Modelling Summary for the paper "Production inefficiency of electricity markets with hydro generation"

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Contents

This document summarizes the modelling assumptions, data estimation and formulation of the central plan model, EP model, INTER model and DOASA model presented in the paper Production inefficiency of electricity markets with hydro generation. The models and those in the paper that they refer to are listed in the following table.

Model	In the paper
Daily Central	CP48
Weekly Central	CP336
Yearly Central	YEAR
EP	EP
Daily Inter	Daily INTER
Weekly Inter	Weekly INTER
DOASA	DOASA

1 Central Model

1.1 Modelling assumptions and data estimation

The Central model is an approximation of the SPD model. The SPD model includes 244 nodes and constraints for voltage support, $N-1$ security, spinning reserve and frequency keeping. Since we need to solve such a model many times in simulation, we have chosen to use 18 nodes and ignore the above constraints. The representations of the 18 node transmission network with major thermal generators and hydro systems is shown in Figure 1 and 2.

The main data source is the Centralised Dataset (CDS) 2008 October version [1], provided by the New Zealand Electricity Commission (now by the Electricity Authority). It records the offer curves and historical dispatches for every generator in the wholesale market, historical nodal prices and power flows in the transmission network, and the daily reservoir inflows and lake levels. The other data sources include COMIT Hydro [2], Ministry of Economic Development (MED) [4], industrial

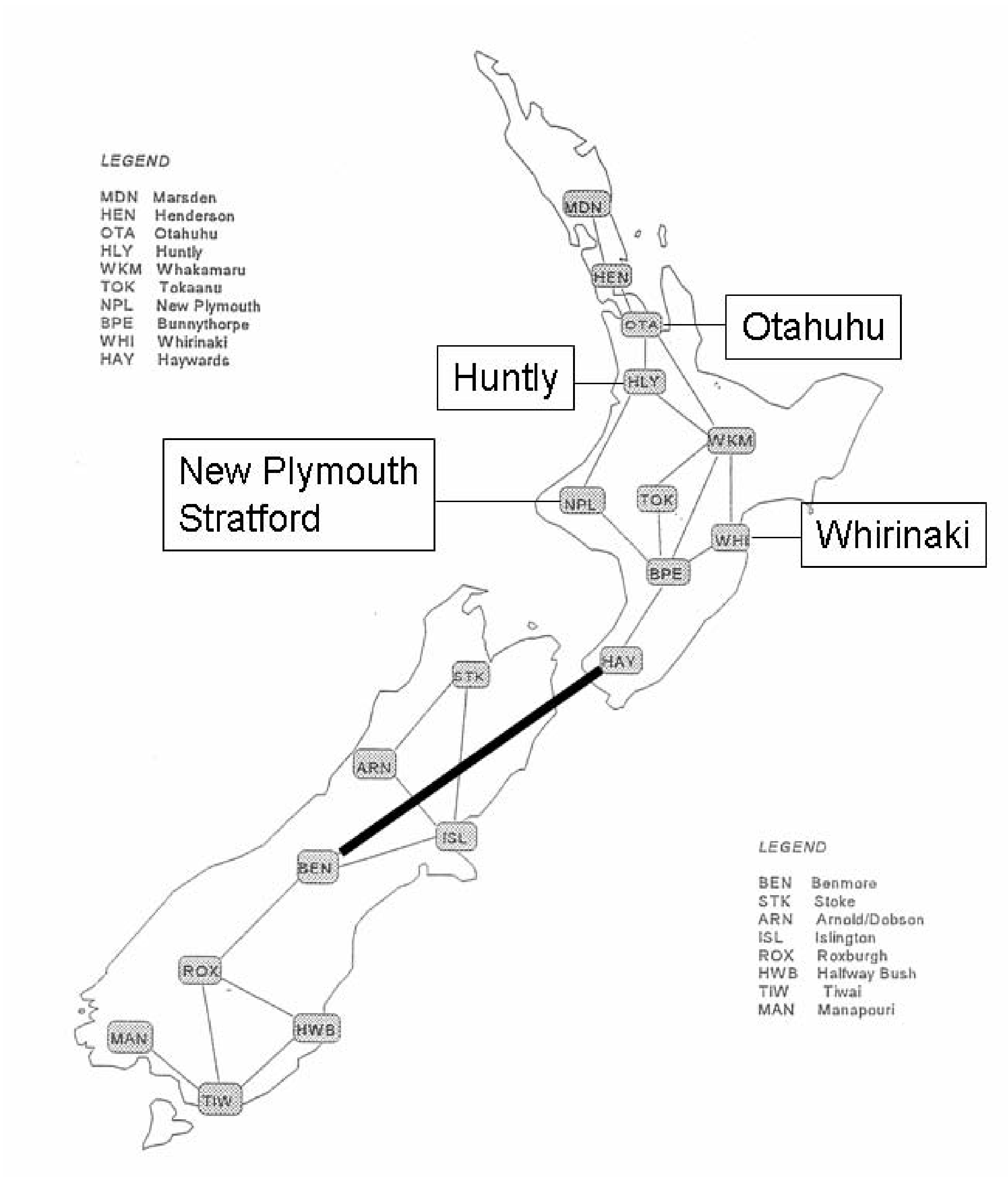

Figure 1: Approximation of New Zealand transmission network showing location of major thermal generators. The bold line represents a HVDC cable connecting the South and North islands.

