

Stochastic optimization models for electricity systems with stored hydroelectricity

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joint work with

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with acknowledgements to

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Outline

What is JADE?

Inflow modelling for testing policies

FUELJADE: Modelling fuel constraints

Capacity expansion modelling

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What is JADE?



JADE.jl is a hydrothermal reservoir optimization model of the New Zealand electricity system (developed by Downward and Kapelevich) that minimizes expected discounted social cost. It applies the **stochastic dual dynamic programming** algorithm as implemented in the Julia package **SDDP.jl** developed by Oscar Dowson.

JADE.jl is made available by NZ Electricity Authority at

<https://www.emi.ea.govt.nz/Wholesale/Tools/JADE>

and source is on github at

<https://github.com/EPOC-NZ/JADE.jl>



What is SDDP.jl ?



- ▶ open source Julia implementation of SDDP maintained by Oscar Dowson.
- ▶ built on `JuMP.jl` modelling language.
- ▶ supports a number of open-source and commercial solvers.
- ▶ support for:
 - ▶ infinite horizon problems
 - ▶ convex risk measures
 - ▶ mixed-integer state and control variables
 - ▶ partially observable stochastic processes

<https://odow.github.io/SDDP.jl/stable/>



Advantages of JADE

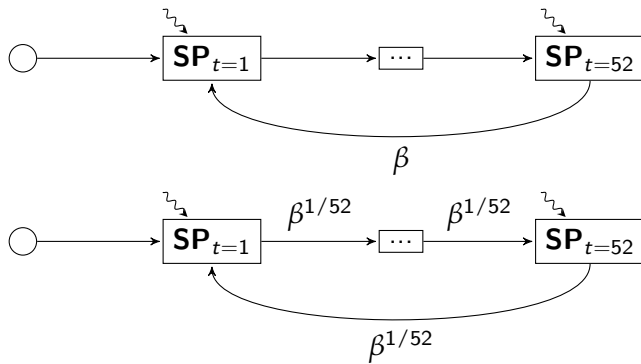
- ▶ JADE is used by the Electricity Authority (EA) to monitor the wholesale market and security of supply.
- ▶ Wholesale market input data is curated by the EA.
- ▶ Open source versus proprietary software.
- ▶ No software licence fees (except possibly for LP solver).
- ▶ SDDP.jl algorithm is based on published research.
- ▶ JuMP model makes changes easy to implement.
- ▶ Policy graph features of SDDP.jl make JADE very flexible.

JADE policy graph for a planned year



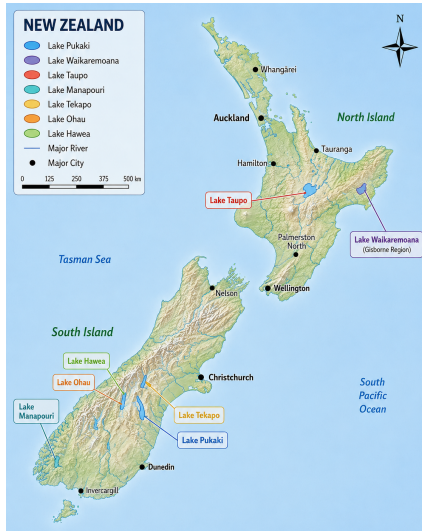
- ▶ 52 weekly stages
- ▶ $\mathbf{SP}_{t=52}$ has terminal future-cost function

JADE steady-state policy graph



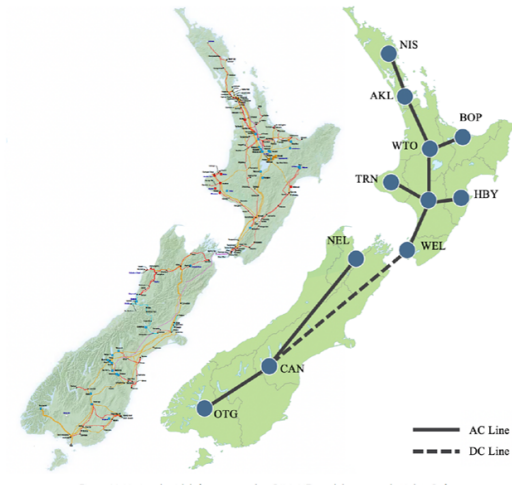
- ▶ 52 weekly stages
- ▶ annual (weekly) discount factors β ($\beta^{1/52}$) on edges

JADE reservoirs



New Zealand major hydro reservoirs (map by ChatGPT).

