Modelling counter-intuitive effects on cost and air pollution from intermittent generation

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Abstract

In this paper, we first present a market environment with a conventional two settlement mechanism. We show that when we add some wind generation to the system, the steady-state market conditions yield lower social and consumer welfare and higher use of fossil fuels. We also present results of a counterfactual stochastic settlement market which improves social and consumer welfare after the introduction of new intermittent generation. Thus, we conclude that the choice of market mechanism is a critical factor for capturing the benefits of large-scale wind integration.

We also introduce a method to compute analytical equilibria of games in which the payoff functions of players depend on the optimal solution to an optimization problem with inequality constraints.

1 Introduction

The benefits of renewable energy are becoming clearer as economies of scale begin to drive down the capital costs and charges are applied to carbon emissions. It is likely that renewables will further increase their competitiveness and soon surpass traditional generation technology. Many papers have investigated the benefits resulting from using renewable resources; for example, [1], [2], [3], [4], and [5] discuss the economic and environmental incentives for using renewable energies. Two of the key drivers for the current interest in renewable energy are: the reduction in CO_2 emissions, and the desire to reduce dependence on imported oil and gas. These provide a strong incentive for many countries to turn to wind and other renewable energy resources to meet some of their energy needs.

Penetration of renewable resources has occurred much faster than anticipated and 16.7% of global final energy consumption was supplied by renewable sources in 2011 [3, 6]. Worldwide, the wind penetration growth rate has been 27.6% each year, and in fact, some countries have developed their wind generation capacity much more rapidly than this. For example, the growth-rate for Mexico was 373% in 2009 [7].

The uncertainty in wind makes its integration with the electricity grid a difficult task, as far as the operation and planning of the power system is concerned [8, 9]. Increasing wind penetration calls for more accurate wind prediction systems and more short-term and long-term reserves [8]. Readers should be aware that the unpredictability of wind production is much higher than the uncertainty surrounding demand forecasts. Demand can be predicted with a good accuracy, as it depends on (mostly) predictable parameters such as temperature and time of day/year [10]. Thus, introducing wind turbines (which are dispatched to the extent that the wind blows) to the electricity grid calls for fast-responding (usually expensive) generators to make up for the lack (or surplus) of wind generation in some scenarios. Compounding this issue is the fact that wind generation, in periods of low demand (e.g. during night), cannot be stored to make up shortage of electricity in the other time periods. Increasing intermittent generation penetration exacerbates these concerns.

One approach towards integration of intermittent generation is treating these generators as conventional generators, while another approach could be to consider intermittent generation as negative demand. Since some intermittent generation (e.g. wind) is uncontrollable, such generators cannot be integrated as conventional generators into the electricity grid. Moreover, intermittency prevents them from being used strategically, and therefore, may cause a bias towards investment in other generators. On the other hand, fully dispatching wind generation (i.e. considering it as negative demand) creates uncertainty for the other generators (i.e. wind generators are not paying for the cost they impose on the system) [11]. Several remedies have been suggested for integrating intermittent generators into the electricity network (for example, see [12, 13]). However, these remedies are not very effective for large-scale penetration and new support schemes and mechanisms are called for [13]. One scheme, used in the Nordic market, involves wind generators being penalised (through a lower price) if their production imbalance contributes to the system imbalance; this is a way of making the generators pay for the costs they impose on the system [14].

Green and Vasilakos [15] provide a numerical supply function equilibrium model for the UK electricity market in 2020. They investigate the impact of large-scale wind generation on electricity prices, and conclude that the amount of hourly wind generation will dramatically affect electricity prices. In Addition, market power will increase the level and the volatility of prices. Negrete-Pincetic et al. [11] have examined the negative effects of introducing a large-scale intermittent generation into the electricity grid. Their paper uses a simulation approach to quantify these effects, and emphasizes the need for designing new market-clearing mechanisms for dealing more effectively with the uncertainty introduced by intermittent generation. However, one important point lacking from their analysis is the modelling of the strategic behaviour of firms after the introduction of wind. They assume that the supply function offers of the generating firms will not change after the integration of wind farms into the system. This may not necessarily be the case in an electricity market with only a few participants in absence of strict monitoring.

In this paper, we consider the strategic behaviour of firms, in a market with and without wind generation. We aim to investigate the impacts of large-scale wind integration on criteria such as expected social welfare and generation from fossil fuels. To analyse this, we construct a stylized analytical model to understand the changes in incentives of firms when stochastic wind generation is present in the market. Specifically, we construct linear supply function equilibria for markets with elastic demand and compare the equilibrium prices and welfare before and after the integration of wind. As shown in [16] and [17], linear SFE models are good alternatives to general SFE models, as firstly under linear marginal cost and demand functions, a linear SFE is also an equilibrium to the unrestricted general SFE model, and secondly they are more tractable for complex markets.

In order to compute these equilibria, we relax the non-negativity inequalities in the optimization problem solved by the Independent System Operator (ISO), which would otherwise lead to potential non-convexities. To do this we construct an equivalent game, whose equilibrium is an equilibrium to the original game, but one which permits an analytical solution. The methodology we employ to solve this problem is reasonably general and can be used in other contexts.

We then discuss some of our results in the context of CO_2 emissions and demonstrate that the touted benefits of renewable generation (such as wind) may not be realised due to the intermittent nature of such generation technology.

Finally, we present an alternate market mechanism which under the same circumstances does not exhibit the same outcomes.

2 Market environment

We consider a single-node market with two types of generators: m cheap generators (e.g. nuclear generators) with a low short run marginal cost (SRMC) and one more expensive thermal generator with high SRMC (later we also add a wind generator to our model). Our expensive generator has the capability to change output rapidly without incurring considerable extra cost for this change, i.e. it is flexible. A further assumption is that the thermal generator is a non-strategic generator. (For example, in the New Zealand Electricity Market (NZEM), a diesel plant at Whirinaki is owned by the government and is offered in as a non-strategic peaking plant.) The m cheap generators need to know their proposed production for some time beforehand. They cannot increase their production instantaneously, nonetheless they can cut down their generation.

In order to assess the cost and operating implications of adding intermittent generation to such a market, we add a single wind generator to our model. We assume that the short-run marginal cost of production from the wind generator is \$0. In addition, it is going to be added to the system as a price taker generator. Due to market regulations it must offer at price \$0, at all times. This happens in currently active markets such as NZEM, and the Pennsylvania-New Jersey-Maryland Interconnection (PJM) [18, 19].

After the introduction of this new generator, which introduces uncertainty into the market, a pre-dispatch market is insufficient, and a real-time market clearing is required for dealing with the sudden deviations in wind production.

In this paper, we aim to understand the possible implications of adding this type of generator to the market by comparing the steady-state (equilibrium) behaviour of firms before and after this modification.

Let q_i denote the quantity to be generated by generator i. We assume that the cost of generating q_i for generator i is given by $\Psi(q_i) = \alpha_i q_i + \frac{\beta_i}{2} {q_i}^2$.

We index the hydro generators $i = 1, \ldots, m$, and the thermal and wind generators are indexed $i = t$, and $i = w$, respectively. The marginal cost of generation is $\alpha_i + \beta_i q_i$, with

$$\alpha_i = \begin{cases} \alpha, & i \in \{1, \ldots, m\}, \\ \bar{\alpha}, & i = t, \\ 0, & i = w, \end{cases}$$

$$\beta_i = \begin{cases} \beta, & i \in \{1, \ldots, m\}, \\ \bar{\beta}, & i = t, \\ 0, & i = w. \end{cases}$$

Moreover, we assume $0 \leq \alpha \leq \bar{\alpha}$ and $0 < \beta \leq \bar{\beta}$.

Let Q and $p(Q)$ denote total market generation and market price, respectively. Demand is assumed to be elastic and deterministic, with a linear inverse demand function

$$p(Q) = Y - ZQ.$$

This demand function can also be used to investigate the case of (nearly) inelastic demand by choosing the appropriate Y and Z parameters.

3 Market before the introduction of wind

3.1 Market description

Prior to the addition of the wind generator, conditions are deterministic and a forward (pre-dispatch) market is enough to determine dispatch quantities and prices (i.e. to clear the market). Firms offer a linear supply function ($a_i + b_i q_i$) as an indication of their marginal cost function ($\alpha_i + \beta_i q_i$). We assume $b_i > 0$.

In this setting, ISO's deterministic problem takes the following form:

ISODP :

$$\begin{aligned} \min_{q,Q} z = \quad & \sum_{i=1}^{m} \left(a_i q_i + \frac{b_i}{2} q_i^2 \right) + a_t q_t + \frac{b_t}{2} q_t^2 - \left(YQ - \frac{Z}{2} Q^2 \right) \\ s.t. \quad & \sum_{i=1}^{m} q_i + q_t - Q = 0 \qquad (1) \\ & q_t \geq 0. \end{aligned}$$

In ISODP, Q is the total dispatched quantity (i.e. met demand), and the ISO's objective is to maximize observed social welfare, assuming firms offer their true marginal cost. Note that we assume a single node market as reflected in the ISO's problem.

Remark One implicit (and logical) assumption we have made, is that both α and $\bar{\alpha}$ (i.e. minimum marginal cost of generation), are less than or equal to Y (i.e. maximum price). (Otherwise, they would not participate in the market.) This assumption ensures a positive total generation in the equilibrium. Therefore, the cheaper generators (with $\alpha \leq \bar{\alpha}$ and $\beta \leq \bar{\beta}$) will have a total positive dispatch in the equilibrium, even without imposing a constraint in ISO's problem. In this paper, we have omitted the non-negativity constraints of the cheap generators from ISODP for simplicity of computation. However, the final symmetric equilibrium of this game in our examples satisfies these constraints.

The dual variable corresponding to constraint (1) determines the market price f, and firm i is paid fq_i, regardless of its offered supply function (under the assumption of uniform pricing). Therefore, the utility (i.e. profit) function of firm i is

$$u_i = fq_i - \left(\alpha_i q_i + \frac{\beta_i}{2} q_i^2\right).$$

3.2 A simpler equivalent game

As the price a firm is paid is determined through ISODP, and the firm's offer strategy affects its dispatched quantity and final prices, each firm wishes to maximize profit solving an optimization problem subject to ISODP. Due to the constraint $q_t \geq 0$ in ISODP, it is not easy to find a closed-form solution to ISODP. Therefore, in presence of this constraint, the profit optimization problem of the firms becomes a non-convex problem. This would render the equilibrium problem a tri-level optimization problem to which it is difficult to find a closed form solution.

To simplify the computations, we present an equivalent game which uses a relaxation to the ISODP by omitting the non-negativity constraint:

$$\begin{aligned} &\text{ISODP}^r: \\ &\min_{q,Q} z \;=\; \sum_{i=1}^m \left(a_i q_i + \frac{b_i}{2} q_i^2\right) + a_t q_t + \frac{b_t}{2} q_t^2 - \left(YQ - \frac{Z}{2} Q^2\right), \\ &s.t. \quad \sum_{i=1}^m q_i + q_t - Q = 0. \end{aligned}$$

In order to find the steady-state behaviour of participants in the equivalent game, we need an analytic solution to ISODP^r, described above. In order to make the closed form solution analytically tractable, we use the one-to-one transformation described in lemma 3.1.

