# Long-term models

Andy Philpott

July 27, 2025

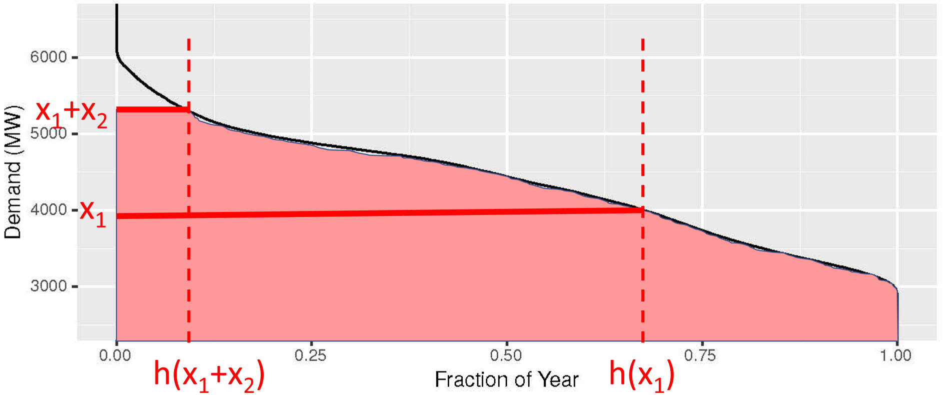
# This tutorial in three parts

1. Short-term models (hours/days)
   - social plan minimizing cost
   - maximizing profit given prices
2. Multistage and medium-term models (weeks/months)
   - social plan minimizing cost
   - maximizing profit given prices
3. Long-term models (years/decades)
   - social plan minimizing cost
   - maximizing return on investment given prices

# Summary

# Summary

# Screening Curves




Load duration curve showing optimal capacities of conventional generation with annual fixed costs $a_1$, $a_2$, and marginal costs $c_1$, $c_2$. Here $h(x_1 + x_2) = \frac{a_2}{V - c_2}$, $h(x_1) = \frac{a_1 - a_2}{c_2 - c_1}$.

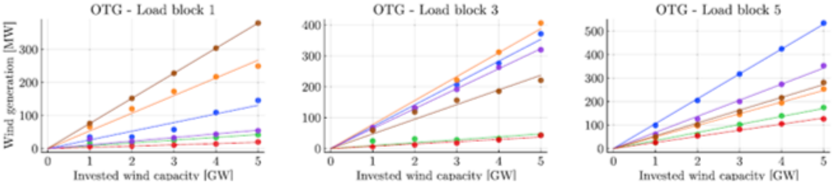
# With intermittent renewables

- Wind and solar are not dispatchable and disrupt the merit order.
- How to do screening?
- Subtract wind and solar from demand and create net load duration curve.
  - Suitable if wind/solar investment is exogenous ...
  - ... but difficult to optimize short-term storage.
  - If planning wind/solar investment need to approximate this process for different capacity choices.
- Use representative days and solve two-stage stochastic program.
  - Represents intraday variation e.g. for battery investment.
  - Suffers from perfect foresight bias.

# Wind adjusted load duration curve

[Hole et al, 2024]

- Load duration curve piecewise constant with decreasing load blocks.
- Increased wind investment decreases net load across all load blocks.
- Fix the set of hours in each block and fit a linear curve that defines how increased wind capacity decreases load in that block.




(a) Linear wind representation in Otago for load block 1

(b) Linear wind representation in Otago for load block 3

(c) Linear wind representation in Otago for load block 5

# Wind investment using SDDP

[Hole et al, 2024]