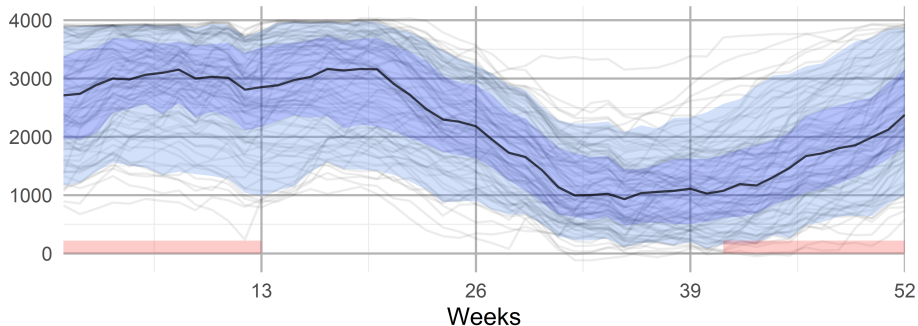
JADE approximate transmission network



In JADE.jl, 250-node network on left is approximated by an 11-node transmission system.

JADE in steady state

- ▶ Infinite horizon JADE computes **End of Horizon (EOH)** cuts defining a **terminal future cost function**.
- ▶ EOH cuts are added to finite horizon model.
- ▶ Any infinite horizon policy can be **simulated** with 92 consecutive years of inflows (1932-2023)



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Inflow modelling for testing policies

FUELJADE: Modelling fuel constraints

Capacity expansion modelling

Inflow modelling in SDDP

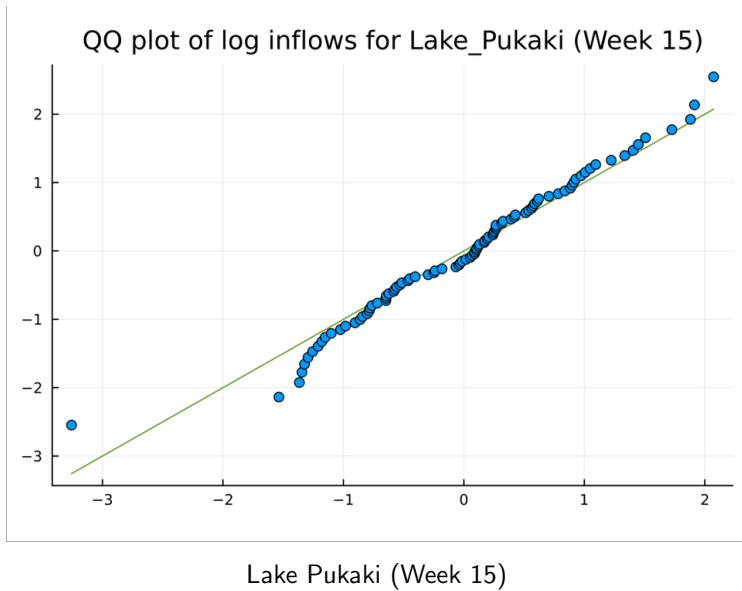
- ▶ Inflows are modelled so we can compute a **policy** using **SDDP**.
- ▶ PSR SDDP takes historical inflows and fits a PAR model.
 - ▶ PAR model is linear in inflows and needs adjustments to prevent negative inflows.
 - ▶ Some evidence that this gives optimistic policies.
- ▶ Academic versions of JADE by EPOC researchers have tried...
 - ▶ independent inflows with DIA (doasa);
 - ▶ an AR1 model in the logs of inflows (Adam Clifford, 2022);
 - ▶ a model with Markov ENSO states (Xander Butler, 2023);

In-sample versus out-of-sample performance

[Wendy Liou and Shahed Ismael, 2025]

- ▶ Standard **convergence of SDDP** simulates the policy with the same distribution that we used to train it.
- ▶ In its simplest form this works with **stagewise independent inflows**.
- ▶ How will it work if we assume historical inflows 1932-2023?
- ▶ If this performance is our KPI, then should we train (overfit?) SDDP on this history?
- ▶ How will it work in practice?
- ▶ To test we need a set of **synthetic** inflows that is **not history** but has the **same distribution**.

