A single-settlement, energy-only electric power market for unpredictable and intermittent participants

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Abstract

We discuss a stochastic-programming-based method for scheduling electric power generation subject to uncertainty. Such uncertainty may arise from either imperfect forecasting or moment-to-moment fluctuations, and on either the supply or the demand side. The method gives a system of locational marginal prices which reflect the uncertainty, and these may be used in a market settlement scheme in which payment is for energy only. We show that this scheme is revenue-adequate in expectation.

Key words and phrases: electricity market, stochastic programming, locational pricing, wind power, regulation.

1 Introduction

In response to efforts to encourage more sustainable energy use and to discourage carbon emissions, many electricity systems are seeing an increase in growth of generation from renewable resources such as solar energy and wind. To varying degrees power generation from these sources is intermittent and so it is difficult to predict in advance. This makes scheduling and dispatching their electricity generation a challenging task.

In most industrialised countries, scheduling and dispatching electricity generation is carried out by solving an optimal power flow model. When one attempts to solve this model in practice, a difficulty arises since intermittent generators, as well as many consumers, cannot accurately predict the quantity of power they will produce or consume in advance. A solution that accurately represents reality can thus be computed only in real time, or in hindsight. On the other hand, some types of power plant are so slow to change their output that they have no hope of implementing the optimal solution to this model unless it is computed several hours (at least) beforehand.

To deal with this, much attention is paid to forecasting the intermittent sources of generation and load accurately, so that a (near) optimal generation schedule can be computed in advance. However, with high levels of uncertainty in such forecasts, such a pre-dispatch of generation is likely to be less efficient than anticipated, because it must later make expensive (or at worst physically impossible) adjustments to meet variations from the forecast.

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A related problem is that of variability. Each computed dispatch must correspond to some interval of time (in practice usually between 5 and 60 minutes in duration). But intermittent generators, as well as many consumers, will produce or consume a fluctuating quantity of power during this period. Even if the forecasts for the period prove correct in some average sense, there will still be departures from the forecast from moment to moment. These, too, may make the computed dispatch less efficient than anticipated by the optimal power flow problem. An exacerbating factor is that some power plants cannot change their output quickly enough to respond to the fluctuations due to load or other, intermittent, generation.

A traditional way to deal with variability is by contracting a frequency-keeping station that adjusts its output in real time to compensate. Such a solution is poorly integrated with the optimal-power-flow calculation, but the resulting inefficiency may be tolerable provided fluctuations are small in magnitude. Difficulties may arise, though, when the required adjustments become larger than can be accommodated by a single flexible unit.

Uncertainties in scheduling and dispatch may even arise from transmission parameters. The capacity (thermal limit) of an overhead transmission line depends on the temperature and velocity of the ambient air, and so is also somewhat uncertain in advance. This situation can be dealt with by adopting conservative capacity values, at some cost in system performance.

In this paper we explore an alternative scheduling and dispatch mechanism for intermittent generation that is based on a stochastic programming model. Our approach resembles that of [2, 3, 6, 8], all of which apply similar models to the problem of ensuring system security. Such a model is formulated in [2, 3] and applied to some standard test problems, while [6] suggests a computational technique. Some generator compensation (or settlement) schemes are considered in [8]. Also of interest is the recent paper [4], which applies a similar viewpoint to the topic of wind power integration.

We employ a model in which the contingencies represent forecast errors (e.g. in load, or in wind generation), or fluctuations, rather than plant failures. Our objectives are thus primarily economic, rather than security-related. This leads us to more explicitly consider how "flexibility" can be offered in a market context, and the role of the dual variables. The subject of revenue adequacy for such models (section 3 of the present paper) does not appear to have been considered previously.

In a deregulated power system, the cost coefficients in the optimal power flow model arise from offers (or bids) made by participants in a market. The dual variables in the model then give locational marginal prices at which energy is traded by the participants. Our model also produces such a nodal pricing scheme, which extends in a natural way to a market settlement mechanism in which payment is for energy only.

Unpredictability and variability are sometimes interpreted as "missing market" problems. A basic electricity market trades in only one commodity (energy), treated as an undifferentiated good even though some of its sources (and uses) are predictable and constant while others are unpredictable and fluctuating. A natural way to try to address the issue is by inventing a new market in "firm capacity" or "regulation", or something similar. Indeed, just such ancillary markets exist in many real-world deregulated power systems. But having two completely separate markets may create new opportunities for participants to engage in arbitrage or to exploit their market power. The approach in this paper attempts to avoid such problems by tying the markets together. Participants need not decide which market to trade in; that is, in effect, decided for them by a system optimization.

The paper is organized as follows. The next section will introduce scheduling models, and discuss pricing and settlement. Section 3 is devoted to the important topic of revenue adequacy, and includes a result showing that our settlement scheme is revenue adequate in expectation. Section 4 contains illustrative numerical examples, and Section 5 some additional comments.

2 Statement of the market model.

2.1 The conventional dispatch model.

We begin by defining a standard dispatch (optimal power flow) problem, without regulation or reserve. This will serve to introduce notation, and provide a starting point for the developments in the rest of the paper.

As with other problems in this paper, the setting will be an electric power system comprising a collection N of nodes (locations) connected by a collection L of transmission lines. Let $\tau_n(f)$ denote the net power imported via the transmission system to node $n \in N$, when $f = (f_\ell)_{\ell \in L}$ is the vector of line flows. If the lines are lossless, then the τ_n are linear functions:

$$\tau_n(f) = \sum_{\ell:\nu_1(\ell)=n} f_\ell \; - \sum_{\ell:\nu_0(\ell)=n} f_\ell,$$

where $\nu_0(\ell)$, $\nu_1(\ell)$ are the endpoints of ℓ, and f_ℓ is taken to be positive in the direction from $\nu_0(\ell)$ to $\nu_1(\ell)$.