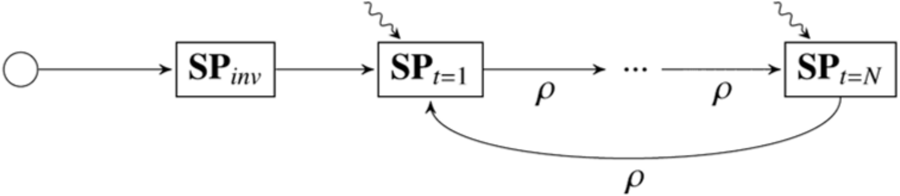
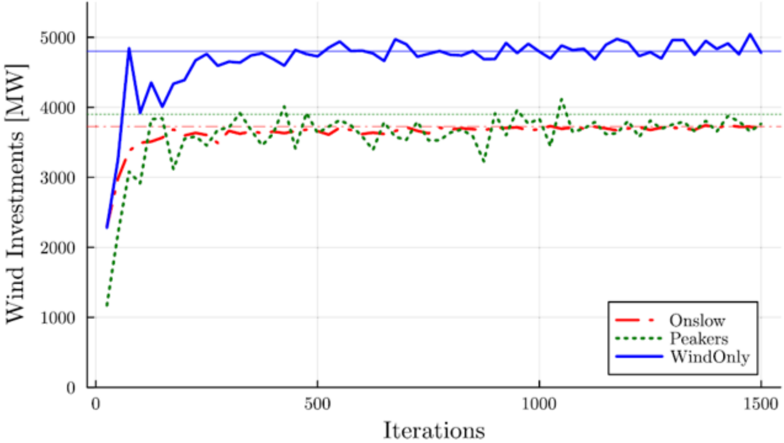



**Fig. 3.** The policy graph structure for **INV – HTP –** $\infty$.

# New Zealand Case study

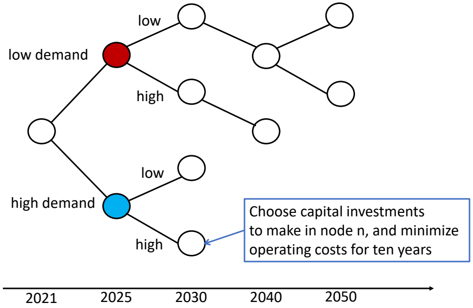
[Hole et al, 2024]




Investment decisions plotted every 20 iterations of SDDP.

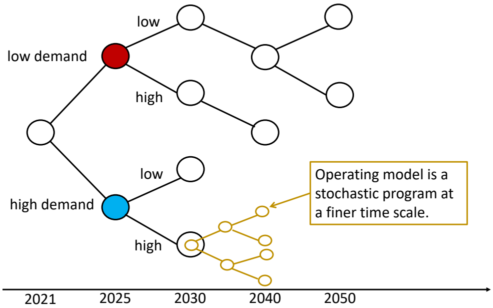
# Summary

# Multi-horizon planning

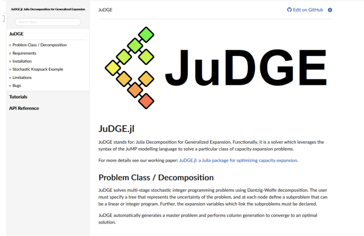



- Capacity-expansion decisions over longer time scale (5 years or 10 years)
- Use a scenario tree to model uncertainty.

# Multi-horizon scenario trees




- Operational uncertainty (brown) modeled with a finer time scale.
- Can model this using
  - a fine scenario tree;
  - a Markov Decision Process;
  - representative days/weeks/seasons.




`https://github.com/EPOC-NZ/JuDGE.jl`

JuDGE stands for Julia Decomposition for Generalized Expansion.).

- allows users to easily implement multi-horizon optimization models using the JuMP modelling language;
- can apply end-of-horizon risk-measures in objective function and/or the constraints; and
- outputs an interactive view of the results over the scenario tree, enabling decision makers to explore the optimal expansion plan.

# Summary

# Example: New Zealand decarbonization model

## Expansions and shutdowns

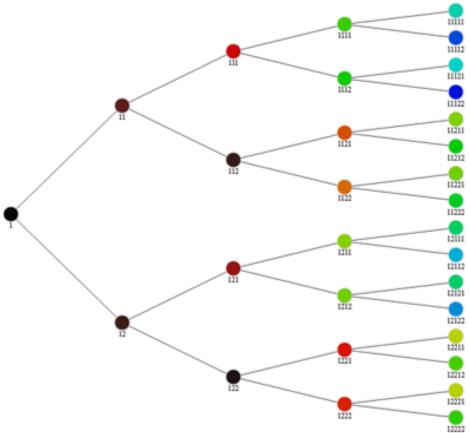
Optimize capacity expansion under uncertainty represented by a scenario tree.