Marginal distribution of log inflows are normal

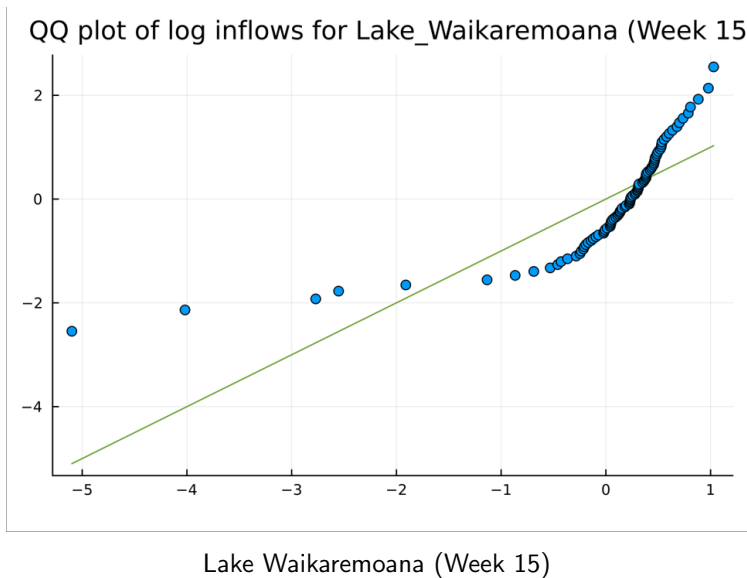


Temporal correlation of Pukaki

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
1	0.04	0.17	0.16	0.17	0.07	0.07	0.12	0.11	0.05	0.08	0.05	0.06	0.08	0.05	0.10	0.08	0.09	0.11	0.10	0.08	0.02	0.07	0.02	0.09	0.01	0.11	0.13	0.07	0.02	0.06	0.10	0.08	0.13	0.01	0.09	0.03	0.02	0.11	0.12	0.11	0.13	0.06	0.03	0.02	0.07	0.01	0.06	0.02	0.02		
2	0.04	0.17	0.16	0.17	0.08	0.08	0.11	0.01	0.02	0.02	0.02	0.08	0.08	0.05	0.10	0.08	0.09	0.11	0.04	0.03	0.01	0.01	0.04	0.01	0.04	0.01	0.03	0.07	0.02	0.06	0.10	0.08	0.13	0.01	0.09	0.03	0.02	0.11	0.12	0.11	0.13	0.06	0.03	0.02	0.07	0.01	0.06	0.02	0.02		
3	0.04	0.17	0.16	0.17	0.08	0.08	0.11	0.01	0.02	0.02	0.02	0.08	0.08	0.05	0.10	0.08	0.09	0.11	0.04	0.03	0.01	0.01	0.04	0.01	0.04	0.01	0.03	0.07	0.02	0.06	0.10	0.08	0.13	0.01	0.09	0.03	0.02	0.11	0.12	0.11	0.13	0.06	0.03	0.02	0.07	0.01	0.06	0.02	0.02		
4	0.04	0.17	0.16	0.17	0.08	0.08	0.11	0.01	0.02	0.02	0.02	0.08	0.08	0.05	0.10	0.08	0.09	0.11	0.04	0.03	0.01	0.01	0.04	0.01	0.04	0.01	0.03	0.07	0.02	0.06	0.10	0.08	0.13	0.01	0.09	0.03	0.02	0.11	0.12	0.11	0.13	0.06	0.03	0.02	0.07	0.01	0.06	0.02	0.02		
5	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03	0.02	0.06	0.01	0.03	0.07	0.02	0.01	0.11	0.17	0.08	0.04	0.03	0.02	0.03	0.03	0.07	0.08	0.08	0.11	0.19	0.06	0.08	0.04	0.04	0.02	0.14				
6	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03	0.02	0.06	0.01	0.03	0.07	0.02	0.01	0.11	0.17	0.08	0.04	0.03	0.02	0.03	0.03	0.07	0.08	0.08	0.11	0.19	0.06	0.08	0.04	0.04	0.02	0.14				
7	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
8	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
9	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
10	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
11	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
12	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
13	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
14	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
15	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
16	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
17	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
18	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
19	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
20	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
21	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
22	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
23	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
24	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
25	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
26	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
27	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
28	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
29	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
30	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
31	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
32	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
33	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
34	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
35	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
36	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
37	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
38	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
39	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
40	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
41	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
42	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
43	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
44	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
45	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
46	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
47	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
48	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
49	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
50	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
51	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.04	0.01	0.03																														
52	0.01	0.08	0.11	0.11	0.12	0.16	0.29	0.36	0.21	0.39	0.16	0.18	0.13	0.02	0.06	0.11	0.04	0.04	0.0																																