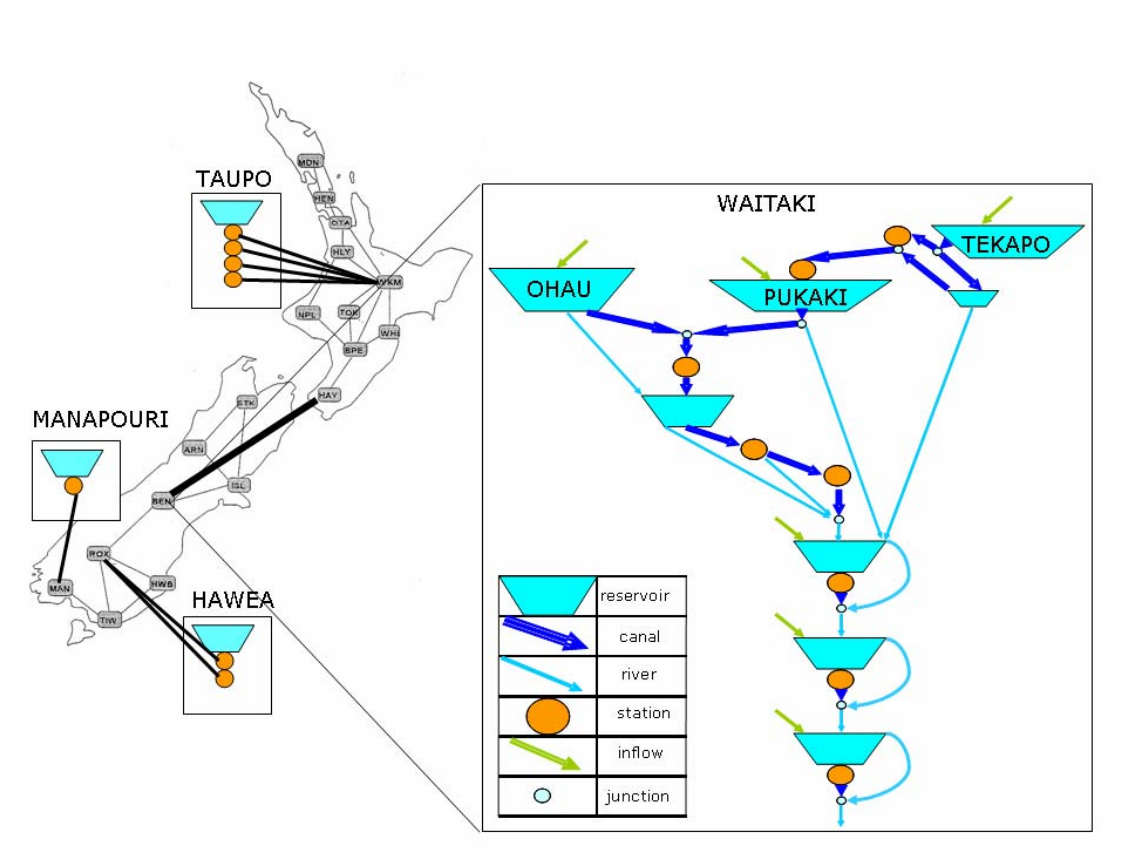

Figure 2: Approximate network representation of New Zealand electricity network showing main hydro-electricity generators

companies such as Meridian Energy and Contact Energy, and research publications.

Most of the weekly data are presented in spreadsheets by weeks in subfolders on the EPOC server, including demands, demand scales, historical dispatches, generator de-rating, historical nodal prices, historical HVDC power flows, line de-rating, HVDC de-rating, inflows, inflow scales, and reservoir storage at the beginning and the end of days (weeks) and flows in transit in the boundary conditions for daily (weekly) Central. The other data are presented in a spreadsheet on the website. The units of data and variables related to demand, dispatch and power flow are MW, those related to water flows are cumecs (cubic metres per second), and those related to storage are cubic metres.

Time

The model uses weekly stages from 2005 to 2007. Each calendar year is divided into 52 weeks, with 7 days in each of the first 51 weeks and the remaining days in the last week.

The time unit for decisions is typically a trading period (TP), which is a half hour time period. There are 48 TPs in each day. Due to the daylight savings scheme, the data in TP 5 and 6 do not exist in the short day, which are March 20, 19 and 18 in the three years respectively, and thus TP 5 and 6 are added with zero values, and TP 4.5 and 5.5 in the long day, which are October 1 and 2 and September 30 respectively, are ignored.

For daily and weekly Central, we set a decision horizon to be one week. For yearly Central, we set a decision horizon to be one week plus one day in the next week, as we need to compute flows in transit (the description of flow gives the details). We assume perfect information about data over the decision horizon.

Location

We model New Zealand as two islands, the North Island and the South Island.

Demand

Demand (or called load, in MW) at demand points is extracted from the CDS. Some small negative values represent embedded generation and we set them to zero (the description of generation gives the details). Then the demands are aggregated into regions around each of the 18 nodes according to a SPD transmission diagram and their geographical positions. This implies that we ignore in our approximation the thermal line losses that occur in lines that join the points within each region, which means that the regional totals of historical demand will underestimate the true demand. Moreover, since the aggregation has shifted the positions of demand points in the transmission network and we take into account losses in transmission between nodes, the demand in a demand point may now be satisfied by more/less power supply than in the SPD model. This raises the necessity of demand scaling. One option is to apply a uniform scaling to demand, but this does not reflect the fact that demand is concentrated in some nodes (e.g. TIW which contains a large aluminium smelter) and not in others (e.g. WKM that meets the needs of a dispersed region in our model). To overcome this, we estimate demand values for each node by solving the EP model for each TP being studied, which is presented in the EP model section. This model treats the transmission network as a transportation problem, maximizing the sum of historical nodal price and net flow at demand nodes with historical dispatches of generators, historical HVDC flows and bounds on the demand scales. Most of the demand scales are between 0.9 and 1.1, with a very small number of them between 0.8 and 1.2. Note that TIW has a demand of 500 MW that is not to be scaled, along with the demand in BEN.

In meeting demand, in case of supply shortages, load shedding (in MW) is allowed at high costs. The costs depend on the type of customers and amount of reduction (in $/MWh). Load in each node is divided into three sectors to represent different types of customers, which are industrial, commercial and residential, and each sector has the same distribution in each island. Their proportions are the proportions of consumption in 2003 adjusted to higher commercial and residential proportions in the North Island due to dense population, and to a higher industrial proportion in the South Island due to an aluminium smelter. Although these proportions change over the years, for simplicity we assume they are constant. Each sector is then divided into three segments to represent the amount of reduction, namely 5%, 5% and 90%. The third segment represents unplanned interruption of power supply. The cost for load shedding is called the *value of loss load*, or *VOLL*, in the electricity industry. The VOLLs for the industrial sector are set to be lower than the other two and the VOLLs increase over segments in each sector. We assume that up to 10% reduction in load can be achieved at a relatively low cost, but the value of unplanned interruption is very high ($10,000/MWh). The costs are displayed in Table 1.

	Up to 5%	Up to 10%	VOLL	North Is	South Is
Industrial	$1,000	$2,000	$10,000	0.34	0.58
Commercial	$2,000	$4,000	$10,000	0.27	0.15
Residential	$2,000	$4,000	$10,000	0.39	0.27

Table 1: Load reduction costs ($/MWh) and proportions of each load that is industrial, commercial, and residential load.

Generation

In New Zealand, dispatches of some generators have limited control, such as those from cogeneration, geothermal plant, run-of-river hydro and wind. Although these have low marginal cost, their availability is subject to the vagaries of inflows and wind, and so we cannot centrally dispatch these in our model. We choose to fix all cogeneration, geothermal generation, wind generation, embedded generation, run-of-river generation and small hydro plant at their historical dispatches. We call generators with fixed generation *fixed generators*, except those with embedded generation called *embedded generators*. Then this leaves the large hydro systems (Clutha, Manapouri, Waikato and Waitaki) available for control along with the major thermal stations (Huntly (4 units plus e3p and P40), Otahuhu, New Plymouth, Stratford, and Whirinaki). These are the only generators that we allow to offer energy within our model; we call them *offering generators*.