If we wish to model line losses, then the τ_n are nonlinear. A physically plausible choice is a quadratic loss $\rho_\ell f_\ell^2$ on each line:

$$\tau_n(f) = \sum_{\ell:\nu_1(\ell)=n} (f_\ell - \frac{1}{2}\rho_\ell f_\ell^2) \; - \sum_{\ell:\nu_0(\ell)=n} (f_\ell + \frac{1}{2}\rho_\ell f_\ell^2).$$

Alternatively, the losses may be modelled as piecewise linear; this is convenient for computational purposes. This paper will assume that the τ_n are concave functions with $\tau_n(0) = 0$; this holds under almost any reasonable loss model.

The capabilities of the transmission system are represented by the requirement $f \in U$. The set U incorporates the maximum capacities of individual lines, loop flow constraints, and possibly other constraints as well. The only assumptions we need regarding U are that it be a convex compact set with $0 \in U$.

In the standard dispatch problem, a system operator (SO) must consider a collection T of offers to supply or consume electricity. Offer $i \in T$ is for a tranche of quantity q_i at a local node $\nu(i)$; the SO must decide the quantity x_i of this to accept. Both q_i and x_i are taken to be positive in the sense of injecting power to the local node, so we must have

$$x_i \in C_i := \begin{cases} [0, q_i], & \text{if } q_i \geq 0 \text{ (supply-side offer)} \\ [q_i, 0], & \text{if } q_i \leq 0 \text{ (demand-side bid)} \end{cases}.$$

Offer i also has an associated ask or bid price p_i; this is taken to be positive when the corresponding cashflow is opposite in direction to the energy flow (which is usually the case). Inelastic loads can be handled by setting p_i =VOLL, the value of lost load.

The SO's problem can then be stated as:

Problem P0:

$$
\begin{array}{llll}
\min & \sum_{i\in T} p_i x_i & & \\
\text{s.t.} & \tau_n(f) + \sum_{i\in T(n)} x_i = 0 & \forall n \in N & [\pi_n] \\
& x_i \in C_i & \forall i \in T & \\
& f \in U & &
\end{array}
$$

Here $T(n) = \{i : \nu(i) = n\}$. This is an optimization problem in the variables $x = (x_i)_{i\in T}$ and f. The problem is convex (i.e. involves minimizing a convex function over a convex set) if the transmission lines are lossless, but not if losses are modelled. The most important constraints are those requiring energy balance at node n ($\tau_n(f)+\sum_{i\in T(n)} x_i = 0$). The dual variable π_n associated with such a constraint gives the marginal cost of creating a small power surplus at node n, and is thus an appropriate price at which to trade power generated or consumed at n.

2.2 A stochastic dispatch model.

We now elaborate the problem of the previous section to include real-time regulation in the presence of uncertainties in the offers.

Our new model will be a two-stage stochastic program. The first stage represents an initial dispatch computed in advance, with only probabilistic estimates of some quantities available. This could be thought of as a "day-ahead" dispatch, although the same ideas may apply to shorter time scales. Inasmuch as this initial dispatch may be modified later, it could be described as "non-physical"; however, it is important to have such a dispatch for planning purposes. As we shall see later, it can also play a role in pricing and settlement.

The second stage represents "real time", i.e. the actual dispatches over a short period. This period is meant to coincide with the market trading period or perhaps a sub-interval thereof; its duration might thus be somewhere between a minute and an hour. During this period some quantities (loads and the output of wind farms) will take on realized values unknown at the first stage; these quantities may also fluctuate during the period. Adapting to these changes will require re-dispatch; we use the term "regulation" for differences between first-stage and second-stage dispatches.

From the point of view of the first stage, quantities relating to the second stage are random variables defined on some probability space $(\Omega, \mathcal{F}, \mathcal{P})$, which we assume to be finite.

The general form of the objective for such a problem should be

$$\text{minimize} \qquad c_1 + E\left[\frac{1}{\theta}\int_0^\theta c_2(t)\ dt\right]$$

where c_1 represents costs associated with the first stage, and $c_2(t)$ represents (random) instantaneous costs associated with regulation in the second-stage period $[0, \theta]$. That is, decisions at the first stage should be made with regard for their implications for the expected costs of regulation in real-time.

Note that this expression involves averaging over both time (the integral) and the probability space (the expectation). We can make the notation less cumbersome by defining V to be a random variable distributed uniformly on $[0, \theta]$, independently of the other random variables in the problem. Then the objective can be written

$$\text{minimize} \qquad c_1 + E\left[c_2(V)\right]$$

The conceptual purpose of this trick is that it enables us to think of the second stage as representing a single point in time. The associated random quantities can then be modelled as random variables, rather than random functions of time.

Again we have a collection T of offers. For each $i \in T$ we require a solution (x_i, X_i), where the first-stage dispatch x_i is a real number, and the second-stage dispatch X_i may be a random variable. We must have

$$(x_i, X_i(\omega)) \in C_i(\omega) \qquad \forall \omega \in \Omega,$$

where C_i is a random convex set which may take several different forms, depending on the kind of offer involved. Some examples of different types of offers include:

- Completely inflexible generation: A fixed (firm) quantity q_i of power is offered. First-stage dispatched quantity is x_i; this cannot be varied at the second stage.

$$\begin{aligned} x_i &\in [0, q_i] \\ X_i(\omega) &= x_i \qquad \forall \omega \in \Omega. \end{aligned}$$

- Completely flexible generation: A firm quantity q_i is offered. First-stage dispatched quantity is x_i; this may be varied at the second stage.

$$\begin{aligned} x_i &\in [0, q_i] \\ X_i(\omega) &\in [0, q_i] \qquad \forall \omega \in \Omega. \end{aligned}$$

- Unpredictable or intermittent generation (e.g. wind farm): A generator with maximum capacity q_i offers a random quantity S_i. First-stage dispatched quantity is x_i; this may be varied at the second stage.

$$\begin{aligned} x_i &\in [0, q_i] \\ X_i(\omega) &\in [0, S_i(\omega)] \qquad \forall \omega \in \Omega. \end{aligned}$$

- Demand-side bid: A quantity $-q_i \geq 0$ is bid for. First-stage dispatched quantity is x_i; this may be varied at the second stage. To the extent that $X_i \neq q_i$, the bid goes unsatisfied.

$$\begin{aligned} x_i &\in [q_i, 0] \\ X_i(\omega) &\in [q_i, 0] \qquad \forall \omega \in \Omega. \end{aligned}$$

- Unpredictable load: A random load of size $D_i \geq 0$. First-stage dispatched quantity is x_i; this may be varied at the second stage. To the extent that $X_i \neq -D_i$, the load is shed.

$$\begin{aligned} x_i &\leq 0 \\ X_i(\omega) &\in [-D_i(\omega), 0] \qquad \forall \omega \in \Omega. \end{aligned}$$

We will assume $(0, 0) \in C_i(\omega)$ to ensure feasibility.