Log(Lake Pukaki weekly inflow) correlation

Some marginal distributions of log inflows are not normal



Spatial correlation of NZ inflows

1

	Matahina	Mangahao	Moawhango	Rotoaira	Taupo	Arapuni	Vaikaremoana	Cobb	Coleridge	Tekapo	Pukaki	Ohau	Benmore	Hawea	Wanaka	Dunstan	Manapouri
Matahina	1.000	0.177	0.556	0.708	0.731	0.693	0.401	0.280	0.174	-0.033	-0.113	-0.023	0.059	0.053	-0.019	0.019	-0.043
Mangahao	0.177	1.000	0.275	0.373	0.360	0.235	0.069	0.294	0.257	0.099	0.032	0.140	0.172	0.189	0.089	0.128	0.225
Moawhango	0.556	0.275	1.000	0.855	0.760	0.528	0.239	0.336	0.294	-0.008	-0.129	0.022	0.125	0.136	0.030	0.110	0.044
Rotoaira	0.708	0.373	0.855	1.000	0.965	0.711	0.334	0.446	0.317	0.029	-0.099	0.055	0.156	0.170	0.047	0.121	0.055
Taupo	0.731	0.360	0.760	0.965	1.000	0.722	0.350	0.426	0.298	0.036	-0.079	0.058	0.159	0.156	0.063	0.124	0.045
Arapuni	0.693	0.235	0.528	0.711	0.722	1.000	0.342	0.354	0.199	-0.072	-0.169	-0.042	0.050	0.069	-0.064	0.021	0.009
Vaikaremoana	0.401	0.069	0.239	0.334	0.350	0.342	1.000	0.084	0.000	-0.204	-0.247	-0.211	-0.139	-0.141	-0.197	-0.160	-0.199
Cobb	0.280	0.294	0.336	0.446	0.426	0.354	0.084	1.000	0.306	0.227	0.097	0.264	0.330	0.367	0.188	0.229	0.289
Coleridge	0.174	0.257	0.294	0.317	0.298	0.199	0.000	0.306	1.000	0.246	0.099	0.284	0.358	0.334	0.279	0.338	0.258
Tekapo	-0.033	0.099	-0.008	0.029	0.036	-0.072	-0.204	0.227	0.246	1.000	0.886	0.900	0.795	0.751	0.726	0.589	0.464
Pukaki	-0.113	0.032	-0.129	-0.099	-0.079	-0.169	-0.247	0.097	0.099	0.886	1.000	0.850	0.677	0.633	0.651	0.468	0.404
Ohau	-0.023	0.140	0.022	0.055	0.058	-0.042	-0.211	0.264	0.284	0.900	0.850	1.000	0.899	0.877	0.772	0.662	0.618
Benmore	0.059	0.172	0.125	0.156	0.159	0.050	-0.139	0.330	0.358	0.795	0.677	0.899	1.000	0.862	0.802	0.767	0.583
Hawea	0.053	0.189	0.136	0.170	0.156	0.069	-0.141	0.367	0.334	0.751	0.633	0.877	0.862	1.000	0.690	0.634	0.711
Wanaka	-0.019	0.089	0.030	0.047	0.063	-0.064	-0.197	0.188	0.279	0.726	0.651	0.772	0.802	0.690	1.000	0.875	0.464
Dunstan	0.019	0.128	0.110	0.121	0.124	0.021	-0.160	0.229	0.338	0.589	0.468	0.662	0.767	0.634	0.875	1.000	0.459
Manapouri	-0.043	0.225	0.044	0.055	0.045	0.009	-0.199	0.289	0.258	0.464	0.404	0.618	0.583	0.711	0.464	0.459	1.000

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Spatial correlation of historical log(inflow) 1932-2025.

Modified Fractional Gaussian Noise

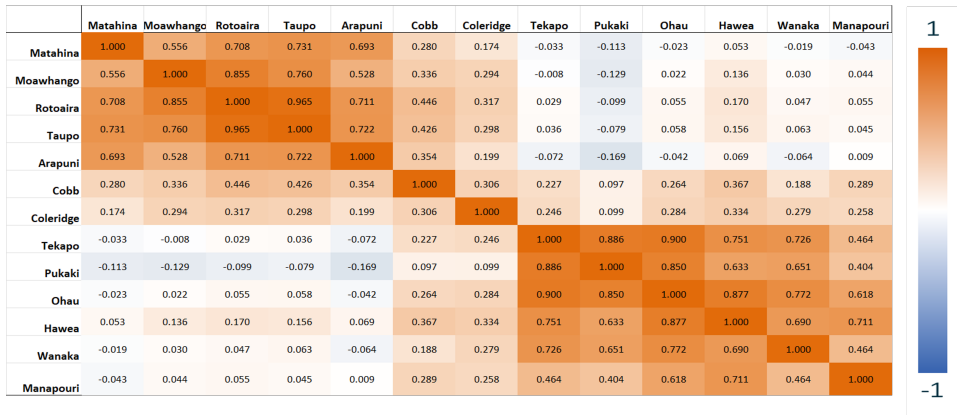
[Kirsch, B. R. et al, 2013, Chadwick, C. et al, 2024]

- ▶ Transform logarithms of inflows to have mean 0, variance 1.
- ▶ Establish correlations between transformed log inflows in space and time. Remove uncorrelated reservoirs.
- ▶ Create synthetic multivariate normal random variables of scaled log inflows with $\mathcal{N}(0, 1)$ marginals.
- ▶ **SmFGN**: spatial then temporal; **mFGNS**:temporal then spatial.
- ▶ **Weighted** mFGN analysis [Chadwick et al, 2024] resulted in us using mFGNS.
- ▶ For simulation we either ...
 - ▶ **Transform marginals** to $\mathcal{N}(\mu, \sigma^2)$ and **exponentiate**, or
 - ▶ Recreate marginal inflows from $\mathcal{N}(0, 1)$ using **NORTA**².