Note that the historical dispatches of embedded generators are aggregated into demands in the CDS. Their dispatches offset some demands and thus if the dispatches are larger than demand then negative values are presented. But as described in the CDS, some negative values have been set to zero, but no information is given on how much and where, and thus we cannot obtain all historical dispatches for embedded generators. However, since the CDS says that there is only a small number of occurrences of negative values, we choose to set all historical dispatches of embedded generators to be zero (as mentioned in describing demand).

Among the offering generators, there are 21 hydro generators sited in 19 stations in the 4 hydro systems supplying electricity in both islands (from here, we define a station by a collection of generators in the same site, and a generator by a collection of units.) Each generator has a nominal dispatch capacity (in MW), and a nominal conversion factor (in MW/cumec) to compute energy generated from flow through the generator at the average storage levels. The offering hydro generators are presented in Table 2.

Generator	System	Capacity	Conversion factor	Node
GEN.Hydro.Arapuni	Waikato	196.7	0.4348	WKM
GEN.Hydro.Aratiatia	Waikato	78	0.2703	WKM
GEN.Hydro.Atiamuri	Waikato	84	0.2041	WKM
GEN.Hydro.Aviemore	Waitaki	230.2	0.3175	BEN
GEN.Hydro.Benmore.162	Waitaki	270.2	0.8214	BEN
GEN.Hydro.Benmore.163	Waitaki	270.2	0.8214	BEN
GEN.Hydro.Clyde.220KV	Clutha	464	0.5128	ROX
GEN.Hydro.Karapiro	Waikato	100	0.2632	WKM
GEN.Hydro.Manapouri	Manapouri	885	1.5180	MAN
GEN.Hydro.Maraetai	Waikato	360	0.5000	WKM
GEN.Hydro.Ohakuri	Waikato	112	0.2778	WKM
GEN.Hydro.Ohau.A	Waitaki	264.2	0.4790	BEN
GEN.Hydro.Ohau.B	Waitaki	212.2	0.4251	BEN
GEN.Hydro.Ohau.C	Waitaki	212.2	0.4251	BEN
GEN.Hydro.Pukaki	Waitaki	160.1	1.2970	BEN
GEN.Hydro.Roxburgh.110KV	Clutha	124	0.3876	ROX
GEN.Hydro.Roxburgh.220KV	Clutha	210	0.3876	ROX
GEN.Hydro.Tekapo	Waitaki	25.1	0.2436	BEN
GEN.Hydro.Waipapa	Waikato	55	0.1429	WKM
GEN.Hydro.Waitaki	Waitaki	105	0.1751	BEN
GEN.Hydro.Whakamaru	Waikato	100	0.3125	WKM

Table 2: Offering thermal generators and their locations, capacities, heat rates, fuel types and supplying demand nodes.

We observe that with the nominal conversion factors, the water usage computed from the historical dispatches and historical inflows does not match the historical reservoir storages. Given that head levels vary over time, the conversion factors to compute water usage for historical dispatches should also vary. In order to match the water usage, we use a scaling factor δ_p for each day to scale the nominal conversion factor γ_h, which gives a daily conversion factor for each generator computed as $\delta_p \gamma_h$, where p is a particular day and h is a particular hydro generator. The scaling factor is set to be between 0.9 and 1.1. Note that generators in the same station have the same nominal conversion factor and scaling factor.

Ideally we should use historical data for reservoir storages, inflows, dispatches of hydro generators, flows and spills in each TP to compute a scaling factor for that TP, but we do not have all of these data to this resolution. We only have daily reservoir storages, daily inflow averages and historical dispatches by TPs, and thus use the daily Inter model (see Section 3) with these data to estimate the daily scaling factors.

There are 12 thermal generators sited in 5 stations feeding energy into the North Island. They run on different types of fuel, either coal, gas, or diesel, and have different heat rates (in GJ/MWh). Each station has a nominal capacity (in MW). We assume that generator can supply any quantity at cost per MWh. The running cost is the product of heat rate and the wholesale cost for fuel (in $/GJ) which is in 2008 dollars, and the fuel prices for coal are 4$/GJ and for gas and diesel are quarterly averages obtained from [4]. The offering thermal generators are presented in Table 3, and the fuel costs for gas and diesel are presented in Table 4.

Generator	Station	Capacity	Heat rate	Fuel	Node
GEN.Stratford.220KV	Stratford	387	7.3	Gas	NPL
GEN.Thermal.Huntly.gas	Huntly	430	6.8	Gas	HLY
GEN.Thermal.Huntly.main.g1	Huntly	260	10.5	Coal	HLY
GEN.Thermal.Huntly.main.g2	Huntly	260	10.5	Coal	HLY
GEN.Thermal.Huntly.main.g3	Huntly	260	10.5	Coal	HLY
GEN.Thermal.Huntly.main.g4	Huntly	265	10.5	Coal	HLY
GEN.Thermal.Huntly.Peak	Huntly	50	9.5	Gas	HLY
GEN.Thermal.NewPlymouth.110KV.g1	NewPlymouth	120	11	Gas	NPL
GEN.Thermal.NewPlymouth.110KV.g2	NewPlymouth	120	11	Gas	NPL
GEN.Thermal.NewPlymouth.220KV.g3	NewPlymouth	120	11	Gas	NPL
GEN.Thermal.Otahuhu.B	Otahuhu	396	7.05	Gas	OTA
NI.Whirinaki.220KV	Whirinaki	159	11	Diesel	WHI

Table 3: Offering thermal generators and their locations, capacities, heat rates and fuel types and supply demand nodes.

		Gas	Diesel
2005	Mar	4.49	22.78
	Jun	4.21	24.60
	Sep	4.13	26.43
	Dec	5.14	25.68
2006	Mar	5.12	26.73
	Jun	5.07	31.64
	Sep	5.18	30.07
	Dec	5.67	25.23
2007	Mar	6.00	24.07
	Jun	5.97	25.39
	Sep	6.01	25.81
	Dec	5.57	28.76
2008	Mar	4.11	30.66
	Jun	5.13	37.22
	Sep	5.36	37.40
	Dec	5.77	28.07

Table 4: Quarterly real gas and diesel wholesale prices in (2008 NZ)$/GJ.

