Line Capacity Expansion and Transmission Switching in Power Systems with Large-Scale Wind Power

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Abstract

In 2025 electricity production from wind power should constitute nearly 50 % of electricity demand in Denmark. In this paper we look at optimal expansion of the transmission network in order to integrate 50 % wind power in the system, while minimising total fixed investment cost and expected cost of power generation. We allow for active switching of transmission elements to eliminate negative effects of Kirchhoffs voltage law. Results show that actively switching transmission lines may yield a better utilisation of transmission networks with large-scale wind power and increased wind power penetration. Furthermore, transmission switching is likely to affect the optimal line capacity expansion plan.

1 Introduction

In 2025 electricity generation from wind power is planned to constitute nearly 50 % of demand for electricity in Denmark. This will primarily be achieved through a huge increase in the number of offshore wind farms.

As a consequence, massive changes are expected in the Danish electricity system in the years to come. Conventional power plants will close down, new market structures are expected to emerge as balancing needs and the requirement for flexible demand are going to increase. Transmission flows will change and the need for transmission capacity will increase.

In addition to the wind power development, it has been decided that all 132/150 kV overhead lines in Denmark shall be replaced by underground cables during the next 30 years. This constitutes a huge challenge for the Danish transmission system operator, Energinet.dk, however, it is also a great opportunity to redesign this part of the transmission grid as a whole.

Energinet.dk, is state owned and operates the grid on a non-profit basis. In general, transmission investments are carried out at lowest cost while maintaining a certain high level of security of supply. Connections abroad and large domestic grid investments are considered to have significant societal economical impact, and hence the socio-economic welfare effect of such investments must be evaluated. Only investments that provide positive overall socio-economic impacts are promoted. To ensure robust decisions, the impact of any larger investment is evaluated in a context of different future scenarios, each describing likely developments of the society twenty years ahead in time.

Traditionally, in Denmark, investments have mainly been considered incrementally. However, considering the underground cabling and the rapid wind power development there might be huge gains by coordinating investments to find an optimal future grid. In this paper, we consider the stochastic line capacity expansion problem with transmission switching. This model can be applied to the development of the Danish grid and we show that active switching of transmission lines increases the utilisation of the transmission grid.

The combinatorial complexity of the line capacity expansion problem makes it hard to solve. The deterministic version of this problem has been solved succesfully using Benders decomposition [17, 3] and a commercial MIP solver (CPLEX) [2]. In [8] a stochastic scenario based formulation of the line capacity expansion problem for competitive markets is presented.

Adding new lines to an electricity network may increase the cost of power generation. This paradox is due to Kirchhoff's *voltage law* and is demonstrated in [4]. Transmission switching [14] may alleviate such effects by switching out transmission lines. In general transmission switching may increase security of a network [18] and decrease cost of generation [12, 15]. Khodaei and Shahidehpour [16] propose a Benders decomposition approach for solving the dynamic line capacity expansion problem with transmission switching.

In this paper we apply the model proposed in [21] for the two-stage stochastic line capacity and switch investment problem with uncertain future generation capacities and demand. In the first stage decisions on line capacity expansions and switch investments are made. Line capacity expansions and switch investments are chosen from a candidate set of potential line upgrades. In the second stage operational decisions on power generation, line flows, and switching decisions are made in a number of scenarios reflecting variations in demand (e.g. peak/off-peak) and generation capacity (e.g. level of wind).

The contribution of this paper is two-fold. Firstly, we demonstrate that the line capacity expansion problem with transmission switching modeled as a two-stage stochastic mixed integer program can be solved efficiently for realistically sized networks using column generation. Secondly, we show that actively switching transmission elements in congested networks with large-scale wind power may reduce generation cost and increase the amount of wind power that can be integrated into the system. Furthermore, actively switching transmission lines may alter the optimal line capacity expansion plan.

We begin the paper by recalling a mixed integer progamming formulation of the direct current approximated optimal power flow problem with transmission switching [12]. In section 3 we state the line capacity expansion and switch investment problem as a two-stage stochastic program and provide the Dantzig-Wolfe reformulation [7] as proposed in [21]. Section 4 provides computational results for the IEEE 118-bus network and the Danish transmission network with large-scale wind power. Finally, concluding remarks and directions for future research is given in section 5.