²Cario, M.C. and Nelson, B.L., 1997. We do not try and match empirical correlation matrix.

Historical correlation: minus Mangahao and Waikaremoana

Historical Inflow Correlations



Spatial correlation from historical data

SmFGN versus mFGNS: SmFGN

SmFGN Synthetic Inflows

Lakes	Matahina	Moawhango	Rotoaira	Taupo	Arapuni	Cobb	Coleridge	Tekapo	Pukaki	Ohau	Hawea	Wanaka	Manapouri
Matahina	1.000	0.063	0.482	0.560	0.555	0.200	0.098	-0.026	-0.120	0.027	0.149	0.018	0.049
Moawhango	0.063	1.000	0.151	0.130	0.055	0.042	0.046	-0.032	-0.051	-0.014	0.028	-0.001	-0.003
Rotoaira	0.482	0.151	1.000	0.898	0.435	0.312	0.207	0.060	-0.078	0.066	0.191	0.098	0.084
Taupo	0.560	0.130	0.898	1.000	0.498	0.302	0.237	0.050	-0.086	0.058	0.181	0.096	0.090
Arapuni	0.555	0.055	0.435	0.498	1.000	0.205	0.076	-0.072	-0.191	-0.021	0.117	0.000	0.110
Cobb	0.200	0.042	0.312	0.302	0.205	1.000	0.107	0.121	-0.006	0.131	0.187	0.121	0.147
Coleridge	0.098	0.046	0.207	0.237	0.076	0.107	1.000	0.146	0.034	0.172	0.184	0.175	0.138
Tekapo	-0.026	-0.032	0.060	0.050	-0.072	0.121	0.146	1.000	0.740	0.775	0.549	0.588	0.252
Pukaki	-0.120	-0.051	-0.078	-0.086	-0.191	-0.006	0.034	0.740	1.000	0.712	0.405	0.468	0.204
Ohau	0.027	-0.014	0.066	0.058	-0.021	0.131	0.172	0.775	0.712	1.000	0.725	0.630	0.418
Hawea	0.149	0.028	0.191	0.181	0.117	0.187	0.184	0.549	0.405	0.725	1.000	0.497	0.477
Wanaka	0.018	-0.001	0.098	0.096	0.000	0.121	0.175	0.588	0.468	0.630	0.497	1.000	0.283
Manapouri	0.049	-0.003	0.084	0.090	0.110	0.147	0.138	0.252	0.204	0.418	0.477	0.283	1.000

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Spatial correlation from SmFGN model

SmFGN versus mFGNS: mFGNS

mFGNS Synthetic Inflows

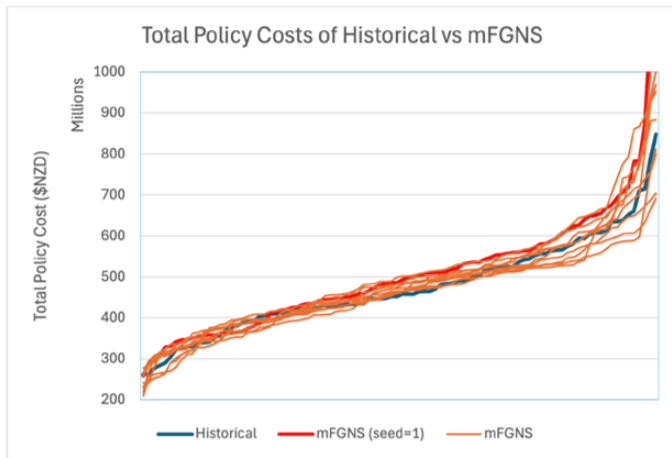
Lakes	Matahina	Moawhango	Rotoaira	Taupo	Arapuni	Cobb	Coleridge	Tekapo	Pukaki	Ohau	Hawea	Wanaka	Manapouri
Matahina	1.000	0.270	0.681	0.711	0.722	0.270	0.208	-0.014	-0.117	-0.016	0.077	-0.058	-0.047
Moawhango	0.270	1.000	0.345	0.322	0.250	0.116	0.099	-0.029	-0.094	-0.027	0.042	-0.052	-0.014
Rotoaira	0.681	0.345	1.000	0.960	0.659	0.403	0.306	0.053	-0.087	0.059	0.171	-0.016	0.031
Taupo	0.711	0.322	0.960	1.000	0.685	0.395	0.323	0.048	-0.092	0.059	0.164	-0.001	0.022
Arapuni	0.722	0.250	0.659	0.685	1.000	0.341	0.171	-0.075	-0.187	-0.050	0.059	-0.125	-0.018
Cobb	0.270	0.116	0.403	0.395	0.341	1.000	0.295	0.133	0.019	0.148	0.235	0.082	0.171
Coleridge	0.208	0.099	0.306	0.323	0.171	0.295	1.000	0.244	0.078	0.251	0.295	0.231	0.213
Tekapo	-0.014	-0.029	0.053	0.048	-0.075	0.133	0.244	1.000	0.848	0.890	0.708	0.744	0.446
Pukaki	-0.117	-0.094	-0.087	-0.092	-0.187	0.019	0.078	0.848	1.000	0.821	0.548	0.661	0.353
Ohau	-0.016	-0.027	0.059	0.059	-0.050	0.148	0.251	0.890	0.821	1.000	0.816	0.805	0.571
Hawea	0.077	0.042	0.171	0.164	0.059	0.235	0.295	0.708	0.548	0.816	1.000	0.679	0.682
Wanaka	-0.058	-0.052	-0.016	-0.001	-0.125	0.082	0.231	0.744	0.661	0.805	0.679	1.000	0.463
Manapouri	-0.047	-0.014	0.031	0.022	-0.018	0.171	0.213	0.446	0.353	0.571	0.682	0.463	1.000

