two different choices of α are plotted in figure 7 below.

We can plot the values of Z^L, as defined in (16), to see that the conditions 1, 2 and 4 of Corollary 1 are indeed satisfied for the equilibrium with $\alpha = \frac{1}{4}$. In figure 8, we see that $Z_q < 0$ everywhere and that $Z = 0$ along the supply curve. Note that the discontinuity in the integrand of the objective occurs along $h(q, p) = J - q - S(p) = 0$, and that the supply curve intersects this locus once, at the kink.

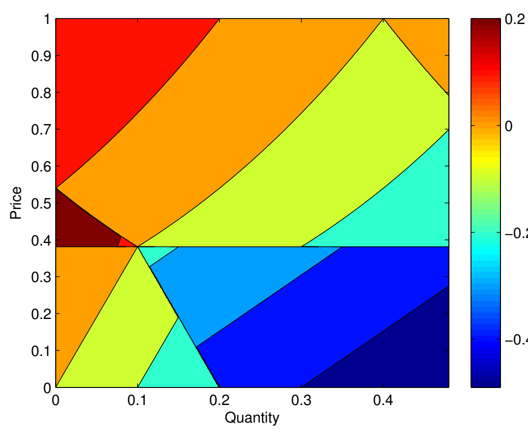

Figure 8 Plot of $Z^L(q, p)$ when competitor is playing the $\alpha = \frac{1}{4}$ curve from figure 7

The degree to which the taxed equilibrium is marked up above the untaxed equilibrium depends on the range of the demand shock. If the range of the demand shock is large, then there is a high probability that the expanded line will be congested. This means that the equilibrium offers try to avoid taxation of these by flattening the offer curve. This can be observed in Figure 9. As the probability of lost load increases, small quantities are marked up more, as their contribution to tariffs charged rises relative to profits earned when they are at the margin.

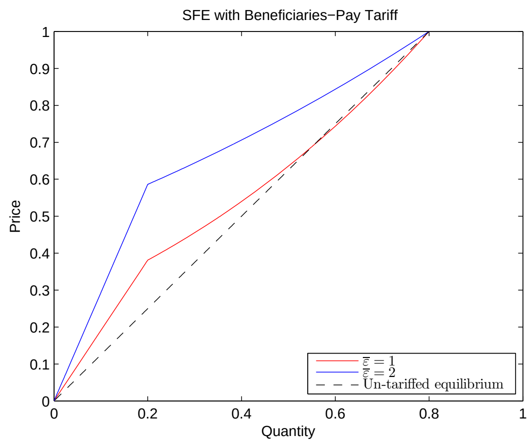

Figure 9 Plot of untaxed equilibrium offer (dashed) and taxed equilibrium offer (solid) when the maximum demand is 1 (red) and 2 (blue)

We finish this example by computing equilibria for $\alpha = \frac{1}{4}$, $\bar{\varepsilon} = 1$, and different values of J. These are shown in Figure 10. Observe that for small increases in line capacity (from $J = 0.6$ to $K = 0.8$) the blue and dashed curves almost coincide, so there is minimal change in offer strategy to avoid the tax.

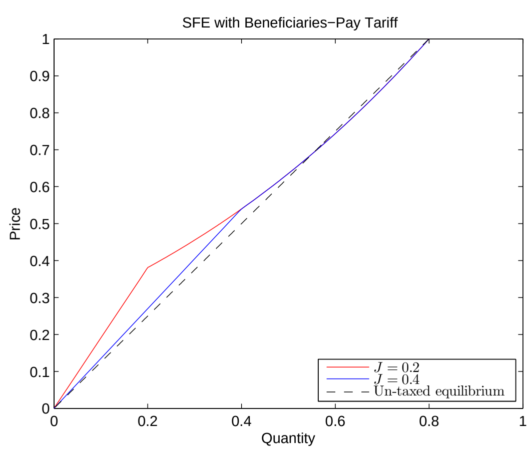

Figure 10 Plot of untaxed equilibrium offer (dashed) and taxed equilibrium offer (solid) when $J = 0.2$ (red) and $J = 0.4$ (blue)

4.2.1. Welfare calculations We may calculate consumer and producer welfare for different levels of tax. Taking the base level parameters (17) and repeating the analysis of section 2, we obtain the values of Table 2. Again, the total surplus does not change.

Curve	α	CS	Π^U	Π^A	Tax per firm	Social Surplus
S	0.25	0.1067	0.1067	0.0833	0.0233	0.32
S^A	0.25	0.1003	0.1098	0.0887	0.0211	0.32

Table 2 Benefits and taxes under a tariff on line-expansion benefits.