Offer i has an associated ask or bid price p_i, which applies to power dispatched at the first stage. In addition, the participant making the offer also offers to, in real time,

- sell additional power (or sell back power) to the system at an asking price $p_i^+ > p_i$, or
- buy back power (or buy additional power) from the system at a bid price $p_i^- < p_i$,

wherever this is permitted or required by the constraints C_i. For supply-side offers, the use of p_i^+ and p_i^- effects an offer of regulation: the generator offers to help make real-time adjustments in return for a price premium on the energy used (or in the case of down-regulation, not used). This possibility should be attractive to the owner of (for example) dispatchable hydropower plant: not only can stored water be converted to energy and sold, it may also earn regulation revenue *without* being released.

Demand-side bidders may also offer regulation by using p_i^+ and p_i^-. For an inelastic load, we should set p_i =VOLL; the magnitudes of $p_i^+ - p_i$ and $p_i - p_i^-$ should not be important unless load-shedding is a real possibility.

The strict inequality between p_i^-, p_i, and p_i^+ is required to avoid degeneracy in the solution. For example, if co-located consumers i and j had $p_i^- = p_i = p_i^+ = p_j^- = p_j = p_j^+$ then x_i and x_j would not be individually determined (since dispatch may be re-allocated between them at no cost), although $x_i + x_j$ would be.

It is helpful in what follows to use $(y)_+$ to denote $\max\{y, 0\}$ and $(y)_-$ to denote $\max\{-y, 0\}$. Observe that both of these are non-negative quantities.

The SO's problem can then be stated as:

Problem P1:

$$\begin{array}{llll}
\min & \sum_{i\in T}\left(p_i x_i + E\left[p_i^+(X_i - x_i)_+ - p_i^-(X_i - x_i)_-\right]\right) & & \\
\text{s.t.} & \tau_n(f) + \sum_{i\in T(n)} x_i = 0 & \forall n & [\pi_n] \\
& \tau_n(F) - \tau_n(f) + \sum_{i\in T(n)}(X_i - x_i) = 0 & \forall n\ \forall \omega \in \Omega & [P(\{\omega\})\lambda_n(\omega)] \\
& (x_i, X_i) \in C_i & \forall i\ \forall \omega \in \Omega & \\
& f \in U & & \\
& F \in U & \forall \omega \in \Omega. &
\end{array}$$

This is a stochastic optimization problem in the variables x, f, X, and F. The last two of these represent dispatches and line flows at the second stage, and so are random variables. (We have followed the usual notational convention for random variables of suppressing the dependence on ω; thus X_i rather than $X_i(\omega)$.) With respect to these, it is important to realize that the SO's job is to choose the whole random variable (i.e. all of its possible values), rather than just a single value. As with the standard dispatch problem, problem **P1** is convex if the transmission lines are lossless, but not if losses are modelled.

Note that the existence of a (primal) optimal solution to **P1** is guaranteed, since we are optimizing a continuous function over a non-empty compact set. (A feasible solution can be obtained by setting all the variables to zero.) For the existence of the dual variables, an additional condition may be required. In the lossless case, the problem will usually reduce to a linear program, for which duals always exist. Otherwise, we assume

For all sufficiently small $|\epsilon|$, **P1** remains feasible if the zeros on the right-hand-sides of its first two constraints are replaced by ϵ. (1)

That is, it is possible to deliver marginal additional power to any node, in any or all scenarios – a condition likely to be satisfied in problems of practical interest. Condition (1) is a form of constraint qualification (a Slater condition) which guarantees the existence of the dual variables π_n and λ_n (see [7]).

The reader may be wondering why the second-stage energy balance constraint of **P1** was not written in the apparently simpler way

$$\tau_n(F) + \sum_{i\in T(n)} X_i = 0 \quad \forall n\ \forall \omega \in \Omega.$$

This formulation of the primal problem would be equivalent to the one we have used. But the dual variables would be different, and less convenient for our purposes (see section 2.3).

In the special case where $p_i^+ - p_i = p_i - p_i^- = r_i$, say, the objective of **P1** may also be written

$$\min \quad E\left[\sum_{i\in T} (p_i X_i + r_i \left|X_i - x_i\right|)\right]. \tag{2}$$

This indicates that the initial dispatch x_i can be thought of as a kind of forecast of the second-stage dispatch X_i, in the sense that the SO's objective penalizes deviations from it.

2.3 Nodal pricing with regulation.

Suppose that problem **P1** is solved and an optimal solution (x^*, f^*, X^*, F^*) obtained. The question then arises as to what the market participants should pay or be paid for the power they have consumed or provided.

A conventional energy-only power market (with dispatch as in section 2.1) already encompasses the idea that the market price of electricity may vary with both location (in the network) and time (of day). In a two-stage market (with dispatch as in 2.2), we need a further distinction between the prices of electricity traded at the first stage and at the second stage.

The first-stage nodal price π_n is the dual variable corresponding to a first-stage energy balance constraint in **P1**. This can be interpreted as the marginal cost of serving a deterministic additional load at node n which is present in the first stage and in every second-stage scenario. It is thus an appropriate price at which to trade non-random (i.e. notified in advance) quantities of electricity at node n.