Note that we assume fuel can be purchased on demand. Natural gas is typically acquired under a take-or-pay contract that gives a different operating imperative from that faced by a purchaser with more flexibility. Similarly coal is typically used from a stockpile that is periodically restocked; in this setting, supply shortages can lead to high opportunity costs. We argue, however, that a central planner might avoid many of the contractual problems in obtaining thermal fuel that a number of competing generators might face, which would make our assumption less important. In any case including these effects leads to a more complicated optimization problem than we would want to study here, so we ignore them.

Some thermal generators start commissioning or became decommissioned in the period of interest. The Huntly unit 5, also known as e3p and Huntly gas, was officially commissioned in June 2007. However, the historical data shows that it was dispatched at zero price from February 2 which is day 5 in week 5 until June 2 when it was dispatched at normal prices. Thus, this generator is set to be unavailable before week 5. For the period from week 5 to June 1, its capacity is set to be the historical dispatches, which is computed by the nominal capacity minus de-rating (as described below), and the running cost is set to be zero. On the other hand, the New Plymouth station was decommissioned in 2007. The historical data shows that the station had no offers after 26 September 2007, which is day 2 in week 39, except one generator was temporarily available in May 2008. Since the station is inefficient, it is unlikely to be used in Central and thus the generators in the station are set to be unavailable from week 39.

In practice, there is ramping for thermal generators, which constrains the change in dispatching between consecutive TPs. The ramping up and down rates (MW/TP) for most of the thermal generators are lower than their capacities and thus may limit their dispatches. There are also unit commitments for some thermal stations, e.g., a minimum dispatch of 110 MW for the generators in the Huntly station combined if any of them is dispatched, which may also limit their dispatches. However, a test on a particular week, week 25 in 2005, with these two features shows that they have no effect, and thus we do not include these in the model.

For the offering generators, we observe that some historical offers or dispatches are higher than the nominal capacities, particularly for the hydro generators (due to varying head rates). In our model, we assume constant capacities for generators, and we set the nominal capacities to be the maximum of the capacities presented in the CDS and the historical offers from 2004 to 2009. The historical dispatches are not taken into account as they are not anticipated to exceed the capacities (unless the offer already has), although they did by a very small amount sometimes.

To enable a fair comparison with the market outcomes, we have de-rated generators (in MW) at which units have been removed for planned maintenance, as outlined in the POCP database [12]. The schedule in POCP defines the starting and end time of scheduled maintenance for generators, which includes the offering generators and three generators that we treat as fixed (Tokaanu, Rangipo and Waikaremoana). We have observed that in some declared maintenance periods, the generators still offered or were dispatched energy. We have also observed that reduction of capacities displayed in the schedules are not consistent with the actual reduction in historical data. Thus for the offering generators, we define the capacity loss to be the nominal capacity minus the maximum of offer and dispatch (which may exceed the offer due to block dispatching) in the period of interest.

For the three fixed generators, for which only dispatch data are available, the capacity loss is defined to be the nominal capacity minus their dispatch (these data are not needed in Central but used in DOASA). Note that since a generator may not offer or dispatch at its maximum available capacity in any period, our model overestimates the capacity loss due to maintenance.

Transmission

The transmission network consists of 20 demand nodes, AC lines connecting nodes in each island, and two HVDC lines connecting the two islands. The 20 demand nodes consists of the 18 demand nodes and 2 nodes with zero demands at the middle of the HVDC lines. There are pairs of lines for power flow in opposite directions. Each line has a nominal capacity (in MW), and is subject to de-rating due to transmission outage. The transmission lines are presented in Table 5, with the HVDC lines highlighted.

Start	End	Capacity	Reactance	Breakpoint1	Breakpoint2	Slope1	Slope2	Slope3
MDN	HND	1020	0.0386	340	680	0.0238	0.0714	0.119
HND	MDN	1020	0.0386	340	680	0.0238	0.0714	0.119
HND	OTA	1360	0.0074	453.3	906.7	0.0014	0.0041	0.0068
OTA	HND	1360	0.0074	453.3	906.7	0.0014	0.0041	0.0068
OTA	HLY	1760	0.0217	586.7	1173.3	0.0056	0.0167	0.0279
OTA	WKM	800	0.027	266.7	533.3	0.0277	0.0832	0.1387
HLY	OTA	1760	0.0217	586.7	1173.3	0.0056	0.0167	0.0279
HLY	WKM	400	0.0714	133.3	266.7	0.0447	0.134	0.2233
HLY	NPL	920	0.1	306.7	613.3	0.0699	0.2098	0.3496
WKM	OTA	800	0.027	266.7	533.3	0.0277	0.0832	0.1387
WKM	HLY	400	0.0714	133.3	266.7	0.0447	0.134	0.2233
WKM	TOK	600	0.0293	110	220	0.0091	0.0274	0.0456
WKM	WHI	600	0.0368	306.7	613.3	0.011	0.0331	0.0552
WKM	BPE	230	0.2174	76.7	153.3	0.0338	0.1014	0.1691
NPL	HLY	920	0.1	306.7	613.3	0.0699	0.2098	0.3496
NPL	BPE	690	0.0476	230	460	0.0002	0.0007	0.0011
TOK	WKM	600	0.0293	110	220	0.0091	0.0274	0.0456
TOK	BPE	400	0.0741	133.3	266.7	0.0223	0.0668	0.1113
WHI	WKM	600	0.0368	306.7	613.3	0.011	0.0331	0.0552
WHI	BPE	100	3.3333	33.3	66.7	0.0251	0.0753	0.1255
BPE	WKM	230	0.2174	76.7	153.3	0.0338	0.1014	0.1691
BPE	NPL	690	0.0476	230	460	0.0002	0.0007	0.0011
BPE	TOK	400	0.0741	133.3	266.7	0.0223	0.0668	0.1113
BPE	WHI	100	3.3333	33.3	66.7	0.0251	0.0753	0.1255
BPE	HAY	1000	0.0244	333.3	666.7	0.0117	0.035	0.0583
HAY	BPE	1000	0.0244	333.3	666.7	0.0117	0.035	0.0583
HAY	A	520	0	173.3	346.7	0	0	0
HAY	B	520	0	173.3	346.7	0	0	0
A	HAY	520	0	173.3	346.7	0	0	0
A	BEN	520	0	173.3	346.7	0.0251	0.0754	0.1257
B	HAY	520	0	173.3	346.7	0	0	0
B	BEN	520	0	173.3	346.7	0.017	0.051	0.0849
BEN	A	520	0	173.3	346.7	0.0251	0.0754	0.1257
BEN	B	520	0	173.3	346.7	0.017	0.051	0.0849
BEN	ISL	1000	0.04	333.3	666.7	0.0167	0.05	0.0833
BEN	ROX	980	0.0351	326.7	653.3	0.0082	0.0245	0.0408
STK	ARN	20	1	6.7	13.3	0.0016	0.0047	0.0078
STK	ISL	300	0.1111	100	200	0.0002	0.0006	0.001
ISL	STK	300	0.1111	100	200	0.0002	0.0006	0.001
ISL	ARN	46	1	16.7	33.3	0.0007	0.002	0.0033
ISL	BEN	1000	0.04	333.3	666.7	0.0167	0.05	0.0833
ARN	STK	20	1	6.7	13.3	0.0016	0.0047	0.0078
ARN	ISL	46	1	16.7	33.3	0.0007	0.002	0.0033
ROX	BEN	980	0.0351	326.7	653.3	0.0082	0.0245	0.0408
ROX	HWB	780	0.0217	260	520	0.0117	0.0351	0.0585
ROX	TIW	460	0.049	153.3	306.7	0.0152	0.0455	0.0759
HWB	ROX	780	0.0217	260	520	0.0117	0.0351	0.0585
HWB	TIW	680	0.0901	226.7	453.3	0.0521	0.1564	0.2607
TIW	ROX	460	0.049	153.3	306.7	0.0152	0.0455	0.0759
TIW	HWB	680	0.0901	226.7	453.3	0.0521	0.1564	0.2607
TIW	MAN	1260	0.0272	420	840	0.0181	0.0542	0.0903
MAN	TIW	1260	0.0272	420	840	0.0181	0.0542	0.0903