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Spatial correlation from mFGNS model

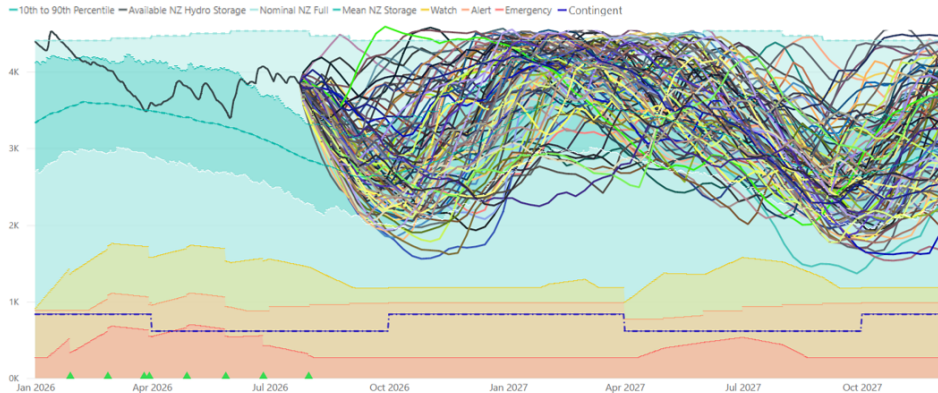
Synthetic mFGNS/NORTA versus historical



Cost duration curves from JADE policy simulated with 940 synthetic mFGN/NORTA inflows and historical inflows 1932-2025.

Risk curve analysis

Fuel Capability Case - New Zealand SST Electricity Risk Status Curves (Available GWh)



System operator uses historical inflow sequences to estimate hydro shortage risk.

Outline

What is JADE?

Inflow modelling for testing policies

FUELJADE: Modelling fuel constraints

Capacity expansion modelling

Fuel constraints

- ▶ What about gas availability?
- ▶ EMI version of JADE.jl assumes **exogenous forecast of gas and coal prices** for electricity generation.
- ▶ These prices will actually be **uncertain** and **endogenous**, i.e., depend on how much is still in the ground and what thermal electricity generation has occurred.
- ▶ But electricity generation depends on the JADE policy and on the inflows observed.