We see a slight decrease in consumer surplus as the very slight discounting at the top of the offer curve is not sufficient to offset the heavy markups around $q = \frac{J}{2}$. For small expansions in line capacity this effect diminishes.

We can measure the change in consumer surplus, profits and tax collected as the magnitude of the line expansion varies. We will make two parameter variations to illustrate the effects of the size of the expansion on strategic behavior.

First, we keep K constant at 0.8 and vary J from 0 to K, we cover a range of scenarios, from a completely new line to a minuscule (zero) increase in line capacity. In this variation the system operator chooses J, the baseline network capacity. The change in welfare after the tariff is imposed depends on the size of the counter-factual line J, as well as the probability of line congestion $1 - \frac{K}{\bar{\varepsilon}}$. The plots for a low probability of line congestion ($\bar{\varepsilon} = 1$, giving 20% probability) are shown in figure 11. Solid curves represent values pertaining to equilibria where producers take the tax into account and dashed curves measure the same thing for equilibria where agents ignore the tax. Note that when $J \approx K = 0.8$, the mark-down effect dominates so that there is actually a reduction in price levels in the post-tariff SFE, leading to a slight gain in consumer surplus and slight reductions in producer profits and transmission charges collected, compared to the equilibrium when no tariff is charged.

Second, the size of the existing line J and range of demand $[0, \bar{\varepsilon}]$ are taken as given , and we would like to know how the gain in welfare from expanding the line to K will depend on K, given that the producers will bid SFE. We fix all parameters except K as in (17) and vary K from

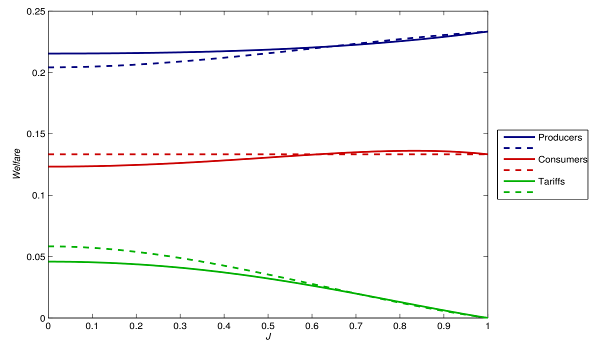

Figure 11 Welfare of distribution as J varies, with (solid) and without (dashed) strategic reaction to tariffs

$J = 0.2$ (no expansion) to $\bar{\varepsilon} = 1$ (fully covering all possible demand realizations). In figure 12 we see that, in absolute terms, the cost of the tariffs fall evenly on producers and consumers. Producer surplus is maximized at a point where there is positive probability of lost load. This is because the line expansion allows more power to be sold but also increases competitiveness in the market, and also because lost load means there is a point mass in the distribution of prices at the pricecap. As lost load reduces, this upward skew in the price distribution reduces too. Tariff revenue is also maximized at a point where there is a small amount of lost load, this is because the producer surplus starts to drop as the probability of lost load nears zero.

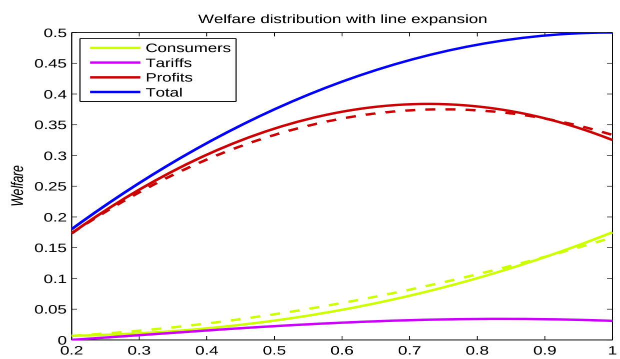

Figure 12 Welfare of distribution as K varies, with (solid) and without (dashed) strategic reaction to tariffs

4.3. Asymmetric cost functions

Using numerical methods to solve the ODE, we can find the supply function equilibrium for a duopoly where the two suppliers are asymmetric in their cost functions. Suppose that all the parameters of the market are as in (17), except for marginal cost.

We compare two linear marginal cost functions, chosen so that industry marginal cost is invariant. One is symmetric

$$C'_S(q_1, q_2) = \left(\frac{q_1}{2}, \frac{q_2}{2}\right)$$

and the other is asymmetric

$$C'_A(q_1, q_2) = \left(\frac{3q_1}{4}, \frac{q_2}{3}\right).$$

Figure 13 shows the industry supply curves for SFE under these two divisions of marginal cost.