The second-stage nodal price λ_n (or rather, $\lambda_n(\omega)$) is the (probability-removed) dual variable corresponding to a second-stage energy balance constraint in **P1**. This can be interpreted as the marginal cost of serving an additional load at node n which is present at the second stage in scenario ω only. This price is itself a random variable. It is an appropriate price at which to trade random (i.e. not foreseen in advance) quantities of electricity in real time at node n.

The reader may object that λ_n is the wrong marginal cost to consider, as it allows the perturbation in load in scenario ω to be met by perturbing the first-stage variables x, f as well as $X(\omega)$, $F(\omega)$. It could be argued that one should instead consider the real-time problem for the particular ω:

Problem RT(ω)

$$\begin{array}{llll} \min & \sum_{i\in T}\left(p_i^+(y_i - x_i^*)_+ - p_i^-(y_i - x_i^*)_-\right) & & \\ \text{s.t.} & \tau_n(g) + \sum_{i\in T(n)} y_i = 0 & \forall n & [\pi_n^R(\omega)] \\ & (x_i^*, y_i) \in C_i(\omega) & \forall i & \\ & g \in U, & & \end{array}$$

where x^* is the already-determined optimal solution of **P1**. However, it can be shown ([8]) that $(\lambda_n(\omega))_{n\in N}$ are also dual-optimal for **RT**(ω), so it is valid to use them as second-stage prices under this interpretation also. In the exceptional situation where $P(\{\omega\}) = 0$ (see section 5), $\lambda_n(\omega)$ is undefined by **P1**; for completeness, we then define $\lambda_n(\omega) = \pi_n^R(\omega)$, the dual variable in **RT**(ω).

When we combine the effects of first- and second-stage trading, we see that an appropriate sum to pay to the market participant responsible for offer i is

$$x_i^*\pi_{\nu(i)} + (X_i^* - x_i^*)\lambda_{\nu(i)}. \tag{3}$$

This can also be written

$$x_i^*(\pi_{\nu(i)} - \lambda_{\nu(i)}) + X_i^* \lambda_{\nu(i)},$$

which illustrates another way to view the two-stage market. The first stage may be viewed as a market for hedges (contracts for differences); the second stage as a spot market in which all power is ultimately traded. Note, though, that these are not two separate markets: they are tightly linked, with the same set of offers driving both.

3 Revenue adequacy

An important requirement for a market settlement scheme is that it be revenue adequate. This means that the payments that the SO must make to (or receive from) the participants do not leave it in financial deficit. (It is permissible for the SO to run a surplus, as this in itself does not preclude the operation of such a market.)

In this section, we present several revenue adequacy results for the stochastic programming model and settlement scheme proposed above. We note, though, that the implication of revenue adequacy in a practical setting relies on the fidelity of the model, a point to be discussed further in section 5.

3.1 Revenue adequacy in expectation

In this subsection, we establish that the settlement scheme (3) is revenue adequate in expectation. That is,

$$E\left[\sum_i \left(x_i^* \pi_{\nu(i)} + (X_i^* - x_i^*)\lambda_{\nu(i)}\right)\right] \leq 0. \tag{4}$$

This means that, if this type of market is used repeatedly over many trading periods, the SO will not run a financial deficit over time. There may, however, be a deficit in an individual trading period. In an actual implementation, the SO would need to maintain a financial reserve to buffer fluctuations in the period-by-period surplus.

Since the energy-balance constraints of **P1** are satisfied at optimality, (4) may also be written

$$E\left[\sum_n \left(\pi_n \tau_n(f^*) + \lambda_n(\tau_n(F^*) - \tau_n(f^*))\right)\right] \geq 0. \tag{5}$$

In general, such revenue adequacy results require a convex optimization problem. Our problem **P1** is convex if the transmission lines are lossless (τ_n linear), but not otherwise. We therefore distinguish two cases.

Theorem 1. If the functions τ_n in **P1** are linear, then (5) holds.

Proof. Consider the following Lagrangian for problem **P1**:

$$\begin{aligned} L \;=\; & \sum_{i \in T} \left(p_i x_i + E\left[p_i^+ (X_i - x_i)_+ - p_i^- (X_i - x_i)_-\right]\right) \\ & - \sum_n \pi_n \left(\tau_n(f) + \sum_{i \in T(n)} x_i\right) \\ & - E\left[\sum_n \lambda_n \left(\tau_n(F) - \tau_n(f) + \sum_{i \in T(n)} (X_i - x_i)\right)\right]. \end{aligned} \tag{6}$$

Since **P1** is convex, L is minimized subject to the remaining constraints

$$\begin{aligned} (x_i, X_i) &\in C_i \quad \forall i\ \forall \omega \in \Omega \\ f &\in U \\ F &\in U \qquad \forall \omega \in \Omega \end{aligned}$$

at the optimum (x^*, f^*, X^*, F^*) of **P1**. In particular, consideration of the terms in L involving f or F gives that

$$\sum_n (\pi_n \tau_n(f) + E[\lambda_n(\tau_n(F) - \tau_n(f))]) \tag{7}$$

is maximized over $\{(f, F) : f \in U, F \in U\ \forall \omega \in \Omega\}$ at (f^*, F^*). Since $0 \in U$, the result follows. ∎

To cover the case of lossy networks (τ_n nonlinear), we define a new problem.