Table 5: Transmission lines' starting and end nodes, capacities, reactances, and break points and slopes for loss functions.

We assume that transmission outages are known at the time of dispatch. The time periods of HDVC line outage, and the HVDC flows are available in the CDS. We define the HVDC line de-rating in each such TP to be the nominal capacity minus the HVDC flow. Other line outages can be detected by examining historical nodal prices. For each line, the ratio of historical nodal prices at the ends of the line and the ratio of power sent and received along this line are computed. If the former exceeds the latter, then this line is deemed to be constrained by some contingency. Some care is needed in treating lines in loops, as a contingency in one line can affect price differences around the loop. If this is the case, then the line in the loop with the highest ratio of nodal prices between its endpoints is assumed to be the one with the contingency. The capacity loss is then defined to be the difference between the nominal capacity and the power sent in the EP model.

Note that the transmission network data that we are using are taken from [13], a thesis dating back to 1997, which needs to be updated for future projects. Using these data, the line between TOK and WKM is constrained in solving the EP model for many TPs. We resolve this by increasing this to 600 MW, which is approximately the N-1 line capacity at the current time.

The quadratic losses are modelled as piecewise linear functions of power flow which enables models to be solved as a linear program (at least when losses are minimized by the optimal solution). There are three segments and increasing slopes in each line for the function.

For AC lines, there are line reactances, and in transmission these along with power flows and voltage angles at the nodes satisfy Kirchhoff's laws.

Reservoir, junction and inflow

There are 23 reservoirs in 4 hydro systems. There are 9 lakes, with Taupo in the Waikato system, Tekapo, Pukaki, Ohau, Benmore, Aviemore and Waitaki in the Waitaki system, Hawea in the Clutha system and Manapouri in the Manapouri system. Benmore, Aviemore and Waitaki are relatively small in storage. The other reservoirs are head ponds for hydro stations, including valleys and canals, which have limited flexibility in storage. Note that the lakes referred to here are controllable lakes in New Zealand's hydro system, and uncontrollable lakes are treated as junctions. The reservoirs are presented in Table 6.

Reservoir	Type	System	Capacity	Specific energy
AVI	Small lake	Waitaki	89,194,289	0.4556
BEN	Small lake	Waitaki	423,451,076	1.2522
HAW	Large lake	Clutha	1,378,764,328	0.9004
MAN	Large lake	Manapouri	1,501,878,016	1.5180
OHU	Large lake	Waitaki	57,245,219	2.5203
PKI	Large lake	Waitaki	2,425,440,000	2.5203
TAUPO	Large lake	Waikato	848,624,230	2.4056
TEK	Large lake	Waitaki	823,190,000	3.9927
WTK	Small lake	Waitaki	19,466,601	0.1626
DUN	Head pond	Clutha	25,200,000	
LSC	Head pond	Waitaki	79,920	
R_PKI	Head pond	Waitaki	10,751,132	
RTH	Head pond	Waitaki	1,454,225	
R_ARAP	Head pond	Waikato	9,547,200	
R_ARAT	Head pond	Waikato	717,120	
R_ATIA	Head pond	Waikato	2,877,120	
R_KARA	Head pond	Waikato	13,936,320	
R_MARA	Head pond	Waikato	8,208,000	
R_OHAK	Head pond	Waikato	13,504,320	
R_OHC	Head pond	Waitaki	43,215,676	
R_ROX	Head pond	Clutha	10,324,800	
R_WAIP	Head pond	Waikato	1,105,920	
R_WHAK	Head pond	Waikato	10,549,440	

Table 6: Reservoirs and their types, locations, capacities and specific energy.

We use minimum and maximum control levels and historical levels to compute storage capacity and historical storage for the lakes. These levels are available in the CDS. The volumetric storage is set to be zero at the minimum control levels, and the capacities are set to be the volumes between the maximum and minimum control levels. The capacities and historical storage for the lakes in the Waitaki system are computed from a level-storage table provided by the Meridian company [5], and those for the other lakes are computed using a formula described in a COMIT document [2]. The formula is

$$\begin{aligned} A &= A_{\min} + \frac{1}{2}B(L - L_{\min}), \\ V &= A(L - L_{\min}), \end{aligned}$$

where L and $L_{\min}$ are the lake level of interest and that at the minimum control level in meters, A and $A_{\min}$ are the lake areas at L and $L_{\min}$ measured in $(km)^2$, B is the beach slope in $(km)^2/m$, and V is the volumetric storage at L in hm^3 (i.e., $10^6 m^3$). $A_{\min}$, $L_{\min}$ and B are available in the document [2].

However, there is a further treatment for Manapouri. Manapouri in our model is an aggregation of Lake Te Anau and Lake Manapouri. For both lakes, the historical storages computed with the formula using the levels from the CDS are sometimes below zero or above capacities. For example, the capacities of Lake Te Anau and Lake Manapouri are 200 and 120 million cubic metres, the maximum storage are 800 and 340 million cubic metres, and the minimum are -110 and -40 million cubic metres. Thus we have re-computed the historical storages using the lower minimum control levels that are available in the COMIT document, and then set the capacity of Manapouri to be the sum of the maximum historical storages of the two lakes. This has increased the historical storage of the two lakes by 220 and 130 million cubic meters, and gives a minimum of 220 million cubic metres in historical storage and a capacity of 1500 million cubic metres for Manapouri in our model.