2 Operational Dispatch

We assume a linear DC-approximation of the optimal power flow (see e.g. [6], [20]) with linear generation costs and no line losses.

Consider the directed graph $G = (\mathcal{N}, \mathcal{A})$ with a source/sink node s. For each arc $a \in \mathcal{A}$, the cost, capacity, and reactance coefficients are given by c_a, u_a, and r_a, respectively. The flow on each arc $a \in \mathcal{A}$ is denoted by x_a, while w_i denotes the voltage phase angle for each node $i \in \mathcal{N}$. Let the set of arcs $\mathcal{F}(i)$, respectively, $\mathcal{T}(i)$ denote the set of arcs with tail, resp. , head i. Let the set of supply and demand arcs $\mathcal{S} = \mathcal{F}(s) \cup \mathcal{T}(s) \subseteq \mathcal{A}$ be defined by having s as the tail or head, and let $r_a = 0$ for all $a \in \mathcal{S}$. The supply arcs $\mathcal{F}(s)$ represent generation units with marginal generation cost c_a and generation capacity u_a, while the demand arcs $\mathcal{T}(s)$ represent flexible demand with value of consumption $-c_a$ and maximum consumption u_a. Inflexible demand d_i must be met at all nodes i in $\mathcal{N}$.

Furthermore, let z_a be a binary variable indicating whether line e is active ($z_a = 0$) or not. An active line is one which is installed and not switched out. Define by $\mathcal{D}$ the set of direct current (DC) lines, while $\mathcal{E}$ denotes the set of existing lines and $\mathcal{H}$ the set of switchable lines.

For a single time period (snapshot) the globally optimal state (with maximum social surplus) of the system may be found by solving the mixed integer linear program

$$\min \sum_{a \in \mathcal{A}} c_a x_a \tag{1}$$

subject to

$$
\begin{aligned}
x_a &\leq u_a z_a && \forall a \in \mathcal{A} && (2)\\
x_a &\geq l_a z_a && \forall a \in \mathcal{A} && (3)\\
\sum_{a \in \mathcal{T}(i)} x_a - \sum_{a \in \mathcal{F}(i)} x_a &= d_i && \forall i \in \mathcal{N} \setminus \{s\} && (4)\\
z_a = 0 \Rightarrow r_a x_a - w_i + w_j &= 0 && \forall a = (i,j) \in \mathcal{A} \setminus (\mathcal{S} \cap \mathcal{D}) && (5)\\
z_a &\in \{0,1\} && \forall a \in \mathcal{A} && (6)\\
z_a &= 0 && \forall a \in \mathcal{A} \setminus \mathcal{H} && (7)
\end{aligned}
$$

where the objective (1) minimises the total cost of operation subject to the following constraints. Capacity limits on arc flows (2)-(3). Conservation of flow (4) at each node except the source node s. Kirchhoffs *voltage law* (5) for all active arcs. The *switching variables* z_a are binary (6) and fixed for lines that are not switchable (7).

Furthermore, we may restrict the number of *existing* lines that are being switched.

$$\sum_{a \in \mathcal{E}} z_a \leq k \tag{8}$$

Note, that constraints (5) may be linearised using a big-M construction

$$-M z_a \leq r_a x_a - w_i + w_j \leq M z_a, \qquad \forall a = (i,j) \in \mathcal{A} \setminus (\mathcal{S} \cap \mathcal{D}) \tag{9}$$

where M is some sufficiently large number.

3 A Two-Stage Stochastic Model with Transmission Switching

We now consider a two-stage stochastic model, where the first stage decisions involve investment in switching equipment y_S and line capacity y_L, while the second stage models operational decisions (x, w, z) for dispatch and switching in a number of scenarios $\omega \in \Omega$ each occuring with probability $p(\omega)$. For each scenario $\omega \in \Omega$, let

$$\mathcal{Q}(\omega) = \{(x(\omega), w(\omega), z(\omega)) \mid (2) \text{ - } (8)\}.$$

The model may now be formulated as

$$\min \; f_S^\top y_S + f_L^\top y_L + \sum_{\omega \in \Omega} p(\omega) c(\omega)^\top x(\omega) \tag{10}$$

$$\text{s.t.} \quad y_L - y_S + z(\omega) \leq 1 \qquad \forall \omega \in \Omega \tag{11}$$
$$y_L + z(\omega) \geq 1 \qquad \forall \omega \in \Omega \tag{12}$$
$$(x(\omega), w(\omega), z(\omega)) \in \mathcal{Q}(\omega) \qquad \forall \omega \in \Omega \tag{13}$$
$$y_L, y_S \in \{0,1\}^{|\mathcal{A}|} \tag{14}$$