Problem P2:

$$\begin{array}{llll} \min & \sum_{i\in T}\left(p_i x_i + E\left[p_i^+(X_i - x_i)_+ - p_i^-(X_i - x_i)_-\right]\right) & & \\ \text{s.t.} & \tau_n(f) + \sum_{i\in T(n)} x_i - z_n = 0 & \forall n & [\pi_n] \\ & \tau_n(F) - \tau_n(f) + \sum_{i\in T(n)}(X_i - x_i) - Z_n = 0 & \forall n\ \forall \omega \in \Omega & [P(\{\omega\})\lambda_n(\omega)] \\ & (x_i, X_i) \in C_i & \forall i\ \forall \omega \in \Omega & \\ & f \in U & & \\ & F \in U & \forall \omega \in \Omega & \\ & z_n \geq 0 & \forall n & \\ & Z_n \geq 0 & \forall n \forall \omega \in \Omega. & \end{array}$$

This introduces new variables z and Z which have the effect of allowing free disposal of power at any node. If the τ_n are concave, then **P2** is a convex problem. Of course, mathematical results concerning **P2** will have physical meaning only when its optimal solution $(x^*, f^*, z^*, X^*, F^*, Z^*)$ is also an optimal solution of **P1**, i.e. has $z^* = 0$ and $Z^* = 0$. As it is rarely beneficial to waste power, this will usually be the case.

Theorem 2. Suppose the functions τ_n in **P1** are concave, the problem **P2** satisfies condition (1), and the (primal and dual) optimal solutions (x^*, f^*, X^*, F^*) and (π, λ) of **P1** are also optimal for **P2** (with $z^* = 0$, $Z^* = 0$). Then (5) holds.

Proof. Form a Lagrangian for **P2** analogous to (6). Since **P2** is convex, this Lagrangian is minimized at $(x^*, f^*, z^* = 0, X^*, F^*, Z^* = 0)$ with respect to the remaining constraints. In particular, the terms of the Lagrangian involving f or F are the same as in (7), and so the result follows as before. ∎

Remark. The Lagrangian used to prove Theorem 2 also contains the following terms in z, Z:

$$\sum_n \pi_n z_n + E\left[\sum_n \lambda_n Z_n\right].$$

Since this expression must be minimized over $\{(z, Z) : z \geq 0, Z \geq 0\ \forall \omega \in \Omega\}$ at $(0, 0)$, we must have $\pi_n \geq 0$ and $\lambda_n \geq 0$ with probability 1. That is, revenue adequacy is obtained only if there are no negative nodal prices. The conventional dispatch problem **P0** also has this property; see, for example, [5]. Consequently, the SO risks losing money in periods with negative nodal prices. There is no easy way to bound such losses. However, it is empirically observed in real power markets that this kind of loss is not large or frequent enough to be of practical concern.

3.2 Revenue adequacy in individual scenarios

Although (3) is revenue adequate in expectation, there may be individual scenarios $\omega \in \Omega$ for which revenue is inadequate, i.e.

$$\sum_i \left(x_i^* \pi_{\nu(i)} + (X_i^* - x_i^*)\lambda_{\nu(i)} \right) > 0,$$

or equivalently

$$\sum_n \left(\pi_n \tau_n(f^*) + \lambda_n(\tau_n(F^*) - \tau_n(f^*)) \right) < 0.$$

If such a scenario occurs, the SO will experience negative cashflow for the period. In this subsection, we describe some limitations on when this can occur.

We first observe that the second-stage cashflows, at least, are always revenue adequate.

Theorem 3. Suppose that either the hypotheses of Theorem 1 or the hypotheses of Theorem 2 hold. Then for each $\omega \in \Omega$,

$$\sum_i (X_i^*(\omega) - x_i^*)\lambda_{\nu(i)}(\omega) \leq 0,$$

or equivalently

$$\sum_n \lambda_n(\omega)(\tau_n(F^*(\omega)) - \tau_n(f^*)) \geq 0.$$

Proof. As remarked in section 2.3, the optimal solutions $(X^*(\omega), F^*(\omega))$ and $\lambda(\omega)$ of **P1** are also optimal for $\mathbf{RT}(\omega)$. In the lossless (τ_n linear) case, $\mathbf{RT}(\omega)$ is convex. So the following Lagrangian of $\mathbf{RT}(\omega)$

$$L_R = \sum_{i \in T} \left(p_i^+(y_i - x_i^*)_+ - p_i^-(y_i - x_i^*)_- \right) - \sum_n \lambda_n(\omega) \left(\tau_n(g) + \sum_{i \in T(n)} y_i \right)$$

is minimized subject to the remaining constraints $(x_i^*, y_i) \in C_i(\omega)\ \forall i$, $g \in U$ at $y = X^*(\omega)$, $g = F^*(\omega)$. In particular,

$$\sum_n \lambda_n(\omega)\tau_n(g)$$

is maximized over $\{g : g \in U\}$ at $F^*(\omega)$. Since $f^* \in U$, we obtain

$$\sum_n \lambda_n(\omega)\tau_n(F^*(\omega)) \geq \sum_n \lambda_n(\omega)\tau_n(f^*),$$

from which the result follows.

In the case of a lossy network, $\mathbf{RT}(\omega)$ can be made convex by the same "free disposal" technique used to prove Theorem 2. The rest of the proof is then similar. ∎

The first-stage part of the settlements need not be revenue adequate. That is, we may have

$$\sum_n \pi_n \tau_n(f^*) < 0.$$

For an example, see section 4.1.

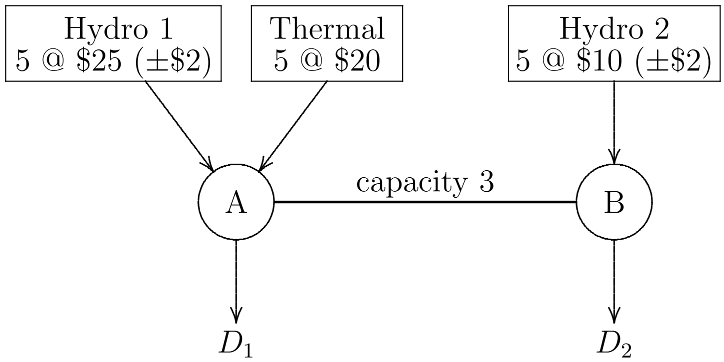

Figure 1: A two-node system with random loads.

Scenario	probability	D_1	D_2
ω_1	0.6	2	6
ω_2	0.4	7	1

Table 1: Scenarios for the two-node problem.