Lemma 3.1 *Transformation* $H : \begin{pmatrix} \mathbb{R} \\ \mathbb{R}^+ \setminus \{0\} \end{pmatrix} \to \begin{pmatrix} \mathbb{R} \\ \mathbb{R}^+ \end{pmatrix}$, *where* $H\begin{pmatrix} a_i \\ b_i \end{pmatrix} = \begin{pmatrix} A_i \\ B_i \end{pmatrix} = \begin{pmatrix} a_i/b_i \\ 1/b_i \end{pmatrix}$, *is a one-to-one transformation.*

Proof We show that if $\begin{pmatrix} A_i^1 \\ B_i^1 \end{pmatrix} = \begin{pmatrix} A_i^2 \\ B_i^2 \end{pmatrix}$, we can conclude $\begin{pmatrix} a_i^1 \\ b_i^1 \end{pmatrix} = \begin{pmatrix} a_i^2 \\ b_i^2 \end{pmatrix}$. $B_i^1 = B_i^2$ implies $b_i^1 = b_i^2$. Therefore, $A_i^1 = A_i^2$ simplifies to $a_i^1 = a_i^2$. ∎

In the following computations we denote

$$A = \sum_{i=1}^m A_i + A_t,$$

$$B = \sum_{i=1}^m B_i + B_t.$$

Original Game (OG)	Equivalent Game (EG)
Thermal generator offers: $$\begin{aligned} B_t &= 1/\bar{\beta} \\ A_t &= \bar{\alpha} B_t \end{aligned}$$	Thermal generator offers: $$B_t = \begin{cases} 1/\bar{\beta} & \frac{(Y+Z\sum_{i=1}^m A_i)}{(1+Z\sum_{i=1}^m B_i)} - \bar{\alpha} > 0 \\ \hat{B}_t \in [0, 1/\bar{\beta}] & \frac{(Y+Z\sum_{i=1}^m A_i)}{(1+Z\sum_{i=1}^m B_i)} - \bar{\alpha} = 0 \\ 0 & \text{Otherwise} \end{cases}$$ $$A_t = \bar{\alpha} B_t$$
ISO solves ISODP	ISO solves ISODP^r

Table 1: Differences in conditions of the Original Game (OG) and the Equivalent Game (EG)

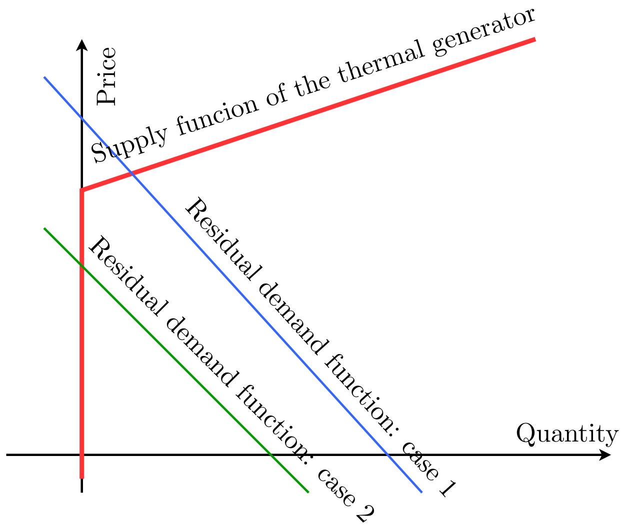

Figure 1: Supply function of the thermal generator

Using this relaxed formulation, we change the game setting in order to obtain an equivalent game (EG), as opposed to our original game (OG). We will show that any equilibrium to EG is also an equilibrium to OG. Table 1 shows the differences between OG and EG. Note that in EG, we have dropped the non-negativity constraint $q_t \geq 0$ from the ISO's dispatch. Yet by specifying a different strategy for the thermal generator, we achieve the same outcome as OG. Readers should be aware that EG is a simultaneous game, i.e. the thermal generator and cheap generators offer simultaneously.

We mathematically prove that both clearing mechanisms yield the same prices and dispatch quantities, and moreover, any Nash equilibrium of EG is an equilibrium point for OG. However, fig. 1 intuitively shows this. Figure 1 shows two different cases of residual demand function as seen by the thermal generator (the blue and green lines). The bold red line represents the supply function of the thermal generator, which consists of two pieces. In EG, only the piece of the supply function which is active (i.e. intersects the residual demand function) is offered. Thus, if $\bar{\alpha}$ is less than the price intercept of its residual demand function (i.e. $\frac{(Y+Z\sum_{i=1}^{m} A_i)}{(1+Z\sum_{i=1}^{m} B_i)}$), the thermal generator offers $\bar{\alpha}+\bar{\beta}q_t$ (i.e. $A_t=\frac{\bar{\alpha}}{\bar{\beta}}, B_t=\frac{1}{\bar{\beta}}$). If $\bar{\alpha}$ is equal to the price intercept, the slope of the thermal generator does not affect the solution, so any slope gives the same q and f. In this case, B_t is chosen $B_t \in [0, 1/\bar{\beta}]$. When $\bar{\alpha}$ is more than the price intercept, the thermal generator offers no generation (i.e. $B_t = A_t = 0$).

Using this equivalent game formulation it is now possible to find a closed-form solution. In the propositions and lemmas that follow we show that the EG formulation yields the same optimal solution as the OG formulation. To this end, we need to find a closed-form optimal solution to ISODP^r.

Proposition 3.2 *The solution to ISODP^r can be represented as follows.*

$$\begin{aligned} f &= \frac{Y+AZ}{1+BZ} \\ q_i &= fB_i - A_i \qquad i \in \{1,\ldots,m\} \cup \{t\} \end{aligned}$$

Proof The objective function of ISODP^r is a convex function and constraints are affine, thus solving KKT conditions provides the optimal solution. The Lagrangian function is

$$L=\sum_{i=1}^{m}\left(a_i q_i+\frac{b_i}{2}q_i^2\right)+a_t q_t+\frac{b_t}{2}q_t^2-\left(YQ-\frac{Z}{2}Q^2\right)-f\left(\sum_i q_i-Q\right).$$

Therefore the solution to ISODP^r can be obtained from solving the following system of equations.

$$\begin{aligned} \forall i: \quad \frac{\partial L}{\partial q_i} &= 0 \\ \frac{\partial L}{\partial Q} &= 0 \\ \frac{\partial L}{\partial f} &= 0 \end{aligned}$$

∎

The following proposition shows that EG clears the market the same way as OG does.

Proposition 3.3 *Let A_i and B_i ($i \in \{1,\ldots,m\}$) be arbitrary but fixed. The optimal dispatch and price resulting from OG is equal to the optimal dispatch and price resulting from EG.*

Proof Proposition 3.2 gives the solution to ISODP^r. First, we show that the optimal dispatch of ISODP^r (q_t^r) is also optimal to ISODP. Let $\mathfrak{d}$ denote the difference between y-intercepts of the residual demand function and the supply function of the thermal generator (i.e. $\frac{(Y+Z\sum_{i=1}^{m} A_i)}{(1+Z\sum_{i=1}^{m} B_i)}-\bar{\alpha}$). Replacing the value of the optimal q_t^r into the expression $q_t^r \mathfrak{d}$ simplifies it to the following non-negative equation.

$$q_t^r \mathfrak{d} = \frac{((1+BZ)A_t - (Y+AZ)B_t)^2}{(1+BZ)B_t(1+Z(B-B_t))} \geq 0 \tag{2}$$

Case 1 : when $\mathfrak{d} > 0$, inequality (2) implies $q_t^r \geq 0$. Therefore, q_t^r (which is the optimal solution to the relaxed problem) satisfies the non-negativity constraint. As the optimal solution to the relaxed problem ISODP^r is feasible in ISODP it is also the optimal solution to ISODP. Note that in this case $B_t^r = B_t$ which makes this comparison valid.

Case 2: when $\mathfrak{d} = 0$, we have

$$Y + Z(A - A_t) = \bar{\alpha}(1 + Z(B - B_t)). \tag{3}$$

Also, from the strategy set given in table 1, we know that

$$A_t = \bar{\alpha} B_t, \tag{4}$$

thus (3) can be written as

$$\frac{Y + ZA}{1 + ZB} = \bar{\alpha}. \tag{5}$$

Therefore, according to proposition 3.2, $f^r = \bar{\alpha}$ and $q_t^r = \bar{\alpha}B_t - A_t$. Furthermore, we can use equation (4) to obtain $q_t^r = 0$.

Similar to the previous case, this point is a feasible solution to ISODP. Thus, it is also the optimal solution to ISODP.

Case 3: when $\mathfrak{d} < 0$, inequality (2) implies $q_t^r \leq 0$. As ISODP is convex and the optimal solution to its relaxation (ISODP^r) is not an internal point of ISODP, the optimal solution to ISODP is on the boundary (i.e. $q_t = 0$). Thus, ISODP is equivalent to ISODP^r if firm t offers nothing (i.e. $B_t^r = 0$).

We demonstrated that $\boldsymbol{q} = \boldsymbol{q}^r$, and $Q = Q^r$. Furthermore, KKT conditions for each ISODP and ISODP^r imply $f = Y - ZQ$ and $f^r = Y - ZQ^r$, respectively. As $Q = Q^r$, we conclude $f = f^r$. ∎

Proposition 3.3 shows the equivalence of OG and EG for fixed offer parameters. However, it does not take into account the strategic behaviour of generators. Proposition 3.5 shows that any equilibrium to EG is also an equilibrium to OG. We first prove a necessary lemma, which states that for any strategy for firm i in OG, there exists another strategy in EG which yields higher profit for that firm.

Lemma 3.4 *Let $A_{-i}^{(m)}$ denote $\sum_{\substack{j=1 \\ j \neq i}}^{m} A_j$. For any A_i and B_i and for any B_t, there exist parameters $\hat{A}_i$ and $\hat{B}_i$ for generator $i \in \{1, \ldots, m\}$ that yield a higher (or the same) profit for generator i from EG clearing mechanism (i.e. ISODP^r problem) in comparison with its profit under the OG clearing mechanism (i.e ISODP). In other words,*

$$u_i^E(\hat{A}_i, \hat{B}_i, A_{-i}^{(m)}, B_{-i}^{(m)}, B_t) \geq u_i^O(A_i, B_i, A_{-i}^{(m)}, B_{-i}^{(m)})$$

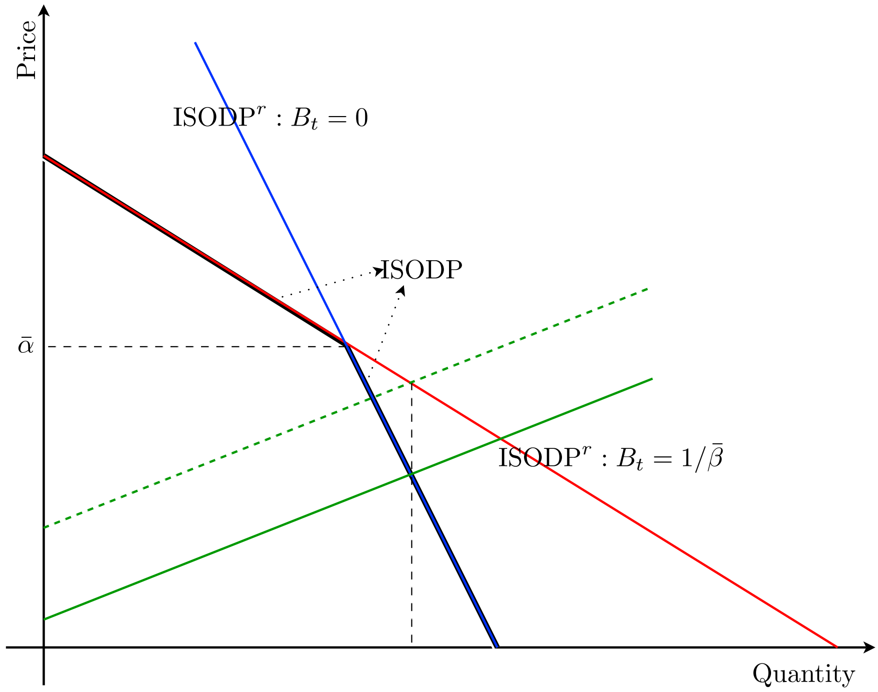

Figure 2: Residual demand function for one generator for ISODP and ISODP^r problems