The objective (10) minimizes the hourly fixed and operational cost, while (11) ensures that switching of installed lines is only possible if a switch is also installed. Constraint (12) allows lines to be switched in only if they are also installed. We set $e_a^\top y_L = 1$ and $e_a^\top f_L = 0$ for existing lines a in $\mathcal{E}$, where e_a is the binary unit vector of $|\mathcal{A}|$ elements with the ath element being 1.

Note, that not installing a line corresponds to having the line switched out (i.e. $z(\omega) = 1$) in all scenarios $\omega \in \Omega$.

3.1 Dantzig-Wolfe Reformulation

The mathematical program (10)-(14) may be reformulated using Dantzig-Wolfe [7] and a branch-and-price algorithm may be applied to obtain optimal solutions to this reformulation.

The idea is to decompose the stochastic problem into a master problem and a number of sub-problems — one for each scenario. We let the binary vector $z(\omega)$ define a *feasible switching plan* (FSP) for scenario ω if there exists $x(\omega), w(\omega)$ such that $(x(\omega), w(\omega), z(\omega)) \in \mathcal{Q}(\omega)$. Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of all FSP's for scenario ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can write any element in $Z(\omega)$ as

$$\begin{aligned} z(\omega) &= \sum_{j \in J(\omega)} \varphi^j(\omega) \hat{z}^j(\omega) \\ \sum_{j \in J(\omega)} \varphi^j(\omega) &= 1, \quad \varphi^j(\omega) \in \{0,1\},\ \forall j \in J(\omega). \end{aligned}$$

Assume that for each feasible switching plan $\hat{z}^j(\omega)$ the corresponding optimal dispatch of generation and load shedding is given by $\hat{x}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{x}$ as

$$\text{MP:} \ \min \ f_L^\top y_L + f_S^\top y_S + \sum_{\omega \in \Omega} \sum_{j \in J_\omega} p(\omega) c(\omega)^\top \hat{x}^j(\omega) \varphi^j(\omega) \tag{15}$$

$$\text{s.t.} \quad \sum_{j \in J(\omega)} \varphi^j(\omega) = 1 \qquad [\mu(\omega)],\ \forall \omega \in \Omega \tag{16}$$
$$y_L - y_S + \sum_{j \in J(\omega)} \hat{z}^j(\omega) \varphi^j(\omega) \leq 1 \qquad [\pi(\omega)],\ \forall \omega \in \Omega \tag{17}$$
$$y_L + \sum_{j \in J(\omega)} \hat{z}^j(\omega) \varphi^j(\omega) \geq 1 \qquad [\rho(\omega)],\ \forall \omega \in \Omega \tag{18}$$
$$\varphi^j(\omega) \in \{0,1\}, \qquad \forall j \in J(\omega) \tag{19}$$
$$y_L, y_S \in \{0,1\}^{|E|} \tag{20}$$

where $\mu(\omega)$, $\pi(\omega)$ and $\rho(\omega)$ denote the dual prices associated with the respective constraints.

The master problem MP is a two-stage stochastic integer program with integer variables in both stages. Although in general these are difficult to solve, the structure of MP is amenable to a branch-and-bound procedure by virtue of the following result.

Proposition 1 *If y_L and y_S are chosen to be fixed binary integers, then the linear programming relaxation of MP has integer extreme points.*

For a proof we refer the reader to [21].

It is convenient to consider only a subset $Z(\omega)' \subseteq Z(\omega)$ of feasible switching plans for each scenario ω in the master problem. We define this restricted master problem (RMP) by (15) - (20) with $J(\omega)$ replaced by $J(\omega)'$ the index set of $Z(\omega)'$. A column generation algorithm is applied to dynamically add feasible switching plans to the linear relaxation of the master problem. The algorithm is initialised by letting $Z(\omega)' = \{\hat{z}^0(\omega)\} = \{\mathbf{0}\}$, for all scenarios $\omega \in \Omega$. That is, initially no line may be switched out in either scenario. The corresponding operational costs $c(\omega)^T \hat{q}^0(\omega)$ can easily be found by solving a linear program for each scenario. In each iteration of the algorithm, the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices μ, π, and ρ. A new column $(p(\omega)c(\omega)^T \hat{x}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = p(\omega)c(\omega)^\top \hat{x}^j(\omega) + \pi(\omega)^\top \hat{z}^j(\omega) - \rho(\omega)^\top \hat{z}^j(\omega) - \mu(\omega)$ is negative.