Model is a risk-averse central-planning model minimizing discounted disbenefit $Z$ summed from 2021-2050.

End-of-horizon risk is a convex combination of expected value and average value at risk, so Risk$(\lambda, \alpha)$ is

$$(1-\lambda)\mathbb{E}[Z] + \lambda AVaR_{1-\alpha}[Z]$$

.



31-node scenario tree.

# Example: New Zealand decarbonization model

## Defining the subproblems

Sets:

- seasons $t \in \mathcal{T}$;
- load blocks $b \in \mathcal{B}_t$, $t \in \mathcal{T}$;
- hydrological years $h \in \mathcal{H}$;
- technologies $k \in \mathcal{K}$.

Variables:

- $x_k$ capacity to build for technology $k$;
- $g_k^{bh}$ generation from technology $k$ in load block $b$, with hydrological year $h$.

Parameters:

- $d^b$ demand in load block $b$;
- $u_k$ initial capacity of technology $k$;
- $U_k$ maximum capacity increment of each new technology $k$;
- $\theta_k^b$ is the capacity factor for technology $k$, in load block $b$.

# Medium-term Operational Model

## Subproblem objective function

Subproblem at node $n$ minimizes the operational costs of the electricity system:

$$\min \quad \sum_{t\in\mathcal{T}} \sum_{b\in\mathcal{B}_t} \Delta_b \sum_{h\in\mathcal{H}} \rho_h \sum_{k\in\mathcal{K}} (c_k + \tau e_k) g_k^{bh},$$

where $\Delta_b$ is the number of hours corresponding to load block $b$;
$\rho_h$ is the probability of hydrological year $h$;
$c_k$ is the marginal cost of technology $k$;
$e_k$ gives the emissions factor of technology $k$;
$\tau$ is the carbon tax.

Cost of investments over the tree:

$$\min \quad \sum_{n\in\mathcal{N}} \phi_n \sum_{k\in\mathcal{K}} C_k x_k,$$

$\phi_n$ is the (discounted) probability of reaching node $n$;
$C_k$ is the capital cost (per unit) of technology $k$;
$x_k \in [0, 1]$ represents investment in technology $k$.

# Medium-term operations

## Subproblem constraints

Load balance:

$$\sum_{k\in\mathcal{K}} g_k^{bh} = d^b, \quad \forall b \in \mathcal{B}, h \in \mathcal{H},$$

Generation capacity:

$$0 \leq g_k^{bh} \leq \theta_k^b (u_k + x_k U_k) \quad \forall b \in \mathcal{B}_t, t \in \mathcal{T}, h \in \mathcal{H}, k \in \mathcal{K},$$

Stored hydro generation:

$$\sum_{b\in\mathcal{B}_t} g_{\text{hydro}}^{bh} \times \Delta_b = \mu_t^h \quad \forall h \in \mathcal{H}, t \in \mathcal{T},$$

Expansions:

$$x_k \in [0,1], \quad \forall k \in \mathcal{K}, i \in \{1,\ldots,N\}.$$

# EMERALD demonstration

EMERALD case study uses...

- Three regions (NI, HAY, SI).
- Four seasons with 10 load blocks each.
- 16 load growth scenarios.
- 13 historical years model seasonal hydrological inflows.
- Data based on two-stage model of NZ system.[1]

[1] Ferris & Philpott, 100% renewable electricity with storage (2019) http://www.epoc.org.nz.