Proof The residual demand function of generator $i \in \{1, \ldots, m\}$ under ISODP^r is equal to the total demand ($\frac{Y}{Z} - \frac{1}{Z}p$) minus dispatch of the other generators ($-\sum_{j \neq i} A_j + \sum_{j \neq i} B_j p$):

$$q_i^{\text{ISODP}^r}(p) = \frac{Y}{Z} - \frac{1}{Z}p + \sum_{\substack{j=1 \\ j \neq i}}^{m} A_j - \sum_{\substack{j=1 \\ j \neq i}}^{m} B_j p + A_t - B_t p.$$

Similarly, the residual demand function of generator i under ISODP is:

$$q_i^{\text{ISODP}}(p) = \begin{cases} \frac{Y}{Z} - \frac{1}{Z}p + \sum_{\substack{j=1 \\ j \neq i}}^{m} A_j - \sum_{\substack{j=1 \\ j \neq i}}^{m} B_j p, & p \leq \bar{\alpha}, \\ \frac{Y}{Z} - \frac{1}{Z}p + \sum_{\substack{j=1 \\ j \neq i}}^{m} A_j - \sum_{\substack{j=1 \\ j \neq i}}^{m} B_j p + \frac{\bar{\alpha}}{\bar{\beta}} - \frac{1}{\bar{\beta}}p, & p > \bar{\alpha}. \end{cases}$$

Observe that $q_i^{\text{ISODP}}(p)$ is a continuous function and its derivative with respect to p (where defined) does not increase when p increases. Thus, it is a concave piecewise linear function (the inverse function of what is shown in fig. 2 i.e. the bold black piecewise linear function). Also, $q_i^{\text{ISODP}^r}(p)$ is a linear function that lies on one of the pieces of $q_i^{\text{ISODP}}(p)$ for $B_t = 1/\bar{\beta}$ or $B_t = 0$ (see fig. 2). For $B_t \in (0, 1/\bar{\beta})$, $q_i^{\text{ISODP}^r}(p)$ intersects $q_i^{\text{ISODP}}(p)$ only at $p = \bar{\alpha}$ so that we have $q_i^{\text{ISODP}^r}(p) \geq q_i^{\text{ISODP}}(p)$. Here, we prove the lemma only for $B_t = 1/\bar{\beta}$. Proof of the other cases follows similarly.

As each piece of $q_i^{\text{ISODP}}(p)$ (and $q_i^{\text{ISODP}^r}(p)$) is a linear function with negative (non-zero) slope, we can find the inverse demand function $p^{\text{ISODP}}(q_i)$ (and

$p^{\mathrm{ISODP}^r}(q_i))$. $p^{\mathrm{ISODP}}(q_i)$ will also be a concave piecewise linear function as $q_i^{\mathrm{ISODP}}(p)$ (see fig. 2). As $p^{\mathrm{ISODP}^r}(q_i)$ is a linear function tangent to the concave function $p^{\mathrm{ISODP}}(q_i)$, we obtain $p^{\mathrm{ISODP}^r}(q_i) \geq p^{\mathrm{ISODP}}(q_i)$.

The profit of generator i in ISODP is dependent on the intersection of its supply function (e.g. the green line with parameters A_i and B_i) and its residual demand function (i.e. the bold black function). As shown earlier, generating the same quantity under ISODP^r yields a higher price ($p^{\mathrm{ISODP}^r}(q_i) \geq p^{\mathrm{ISODP}}(q_i)$) and therefore a higher profit (this is the intersection of the red line with the dashed green line in fig. 2). Note that there always exists an alternative linear supply function (e.g. the dashed green line) that passes through $(q_i, p^{\mathrm{ISODP}^r}(q_i))$. In other words, there exist parameters $\hat{A}$ and $\hat{B}$ that solve

$$q_i = -\hat{A}_i + \hat{B}_i p^{\mathrm{ISODP}^r}(q_i).$$

Thus,

$$\text{there exist } \hat{A}, \hat{B} : u_i^E(\hat{A}_i, \hat{B}_i, A_{-i}^{(m)}, B_{-i}^{(m)}, B_t = 1/\bar{\beta}) \geq u_i^{\mathrm{O}}(A_i, B_i, A_{-i}^{(m)}, B_{-i}^{(m)}).$$

A similar argument can be applied to any $B_t \in [0, 1/\bar{\beta}]$. ∎

Theorem 3.5 *Any equilibrium to EG is also an equilibrium to OG.*

Proof To prove the proposition, we use the property that any action profile results in an identical utility (profit) for generators in OG and EG (proposition 3.3).

Assume $\nu^E = (\boldsymbol{A}^E, \boldsymbol{B}^E, B_t^E, \boldsymbol{q}^E, Q^E, f^E)$ is an equilibrium for EG. This means for any $i \in \{1, \dots, m\}$

$$\forall \hat{A}_i \text{ and } \hat{B}_i : u_i^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_t^E) \geq u_i^E(\hat{A}_i, \hat{B}_i, A_{-i}^E, B_{-i}^E, B_t^E). \quad (6)$$

On the other hand, proposition 3.3 implies

$$u_i^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_t^E) = u_i^O(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E). \quad (7)$$

From lemma 3.4 for any A_i and B_i we have that

$$\text{there exist } \hat{A}_i \text{ and } \hat{B}_i : u_i^E(\hat{A}_i, \hat{B}_i, A_{-i}^E, B_{-i}^E, B_t^E) \geq u_i^O(A_i, B_i, A_{-i}^E, B_{-i}^E). \quad (8)$$

Following (6), and (8) we obtain

$$u_i^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_t^E) \geq u_i^O(A_i, B_i, A_{-i}^E, B_{-i}^E). \quad (9)$$

Then, replacing the value of $u_i^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_t^E)$ from (7) into (9) gives

$$\forall i \in \{1, \dots, m\} : u_i^O(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E) \geq u_i^O(A_i, B_i, A_{-i}^E, B_{-i}^E),$$

which proves that ν^E is an equilibrium for OG. ∎

We now have a way of constructing equilibria for OG through EG. In the remainder of this paper, we will concentrate on EG and appeal to theorem 3.5. The game under consideration throughout the rest of this paper is EG.

3.2.1 Firms' computations

As discussed previously, firm i's revenue and cost are fq_i and $\alpha_i q_i + \frac{\beta_i}{2} q_i^2$, respectively. Consequently, the utility (profit) of firm i can be represented as

$$u_i(A_i, B_i) = fq_i - \left(\alpha_i q_i + \frac{\beta_i}{2} q_i^2\right).$$

When firm i submits its supply function, it knows that its utility is a function of its own and the other firms' offered supply functions, as stated in proposition 3.2. Before proceeding to the best response functions, we prove a necessary lemma.

Lemma 3.6 *Assume that $f(x, y) : R^2 \to R$ is defined on a $D_x \times D_y$ with $D_x, D_y \subseteq R$. Furthermore, assume that $x^*(y) \in D_x$, maximizes $f(x, y)$ for any arbitrary but fixed y. Also assume $g(y) = f(x^*(y), y)$ is maximized at $y^* \in D_y$. Then, $f(x, y)$ is maximized at $(x^*(y^*), y^*)$.*

Proof Note that for any $(x, y) \in D_x \times D_y$,

$$f(x, y) \leq f(x^*(y), y)$$

by the assumption on $x^*(y) \in D_x$. Moreover, $f(x^*(y), y) \leq f(x^*(y^*), y^*)$. Clearly then

$$f(x, y) \leq f(x^*(y^*), y^*) \quad \text{for any } (x, y) \in D_x \times D_y.$$

■

Proposition 3.7 *Let A_{-i} and B_{-i} be arbitrary but fixed, $u_i(A_i, B_i)$ for $i \in \{1, \dots, m\}$ is maximized over all A_i and B_i if*

$$A_i^*(B_i) = \frac{(1 + ZB_{-i})(\alpha_i - ZA_{-i} + Z\alpha_i B - Y) + (Y + ZA_{-i})(Z + \beta(1 + ZB_{-i}))B_i}{(1 + ZB_{-i})(2Z + \beta(1 + ZB_{-i}))}. \tag{10}$$

Proof To start out, assume that B_i is a fixed parameter, we can then show

$$\frac{d^2 u_i}{dA_i^2} = -\frac{(1 + ZB_{-i})(2Z + \beta + Z\beta B_{-i})}{(1 + ZB)^2} \leq 0.$$

This shows that u_i is a concave function of A_i for any fixed B_i. Thus, we can use first order conditions to obtain the optimizer A_i for any B_i:

$$A_i^*(B_i) = \frac{(1 + ZB_{-i})(\alpha_i - ZA_{-i} + Z\alpha_i B - Y) + (Y + ZA_{-i})(Z + \beta + Z\beta B_{-i})B_i}{(1 + ZB_{-i})(2Z + \beta + Z\beta B_{-i})}.$$

To find the optimal value for u_i we can use lemma 3.6 and substitute $A_i^*(B_i)$ in $u_i(A_i^*(B_i), B_i)$. Surprisingly, $u_i(A_i^*(B_i), B_i)$ simplifies to a constant independent of B_i ($\frac{du_i(A_i^*(B_i), B_i)}{dB_i} = 0$). Therefore,

$$\begin{aligned}\max_{A_i, B_i} u_i(A_i, B_i) &= \max_{B_i} u_i(A_i^*(B_i), B_i) \\ &= u_i(A_i^*(B_i), B_i).\end{aligned}$$

■

Proposition 3.7, indicates that there are infinitely many choices of (A_i, B_i) or (a_i, b_i) for firm i to ensure it maximizes its utility function.

In this particular case, this situation causes multiple equilibria. In order to limit the number of equilibria, we consider the case in which the slope of the supply function is fixed to a constant by the ISO. Note that this is a further modification to game EG. As the supply function slope has an identical value for all generators, we represent it by B_1, (i.e. the inverse of the slope for the first generator)

$$B_i = B_1.$$

Using $B_i = B_1$, equation (10) reduces to the best response function of firm $i \in \{1, \ldots, m\}$ for A_i. Readers should be reminded that the strategy of the thermal generator is as stated in table 1 for EG.