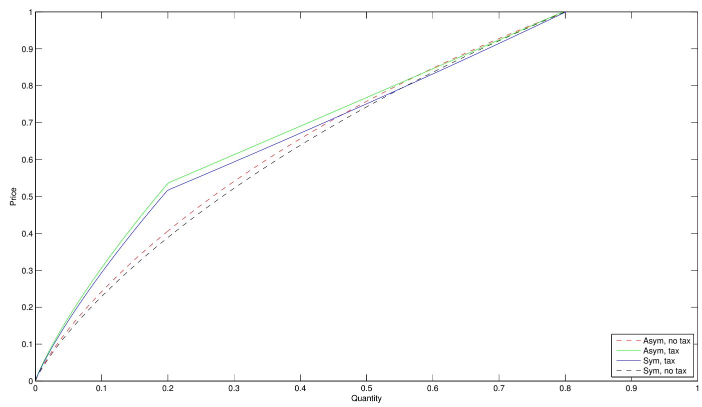

Figure 13 Equilibrium with producers competing in supply functions, having symmetric or asymmetric marginal cost functions

We notice several things. First, asymmetry makes the industry slightly less competitive. This is in line with other models of oligopoly. Second, the magnitude of markup resulting from the beneficiaries-pay tariffs persists. Therefore it is therefore not unreasonable to suppose strategic responses to the beneficiaries-pay tariffs will be of similar magnitude in asymmetric markets and symmetric models.

5. Conclusion

This work has examined the incentives of firms to adjust their offering strategies (in equilibrium) as a charge is applied as a percentage of either perceived profits (where the regulator believes that the firm offers at marginal cost), or perceived benefits of an investment in transmission assets (e.g. a line capacity upgrade). In a deterministic setting one may think that there would be an incentive to conceal one's perceived benefits by increasing the offers up to the dispatch point. However in a setting where the dispatch point is not known in advance (uncertain residual demand), we have shown that a balance must be struck between concealing the benefits and maximizing the (untaxed) profit. This new balance does not always exhibit higher mark-ups than the un-taxed regime.

In regions of quantity-price space where the tax applies, producers' optimize functionals that are a convex combination of uniform and pay-as-bid profit functionals. For a tax rate below a certain threshold a symmetric SFE exists that, compared to the equilibrium without the tax, has generally higher markups at low offer quantities but possibly smaller markups near the capacity constraint.

We discovered a counter-intuitive effect of the 'beneficiary-pays' charge in a duopoly setting. When the size of the line upgrade is small – and the probability of line-congestion is low – the consumer surplus can *increase* when the charge is applied, since firms submit offer curves that are strictly lower than the untaxed curves. Moreover, due to their competition, firms in fact receive a lower profit and actually pay more tax than they would under the un-taxed equilibrium.

References

E. J. Anderson and A. B. Philpott. Optimal offer construction in electricity markets. *Mathematics of Operations Research*, 27(1):82–100, 2002.

E.J. Anderson, P. Holmberg, and A.B. Philpott. Mixed strategies in discriminatory divisible-good auctions. *The RAND Journal of Economics*, 44(1):1–32, 2013.

Giulio Federico and David Rahman. Bidding in an electricity pay-as-bid auction. *Journal of Regulatory Economics*, 24(2):175–211, 2003.

R.J. Green and D.M. Newbery. Competition in the british electricity spot market. *Journal of Political Economy*, 100(5):929–953, 1992.

William W. Hogan. Transmission benefits and cost allocation, 2011. Harvard Electricity Policy Group working paper.

P. Holmberg. Unique supply function equilibrium with capacity constraints. *Energy Economics*, 30(1):148, 2008.

P. Holmberg. Supply function equilibria of pay-as-bid auctions. *Journal of Regulatory Economics*, 36(2): 154–177, 2009.

P. Holmberg and D. Newbery. The supply function equilibrium and its policy implications for wholesale electricity auctions. *Utilities Policy*, 18(4):209–226, 12 2010.

Paul D. Klemperer and Margaret A. Meyer. Supply function equilibria in oligopoly under uncertainty. *Econometrica*, 57(6):1243–1277, Nov. 1989.

NZEA. Transmission pricing methodology: issues and proposal consultation paper: Appendix E. Technical report, New Zealand Electricity Authority, October 2012. https://www.ea.govt.nz/dmsdocument/13801.

NZEA. Transmission pricing methodology review: beneficiaries-pay options. Technical report, New Zealand Electricity Authority, January 2014. https://www.ea.govt.nz/dmsdocument/17482.

Michael Pollitt. Electricity reform in argentina: Lessons for developing countries. *Energy Economics*, 30(4): 1536 – 1567, 2008.

Appendix