A column for scenario ω may therefore be constructed by solving the subproblem:

$$\begin{array}{ll} \min & p(\omega)c(\omega)^\top x + \pi(\omega)^\top z - \rho(\omega)^\top z - \mu(\omega) \\ \text{s.t.} & (x, w, z) \in \mathcal{Q}(\omega), \end{array}$$

where $\mu(\omega)$, $\pi(\omega)$ and $\rho(\omega)$ are the dual prices returned from RMP-LP.

Any feasible solution $(x, w, z) \in \mathcal{Q}(\omega)$ with negative objective function gives rise to a potential candidate column for RMP-LP. If no columns with negative reduced cost exist then we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral then (φ^*, y^*) is an optimal solution to the master problem (15) - (20) and y^* is the optimal switch investment strategy. Otherwise, we may resort to a branch-and-price framework for finding optimal integral solutions. Note that a fractional solution will always have at least one fractional y-value (see Proposition 1). Hence, we branch on one of the fractional y-variables and hope that this will resolve the fractionality. If not one may continue branching on y-variables until the fractionality is resolved.

4 Computational Results

In this section experiments are performed on two different networks — the IEEE 118 bus network and the Danish transmission network. Experiments with the IEEE 118 bus network with four scenarios suggests that transmission switching is beneficial for the integration of large-scale wind power. Also, these results justify the use of stochastic programming. Results for the Danish network with the expected development of off-shore wind power generation by 2025 confirm that allowing switching may reduce generation cost and increase the amount of wind power integrated in the system.

4.1 The IEEE 1118 Bus Network

We will first study the IEEE 118 bus network [1] with network data described in [5]. This network has 185 lines, total peak load of 4519 MW, and a total thermal generator capacity of 5859 MW. We will consider a four scenario instance of the switch investment problem presented in section 3 with uncertain outcomes of demand and wind generation capacity. First stage decisions include only investment decisions in switching equipment. That is we assume $y_L = \mathbf{1}$ to be fixed. The results justify the use of stochastic programming and indicates that transmission switching is particularly beneficial in systems with large-scale wind power.

Four scenarios are defined with respect to the load level (peak/off-peak) and amount of wind power (high/low). The scenarios have equal probabilities and are summarised in Table 1. The fixed amortized switch investment costs are arbitrarily set to \$5/h for each switch.

A 1600 MW intermittent wind power generator with varying supply capacity and 0 marginal cost is located at node 91. Generation from the windpower generator is not fixed so wind generation may be curtailed.

Scenario	Probability	Load		Windpower	
ω	$p(\omega)$		% of peak		capacity, MW
1	0.25	off-peak	59%	high	1600
2	0.25	peak	100%	high	1600
3	0.25	off-peak	64%	low	0
4	0.25	peak	99%	low	0

Table 1: Summary of scenarios for a small instance of the switch investment problem.

Without switches the total generation cost incurred is \$1031.55/h. In the optimal switch investment strategy with $k = 3$ five switches are installed incurring a total investment cost of \$25/h and generation cost \$910.25/h. The total savings from switching is thus approximately 9%. With optimal switching the dispatched windpower is increased from 499 MW to 648 MW in scenario 1 and from 535 MW to 875 MW in scenario 2. Thus by employing active switching one can increase the amount of windpower in the system and decrease system cost. The optimal switching configurations and the corresponding saved operational costs for each scenario are shown in Table 2.

ω	Switching configuration					Saved costs
1	E77-80	E89-90		E89-92		7.6
2	E77-80	E89-90		E89-92		36.0
3	E77-80	E89-90			E94-96	2.2
4	E77-80		E89-91		E94-96	75.4
Total						121.2

Table 2: Optimal switching configurations and saved operational costs for a small four-scenario instance of the switch investment problem on the IEEE 118 bus network.