Proposition 3.8 *The following equations determine a symmetric equilibrium in the market before the introduction of the wind generator.*

$$B_t = \begin{cases} 1/\bar{\beta} & B_t^0 > 1/\bar{\beta} \\ B_t^0 & B_t^0 \in [0, 1/\bar{\beta}] \\ 0 & \textit{Otherwise} \end{cases} \tag{11}$$

$$A_t = \bar{\alpha} B_t \tag{12}$$

$$\forall i \in \{1, \ldots, m\} : A_i = H(B_t) \tag{13}$$

where $H(x)$ and B_t^0 are defined as

$$H(x) = \frac{B_1^2(m-1)Z(\beta(\bar{\alpha}xZ+Y)+mZ\alpha)-(xZ+1)(\bar{\alpha}xZ+Y)+\alpha(xZ+1)^2}{(xZ+1)(xZ\beta+mZ+Z+\beta)+B_1(m-1)Z(\beta(xZ+1)+mZ)} + \frac{B_1(-((m-2)Z(\bar{\alpha}xZ+Y))+\beta(xZ+1)(\bar{\alpha}xZ+Y)+(2m-1)Z\alpha(xZ+1))}{(xZ+1)(xZ\beta+mZ+Z+\beta)+B_1(m-1)Z(\beta(xZ+1)+mZ)},$$

$$B_t^0 = -B_1(m-1) - \frac{1}{Z} - \frac{Y-\bar{\alpha}}{(Y-\bar{\alpha})\beta - mZ(\bar{\alpha}-\alpha)}.$$

Proof Considering symmetry among firms 1 to m, we expect to see the same equilibrium actions from these generators. Let A_{-i} and B_{-i} denote $\sum_{j \neq i} A_j$ and $\sum_{j \neq i} B_j$, respectively. Therefore, to find a symmetric equilibrium we can use

$$\forall i \in \{1, \ldots, m\} : A_{-i} = (m-1)A_i + A_t. \tag{14}$$

On the other hand, $B_{-i} = (m-1)B_1 + B_t$ (B_1 is a fixed parameter chosen by the ISO). Inserting these equations into the best response function (equation (10)) results in an equation in terms of A_i. Solving this equation for A_i, treating B_t as a fixed parameter, gives

$$\forall i \in \{1, \ldots, m\} : A_i = H(B_t).$$

This is what is claimed in the proposition. Now, inserting $A_i = H(B_t)$ into the supply function of firm t (from table 1) leads to

$$B_t = \begin{cases} 1/\bar{\beta} & \frac{Y+ZmH(B_t)}{1+ZmB_1} - \bar{\alpha} > 0, \\ B_t^0 \in [0, 1/\bar{\beta}] & \frac{Y+ZmH(B_t)}{1+ZmB_1} - \bar{\alpha} = 0, \\ 0 & \text{Otherwise.} \end{cases} \tag{15}$$

Note that B_t appears in the conditional expression on the right hand side of parts 1 and 2 of (15). To ensure consistency we replace (15) with (16) bellow:

$$B_t = \begin{cases} 1/\bar{\beta} & \frac{Y+ZmH(1/\bar{\beta})}{1+ZmB_1} - \bar{\alpha} > 0, \\ B_t^0 & \frac{Y+ZmH(B_t^0)}{1+ZmB_1} - \bar{\alpha} = 0, B_t^0 \in [0, 1/\bar{\beta}] \\ 0 & \text{Otherwise.} \end{cases} \tag{16}$$

Here, the only solution to the equation $\frac{Y+ZmH(B_t^0)}{1+ZmB_1} - \bar{\alpha} = 0$ that can be non-negative is given by:

$$B_t^0 = -B_1(m-1) - \frac{1}{Z} - \frac{Y - \bar{\alpha}}{(Y-\bar{\alpha})\beta - mZ(\bar{\alpha} - \alpha)}.$$

Also, note that

$$\frac{Y + ZmH(1/\bar{\beta})}{1 + ZmB_1} - \bar{\alpha} > 0 \Leftrightarrow B_t^0 > 1/\bar{\beta}.$$

Thus from (16), B_t can be written as

$$B_t = \begin{cases} 1/\bar{\beta} & B_t^0 > 1/\bar{\beta}, \\ B_t^0 & B_t^0 \in [0, 1/\bar{\beta}], \\ 0 & \text{Otherwise.} \end{cases}$$

$\blacksquare$

In the following section we will include a intermittent generator in the market and observe the change in equilibrium behaviour.

4 Market after introducing wind

Introducing a new wind generator is equivalent to introducing uncertainty to the system. This uncertainty is realized only in real time, when generators are dispatched. One option to deal with this problem is to introduce an additional clearing mechanism for dealing with this uncertainty in real time. We name this a spot market, as it is somewhat analogous to the conventional spot markets. We also name the first market a pre-dispatch or a day-ahead market.

The two settlement market model that we use in this paper is inspired by NZEM. In the day-ahead market, generators offer their supply functions and are dispatched according to a predicted demand or generation. In NZEM, wind generators are price-taker generators and are dispatched ahead of other generators.

In the spot market, the optimization problem is re-solved based on the realized demand and generation, and generators' actual generation may be different from their pre-dispatch quantities. However, these changes are limited by some constraints. For example, some generators (like generators 1 to m in our model) cannot increase their output instantaneously.

4.1 Market description

In our model, each generator offers a linear supply function $a_i + b_i q_i$ and the pre-dispatch market clears according to the expected demand and generation. Then in the spot market, the ISO solves the same optimization problem with the realized demand and generation.

Let us consider two scenarios for wind generation in the spot market and assume θ_s denotes the probability of scenario s. Let w_s denote the maximum possible wind generation in scenario s. Without loss of generality, we assume $w_1 < w_2$. We show that in our model the wind generator is dispatched up to w_s in scenario s, since in our model the wind generator must offer at price zero (see proposition 4.1). It is important to note that we have assumed the case that wind generation is less than the maximum demand ($D(p = 0)$). Let w denote the expected generation from wind

$$w = \sum_{s=1}^{2} \theta_s w_s.$$

Furthermore, let q_w denote the quantity to be dispatched from the wind generator in the day-ahead model. Also let $y_{i,s}$ and C_s denote generation from generator i in scenario s and total consumption in scenario s respectively. Then, the pre-dispatch and spot clearing optimization problems can be respectively represented as follows.

ISOPP :

$$\begin{aligned}
\min z &= \sum_{i=1}^{m} \left(a_i q_i + \frac{b_i}{2} q_i^2 \right) + a_t q_t + \frac{b_t}{2} q_t^2 - \left(YQ - \frac{Z}{2} Q^2 \right) \\
s.t. \quad & \sum_{i=1}^{m} q_i + q_t + q_w - Q = 0 \qquad [f] \\
& q_w \le w \\
& q_t \ge 0
\end{aligned}$$

ISOSP$_s$:

$$\begin{aligned}
\min z &= \sum_{i=1}^{m} \left(a_i y_{i,s} + \frac{b_i}{2} y_{i,s}^2 \right) + a_t y_{t,s} + \frac{b_t}{2} y_{t,s}^2 - \left(YC_s - \frac{Z}{2} C_s^2 \right) \\
s.t. \quad & \sum_{i=1}^{m} y_{i,s} + y_{t,s} + y_{w,s} - C_s = 0 \qquad [p_s] \\
& y_{w,s} \le w_s \\
& y_{i,s} \le q_i \qquad i \in \{1, \dots, m\} \\
& y_{t,s} \ge 0
\end{aligned}$$

Note that f and p_s are dual variables of the constraints balancing generation with demand and represent pre-dispatch and spot market prices, respectively. Also, q_i is not a decision variable of ISOSP$_s$, but a fixed parameter representing the pre-dispatch quantity of generator i. The constraint $y_{i,s} \le q_i$ represents the deviation constraint of generators $i \in \{1, \dots, m\}$.

4.2 A simpler equivalent game

Similar to the previous model, we find an equivalent formulation to the original model, in order to make the computations easier. The method is very similar to the method we use to simplify the game without wind generation.

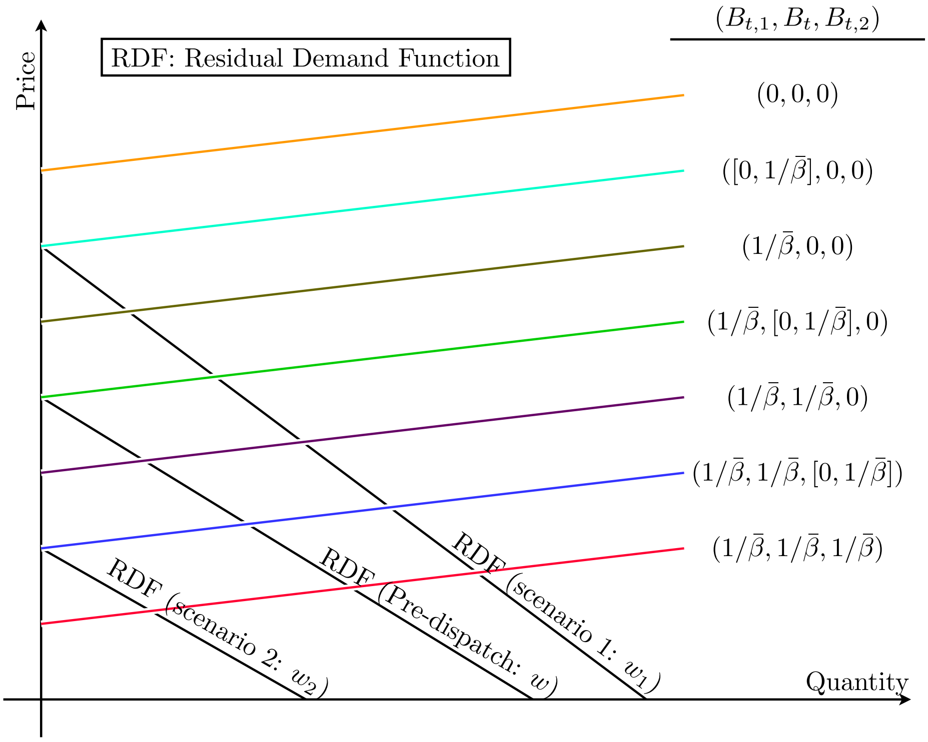

Figure 3: Coloured lines: possible cases that the supply function of the thermal generator can intersect the residual demand functions of the pre-dispatch and spot markets

Table 2 shows the differences in conditions of the original game with wind generation (i.e. OGW) and the equivalent game with wind generation (i.e. EGW). Readers should be aware that EGW is a simultaneous move game.

We mathematically prove why the clearing mechanisms OGW and EGW are equivalent (i.e. for fixed offers A_i and B_i, $i \in \{1, \ldots, m\}$, they give the same dispatch and price). It can also be explained intuitionally. In this mechanism, instead of one residual demand function, the thermal generator is faced with three residual demand functions (i.e. pre-dispatch demand and two spot scenarios). Each of these residual demand functions can intersect the supply function in either of its two pieces (see fig. 3). The intersecting part can be different for each residual demand function. Thus, in EGW we assume the thermal generator offers three supply functions, which actually comply with the original supply function.