For head ponds, no control level or historical level is available. We obtain their capacity from various sources. The capacities of head ponds in the Clutha system are from N. J. de Pont's thesis [9] and Contact Energy's notes on the thesis [3]. Two generators, Tekapo B and Ohau C, are canal-feed, and thus we set their head pond capacities to be the amount of water for them to dispatch at capacity for one day. For the other head ponds, their capacities are the ones presented in COMIT [2]. The initial states of 2005 for all head ponds are set to be 50% of their capacities. The historical storages at the start and end of each day (week) from 2005 to 2007 for daily (weekly) Central are set to be those from daily (weekly) Inter, but they are not needed for yearly Central.

We define the *state* of a reservoir to be its volumetric storage in cubic metres at the end of a time period, and *initial state* to be its storage at the beginning of a time period. For the lakes, the equivalent energy in storage is computed with *specific* *energy*, which is the total of conversion factors of downstream hydro generators. For simplicity, we do not take into account the scaling factors for conversion factors.

There is a set of junctions, which do not have storage, including uncontrollable lakes and small reservoirs that are treated as run-of-river. The junctions are presented in Table 7.

Junction	System	Tributary
J_BEN_TAIL	Waitaki	
J1	Waitaki	
J2	Waitaki	
J3	Waitaki	
J4	Waitaki	
J5	Waitaki	
J6	Waitaki	
J7	Waitaki	
J_ARAP	Waikato	Yes
J_ARAP_TAIL	Waikato	
J_ARAT	Waikato	Yes
J_ARAT_TAIL	Waikato	
J_ATIA	Waikato	Yes
J_ATIA_TAIL	Waikato	
J_KARA	Waikato	Yes
J_KARA_POST	Waikato	
J_MARA	Waikato	Yes
J_OHAK	Waikato	Yes
J_WAIP	Waikato	Yes
J_WAIP_TAIL	Waikato	
J_WHAK	Waikato	Yes
J_CLY	Clutha	Yes
J_CLY_TAIL	Clutha	
J_ROX	Clutha	Yes
J_ROX_2	Clutha	
J_ROX_TAIL	Clutha	
J_WNK	Clutha	Yes

Table 7: Junctions, their locations and whether they have tributary inflows.

The lakes and some junctions have tributary inflows. Daily average inflows in cumecs are available in the CDS. The inflows for Manapouri are computed as the total of those for Lake Te Anau and Lake Manapouri. There are a very small number of negative values in the data, which are due to reservoir leaks, dirty water spill or evaporation. For example, the inflow at the Waikaremoana tributary junction in week 12 in 2007 is -0.13. These values are changed to zero.

We observe that using the daily average inflows as the inflows over the course of a day, along with the water usage of historical dispatches of hydro generators, do not match the historical reservoir storage. Given that the historical inflows vary over the course of a day, the water flow pattern is different from that using the daily average inflow through the day. For example, the reservoirs may be filled up in different TP, and then spill may occur resulting in the loss of water. To resolve this, we estimated the historical inflows in each TP by multiplying the daily average by scaling factors. As we described in scaling factors for hydro generators, we do not have all historical data to compute these exactly, and thus we estimate these scaling factors from daily Inter. We restrict the factors to be between zero and three, and they are chosen so that the average of scaled inflow to matches the observe daily average.

Flow

The water network consists of nodes and arcs. The nodes consist of reservoirs, junctions, hydro generators and the sea which is the exit of water from the hydro systems. Arcs define the waterways connecting nodes for water to flow through.

In some arcs, flows are constrained by lower bounds or upper bounds. The bounds for the Waitaki system are provided by Meridian, those for the Clutha system are from [9] and [3], and the others from the CDS. Some upper bounds are increased to meet the flows from the maximum historical dispatches of the upstream generators.

Some bounds are required to be met by environment consents. In the Waikato system, a minimum flow of 148 cumecs leaving Lake Karapiro is required. In the Waitaki system, a minimum flow of 8 cumecs from Ohau to RTH and that of 150 cumecs leaving Lake Waitaki are required. In the Clutha system, there is a minimum of 120 cumecs for flows leaving Lake Dunstan at night, which is from TP 38 to TP 10 the next day, and a minimum of 250 cumecs and a maximum of 850 cumecs for flows leaving Lake Roxburgh. According to [9], the minimum flows in the Clutha system were not met at times. However, we assume all the bounds to be met, and we penalize violation of the bounds at $500/MWh.

For some arcs, it takes a significant amount of time for flows to go through. This causes delays in flows reaching downstream, and the delay time is expressed in TPs. For other arcs, flows are assumed to enter and leave the arcs instantaneously. For daily (weekly) Central, the flows in transit over days (weeks) are those defined in the boundary conditions obtained from the daily (weekly) Inter model. For yearly Central, the flows in transit between weeks are obtained by using a decision horizon of the current week and the first day of the next week.

Water may spill around stations through some arcs and these arcs are defined as spill arcs. The energy lost depends on the nominal conversion factors of the generators in the stations and flows through the spill arcs. For simplicity we do not use scaling factors for the conversion factors.

The arcs are presented in Table 8. Some arcs in the Waitaki system are not presented in the table due to confidentiality restrictions.

Start	End	System	LB	LB night	UB	Spill rate	Delay
J_KARA_POST	SEA	Waikato	148				
R_WAIP	J_WAIP_TAIL	Waikato				0.1429	
R_ATIA	J_ATIA_TAIL	Waikato				0.2041	
R_KARA	J_KARA_POST	Waikato				0.2632	
R_ARAT	J_ARAT_TAIL	Waikato				0.2703	
R_OHAK	J_ATIA	Waikato				0.2778	
R_WHAK	J_MARA	Waikato				0.3125	
R_ARAP	J_ARAP_TAIL	Waikato				0.4348	
R_MARA	J_WAIP	Waikato				0.5000	
J_ATIA_TAIL	J_WHAK	Waikato					2
J_ARAP_TAIL	J_KARA	Waikato					4
J_WAIP_TAIL	J_ARAP	Waikato					4
J_ARAT_TAIL	J_OHAK	Waikato					22
J_ROX_TAIL	SEA	Clutha	250		850		
J_CLY_TAIL	J_ROX	Clutha		120	1000		5
DUN	J_CLY_TAIL	Clutha				0.5128	
R_ROX	J_ROX_TAIL	Clutha				0.3876	
MAN	GEN.Hydro.Manapouri	Manapouri			584		
MAN	SEA	Manapouri				1.5180	
OHU	RTH	Waitaki	8			0.4790	
J7	SEA	Waitaki	150				
WTK	J7	Waitaki				0.1751	
LSC	J1	Waitaki				0.2436	
AVI	WTK	Waitaki				0.3175	
J5	J6	Waitaki				0.4251	
BEN	J_BEN_TAIL	Waitaki				0.8214	
RTH	J6	Waitaki				0.8502	
PKI	J3	Waitaki				1.3292	
LSC	J3	Waitaki				2.8699	

Table 8: Starting and end nodes and systems of the arcs with lower bounds, upper bound, spill or delay.