Since in general (1)-(8) is a difficult mixed integer program for $k \geq 1$ one might consider to decouple the scenarios and solve each scenario separately with amortized investment costs and subsequently piece the solutions together. While this approach might yield good solutions for some instances, we cannot rely on this in general. Applying this approach to our four-scenario instance described above by solving four smaller mixed integer programs, we obtain an investment strategy with nine switches and total operational cost of \$904/h. The net benefit (including switch installation costs) of switching is only \$82.6/h as opposed to \$96.3/h for the optimal switch investment strategy obtained by solving the integrated model. Hence, the value of switching is clearly lower when decoupling the scenarios completely.

We now consider an instance of the problem where we — in addition to have a wind power park at node 91 — also have wind power parks in node 5 and node 26. All wind power parks are assumed to be relatively large (1600 MW installed capacity). The scenarios and network are unchanged.

Without switching the total expected generation cost is \$881.56/h. With switching ($k = 3$) this is decreased to \$750.45/h with a total of five installed switches leading to a net benefit of \$106.12/h or approximately 12 %. This reduction in costs covers an increase in wind power on the three parks by a total of 430 MW in scenario 1 and 293 MW in scenario 2 (see Table 3). The corresponding optimal switching configuration is shown in Table 4.

Scenario	$k=0$			$k=3$		
ω	91	5	26	91	5	26
1	538	918	383	737.43	937.25	595.08
2	499.7	479	453	633.13	697.49	395.28
3	0	0	0	0	0	0
4	0	0	0	0	0	0

Table 3: Wind power generation in different scenarios without ($k = 0$) and with ($k = 3$) switching.

ω	Switching configuration				
1	E77-80		E94-96	E23-25	
2	E77-80			E23-25	
3				E23-25	
4		E89-91			E38-37

Table 4: Optimal switching configurations for a small four-scenario instance of the switch investment problem on the IEEE 118 bus network.

4.2 The Danish Transmission Network

We will now consider the current Danish transmission network and potential line capacity expansions in a future setting with development of many new off-shore wind power plants.

Potential off-shore windpower development in Denmark in the period 2010 - 2025 is described in [9] and [11]. The projects considered are summarised in table 5. We assume that all projects are realised.

Network and generation data is obtained from the Danish transmission system operator Energinet.dk. This data is confidential, but aggregated values are available in Table 6. Potential line capacity expansion projects are likewise based on confidential data form Energinet.dk. Line investment costs for 400kV lines are based on underground cable costs - overhead lines are not considered. This is in accordance with future expansion guidelines for the Danish electricity transmission grid [10]. The potential candidates for new transmission lines are limited to a set of 10 lines on the 400 kV level and 5 lines on 132 kV level.

Neighbouring areas (Norway, Sweden, Germany, and The Netherlands) are modeled in a very simplistic way with no demand and a generator with fixed marginal cost and capacity. This assumes that there is always excess generation capacity in the respective areas which can be supplied at constant cost.

The ability to switch a particular line incurs a fixed investment cost. This is assumed to cover any equipment needed to perform automatic switching of that line including the switch itself (if it needs to be upgraded) and any communication equipment if necessary. Here, we arbitrarily assumes a relatively small fixed cost of 1 DKK/h. In the experiments swithcing was allowed on high voltage (>100 kV) transmission lines only.

name	capacity (MW)	area
Djursland	400	DK1
Horns Rev	1000	DK1
Læsø	600	DK1
Jammerbugt	800	DK1
Ringkøbing	1000	DK1
Kriegers Flak	800	DK2
Rønne Banke	400	DK2
S. Middelgrund	200	DK2

Table 5

We shall now consider a particular six-scenario instance of the problem. In Table 7 a summary of

no. of nodes (busses)	610
no. of transmission lines	529
no. of transformers	302
no. of generators	418
total gen. capacity (DK)	13530
total peak demand	6945

Table 6: Summary of network data.

the scenarios are given. The scenarios and their probabilities are for illustration only, and do not reflect our true expectations for the future. Nevertheless, they do give some valuable insights – in particular with regard to the value of switching and its impact on investments in line capacity.