We also show that in all ISO's problems (i.e. ISOPP^r and ISOSP^r_s) wind

The Original Game after Wind integration(OGW)
Thermal generator offers: $$B_t = 1/\bar{\beta}$$ $$A_t = \bar{\alpha} B_t$$
ISO solves ISOPP and ISOSP$_s$ (see the next section) with the non-negativity constraints $$q_t \geq 0,$$ $$y_{t,s} \geq 0.$$

The Equivalent Game after Wind integration (EGW)
Thermal generator offers: $$B_{t,2} = \begin{cases} 1/\bar{\beta} & \frac{Y+Z\left(A^{(m)}-w_2\right)}{(1+ZmB_1)} - \bar{\alpha} > 0 \\ [0, 1/\bar{\beta}] & \frac{Y+Z\left(A^{(m)}-w_2\right)}{(1+ZmB_1)} - \bar{\alpha} = 0 \\ 0 & \text{Otherwise} \end{cases}$$ $$B_t = \begin{cases} 1/\bar{\beta} & \frac{Y+Z\left(A^{(m)}-w\right)}{(1+ZmB_1)} - \bar{\alpha} > 0 \\ [0, 1/\bar{\beta}] & \frac{Y+Z\left(A^{(m)}-w\right)}{(1+ZmB_1)} - \bar{\alpha} = 0 \\ 0 & \text{Otherwise} \end{cases}$$ $$B_{t,1} = \begin{cases} 1/\bar{\beta} & Y + Z\left(A^{(m)} - \frac{mB_1\left(Y+Z\left(A^{(m)}-w\right)\right)}{ZmB_1+1} - w_1\right) - \bar{\alpha} > 0 \\ [0, 1/\bar{\beta}] & Y + Z\left(A^{(m)} - \frac{mB_1\left(Y+Z\left(A^{(m)}-w\right)\right)}{ZmB_1+1} - w_1\right) - \bar{\alpha} = 0 \\ 0 & \text{Otherwise} \end{cases}$$ $$A_t = \bar{\alpha} B_t$$ $$A_{t,s} = \bar{\alpha} B_{t,s}$$
ISO solves ISOPP^r and ISOSP^r_s without the non-negativity constraints.

Table 2: Differences in conditions of the Original Game with Wind integration (OGW) and the Equivalent Game with Wind integration (EGW)

generator is dispatched fully (i.e. up to its maximum possible generation):

$$\begin{aligned}
&\text{ISOPP}^r:\\
&\min_{q,Q}\ z \ = \ \sum_{i=1}^{m}\left(a_i q_i + \frac{b_i}{2}q_i^2\right) + a_t q_t + \frac{b_t}{2}q_t^2 - \left(YQ - \frac{Z}{2}Q^2\right)\\
&\quad s.t. \quad \sum_{i=1}^{m} q_i + q_t + q_w - Q = 0 \qquad [f]\\
&\qquad\qquad q_w \le w,
\end{aligned}$$

$$\begin{aligned}
&\text{ISOSP}_s^r:\\
&\min_{y,C}\ z \ = \ \sum_{i=1}^{m}\left(a_i y_{i,s} + \frac{b_i}{2}y_{i,s}^2\right) + a_t y_{t,s} + \frac{b_{t,s}}{2}y_{t,s}^2 - \left(YC_s - \frac{Z}{2}C_s^2\right)\\
&\quad s.t. \quad \sum_{i=1}^{m} y_{i,s} + y_{t,s} + y_{w,s} - C_s = 0 \qquad [p_s]\\
&\qquad\qquad y_{w,s} \le w_s\\
&\qquad\qquad y_{i,s} \le q_i \qquad i \in \{1,\ldots,m\}.
\end{aligned}$$

Note that in addition to the non-negativity constraint, another difference between ISOSP_s and ISOSP_s^r is in the slope of the supply function (b_t vs. $b_{t,s}$). This is because in EGW, the thermal generator offers a different slope for the pre-dispatch market and each scenario in the spot market (see fig. 3).

Proposition 4.1 *The optimal solutions to ISOPP^r and ISOSP$_s^r$ always satisfy*

$$q_w = w \quad or \quad f = 0, \tag{17}$$
$$y_{w,s} = w_s \quad or \quad p_s = 0. \tag{18}$$

Proof We prove the proposition for ISOSP_s^r; the other case can be proven similarly.

The optimal solution to ISOSP_s^r can be determined from its KKT conditions (since its objective function is convex and it has affine constraints). Two of the conditions from the KKT conditions are:

$$0 \le w_s - y_{w,s} \perp \eta_s \le 0 \tag{19}$$
$$p_s + \eta_s = 0 \tag{20}$$

where η_s is the dual of the maximum wind generation constraint, with the associated orthogonality condition 19, and equation 20 is the dual constraint associated with $y_{w,s}$. Combining these two constraints by eliminating η_s we get:

$$0 \le w_s - y_{w,s} \perp p_s \ge 0,$$

which can be rewritten as in the statement of the proposition. ∎

Proposition 4.1 indicates that wind generator is always a price taker generator and is dispatched up to its available capacity[1].

[1] Wind generation may not always be the price taker in reality. For example, an ISO with unit commitment considerations may dispatch wind generators less than their available output.

Substituting $q_w = w$ then results in the following formulation for ISOPP^r:

$$\text{ISOPP}^r:$$
$$\begin{aligned} \min_{q,Q} z &= \sum_{i=1}^{m} \left(a_i q_i + \frac{b_i}{2} q_i^2 \right) + a_t q_t + \frac{b_t}{2} q_t^2 - \left(YQ - \frac{Z}{2} Q^2 \right) \\ s.t. & \quad \sum_{i=1}^{m} q_i + q_t + w - Q = 0. \qquad [f] \end{aligned}$$

The spot problem can also be represented as

$$\text{ISOSP}_s^r:$$
$$\begin{aligned} \min_{y,C} z &= \sum_{i=1}^{m} \left(a_i y_{i,s} + \frac{b_i}{2} y_{i,s}^2 \right) + a_t y_{t,s} + \frac{b_{t,s}}{2} y_{t,s} - \left(YC_s - \frac{Z}{2} C_s^2 \right) \\ s.t. & \quad \sum_{i}^{m} y_{i,s} + y_{t,s} + w_s - C_s = 0 \\ & \quad y_{i,s} \le q_i \qquad i \in \{1, \ldots, m\}. \end{aligned}$$

Let us use the same notation $A_i = \frac{a_i}{b_i}$ and $B_i = \frac{1}{b_i}$. In addition, let A_t, B_t, $A_{t,s}$, $B_{t,s}$, $A^{(m)}$, and $B^{(m)}$ denote $\frac{a_t}{b_t}$, $\frac{1}{b_t}$, $\frac{a_t}{b_{t,s}}$, $\frac{1}{b_{t,s}}$, $\sum_{i=1}^{m} A_i$, and $\sum_{i=1}^{m} B_i$, respectively.

Proposition 4.2 *The optimal solution to ISOPP^r, for the specific market we defined in this section, is as follows.*

$$\begin{aligned} f &= \frac{Y + Z(A^{(m)} + A_t - w)}{Z(B^{(m)} + B_t) + 1} \\ q_i &= fB_i - A_i \end{aligned}$$

Proof The proof is effectively identical to the proof of proposition 3.2. ∎

We can also find the optimal solution to ISOSP$_s^r$.

Proposition 4.3 *The optimal solution to ISOSP$_s^r$, for the specific market we defined in this section, is as follows.*

Case 1: $w_s \ge w$,

$$\begin{aligned} p_s &= \frac{Y + Z(A^{(m)} + A_{t,s} - w_s)}{Z(mB_1 + B_{t,s}) + 1} \\ y_{i,s} &= \begin{cases} p_s B_i - A_i & i \in \{1, \ldots, m\} \\ p_s B_{t,s} - A_{t,s} & i = t \end{cases} \end{aligned}$$

Case 2: $w_s < w$,

$$\begin{aligned} p_s &= \frac{Y + Z(A^{(m)} + A_{t,s} - mB_1 f - w_s)}{ZB_{t,s} + 1} \\ y_{i,s} &= \begin{cases} q_i & i \in \{1, \ldots, m\} \\ p_s B_{t,s} - A_{t,s} & i = t \end{cases} \end{aligned}$$

Proof Let us first optimize the following relaxed version of ISOSP_s^r.

ISOSP_s^{rr} :

$$\begin{aligned} \min z &= \sum_{i=1}^{m}\left(a_i y_{i,s} + \frac{b_i}{2} y_{i,s}^2\right) + a_t y_{t,s} + \frac{b_{t,s}}{2} y_{t,s} - \left(YC_s - \frac{Z}{2} C_s^2\right) \\ s.t. & \quad \sum_{i=1}^{m} y_{i,s} + y_{t,s} + w_s - C_s = 0 \end{aligned}$$

By using the same method used in proposition 3.2, the optimal solution to this optimization problem is as follows.

$$\begin{aligned} p_s^* &= \frac{Y + Z(A^{(m)} + A_{t,s} - w_s)}{Z(mB_1 + B_{t,s}) + 1} \\ y_{i,s}^* &= \begin{cases} p_s^* B_i - A_i & i \in \{1, \ldots, m\} \\ p_s^* B_{t,s} - A_{t,s} & i = t \end{cases} \end{aligned}$$

Case 1 ($w_s \geq w$): The residual demand function of firm t is lower than that of the pre-dispatch market. This is because the demand curve and offer functions of the other generators are fixed, and wind generation is increased. This means producing the same quantity will result in a lower price in comparison with the pre-dispatch market. As the market price is equal to the intersection of the supply function of this generator (see fig. 1) with the residual demand function, the conclusion is $p_s^* \leq f^*$. Consequently, comparing $y_{i,s}^*$ with q_i^* (from proposition 4.2), we conclude $\forall i \in \{1, \ldots, m\} : y_{i,s}^* \leq q_i^*$ (we know that all our generators are able to deviate downward, meaning that this solution is feasible). As the optimal solution to ISOSP_s^{rr} is feasible in the original problem (i.e. ISOSP_s^r), it is also the optimal solution to the original problem.

Case 2 ($w_s \leq w$): We conclude $q_i^* \leq y_{i,s}^*$; this point cannot be feasible for the primal problem. As this optimization problem is a convex optimization problem, and the optimal solution to the relaxed problem is not feasible in ISOSP_s^r, the optimal solution must be on the boundaries:

$$\forall i \in \{1, \ldots, m\}, y_{i,s}^* = q_i^*.$$

Hence this problem can be represented, for this case, as follows:

ISOSP_2^{rr} :

$$\begin{aligned} \min z &= \left(a_t y_{t,s} + \frac{b_t}{2} y_{t,s}^2\right) - \left(YC_s - \frac{Z}{2} C_s^2\right) \\ s.t. & \quad y_{t,s} + \sum_{i=1}^{m} q_i + w_s - C_s = 0. \end{aligned}$$

Next we can solve ISOSP_2^{rr} similar to proposition 4.2, and the solution is

$$\begin{aligned} p_s &= \frac{Y + Z(A_{t,s} - \sum_{i=1}^{m} q_i - w_s)}{ZB_{t,s} + 1}, \\ y_{i,s} &= \begin{cases} q_i & i \in \{1, \ldots, m\} \\ p_s B_i - A_i & \text{Otherwise} \end{cases}. \end{aligned}$$

Replacing $q_i = fB_i - A_i$ into the first equation proves the proposition. ∎

Proposition 4.3 demonstrates why the clearing mechanism of EGW is equivalent to the clearing mechanism of OGW. We also prove that any equilibrium to EGW is an equilibrium to OGW (in proposition 4.6).