# EMERALD input data

## Random demand and carbon prices

- Annual total energy demand increases from
  - Electric vehicles;
  - Industrial load;
  - Consumer load;
  - Aluminium smelter (or replacement).
- NZ carbon prices in target years are assumed.
- Carbon prices affect fossil fuels and electricity prices.
- Electricity demand growth from PEVs.
- Exogeneous decrease in cost of solar panels.

# EMERALD input data

## Random demand and carbon prices

- Annual total energy demand increases from
  - Electric vehicles;
  - Industrial load;
  - Consumer load;
  - Aluminium smelter (or replacement).
- NZ carbon prices in target years are assumed.
- Carbon prices affect fossil fuels and electricity prices.
- Electricity demand growth from PEVs.
- Exogeneous decrease in cost of solar panels.

# EMERALD input data

## Random demand and carbon prices

- Annual total energy demand increases from
  - Electric vehicles;
  - Industrial load;
  - Consumer load;
  - Aluminium smelter (or replacement).
- NZ carbon prices in target years are assumed.
- Carbon prices affect fossil fuels and electricity prices.
- Electricity demand growth from PEVs.
- Exogeneous decrease in cost of solar panels.

# EMERALD input data

## Random demand and carbon prices

- Annual total energy demand increases from
  - Electric vehicles;
  - Industrial load;
  - Consumer load;
  - Aluminium smelter (or replacement).
- NZ carbon prices in target years are assumed.
- Carbon prices affect fossil fuels and electricity prices.
- Electricity demand growth from PEVs.
- Exogeneous decrease in cost of solar panels.

# EMERALD input data

## Random demand and carbon prices

- Annual total energy demand increases from
  - Electric vehicles;
  - Industrial load;
  - Consumer load;
  - Aluminium smelter (or replacement).
- NZ carbon prices in target years are assumed.
- Carbon prices affect fossil fuels and electricity prices.
- Electricity demand growth from PEVs.
- Exogeneous decrease in cost of solar panels.

# EMERALD input data

## Scenario tree for demand and carbon price

```
mytree, data = tree_with_data(myscenariotree.csv)
```

```
n,p,probability,evgrowth,phgrowth,loadgrowth,smelter,carbon
1,-,1,1,1,1,1,50
,1,0.5,1.389,1.261,1.16,1,50
12,1,0.5,1.389,1.35,1.052,1,50
111,11,0.25,5.5,1.44,1.28,1,200
112,11,0.25,5.5,1.317,1.03,1,200
121,12,0.25,5.5,1.542,1.161,1,200
122,12,0.25,5.5,1.411,0.934,1,200
1111,111,0.125,50,1.86,1.427,1,500
1112,111,0.125,50,1.623,1.546,1,500
1121,112,0.125,50,1.702,1.147,1,500
....
```

# EMERALD input data

## Scenario tree for demand and carbon price

```
mytree, data = tree_with_data(myscenariotree.csv)
```

```
n,p,probability,evgrowth,phgrowth,loadgrowth,smelter,carbon
1,-,1,1,1,1,1,50
,1,0.5,1.389,1.261,1.16,1,50
12,1,0.5,1.389,1.35,1.052,1,50
111,11,0.25,5.5,1.44,1.28,1,200
112,11,0.25,5.5,1.317,1.03,1,200
121,12,0.25,5.5,1.542,1.161,1,200
122,12,0.25,5.5,1.411,0.934,1,200
1111,111,0.125,50,1.86,1.427,1,500
1112,111,0.125,50,1.623,1.546,1,500
1121,112,0.125,50,1.702,1.147,1,500
....
```

```
JuDGE.visualize_tree(mytree, data)
```

# Scenario tree

# Creating the JuDGE model

```
model = JuDGEModel(mytree,
         ConditionallyUniformProbabilities,
         sub_problems,
         JuDGE_MP_Solver,
         discount_factor=discfactor)
         risk=Risk(0.95,(1/16)
         )
```

# Running EMERALD

## Solving and producing output

```
JuDGE.solve(model,termination=Termination(reltol=0.001))

resolve_subproblems(model)

solution = JuDGE.solution_to_dictionary(model)

(some code to set up custom_plots using plotly)

JuDGE.visualize_tree(mytree, solution,
custom=custom_plots)
```

# EMERALD results

## What is missing from these planning models?

1. Endogeneous learning;
2. Optimal operational policies for renewables;
3. Revenue stacking for some technologies, e.g. batteries;
4. More sophisticated solution interpretation tools for large scale models;
5. Relationship to generator investment behaviour.