		Demand (MW)		Wind capacity	
ω	$p(\omega)$	DK1	DK2	on-shore	off-shore
0	0.16	4076	2869	0.90	0.95
1	0.16	4076	2869	0.50	0.50
2	0.17	4076	2869	0.00	0.00
3	0.17	1448	934	0.90	0.95
4	0.17	1448	934	0.50	0.50
5	0.17	1448	934	0.00	0.00

Table 7: Summary of scenarios. Wind capacity is the share of installed capacity.

Five instances of the problem with different levels of switching is investigated: *no switch* and $k = 0, 1, ..., 3$. The *no switch* instance has fixed $y_S = 0$ and allows for investments in new lines only. The $k = 0$ instance allows switching on new lines only — no switches on existing lines is allowed. For the instances with $k = 1, 2, 3$ switching on all new lines and at most k existing lines are allowed. Table 8 gives a summary of results for the different instances, while Table 9 gives an overview of the benefit of switching.

For the six scenarios described above, the optimal solution contains investments in 5 of the possible 15 new lines without switching (total cost of 466970 DKK/h of which 141 DKK/h are investment costs). Allowing to switch new lines ($k = 0$), results in a total of 10 new lines installed of which 7 may be switched (Table 8). This gives a net-benefit of 12366 DKK/h (Table 9).

Figure 1 shows part of the 400kV network topology in Eastern Denmark, with proposed network expansions, while Figure 2 shows the partial optimal line capacity expansion plan for that part of the network when no switching is allowed. The network shown in Figure 3 depicts the optimal line capacity expansion plan when switching is allowed on new lines only ($k = 0$).

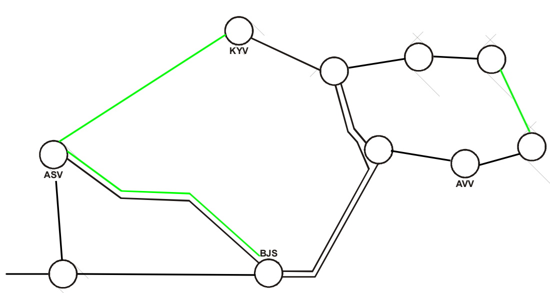

Figure 1: Part of the existing Eastern 400kV topology (black) and potential expansion options (green).

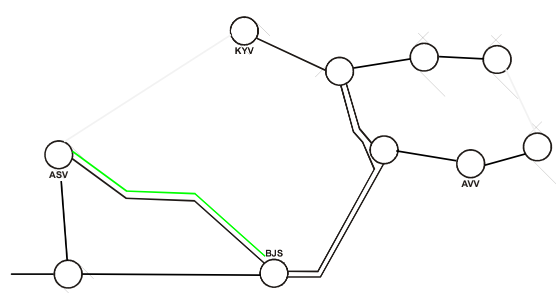

Figure 2: Partial line capacity expansion plan without switching.

In the following experiment, we – in addition to allowing switching on new lines – also allow for switching of one existing line in each scenario ($k = 1$). This yields an investment plan with 8 new lines and 12 switches. The total net benefit of the solution is 32706 DKK/h or 7% compared to the solution without switching. Figure 4 depicts (part of) the corresponding optimal line capacity expansion plan.

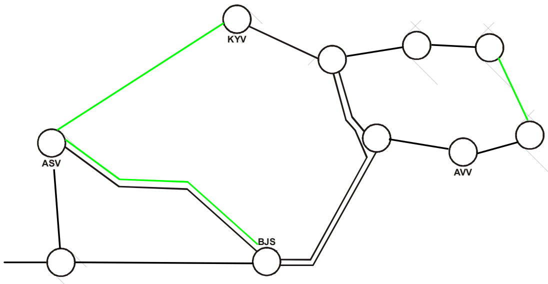

Figure 3: Partial line capacity expansion plan ($k = 0$).

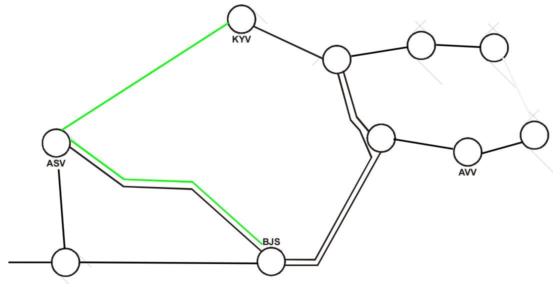

Figure 4: Partial line capacity expansion plan ($k = 1$).