Proposition 4.4 *For fixed offered parameters A_i and B_i for $i \in \{1, \ldots, m\}$, the optimal dispatch and price resulting from OGW mechanism is equal to the optimal dispatch and price resulting from EGW mechanism.*

Proof We prove this proposition only for the spot market of the first scenario. Proof of the other cases follow in a same way.

Proposition 4.3 indicates

$$p_1 = \frac{Y + Z(A^{(m)} + A_{t,1} - mB_1 f - w_1)}{ZB_{t,1} + 1}.$$

The y-intercept of the residual demand function of firm t can be computed as the market price if it offers no generation (i.e. $A_{t,1} = B_{t,1} = 0$), and is equal to

$$p_1(0) = Y + Z(A^{(m)} - mB_1 f - w_1).$$

The value of f from proposition 4.3 can be replaced in this expression. This modifies $p_1(0)$ to the following form.

$$p_1(0) = Y + Z\left(A^{(m)} - \frac{mB_1\big(Y + Z\big(A^{(m)} - w\big)\big)}{ZmB_1 + 1} - w_1\right)$$

As illustrated in fig. 1 and stated in table 2, if this y-intercept ($p_1(0)$) is greater than or equal to $\bar{\alpha}$, the thermal generator offers a positive $B_{t,1}$ (i.e. $B_{t,1} = 1/\bar{\beta}$). Similarly, if $p_1(0) - \bar{\alpha} = 0$, $B_{t,1}$ can be any value in $[0, 1/\bar{\beta}]$ as the value of $B_{t,1}$ does not change the solution in this case. Finally, if $p_1(0) - \bar{\alpha} < 0$, $B_{t,1}$ is chosen to be zero (i.e. no production in this scenario). The rest of the proof is similar to the proof of proposition 3.3. We can further apply a similar argument to prove this proposition for the spot market of the second scenario and the pre-dispatch market. ∎

The following lemma and theorem prove that any equilibrium to EGW is an equilibrium to OGW.

Lemma 4.5 *For any A_i and B_i for $i \in \{1, \ldots, m\}$ and for any $B_{t,s} \in [0, \frac{1}{\beta}]$, there exist parameters $A_i^{(s)}$ and $B_i^{(s)}$ for generator $i \in \{1, \ldots, m\}$ and for any scenario s so that*

$$u^E_{i,s}(A_i^{(s)}, B_i^{(s)}, A_{-i}^{(m)}, B_{-i}^{(m)}, B_{t,s}) \geq u^O_{i,s}(A_i, B_i, A_{-i}^{(m)}, B_{-i}^{(m)}).$$

Proof The proof that we used for lemma 3.4 can also be similarly applied to this lemma.

Theorem 4.6 *Any equilibrium to EGW is also an equilibrium to OGW.*

Proof Assume $\nu^E = (\boldsymbol{A}^E, \boldsymbol{B}^E, B_t^E, B_{t,1}^E, B_{t,2}^E, \boldsymbol{q}^E, Q^E, f^E)$ is an equilibrium for EGW. This means $\forall i \in \{1, \ldots, m\}$:

$$\forall \hat{A}_i \text{ and } \hat{B}_i : u_i^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_t^E, B_{t,1}^E, B_{t,2}^E) \geq u_i^E(\hat{A}_i, \hat{B}_i, A_{-i}^E, B_{-i}^E, B_t^E, B_{t,1}^E, B_{t,2}^E). \tag{21}$$

Note that we have two scenarios. For each scenario, there exists an optimal intersecting point on the residual demand function. The optimal intersection for the low wind scenario yields higher quantity and price than that of the high wind scenario. This means that we can always find parameters A_i and B_i so to form an increasing supply function that passes through both these optimal intersections. Therefore, it also maximizes the expected value of these functions (i.e. u_i^E). In other words, the parameters A_i and B_i that maximize u_i^E, also maximize $u_{i,s}^E$ for both scenarios. From (21) we understand A_i^E and B_i^E maximize $u_i^E(A_i, B_i, A_{-i}^E, B_{-i}^E, B_t^E, B_{t,1}^E, B_{t,2}^E)$. Therefore, they also maximize profit for each scenario and each generator i:

$$\forall \hat{A}_i \text{ and } \hat{B}_i : u_{i,s}^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_{t,s}^E) \geq u_{i,s}^E(\hat{A}_i, \hat{B}_i, A_{-i}^E, B_{-i}^E, B_{t,s}^E). \tag{22}$$

In addition, from lemma 4.5 we know $\forall A_i$ and B_i,

$$\exists \hat{A}_i \text{ and } \hat{B}_i : u_{i,s}^E(\hat{A}_i, \hat{B}_i, A_{-i}^E, B_{-i}^E, B_{t,s}^E) \geq u_{i,s}^O(A_i, B_i, A_{-i}^E, B_{-i}^E). \tag{23}$$

From (22) and (23), we obtain

$$u_{i,s}^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_{t,s}^E) \geq u_{i,s}^O(A_i, B_i, A_{-i}^E, B_{-i}^E).$$

An expectation over all scenarios results in

$$u_i^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_t^E, B_{t,1}^E, B_{t,2}^E) \geq u_i^O(A_i, B_i, A_{-i}^E, B_{-i}^E). \tag{24}$$

On the other hand, proposition 4.4 implies

$$u_i^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_t^E, B_{t,1}^E, B_{t,2}^E) = u_i^O(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E). \tag{25}$$

If we replace the value of $u_i^E(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E, B_t^E, B_{t,1}^E, B_{t,2}^E)$ from (25) into (24), we obtain

$$\forall i \in \{1, \ldots, m\} : u_i^O(A_i^E, B_i^E, A_{-i}^E, B_{-i}^E) \geq u_i^O(A_i, B_i, A_{-i}^E, B_{-i}^E).$$

This proves that ν^E is an equilibrium for OG.

4.2.1 Firms' computations

We remind the reader that we have assumed B_i is a constant (imposed by the ISO) for the first m generators.

We investigate a case with two scenarios: Let w_1 and w_2 denote the low wind generation scenario and the scenario with high wind generation respectively.

The expected utility function of firm i is computed by the following equation.

$$u_i = \sum_{s=1}^{2} \theta_s \left(p_s y_{i,s} - \left(\alpha_i y_{i,s} + \frac{1}{2} \beta_i y_{i,s}^2 \right) \right)$$

Now, we can proceed to find an equilibrium for EGW.

Proposition 4.7 *The following describes a symmetric Nash equilibrium to EGW.*

$$B_{t,2} = \begin{cases} 1/\bar{\beta} & B^0_{t,2} > 1/\bar{\beta} \\ B^0_{t,2} & B^0_{t,2} \in [0, 1/\bar{\beta}] \\ 0 & Otherwise \end{cases}$$

$$B_t = \begin{cases} 1/\bar{\beta} & B^0_{t} > 1/\bar{\beta} \\ B^0_{t} & B^0_{t} \in [0, 1/\bar{\beta}] \\ 0 & Otherwise \end{cases}$$

$$B_{t,1} = \begin{cases} 1/\bar{\beta} & B^0_{t,1} > 1/\bar{\beta} \\ B^0_{t,1} & B^0_{t,1} \in [0, 1/\bar{\beta}] \\ 0 & Otherwise \end{cases}$$

$$A_t = \bar{\alpha} B_t$$

$$A_{t,s} = \bar{\alpha} B_{t,s}$$

$$A_i = G(B_{t,1}, B_t, B_{t,2}) \quad i \in \{1, \ldots, m\}$$

Where $G(B_{t,1}, B_t, B_{t,2})$ is the solution to the following system of equations with variable A_1 (taking B_t, $B_{t,1}$, and $B_{t,2}$ as constant parameters). The closed-form expression for G can be found in [16].

$$\begin{aligned} \frac{\partial u_1}{\partial A_1} &= 0 \\ i \in \{1, \ldots, m\} : A_i &= A_1 \\ A_t &= \bar{\alpha} B_t \\ A_{t,1} &= \bar{\alpha} B_{t,1} \\ A_{t,2} &= \bar{\alpha} B_{t,2} \end{aligned}$$

Also, $B^0_{t,2}$, B^0_t, and $B^0_{t,1}$ are given as follows.

$$\begin{aligned} B^0_{t,2} &\in \{x \geq 0 : \frac{Y + Z\left(mG(1/\bar{\beta}, 1/\bar{\beta}, x) - w_2\right)}{(1 + ZmB_1)} - \bar{\alpha} = 0\} \\ B^0_{t} &\in \{x \geq 0 : \frac{Y + Z\left(mG(1/\bar{\beta}, x, B_{t,2}) - w\right)}{(1 + ZmB_1)} - \bar{\alpha} = 0\} \\ B^0_{t,1} &\in \{x \geq 0 : Y + Z\left(A^{(m)} - \frac{mB_1(Y + Z(mG(x, B_t, B_{t,2}) - w))}{ZmB_1 + 1} - w_1\right) - \bar{\alpha} = 0\} \end{aligned}$$

Proof Using second order conditions, we can show that u_i is a concave function of A_i. Therefore, first order condition $\frac{\partial u_i}{\partial A_i} = 0$ can be used to find a best response A_i for firm i. Considering symmetry among first m generators, we obtain $A_i = G(B_{t,1}, B_t, B_{t,2})$ as a condition for any symmetric equilibrium.

Inserting value of A_i into the equations of B_t, $B_{t,1}$, and $B_{t,2}$ from table 2, determines equilibrium values of these quantities. For example,

$$B_{t,2} = \begin{cases} 1/\bar{\beta} & \frac{Y + Z(mG(B_{t,1}, B_t, B_{t,2}) - w_2)}{(1 + ZmB_1)} - \bar{\alpha} > 0 \\ [0, 1/\bar{\beta}] & \frac{Y + Z(mG(B_{t,1}, B_t, B_{t,2}) - w_2)}{(1 + ZmB_1)} - \bar{\alpha} = 0 \\ 0 & \text{Otherwise} \end{cases} . \tag{26}$$

Note that $B_{t,2}$ appears in the conditional expression on the right hand side of parts 1 and 2 of (26). To ensure consistency it must have the value of $B_{t,2}$

for each condition i.e. $1/\bar{\beta}$ and $B_{t,2} \in [0, 1/\bar{\beta}]$ respectively. Also, note that residual demand function of the thermal generator for scenario 2 has a lower y-intercept than that of pre-dispatch and scenario 1. Therefore, when $B_{t,2} = 1/\bar{\beta}$ or $B_{t,2} \in [0, 1/\bar{\beta}]$, B_t and $B_{t,1}$ must be equal to $1/\bar{\beta}$ (As shown in fig. 3, in all possible cases with $B_{t,2} = 1/\bar{\beta}$ or $B_{t,2} \in [0, 1/\bar{\beta}]$, $B_t = B_{t,1} = 1/\bar{\beta}$). Accordingly, we replace (26) with (27):

$$B_{t,2} = \begin{cases} 1/\bar{\beta} & \frac{Y+Z\left(mG(1/\bar{\beta},1/\bar{\beta},1/\bar{\beta})-w_2\right)}{(1+ZmB_1)} - \bar{\alpha} > 0 \\ B^0_{t,2} & \frac{Y+Z\left(mG(1/\bar{\beta},1/\bar{\beta},B^0_{t,2})-w_2\right)}{(1+ZmB_1)} - \bar{\alpha} = 0, B^0_{t,2} \in [0, 1/\bar{\beta}] \\ 0 & \text{Otherwise} \end{cases} . \tag{27}$$

Note that

$$B^0_{t,2} > 1/\bar{\beta} \Leftrightarrow \frac{Y + Z\left(mG(1/\bar{\beta}, 1/\bar{\beta}, 1/\bar{\beta}) - w_2\right)}{(1 + ZmB_1)} - \bar{\alpha} > 0.$$

Thus, the equilibrium $B_{t,2}$ can be described as

$$B_{t,2} = \begin{cases} 1/\bar{\beta} & B^0_{t,2} > 1/\bar{\beta} \\ B^0_{t,2} & B^0_{t,2} \in [0, 1/\bar{\beta}] \\ 0 & \text{Otherwise} \end{cases} .$$

By applying a similar method, other equations of the proposition can be proved. ■

5 Results of wind integration

In this section, we compare the symmetric equilibrium computed without wind, with the symmetric equilibrium computed after wind is added to the market. One may be tempted to believe that adding additional wind generation to the market (effectively decreasing the demand) will always depress the price; however, if that additional generation is uncertain and the other generators are not sufficiently flexible for the system operator to take advantage of this generation, the outcomes may become worse.