Boundary conditions for daily Central and weekly Central

In order to compare the dispatch of daily (weekly) Central with that of the market we need to ensure that they both have the same boundary conditions. In other words, a market solution may burn more fuel than daily (weekly) Central, while leaving all reservoirs with more water in them at the end of the day (week).

In our daily (weekly) experiments we wish to impose the same boundary conditions on both models in order to compare the efficiency of the dispatch. In daily (weekly) Central, states and flows in transit at the end of days (weeks) are set to be those obtained from daily (weekly) Inter.

Future cost (water value) and cuts for yearly Central

For yearly Central, we maximize the water value that is the value of reservoirs states at the end of decision horizon to the uncertain future. Here we choose only the six large lakes as they are important in the hydro system due to their storage capacities and associated hydro generators' capacities. In minimization, we use the negative of the water value, the future cost. The future cost is computed as the pointwise maximum of a set of cuts, which are linear functions of states, for the given states.

The cuts are obtained by solving the DOASA model, which uses weekly stages, and thus the cuts actually give the future cost at the end of a week. The decision horizon of yearly Central is only one day more than the weekly stages in DOASA, and thus the cuts can give a good estimate on the future cost at the end of decision horizon.

1.2 Formulation

This section presents the formulation of the three Central models. For daily and weekly Central, the constraints for the computation of future cost using cuts should be ignored, and for yearly Central, the constraints for the boundary conditions should be ignored.

Time and location

Sets

$\mathcal{T}$	TPs in decision horizon.
$\bar{\mathcal{T}} \subset \mathcal{T}$	TPs at night time.
$\check{\mathcal{T}}$	the last TPs in the days (weeks) in daily (weekly) Central.
$\tilde{\mathcal{T}}$	TPs for flows in transit in the boundary conditions.
$\mathcal{O}$	islands.

Parameters

$\check{t}$	the last TP in $\mathcal{T}$.
$\bar{T}$	the number of hours in a TP.
$\tilde{T}$	the number of seconds in an hour.

Demand

Sets

$\mathcal{N}$	demand nodes.
$(\mathcal{N}, \mathcal{O})$	nodes in islands.
$\mathcal{U}$	sectors for demand.
$\mathcal{V}$	segments for demand.

Parameters

D_{nt}	demand at node n in TP t.
$\bar{D}_n$	demand that is not to be scaled at node n.
α_{nt}	demand scale for node n in TP t.
ψ_{uv}	cost of load shedding in segment v in sector u.
π_{uo}	proportion of sector u in island o.
$\bar{\pi}_v$	proportion of segment v.

Variables

d_{ntuv}	load shedding in segment v of sector u at node n in TP t.

Generation

Sets

$\mathcal{G}$	generators.
$(\mathcal{G},\mathcal{N})$	generators supplying power to nodes.
$\hat{\mathcal{G}} \subset \mathcal{G}$	offering generators.
$\check{\mathcal{G}} \subset \mathcal{G}$	fixed generators.
$\bar{\mathcal{G}} \subset \mathcal{G}$	embedded generators.
$\mathcal{H} \subset \hat{\mathcal{G}}$	offering hydro generators.
$\mathcal{S}$	hydro stations.
$(\mathcal{H},\mathcal{S})$	offering hydro generators in stations.
$\mathcal{M} \subset \hat{\mathcal{G}}$	offering thermal generators.
$\mathcal{F}$	fuels.
$(\mathcal{M},\mathcal{F})$	offering thermal generators using fuels.
$\tilde{\mathcal{M}} \subset \mathcal{M}$	unavailable offering thermal generators.

Parameters

Q_g	capacity of offering generator g.
$\tilde{Q}_{gt}$	de-rating of offering generator g in TP t.
$\overline{q}_{gt}$	historical dispatch of generator g in TP t.
γ_h	nominal conversion factor for offering hydro generator h.
δ_{st}	scaling factor for hydro station s in TP t.
κ_m	heat rate of offering thermal generator m.
ϕ_f	wholesale cost of fuel f in the current week.

Variables

q_{gt}	dispatch of generator g in TP t.

Transmission

Sets

$\mathcal{L} = (\mathcal{N},\mathcal{N})$	transmission lines, indexed by l or (n,n').
$\hat{\mathcal{L}} \subset \mathcal{L}$	AC lines.
$\check{\mathcal{L}} \subset \mathcal{L}$	HVDC lines.

Parameters

Y_l	nominal capacity of line l.
$\bar{Y}_{lt}$	de-rating of line l in TP t.
$\tilde{Y}_{lt}$	de-rating of HVDC line l in TP t.
Z_l	line reactance of AC line l.

Variables

y_{lt}	power flow in line l in TP t.
$\hat{y}_{lt}$	power sent into line l in TP t.
$\check{y}_{lt}$	power received from line l in TP t.
$\tilde{y}_{lt}(y_{lt})$	power loss in transmission computed from y_{lt} in line l in TP t.
z_{nt}	voltage angle at node n in TP t for an AC line.

Reservoir and junction

Sets

$\mathcal{R}$	reservoirs.
$\hat{\mathcal{R}} \subseteq \mathcal{R}$	reservoirs that are lakes.
$\bar{\mathcal{R}} \subseteq \hat{\mathcal{R}}$	reservoirs that are large lakes.
$\mathcal{J}$	junctions.
$\hat{\mathcal{J}} \subseteq \mathcal{J}$	junctions with tributary inflows.

Parameters

X_r	capacity of reservoir r.
$\bar{\gamma}_r$	specific energy of reservoir r.
$\hat{\gamma}$	the maximum specific energy of the lakes.
$x_{r,0}$	initial state in reservoir r of the current week.
$\bar{x}_{rt}$	synthetic state in reservoir r in TP t in the boundary conditions.

Variables

x_{rt}	state in reservoir r in TP t.