By allowing switching the wind generation is increased by up to 251 MW in scenario 0 ($k = 3$). Also, operational costs are reduced significantly in the peak demand without wind (scenario 2) by switching to lower cost (thermal) generation.

	no switch	$k = 0$	$k = 1$	$k = 2$	$k = 3$
no. of installed lines	5	10	8	11	10
no. of installed switches	-	7	12	13	15
wind (MWh/h)	2649	2661	2687	2687	2689
fixed cost (DKK/h)	141	311	283	320	319
op. cost (DKK/h)	466829	454293	433982	430916	427294
total cost (DKK/h)	466970	454604	434265	431237	427613

Table 8: Summary of results for different levels of switching.

	$k = 0$		$k = 1$		$k = 2$		$k = 3$	
	abs	rel	abs	rel	abs	rel	abs	rel
op. cost, DKK/h	-12536	-3	-32847	-7	-35913	-8	-39535	-8
fixed cost, DKK/h	170	120	142	100	179	127	178	126
total cost, DKK/h	-12366	-3	-32706	-7	-35734	-8	-39357	-8
wind (avg.), MWh/h	12	0.46	38	1.42	38	1.42	40	1.52
wind ($\omega = 0$), MWh/h	76	1.13	235	3.53	236	3.53	251	3.77

Table 9: Benefit of switching. Values are absolute and relative (in %) difference as compared to the non-switched network ($y_S = 0$).

The results obtained from experiments with the Danish transmission network with large-scale wind power suggests that transmission switching may reduce generation cost and increase wind power generation. As transmission switching acts to reduce congestion in the network, this reduction in cost is entirely due to relief of congestion in the peak demand scenarios.

More interestingly, the optimal line expansion plan is highly sensitive to the level of switching allowed. Actively switching transmission elements increases the number of installed transmission lines (roughly by a factor of 2). This is due to the fact that some lines may be beneficial in some scenarios, but restrictive in others.

The previous experiments include only a few *extreme* scenarios with equal probabilities. In addition, we now introduce a *medium* scenario with a high probability in order to investigate the proportionality of cost and wind power generation. The scenarios are summarised in Table 10. Also, the cost of adding a switch has been quadrupled. Both of these changes are expected to discourage the use of transmission switching.

Results for the seven scenario instances are summarised in Table 11. We see that increasing the

ω	$p(\omega)$	Demand (MW)		Wind capacity	
		DK1	DK2	on-shore	off-shore
0	0.0833	4076	2869	0.90	0.95
1	0.0833	4076	2869	0.50	0.50
2	0.0833	4076	2869	0.00	0.00
3	0.0833	1448	934	0.90	0.95
4	0.0833	1448	934	0.50	0.50
5	0.0833	1448	934	0.00	0.00
6	0.5000	2869	1902	0.30	0.30

Table 10: Summary of scenarios for the instances with 7 scenarios. Wind capacity is the share of installed capacity.

cost of switches and introducing a new scenario, results in different investment strategies for $k > 0$ with fewer (or the same) switches and line expansions.

	no switch	$k = 0$	$k = 1$	$k = 2$	$k = 3$
no. of installed lines	6	10	8	8	7
no. of installed switches	-	7	10	10	12
wind (MWh/h)	2860	2861	2874	2874	2875
fixed cost (DKK/h)	254	332	300	300	323
op. cost (DKK/h)	339693	332965	322614	321094	319265
total cost (DKK/h)	339947	333297	322914	321394	319588

Table 11: Summary of results for different levels of switching with seven scenarios.

Table 12 shows the benefit of allowing to switch transmission lines in the instances with seven scenarios. It is seen that increasing the cost of switches and introducing a *medium* scenario does reduce the benefit of switching considerably. However, the benefit is still significant. Switching allows a reduction in total cost of up to 6% and increases wind power generation in scenario $\omega = 0$ by up to 187 MW.

	$k = 0$		$k = 1$		$k = 2$		$k = 3$	
	abs	rel	abs	rel	abs	rel	abs	rel
op. cost, DKK/h	-6728	-1.98	-17080	-5.03	-18599	-5.48	-20428	-6.01
fixed cost, DKK/h	78	30.76	46	18.11	46	18.11	69	27.15
total cost, DKK/h	-6650	-1.96	-17034	-5.01	-18553	-5.46	-20359	-5.99
wind (avg.), MWh/h	1	0.03	14	0.50	14	0.50	16	0.55
wind ($\omega = 0$), MWh/h	12	0.17	171	2.55	172	2.55	187	2.78

Table 12: Benefit of switching. Values are absolute and relative (in %) difference as compared to the non-switched network ($y_S = 0$).