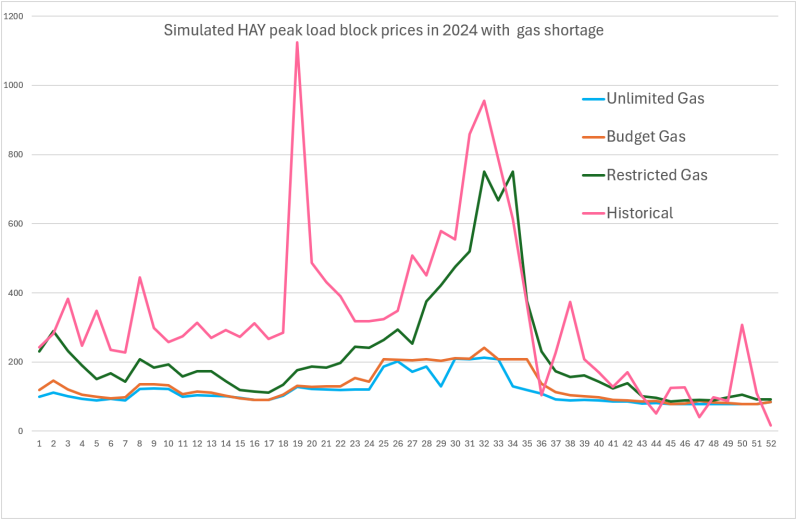
FUELJADE model

- ▶ Fuel is a **state variable** in FUELJADE.
- ▶ Fuel is purchased once in stage 1 and then is consumed over the year from a fuel stockpile at **zero variable cost**.
- ▶ Purchases in stage 1 of FUELJADE will be **dependent on fuel state and other states (reservoir levels)** but not on anticipated future inflows.
- ▶ Policy computes a **marginal fuel value** as well as a marginal water value. These are **opportunity costs** of using fuel and water respectively.

FUELJADE model experiments

- ▶ Compute a FUELJADE policy for 2024.
 - ▶ Train steady-state problem for 2024 with unlimited purchase of fuel in stage 1 at price 0 (5000 iterations).
 - ▶ Train 2024 finite horizon problem with EOH cuts and unlimited fuel purchase in stage 1 (5000 iterations).
 - ▶ Train 2024 finite horizon problem with EOH cuts but 2024 historical fuel purchases in stage 1 (5000 iterations).
 - ▶ Train 2024 finite horizon problem with EOH cuts but more limited fuel purchases in stage 1 (5000 iterations).

Peak forecast prices at Haywards 2024: FUELJADE



Peak prices (\$/MWh) at Haywards in 2024 forecast from FUELJADE

Outline

What is JADE?

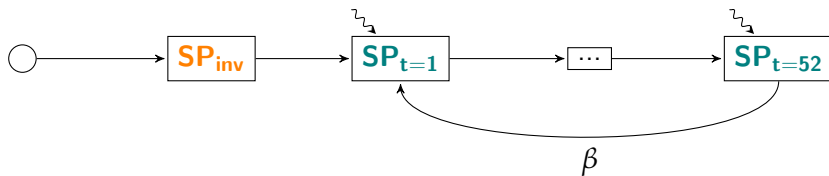
Inflow modelling for testing policies

FUELJADE: Modelling fuel constraints

Capacity expansion modelling

EPOCJADE capacity planning

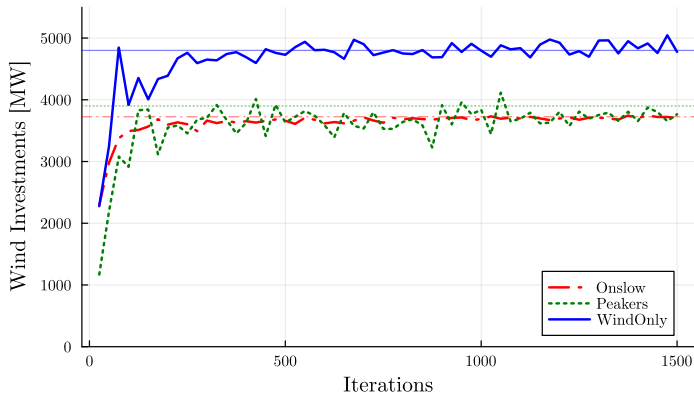
[Hole et al, 2024]



Add capacity investment node **SP_{inv}** to the steady-state policy graph.

Investments in new wind capacity with different backup

Three cases: **Pump storage**, **Peakers**, **None**



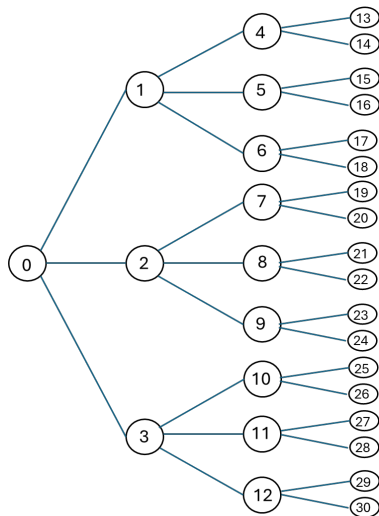
Investment decisions for new wind capacity in the three different cases against the number of SDDP training iterations. The investment decisions are evaluated every 25 iterations.

Cost recovery from investments

[Anderson and P., 2026]

- ▶ Classical screening curve analysis ([Boiteaux, 1960, Stoft, 2002]) shows Ricardian rents exactly cover capital costs in perfect competition.
- ▶ Does this extend to systems with variable renewable electricity, storage, and hydroelectricity?
- ▶ Exact screening curve analysis is difficult with JADE since the electricity prices in each stage problem may not be unique and might also be poor approximations to optimal prices.
- ▶ [Anderson and P.] derive screening curve analysis in a scenario tree with elastic demand.