The next result shows that in some circumstances, we may have revenue adequacy at the first stage (and hence overall, in every scenario).

Theorem 4. Suppose that the network is lossless, and that the optimal solution of **P1** satisfies $F^* - f^* \in U \ \forall \omega \in \Omega$. Then we have first-stage revenue adequacy, i.e.

$$\sum_n \pi_n \tau_n(f^*) \geq 0.$$

Remark. Theorem 4 assures us of revenue adequacy for every ω provided the regulation adjustments $F^* - f^*$ are not so great as to constitute an infeasible flow pattern themselves. Thus, the revenue-inadequate instances of **P1** will lie among those in which some contingencies require very large adjustments to the flows.

Proof. In the Lagrangian argument used in Theorem 1, we saw that

$$\sum_n \left(\pi_n \tau_n(f^*) + E\left[\lambda_n(\tau_n(F^*) - \tau_n(f^*))\right]\right) \ \geq \ \sum_n \left(\pi_n \tau_n(f) + E\left[\lambda_n(\tau_n(F) - \tau_n(f))\right]\right)$$

whenever $f \in U$ and $F \in U \ \forall \omega \in \Omega$. Take $f = 0$ and $F = F^* - f^*$, and use the linearity of τ_n, to obtain the result. ∎

4 Examples

4.1 Two-node example

The example depicted in Figure 1 demonstrates the use of stochastic programming in dispatch. It has two demand scenarios which, although they agree as to the total load, differ markedly in the location of the load. The Thermal generation offer is completely inflexible (requiring $X_i(\omega_1) = X_i(\omega_2) = x_i$), while the Hydros are completely flexible (i.e. may be re-dispatched arbitrarily) and have made regulation offers. The line is lossless. The loads are treated as inelastic, with p_i =VOLL, $p_i^+ = p_i + \epsilon$, and $p_i^+ = p_i - \epsilon$. We have taken VOLL=\$1000 and ϵ =\$0.001, although the values chosen (within reason) do not affect either the primal or dual optimal solutions.

The reader may wish to examine a full presentation of Problem **P1** for this example:

$$
\begin{array}{lll}
\min & 25x_1 + 20x_2 + 10x_3 & \\
& + \sum_{j=1}^{2} P(\omega_j) \Big[27(X_1(\omega_j) - x_1)_+ - 23(X_1(\omega_j) - x_1)_- + 12(X_3(\omega_j) - x_3)_+ - 8(X_3(\omega_j) - x_3)_- & \\
& \quad + (VOLL)(X_4(\omega_j) + X_5(\omega_j)) \; + \; \epsilon\,|X_4(\omega_j) - x_4| + \epsilon\,|X_5(\omega_j) - x_5| \quad \Big] & \\
\text{s.t.} & -f + x_1 + x_2 + x_4 = 0 & [\pi_A] \\
& f + x_3 + x_5 = 0 & [\pi_B] \\
& -F(\omega) + f + X_1(\omega) - x_1 + X_4(\omega) - x_4 = 0 & \omega = \omega_1, \omega_2 \qquad [P(\{\omega\})\lambda_A(\omega)] \\
& F(\omega) - f + X_3(\omega) - x_3 + X_5(\omega) - x_5 = 0 & \omega = \omega_1, \omega_2 \qquad [P(\{\omega\})\lambda_B(\omega)] \\
& 0 \le x_i \le 5 & i = 1, 2, 3 \\
& x_i \le 0 & i = 4, 5 \\
& 0 \le X_i(\omega) \le 5 & i = 1, 3 \quad \omega = \omega_1, \omega_2 \\
& X_2(\omega) = x_2 & \omega = \omega_1, \omega_2 \\
& -2 \le X_4(\omega_1) \le 0, \quad -6 \le X_5(\omega_1) \le 0 & \\
& -7 \le X_4(\omega_2) \le 0, \quad -1 \le X_5(\omega_2) \le 0 & \\
& -3 \le f \le 3 & \\
& -3 \le F(\omega) \le 3 & \omega = \omega_1, \omega_2.
\end{array}
$$

Here the participant indices $i = 1, 2, 3, 4, 5$ refer, respectively, to the Hydro 1, Thermal, and Hydro 2 generators and the loads at A and B. The line flows f, F are positive from A to B. Note that the objective terms relating to the loads have been written in the form suggested by (2).

The primal solution of this problem has $x^* = (0, 3, 5, -2, -6)$ and $f^* = 1$, indicating that the system operator should prepare as if for scenario ω_1. That is, 3 units from the Thermal and all 5 units of Hydro 2 should be initially dispatched. If ω_2 occurs, lack of transmission capacity forces a re-dispatch: $X^*(\omega_2) = (1, 3, 4, -7, -1)$ and $F^*(\omega_2) = -3$.

The first-stage dual solution is $\pi_A = 20$, $\pi_B = 12.4$. Note that these prices differ even though, in the first-stage primal solution, the transmission line between them is not constrained. This is different from the conventional dispatch problem (**P0**) in which (absent loop constraints) a price difference across a line may occur only when the line is at capacity. Here the price difference reflects a contingent transmission constraint that may come into play upon re-dispatch.

Note, too, that the price difference is in the *opposite* direction to that suggested by the initial flow $f^* = 1$: the flow is from the more expensive node to the cheaper one. This is because the contingent transmission constraint, if it occurs, will reverse the direction of flow ($F^*(\omega_2) = -3$). A consequence of this is that the market is not revenue-adequate with respect to the first-stage payments alone. (The SO uses the prices π_A, π_B to buy 3 units from the Thermal and 5 units from Hydro 2, and to sell 2 units at A and 6 units at B, creating a system deficit of \$7.6.) If scenario ω_1 occurs, there will be no further payments at the second stage, and so revenue-inadequacy is possible for the market overall.