We acknowledge that the scenarios described here are not truly representative and that more work is necessary to identify a set of scenarios representing our true expectation of the future.

4.3 Running times

Optimal solutions for the six-scenario instances described above was obtained using column generation. The model was implemented using the COIN-OR DIP framework [13] and instances were solved using default parameters except that each node was solved to optimality before branching (`TailOffPercent = 0`), compression of columns was turned off (`CompressColumns = 0`), and the master problems were solved to optimality (`MasterGapLimit = 0`) using interior point method (CPLEX 12.2 barrier). Subproblems were solved using CPLEX 12.2 MIP-solver. Table 13 gives a summary of running times for different instances of the problem with branch-and-price (DIP) and CPLEX.

Instance		Branch-and-price					CPLEX	
		time (s)		price-	no.			
$\|\Omega\|$	k	total	master	passes	nodes	gap	time (s)	gap (%)
6	-	740	131	134	3	0	8.4	0.00
6	0	126	11	20	1	0	431	0.00
6	1	965	68	35	1	0	2239	† 0.00
6	2	2592	58	32	1	0	8999	† 0.02
6	3	4795	57	31	1	0	10006	† 0.02
12	1	4094	201	62	1	0	-	-
24	1	11982	178	62	1	0	-	-

Table 13: Computational results for solving the Dantzig-Wolfe reformulation using branch-and-price and the compact formulation using CPLEX. Gap is relative (in %) from best known solution.

Except for the instance without switching all instances were solved to optimality in the root node — that is no branching was needed. For all instances with switching column generation seems to be superior to solving the compact formulation using a commercial MIP solver (CPLEX). We were able to solve for 24 scenarios with $k = 1$ in less than 12000 s. Solution times for the column generation approach seem to scale relatively well with the number of scenarios. However, the majority of the solution time is used to solve subproblems and this is prohibitive for the number of scenarios that can be solved in reasonable time — especially for values of k larger than 1.

5 Conclusion

In this paper we have treated the line capacity expansion problem with transmission switching under future uncertainty in demand and wind generation capacity. The problem is formulated as a two-stage stochastic program and the Dantzig-Wolfe decomposition is solved using column generation.

Results indicate that the topology of the transmission network is important for the dispatch of wind energy and that intermittent generation calls for a dynamically optimised topology. This can be achieved by actively switching transmission lines. Our results show that transmission switching may reduce curtailment of wind power with up to 250 MW in peak demand for the Danish network under study. Also, switching of transmission elements may influence the optimal line capacity expansion strategy, making it worthwhile to install more new transmission capacity. Solving the decomposed model makes it possible to solve instances for real networks in reasonable time and is superior to solving the compact formulation using a commercial MIP solver (CPLEX). Furthermore, the decomposition approach seems to scale well with increased number of scenarios.

The Danish network presented in this paper is isolated from the remaining European electricity transmission network. In order to obtain more realistic results further work is needed to represent the neighbouring areas in a better way. This is important as large scale wind power generation is also under way in other parts of Northern Europe.

The results presented here are only for a limited number of scenarios, that may not reflect our true expectation of the future. Further work is needed to identify realistic and representative scenarios. Other stochastic parameters may be relevant such as generation prices (depending on water values of hydro power generation units, oil prices, etc.). Also, geographically dependent wind power generation time series is highly relevant in order to capture periods of high wind power in one part of the network and low wind power in other parts of the network. Even though such outcomes may occur only with low probability (e.g. only for short periods of time), this may increase further the need for a dynamic network topology and the value of transmission switching.

In practice, expansion of transmission line capacity and investment in new off-shore wind power plants is performed over a planning period of many years. At each stage of the planning period the expectation of the future is changed as more information becomes available and so the optimal expansion plan may change as well. This model can be extended to a multi-stage formulation following the approach in [19]. In a multi-stage setting decisions on line capacity expansions may be made at any stage, while the expansion of wind power capacity may be subject to uncertainty.

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