Social welfare before the integration of wind is given by

$$\begin{aligned} \text{SW}^{\text{before}} &= Y\left(\sum_{i=1}^{m} q_i + q_t\right) - \frac{1}{2}Z\left(\sum_{i=1}^{m} q_i + q_t\right)^2 \\ &\quad - \sum_{i=1}^{m}\left(\alpha q_i + \frac{1}{2}\beta q_i^2\right) - \left(\bar{\alpha} q_t + \frac{1}{2}\bar{\beta} q_t^2\right), \end{aligned}$$

and after the integration of wind, social welfare is computed through

$$\begin{aligned} \text{SW}^{\text{after}} = \sum_{s=1}^{S} \theta_s \Bigg(& Y\left(\sum_{i=1}^{m} y_{i,s} + y_{t,s} + w_s\right) - \frac{1}{2}Z\left(\sum_{i=1}^{m} y_{i,s} + y_{t,s} + w_s\right)^2 \\ & - \sum_{i=1}^{m}\left(\alpha y_{i,s} + \frac{1}{2}\beta y_{i,s}^2\right) - \left(\bar{\alpha} y_{t,s} + \frac{1}{2}\bar{\beta} y_{t,s}^2\right)\Bigg). \end{aligned}$$

5.1 Example 1

Consider a market described as follows.

- We consider two markets: one with two and the other with three generators. The following table shows the features of these generators.

	#	Game 1 no wind	Game 2 with wind	Offers Strategically	Cost Function	Dev. Up	Dev. Down
Cheap gen	c	1	1	Yes	$10q_c$	No	Yes
Thermal	t	1	1	No	$90q_t + 2q_t^2$	Yes	Yes
Wind	w	0	1	−Demand	0	−Dem.	−Dem.

Table 3: Market environment for the simple example

  Note that the cheap generator cannot deviate upward in the spot market (i.e. it is not flexible enough to increase output in the spot market).

- We consider 2 equally-likely wind scenarios 0 MW and 20 MW.
- Demand is deterministic and elastic with the demand function $Q = 40 - \frac{1}{100}p$ (or $p(Q) = 4000 - 100Q$). Hence, our demand is nearly 40 MW with low elasticity ($Z = 100$).

5.1.1 The market without wind generation

Figure 4 represents the offering behaviour of the cheap strategic generator, as well as market clearing in the market before wind integration. In this case, the strategic generator undercuts the offer of the non-strategic generator at $90 per MW.

Also the equilibrium dispatch and prices of this market are listed in table 4.

Gen	q	**Profit**
1	39.1	3128
2	0	0

f	**SW**	**CW**	**PW**
90	79568.5	76440.5	3128

Table 4: The equilibrium of the market without wind generation

5.1.2 The two settlement market after wind integration

Figure 5 represents the bidding behaviour of the strategic generator. It also shows how the two settlement market clears. As shown in this picture, the strategic generator cannot be used for the balancing (spot) market in the case that scenario 1 occurs. Therefore, the more expensive generator is used. This is why the price of electricity is higher in scenario 1 (see table 5).

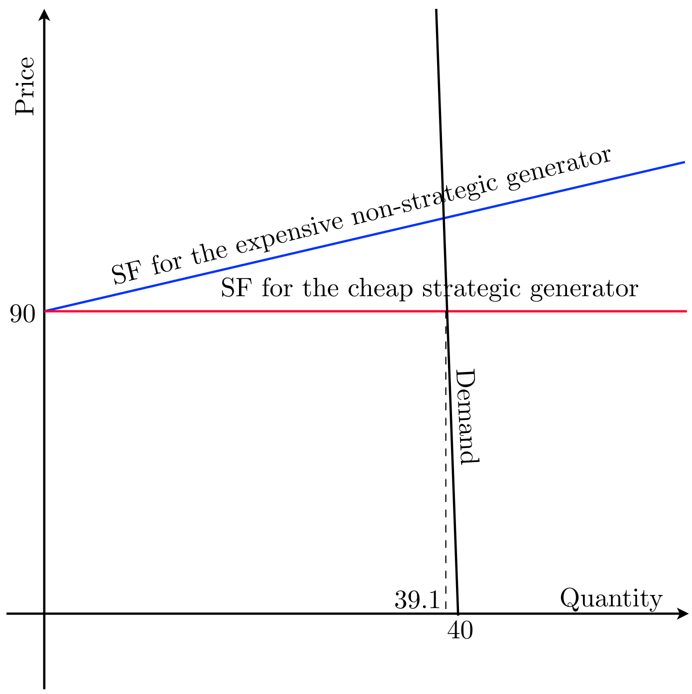

Figure 4: Equilibrium offered supply functions and clearing mechanism without wind generation

Gen	q	**E(Profit)**
1	29.1	2213.3
2	0	48.1

Scen	y**:Gen 1**	y**:Gen 2**	p
1	29.1	10	110
2	19.1	0	90

f	**SW**	**CW**	**PW**
90.0	79219.5	76058.1	3161.4

Table 5: The equilibrium of the two settlement market after wind integration.

5.2 Example 2

In this example, we investigate the effect of demand elasticity on equilibrium prices, the expected social welfare and cost of generation before and after the introduction of a wind generation. The specific parameters are outlined in table table 6. For this example, we inflate the inverse demand function introduced earlier with an elasticity coefficient, e, which gives: $p(Q) = e(Y - ZQ)$ (fig. 6). This means that the maximum demand (when price is 0) is fixed at $\frac{Y}{Z}$, but changing e allows us to analyse the effects that changing the slope of the demand curve has on computed equilibria. Table 6 shows the numerical values of parameters used in this example. Table 7 shows how adding wind can increase the expected thermal generation and generation cost, and decrease expected social welfare. Figure 7 represents the effects of adding a wind generator for different levels of e. Our results for this example show that when demand elas-

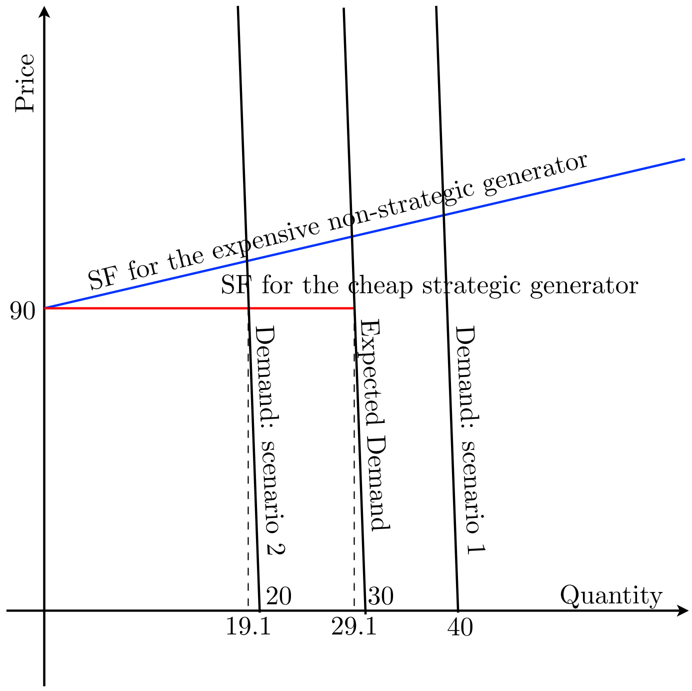

Figure 5: Equilibrium offered supply functions and clearing mechanism using the two settlement mechanism after wind integration

ticity decreases wind integration is less successful, and finally our benchmarks (i.e. the expected social welfare and generation cost) become worse than before wind integration.

The example above demonstrates the counter-intuitive behaviour for a specific range of parameters Y, Z, α, etc[2].

Parameter	Value
m	5
α, β	4, 0.01
$\bar{\alpha}$, $\bar{\beta}$	30, 0.01
θ_1, θ_2	0.5, 0.5
w_1, w_2	0, 10
Y, Z	100, 1
b	0.01

Table 6: Parameter values of the example 1

[2]One can choose these parameters so that there is no e that would lead to such counter-intuitive results.

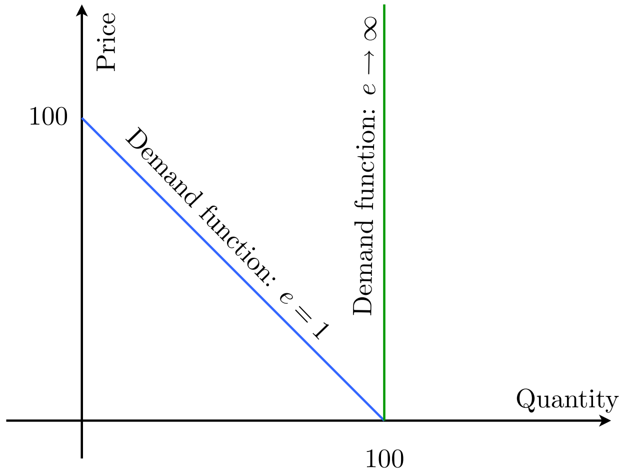

Figure 6: Demand function for different elasticity factors

e	0.01	1	10	100	100000
q_t^{before}	0	0	0	0	0
q_t^{after}	0	0	0	0	0
$y_{t,1}^{\text{after}}$	0	0	0	4.48336	4.99948
$y_{t,2}^{\text{after}}$	0	0	0	0	0
$\text{SW}^{\text{before}} - \text{SW}^{\text{after}}$	-17.4192	-17.7895	10.2979	40.2826	43.6217
$\text{GC}^{\text{after}} - \text{GC}^{\text{before}}$	-14.1573	-20.8743	-20.9593	36.9467	43.6183

Table 7: Effects of demand elasticity on thermal generation, expected social welfare (SW) and generation cost (GC), before and after introduction of the wind generator

6 A stochastic settlement mechanism

In the first part of this paper, we introduced a two settlement market clearing mechanism. In this mechanism, each participant is allowed to bid a linear function as a representative of its marginal cost function. Then, we provided a method that enabled us to analytically find an equilibrium for this complicated game.

Comparing these equilibria, we showed that contrary to one's expectations, integrating wind generation into the market can decrease social welfare and increase cost of generation. It can also increase generation from fossil fuels.