However, if scenario ω_2 occurs, there will be additional payments arising from the re-dispatch. The second-stage duals are $\lambda_A(\omega_1) = \lambda_B(\omega_1) = 46/3$, $\lambda_A(\omega_2) = 27$, $\lambda_B(\omega_2) = 8$. In the event of

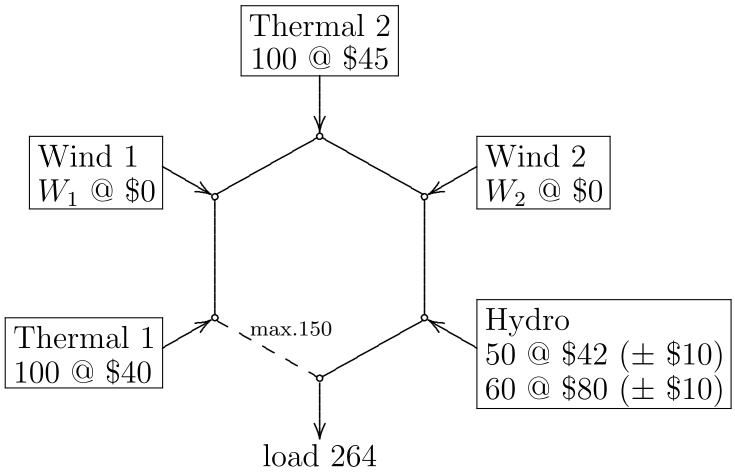

Figure 2: A system with uncertain wind generation.

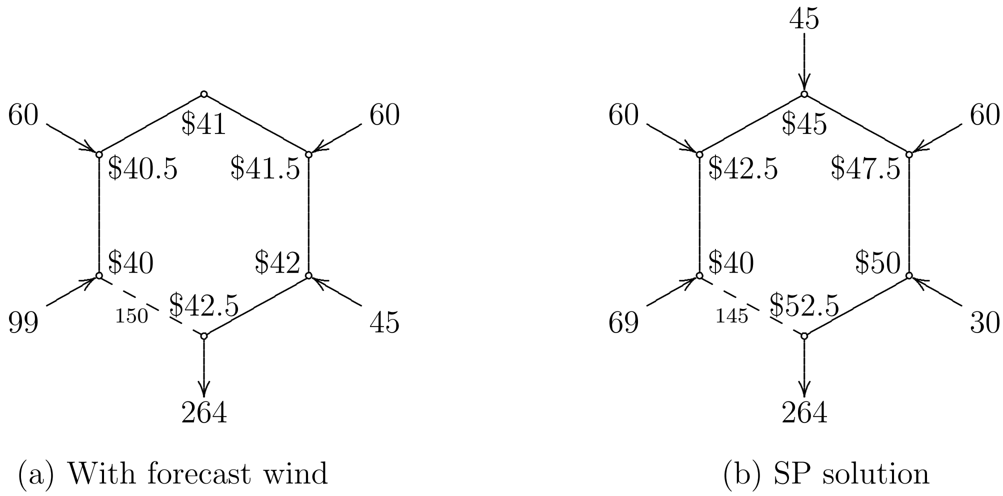

Figure 3: Optimal dispatches.

ω_2, the system operator uses the prices $\lambda_A(\omega_2)$, $\lambda_B(\omega_2)$ to buy 1 unit from Hydro 1, sell back 1 unit to Hydro 2, sell 5 additional units to the load at A, and buy back 5 units from the load at B, creating a surplus of \$76. The expectation of the system surplus for the market overall is thus -7.6 + (0.4)(76) = 22.8 dollars, maintaining revenue adequacy in expectation. The reader may wish to compare this result with Theorems 1, 3, and 4.

4.2 Six-node example

A more complicated example is depicted in Figure 2. Here we have five generators sharing a loop network on which the only transmission constraint of significance is the maximum flow of 150 on the dashed line in the diagram. The six lines are assumed to be lossless, but with equal reactances. This means that $\frac{5}{6}$ of the power generated by Thermal 1 will flow to the load via the limited-capacity line, while $\frac{1}{6}$ flows via the other five lines. For Wind 1, the flows divide in proportion $(\frac{2}{3}, \frac{1}{3})$; for Thermal 2, $(\frac{1}{2}, \frac{1}{2})$; for Wind 2, $(\frac{1}{3}, \frac{2}{3})$. The Hydro has the most advantageous position: only $\frac{1}{6}$ of its power flows over the limited-capacity line, with the other $\frac{5}{6}$ taking the more direct route.

As in the previous example, the Thermals are completely inflexible, while the Hydro is completely flexible and makes a regulation offer. The load is inelastic. The Wind farms have uncertain production: for each wind farm, the output is equally likely to be 30, 50, 60, 70, or 90. The two wind farms are independent, so there are 25 scenarios in total.

Note that the most accurate possible forecast for each wind farm's output is 60; this is both the median and the expected value, as the distribution is symmetric about that value. If these forecasts are used to find a conventional optimal dispatch as in section 2.1, the result is that shown in Figure 3(a). It consists largely of dispatching the cheapest generators first. The transmission constraint binds, preventing full use of Thermal 1 and causing the clearing prices to differ among the nodes in a so-called "spring-washer" pattern. This conventional solution is not very robust to deviations from the forecast. If total wind power production should fall below the 120 forecast, there are only 5 additional units of reasonably-priced hydropower available to make up the deficit; anything beyond that will be dearly bought. On the other hand, if wind power availability exceeds the forecast, it will be difficult to make use of the surplus (by reducing hydro generation) without violating the transmission constraint. Even in the case where the wind farms collectively achieve their forecast, there may be a problem if Wind 1 is above the forecast while Wind 2 is below it (e.g. $(W_1, W_2) = (70, 50)$). A simple re-dispatch from Wind 2 to Wind 1 is prevented by the transmission constraint, so this situation must be dealt with by spilling some wind and increasing hydro output. (A re-dispatch from Wind 1 to Wind 2 would relieve the constraint, so could be done without difficulty.)