# Summary

# Dynamic investment equilibrium = EMERALD solution

[Ralph & Smeers (2015)], [Abada et al, (2017)], [De Maere d'Aertrycke et al (2017)], [Ferris & P. (2022).]

- Suppose each agent in EMERALD has their own nested coherent risk measure with single-stage risk sets (that can vary with node).
- Each agent invests to maximize risk-adjusted return at market prices, where they trade risk in each node in a complete market of Arrow-Debreu securities.
- Suppose planner optimizes welfare using a social risk measure that is nested using the intersection of agent risk sets at each node. (JuDGE uses an end-of-horizon risk measure.)
- Optimal risk-averse plan gives prices and investments that form a partial equilibrium.

# Incomplete risk markets

[Abada et al, (2017)], [Gerard et al, (2018)], [Kok et al, (2018)]

- When markets for risk are incomplete risked equilibrium might not correspond to social plan.
- Can show risked equilibria exist either with no contracts or a complete market.
- There might exist multiple risked equilibria or none.[Gerard et al (2018)]
- If contracts have bounded payoffs (e.g. contracts for differences with price caps) then can prove existence [Kok et al (2018)].

# Summary

# New Challenges

- Is (risked) Walrasian equilibrium the right model?
  - Subgame perfect Nash equilbrium arguably more realistic.
  - Dispatch is a repeated game, so perhaps we should study tacit collusion.
  - Should price-setting behaviour in markets be penalized by regulator? How to detect it.
- How to model constraints on deployment.
  - Raw material constraints;
  - Labour and expertise;
  - Connection queues.
- Prosumers and aggregation.
- System stability with random events.
- System reliability for climate change.

# Stochastic programming, energy and A.I.

- Is A.I. a game-changer in energy optimization?
  - Machine learning can determine reserve requirements using offline optimization.
  - Machine learning can help train operational models to optimize subproblems in multihorizon settings.
  - Will A.I. create a new law of learning rate?
- How will regulators prevent LLMs from enabling collusive outcomes?
- Will demand for A.I. data centres overwhelm the electricity transition?

# References


- Abada, I., Ehrenmann, A. and Smeers, Y., 2017. Modeling gas markets with endogenous long-term contracts. Operations Research, 65(4), pp.856-877.
- Abada, I., de Maere d'Aertrycke, G. and Smeers, Y., 2017. On the multiplicity of solutions in generation capacity investment models with incomplete markets: a risk-averse stochastic equilibrium approach. Mathematical Programming, 165(1), pp.5-69.
- De Maere d'Aertrycke, G., Ehrenmann, A. and Smeers, Y., 2017. Investment with incomplete markets for risk: The need for long-term contracts. Energy Policy, 105, pp.571-583.
- Ferris, M. and Philpott, A., 2022. Dynamic risked equilibrium. Operations Research, 70(3), pp.1933-1952.
- Gérard H, Leclère V, Philpott A. On risk averse competitive equilibrium. Operations Research Letters. 2018 Jan 1;46(1):19-26.

# References


- Kaut, M., Midthun, K.T., Werner, A.S., Tomasgard, A., Hellemo, L. and Fodstad, M., 2014. Multi-horizon stochastic programming. Computational Management Science, 11(1), pp.179-193.
- Kok, C., Philpott, A. and Zakeri, G., 2018. Value of transmission capacity in electricity markets with risk averse agents. Technical Report www. epoc. org. nz.
- Ralph, D. and Smeers, Y., 2015. Risk trading and endogenous probabilities in investment equilibria. SIAM Journal on Optimization, 25(4), pp.2589-2611.

**The End**