The choice of market clearing mechanism might account for these results. To illustrate this point, in addition to a two settlement mechanism similar to the first part of this paper, we analyse a stochastic settlement mechanism. Stochastic settlement mechanisms tend to take possible future scenarios into account in their pre-dispatch decisions. We present examples in which this mechanism is able to overcome the difficulties of dispatching intermittent resources more efficiently.

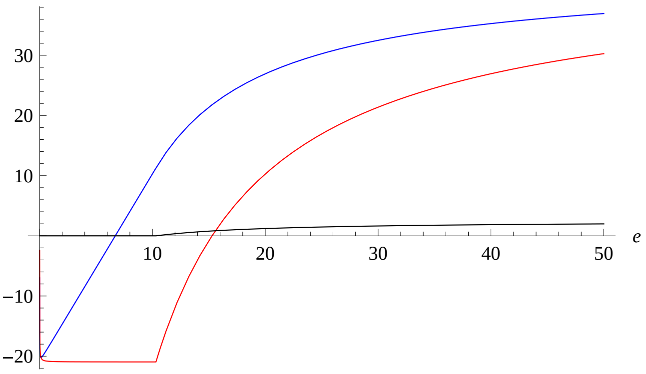

Figure 7: Difference in expected thermal generation, social welfare and generation cost before and after wind introduction. $\mathrm{TG}^{\mathrm{after}} - \mathrm{TG}^{\mathrm{before}}$: black, $\mathrm{SW}^{\mathrm{before}} - \mathrm{SW}^{\mathrm{after}}$: blue, $\mathrm{GC}^{\mathrm{after}} - \mathrm{GC}^{\mathrm{before}}$: red

6.1 The stochastic settlement model

Our stochastic settlement model is a modified version of the stochastic programming market clearing mechanism introduced by Pritchard et al. [20]. A stochastic programming mechanism chooses pre-dispatch taking into account different possible scenarios in the spot market. In other words, it chooses pre-dispatch values that maximize expected social welfare. This is in contrast to our two settlement mechanism that does not consider spot scenarios for clearing the pre-dispatch market.

The model we use for the stochastic settlement mechanism is similar to the model of Khazaei et al. [21]. The market process is as follows.

1. Each generator offers a linear supply function.
2. The ISO uses these offers and solves the following stochastic optimization problem to determine pre-dispatch quantity and prices, and spot dispatch and prices for different scenarios:

$$\text{ISOSS: } \min_{q,y,C} \sum_{s=1}^{S} \theta_s \left(\sum_{i=1}^{n} \left(a_i y_{i,s} + \frac{b_i}{2} {y_{i,s}}^2 \right) - Y C_s + \frac{Z}{2} {C_s}^2 \right)$$

$$\begin{aligned} s.t.\ & w_s + \sum_{i=1}^{n} y_{i,s} - C_s = 0, && \forall s\ [\theta_s\ p_s] \\ & y_{i,s} \geq 0, && \forall i, s\ [\lambda^y_{i,s}] \\ & q_i \geq 0, && \forall i\ [\lambda^q_i] \\ & q_i - y_{i,s} \geq 0, && \forall i \in \mathcal{S}^U, s\ [\lambda^U_{i,s}] \\ & y_{i,s} - q_i \geq 0. && \forall i \in \mathcal{S}^D, s\ [\lambda^D_{i,s}] \end{aligned}$$

Note that we assume some generators may not be flexible in deviating upward or downward. Set $\mathcal{S}^{\mathrm{D}}$ ($\mathcal{S}^{\mathrm{U}}$) is the set of all generators that cannot decrease (increase) output (i.e. their pre-dispatch quantity) in the spot market.

3. Demand is realized.

4. Generators produce their dispatch quantity from ISOSS for the realized scenario.

6.2 Equilibrium computations

To find a Nash equilibrium to the game with stochastic market clearing, we use a dynamic process. The idea is to allow each participant in a row to update its strategy (assuming the strategy set of the other participants is fixed) by solving the profit maximizing problem MaxProfit(i):

max u_i

s.t.

ISOSS optimization problems,

Constraints on the supply function of generator i.

If we continue this process until no participant is willing to deviate from its last strategy, we have actually found a Nash equilibrium [22, 23]. This dynamic process is called fictitious play or tâtonnement, and is explained in detail in [16]. The dynamic procedure is as follows:

1. Set the initial supply functions to the true cost function for each generator.

2. While an equilibrium is not obtained (i.e. there exists at least one generator that has changed its supply function in the last round):

   (a) For generator i in the set of all generators solve MaxProfit(i)

3. Output the equilibrium.

The optimization problem ISOSS is a convex optimization problem, as its objective function is a convex function and all constraints are linear. Thus, we can use KKT conditions to represent this problem. The detailed description of the KKT conditions and MaxProfit(i) can be found in [21] and [16].

We use the global solver of LINGO to solve these optimization problems. Reader can see [16] or [24] for more information about the global optimization method used by LINGO. Also, the LINGO code of our program can be found in [16].

6.3 An extension of example 1

In example 1 (section 5.1), we used our two settlement mechanism, and surprisingly, we observed that adding a wind generator actually increases the cost of generation and the use of fossil fuels. Here, we use a stochastic settlement mechanism, and we observe that this mechanism integrates the wind generator into the market more efficiently, and reduces cost of generation in comparison with the time before wind integration.

6.3.1 The stochastic settlement market after wind introduction

As the stochastic market clearing takes into account the future possibility of shortage in scenario 1, it allocates more pre-dispatch for the cheaper offer. Thus, a cheaper offer can also be used in scenario 1. The bidding behaviour, as shown in fig. 8, is similar to the case without wind generation.

The equilibrium quantities of this market are shown in table 8. In this case, the expensive inefficient generator is not used at all.

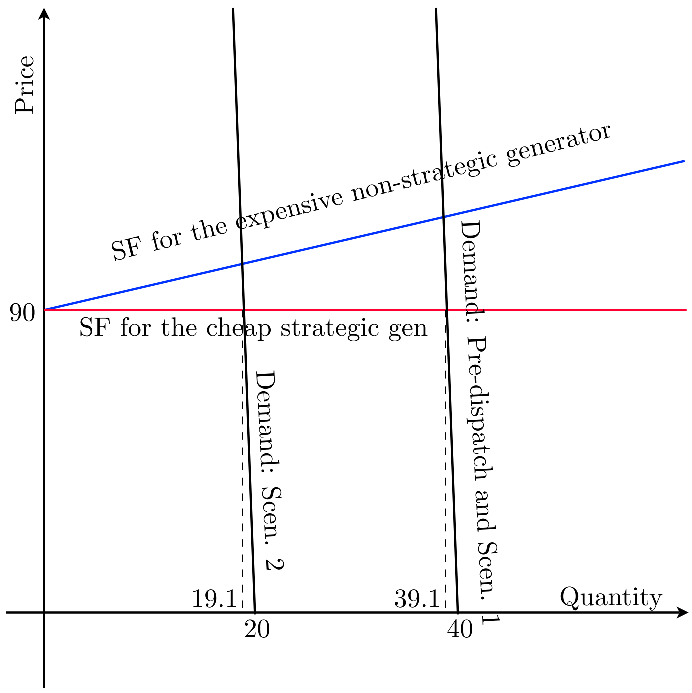

Figure 8: Equilibrium offered supply functions and clearing mechanism using the stochastic settlement mechanism after wind integration

Gen	q	**E(Profit)**
1	39.1	2328
2	0	0

Scen	y**:Gen 1**	y**:Gen 2**	p
1	39.1	0	90
2	19.1	0	90

f	**SW**	**CW**	**PW**
90.0	79668.5	76440.5	3228.0

Table 8: The equilibrium of the single settlement market after wind integration

6.3.2 Comparison

Let us use the following abbreviations for the name of our models and comparison criteria:

- TSBW (as a superscript): the two settlement mechanism before wind integration,
- TSAW (as a superscript): the two settlement mechanism after wind integration,
- SSBW (as a superscript): the stochastic settlement mechanism before wind integration,
- SSAW (as a superscript): the stochastic settlement mechanism after wind integration,
- TG: thermal generation i.e. generation from fossil fuels,
- SW: social welfare,
- CW: consumer welfare,
- PW: producer welfare.

According to the equilibrium values from tables 4, 5 and 8, the following results can be obtained for this example:

- thermal generation: $\mathrm{TG}^{\mathrm{TSAW}} > \mathrm{TG}^{\mathrm{TSBW}} = \mathrm{TG}^{\mathrm{SSBW}} = \mathrm{TG}^{\mathrm{SSAW}} = 0$,
- social welfare: $\mathrm{SW}^{\mathrm{SSAW}} > \mathrm{SW}^{\mathrm{SSBW}} = \mathrm{SW}^{\mathrm{TSBW}} > \mathrm{SW}^{\mathrm{TSAW}}$,
- consumer welfare: $\mathrm{CW}^{\mathrm{SSAW}} = \mathrm{CW}^{\mathrm{SSBW}} = \mathrm{CW}^{\mathrm{TSBW}} > \mathrm{CW}^{\mathrm{TSAW}}$,
- producer welfare: $\mathrm{PW}^{\mathrm{SSAW}} > \mathrm{PW}^{\mathrm{TSAW}} > \mathrm{PW}^{\mathrm{SSBW}} = \mathrm{PW}^{\mathrm{TSBW}}$.

In summary, it appears that for this particular market, adding a wind generator can decrease social and consumer welfare if we use a conventional two settlement clearing mechanism, while it improves these factors under a stochastic settlement mechanism.

7 Conclusion

In this paper, we have considered the effects of uncertainty resulting from intermittent generation on market performance. We have modeled the impacts of adding wind generation to a conventional two settlement market clearing mechanism, as well as a stochastic settlement mechanism under a supply function equilibrium paradigm. Although there are papers that explore cost of wind integration in electricity markets, their approach does not take into account that market participants can behave in a strategic manner in order to increase their profits. Strategic behaviour is a natural part of many electricity markets and it is essential for it to be considered in a market analysis. One important aspect of our analysis, which sets our work apart from other papers on the cost of wind integration, is the fact that we find and analyse a steady state equilibrium for the market that includes the strategic behaviour of market participants.

We started by introducing a two-settlement market clearing mechanism with firms that offer linear supply functions, with an Independent System Operator solving an optimization problem to clear the market.

It is difficult to find an analytical equilibrium for this market, as the payoff (i.e. profit) function of market participants depends on an optimization problem that does not yield a closed-form solution. Our first contribution in this paper is providing a method that enables us to find an analytical equilibrium for this game. This method can be used in similar games, where the payoff of participants is a function of the optimal solution to an optimization problem with inequality constraints.

Using our equilibrium analysis, we showed that increasing wind capacity does not necessarily result in higher social or consumer welfare or lower generation from fossil fuels. We presented examples in which wind integration decreases social and consumer welfare. This is the second contribution of this paper.

We also demonstrated that market clearing mechanism is an important factor in obtaining such results. In other words, we showed that a market reform might be essential if one considers large-scale wind integration into an electricity market. To demonstrate this claim, we provided a stochastic settlement market clearing mechanism, and showed that this mechanism improves social and consumer welfare, while decreasing generation from fossil fuels for the same example that yields lower social welfare under the conventional two settlement mechanism.

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