Figure 3(b) shows the initial dispatch found from a stochastic program as in section 2.2, taking all 25 possible wind scenarios into account. (The re-dispatches required in those scenarios are not shown.) The differences from Figure 3(a) can be interpreted as attempts to increase the robustness of the solution. The unused capacity in the cheaper hydro tranche has increased from 5 to 15, providing a larger cushion against wind shortfalls. The flow on the limited-capacity line has reduced from 150 to 145, providing some much-needed flexibility to adjust dispatch between the two wind farms, or to take advantage of additional wind that may arise.

Figure 3(b) also shows the nodal clearing prices. Although the transmission constraint is not binding, it will become binding after re-dispatch in some scenarios. The prices anticipate this by falling into a spring-washer pattern.

The structure of the Hydro generator's offer plays an important role in this example. The high price placed on the second tranche makes the SO less willing to dispatch the first tranche (preferring to leave it as a kind of reserve), even though the second tranche is itself not used in the initial dispatch. A similar situation would arise if the Hydro were to split off some of its first tranche (say, 40 units) into a separate tranche with a low ask price. The conventional solution is unaffected by this change. But in the SP solution, the SO increases the total (initial) hydro dispatch to 50, to make it more likely that the cheap water will actually be used.

5 Further discussion

To set up a realistic version of **P1** requires an ensemble forecast: a collection Ω of contingencies with attached probabilities. Note that each $\omega \in \Omega$ must describe a realization of the uncertain parameters *for the whole system*. It would be much less useful if (for example) each wind farm on the system had an ensemble forecast produced by its owner, as this would give no information about correlations between wind farms. An exhaustive list of all the scenarios one might want could thus be very long; however, it may be that relatively few of these scenarios are really required for a good model. Of course, **P1** with any number (more than one) of scenarios is likely to be an improvement over conventional dispatch.

We have assumed throughout that the number of second-stage scenarios is finite. This avoids measurability and integrability considerations. More practically, finiteness of Ω allows our problem to be solved by standard computational techniques. In principle, though, it would be straightforward to extend the formulation to general probability spaces. This would open up some new possibilities for modelling contingencies, using continuous random variables. The results on revenue adequacy, etc. in this paper should all have analagous versions for such a generalization, since none of their proofs depend on finiteness of the sample space. However, the computational difficulty of solving the resulting problems would be greatly increased.

Note the importance of attaching probabilities to scenarios. This makes them different from the contingencies usually addressed by an $n-1$ security standard, which occur so rarely that it is not useful to consider their actual probabilities with any accuracy. The contingencies in this paper – forecast errors and fluctuations – need not be rare, but may recur frequently. It is thus appropriate to include them in an objective function via an expectation, as in the stochastic programming approach.

It is nonetheless possible that some scenarios may have zero probabilities attached. To allow for this, the stochastic programs in this paper have been formulated with constraints holding "for all ω" rather than the more usual "with probability 1". Such scenarios represent contingencies (e.g. plant outages) which are so rare as to have negligible contribution to any expectation, but for which the system must have adequate recourse if they occur. For example, constraints of the "$n-1$ security" type can be modelled this way.

In a practical implementation, the finite collection Ω of possible scenarios is inevitably a modelling approximation to the real world. The real-time situation that actually arises is unlikely to correspond exactly to any $\omega \in \Omega$. It will thus be necessary to determine real-time prices by solving a real-time problem similar to $\mathbf{RT}(\omega)$. Questions of revenue adequacy, etc. for the prices so determined remain open.

In itself, this is hardly new: all models contain approximations. Even the simple conventional dispatch model **P0**, for example, relies on modelling transmission line losses by one of the approximations discussed in section 2.1. In practice the line losses (and thus, generation) will be slightly different from the model prediction, and this may occasionally lead to a technical revenue inadequacy. The usefulness of such models relies on the empirical observation that, over time, revenue surpluses easily outweigh any such deficits.

First-stage solutions to our model have some unfamiliar features, at least by comparison with deterministic markets. In particular, prices may vary substantially by location even when the network appears to be uncongested, due to the existence of congestion in some of the second-stage scenarios. Among other things, this can create the illusion of arbitrage opportunities. (Equivalently, individually rational participants might not want to follow the dispatch derived by the SO.) In the two-node model of subsection 4.1, for example, it might appear that an arbitrageur could profit from buying at B (at \$12.4), selling at A (at \$20), and utilizing the free transmission capacity available from B to A. But to use the "free" transmission capacity in this way would lead to a worse outcome in scenario ω_2 at the second stage – meaning that the capacity is not really free after all. In effect, the SO has reserved this capacity for a contingency. Similar effects are often seen in conventional power markets when security-related constraints are included in the optimal power flow problem. This may lead to lines having physical spare capacity, which would be used by arbitrageurs if the SO allowed it, but which cannot be used without compromising system security.

The form of regulation offer considered in this paper (via p_i^+ and p_i^-) effectively sets a constant regulation margin. However, one might consider other possibilities. If one considered the regulation margin to vary linearly in the additional energy used, this would lead to quadratic terms in the objective of **P1**:

$$\min \quad E\left[\sum_{i\in T}\Big(p_iX_i + b_i(X_i - x_i)^2\Big)\right], \tag{8}$$

(*cf.* (2)). Here $b_i > 0$ are "regulation cost parameters" offered by the market participants. This formulation is likely to have mathematical properties similar to those of **P1**. The computational difficulties of this approach, however, may be greater.

Our model is a two-stage stochastic program; a possible generalization would see it replaced by a multi-stage version. There is some practical justification for this. In a power system having several different kinds of inflexible plant, with different lead times, it would be natural to make the commitment decisions sequentially, rather than all at once in a single first stage. Such a model would have different prices at each stage; uncertain participants, such as wind farms, could expect to sell (or buy back) some energy at each stage as their forecast horizon grows shorter. This is reminiscent of electricity futures trading. On the other hand, it is worth noting that real power systems with multiple operational markets (e.g. NordPool, PJM) usually have only two such markets – day-ahead and real-time. This suggests that a two-stage approximation may be acceptable in practice.

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