Uncertain inflows form a scenario tree



A scenario tree $\mathcal{N}$ with 4 stages and 27 scenarios. We let $p(n)$ denote the probability of node n .

Investment planning problem for thermal plant

Let $g(n)$ be fixed generation from hydro plant and $y_j(n)$ be thermal generation with capacity optimized to be x_j .

$$\begin{aligned} \text{SMP: min} \quad & \sum_{j \in \mathcal{J}} a_j x_j + \sum_{n \in \mathcal{N}} p(n) \left(\sum_{j \in \mathcal{J}} c_j(n) y_j(n) - U_n(d(n)) \right) \\ \text{s.t.} \quad & \sum_{j \in \mathcal{J}} y_j(n) + g(n) = d(n), \quad n \in \mathcal{N} \quad [p(n)\pi(n)] \\ & 0 \leq y_j(n) \leq x_j, \quad \text{for all } j \in \mathcal{J}. \quad [p(n)\rho_j(n)] \end{aligned}$$

$$\pi(n) = U'_n(d(n)).$$

Fix optimal capacities at x^* , and solve dispatch problem

$$\begin{aligned} \text{DP: min} \quad & \sum_{j \in \mathcal{J}} a_j x_j^* + \sum_{n \in \mathcal{N}} p(n) \left(\sum_{j \in \mathcal{J}} c_j(n) y_j(n) - U_n(d(n)) \right) \\ \text{s.t.} \quad & \sum_{j \in \mathcal{J}} y_j(n) + g(n) = d(n), \quad \text{for all } n \in \mathcal{N} \quad [p(n)\pi(n)] \\ & 0 \leq y_j(n) \leq x_j^*, \quad \text{for all } j \in \mathcal{J}. \quad [p(n)\rho_j(n)] \end{aligned}$$

This decouples into a dispatch problem at each n , where plant j offers x_j^* at **marginal cost** $c_j(n)$, and demand bids **marginal utility** $U'_n(d)$.

DP has same primal and dual solution as SMP.

Optimality conditions for SMP imply DP rents cover costs

$$\pi(n) = U'(d(n))$$

$$0 \leq c_j(n) + \rho_j(n) - \pi(n) \perp y_j(n) \geq 0$$

$$0 \leq x_j - y_j(n) \perp \rho_j(n) \geq 0$$

$$0 \leq a_j - \sum_{n \in \mathcal{N}} p(n) \rho_j(n) \perp x_j \geq 0$$

$$\begin{aligned} \text{Annual rent } (R_j) \text{ for technology } j &= \sum_{n \in \mathcal{N}} p(n) (\pi(n) - c_j(n)) y_j(n) \\ &= \sum_{n \in \mathcal{N}} p(n) \rho_j(n) y_j(n) \\ &= \sum_{n \in \mathcal{N}} p(n) \rho_j(n) x_j = a_j x_j \end{aligned}$$

KOTUITUI

[Robinson, 2026]



kōtuitui  

1. (verb) (-a) to lace, fasten by lacing, interlace, interlink, connect.

- ▶ Key Optimization Tool for Utility Investment with Tree-based Uncertain Inflows.
- ▶ Kotuitui scenario tree has 5 stages:
 - ▶ Stage 1: Weeks 1-9
 - ▶ Stage 2: Weeks 10-18
 - ▶ Stage 3: Weeks 19-27
 - ▶ Stage 4: Weeks 28-36
 - ▶ Stage 5: Weeks 37-52
- ▶ Inflows from 81 historical years 1941-2021 are clustered into bundles and non-anticipativity constraints applied over each bundle.

KOTUITUI

- ▶ Investment **JADE** is modelled with **quadratic load-shedding costs** (i.e., a **linear demand curve**).
- ▶ **JADE** yields optimal investments and end-of-horizon cuts.
- ▶ **Kotuitui** applies end-of horizon cuts to each leaf node in scenario tree and solves a stochastic quadratic program. This replicates **JADE** investments.
- ▶ **Kotuitui** extracts system marginal prices and water values at each node in the tree.
- ▶ **Kotuitui** computes the rent from energy prices and the inflow rent which is sum of inflows times marginal water value.

KOTUITUI results



Cobb dam and 32-36 MW power station [<https://www.manawaenergy.co.nz/generation-images>]

- ▶ JADE and Kotuitui give optimal investment of 33.1 MW when capital cost/MW/year = \$150K. Annual capital cost = \$5M.
- ▶ $PV(\$5M) = \48.7 M (20 years at discount rate = 7.8%)
- ▶ Annual present value of inflow rent = \$14.3 M.
- ▶ Annual present value of energy price rents = \$19.3 M.

So if your question is "what value did Contact effectively put on Cobb when it bought Manawa?", I would use approximately \$160 million (2025 NZ dollars).

The End

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