Inflow

Parameters

ω_{it}	inflow of lake reservoir or tributary junction i.
λ_{it}	scaling factor on inflow of lake reservoir or tributary junction i.

Flow

Sets

$\mathcal{I} = \{\mathcal{R}, \mathcal{J}, \mathcal{H}, Sea\}$	nodes in the water network.
$\mathcal{A} = (\mathcal{I}, \mathcal{I})$	arcs in the water network.
$\check{\mathcal{A}}$	arcs with lower bounds.
$\check{\mathcal{A}}' \subset \check{\mathcal{A}}$	arcs with lower bounds at night time.
$\hat{\mathcal{A}}$	arcs with upper bounds.
$\bar{\mathcal{A}}$	arcs for spilling water around stations.
$\tilde{\mathcal{A}}$	arcs with delay for flows.

Parameters

$\check{b}_a$	lower bound of arc a.
$\check{b}'_a$	lower bound of arc a at night time.
$\hat{b}_a$	upper bound of arc a.
ρ	penalty cost on violation of flow bounds.
$\bar{t}_a$	the number of TPs for which flow is in transit in arc a.
$\bar{w}_{at}$	flow in transit that will leave arc a in TP t.
$\bar{w}'_{at}$	flow entering arc a in TP t in the boundary conditions.
$\tilde{\gamma}_a$	conversion factor for water spilled through arc a.

Variables

w_{at}	flow entering arc a in TP t.
$\tilde{w}_{at}$	flow leaving arc a in TP t.
$\check{w}_{at}$	penalty variable for flow lower bound violation in arc a in TP t.
$\hat{w}_{at}$	penalty variable for flow upper bound violation in arc a in TP t.

Cut

Sets

$\mathcal{K}$	cuts for computing future cost.

Parameters

α_k	intercept in cut k.
β_{rk}	slope for large lake r in cut k.

Variables

θ	future cost.

Objective:

The objective is to minimize the sum of thermal fuel cost, load shedding cost, penalty cost on violation of flow lower bounds and upper bounds, penalty cost on spill and future cost. Note that given that the maximum specific energy of lakes is much higher than conversion factors for spill arcs, the penalty cost on violation of flow bounds is set to be higher than those on spill, so that these bounds are met with possible spilling.

$$\begin{aligned} \min \quad & \bar{T}\sum_{(m,f)\in(\mathcal{M},\mathcal{F}),t\in\mathcal{T}} \kappa_m \phi_{ft} q_{mt} + \bar{T}\sum_{n\in\mathcal{N},t\in\mathcal{T},u\in\mathcal{U},v\in\mathcal{V}} \psi_{uv} d_{ntuv} + \\ & \rho\hat{\gamma}\bar{T}\sum_{a\in\check{\mathcal{A}},t\in\mathcal{T}} \check{w}_{at} + \rho\hat{\gamma}\bar{T}\sum_{a\in\hat{\mathcal{A}},t\in\mathcal{T}} \hat{w}_{at} + \bar{T}\sum_{a\in\hat{\mathcal{A}},t\in\mathcal{T}} \tilde{\gamma}_a w_{at} + \theta. \end{aligned}$$

Boundary conditions for daily Central and weekly Central:

The boundary conditions for daily (weekly) Central fix the states and flows entering arcs with delay in the corresponding TPs to be the those from daily (weekly) Inter. The future cost is set to be zero as trivial.

$$\begin{aligned} x_{rt} &= \bar{x}_{rt}, \quad \forall r \in \mathcal{R}, t \in \check{\mathcal{T}}, \\ w_{at} &= \bar{w}'_{at}, \quad \forall a \in \bar{\mathcal{A}}, t \in \tilde{\mathcal{T}}, \\ \theta &= 0. \end{aligned}$$

Future cost computed using cuts for yearly Central:

The future cost for yearly Central is computed as the pointwise maximum of cuts of the states of six large lakes at the end of decision horizon.

$$\theta \geq \alpha_k + \textstyle\sum_{r \in \bar{\mathcal{R}}} \beta_{rk} x_{r,\check{t}}, \quad \forall k \in \mathcal{K}.$$

Dispatch:

Dispatches of offering generators are constrained by the nominal capacity minus de-rating. Dispatches for fixed generators and embedded generators are set to be historical dispatches. Dispatch of an offering hydro generator is calculated as a product of total flows through the generator, nominal conversion factor and scaling factor. Dispatches of unavailable thermal generators are set to be zero.

$$\begin{aligned}
q_{gt} &\leq Q_g - \tilde{Q}_{gt}, \quad \forall g \in \hat{\mathcal{G}}, t \in \mathcal{T}, \\
q_{gt} &= \overline{q}_{gt}, \quad \forall g \in \check{\mathcal{G}} \cup \bar{\mathcal{G}}, t \in \mathcal{T}, \\
q_{ht} &= \delta_{st} \gamma_h \textstyle\sum_{(h,i) \in \mathcal{A}} w_{hit}, \quad \forall (h,s) \in (\mathcal{H}, \mathcal{S}), t \in \mathcal{T}, \\
q_{mt} &= 0, \quad \forall m \in \tilde{\mathcal{M}}, t \in \mathcal{T}.
\end{aligned}$$

Meet demand:

Dispatches from generators and net power flow meet demand minus load shedding. Load reduction in each segment and sector is bounded.

$$\begin{aligned}
&\textstyle\sum_{(g,n) \in (\mathcal{G},\mathcal{N})} q_{gt} + \sum_{(n',n) \in \mathcal{L}} (\check{y}_{n'nt} - \hat{y}_{nn't}) = (\alpha_{nt}(D_{nt} - \bar{D}_n) + \bar{D}_n) - \\
&\qquad\qquad\qquad\qquad \textstyle\sum_{u \in \mathcal{U}, v \in \mathcal{V}} d_{ntuv}, \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, \\
&d_{ntuv} \leq \pi_{uo} \bar{\pi}_v (\alpha_{nt}(D_{nt} - \bar{D}_n) + \bar{D}_n), \quad \forall (n,o) \in (\mathcal{N}, \mathcal{O}), u \in \mathcal{U}, v \in \mathcal{V}, t \in \mathcal{T}.
\end{aligned}$$

Transmission:

Power sent into a line is calculated as the power flow plus half of the loss, with the loss being computed from a piecewise linear function of power flow, and power received from a line is the power flow minus half of the loss. Power sent into a line is constrained by the nominal capacity minus de-rating. For an AC line, power flow and reactance of a line and voltage angles at both ends satisfy Kirchhoff's Law.

For a HVDC line, power sent is further constrained by the nominal capacity minus de-rating. Note that in minimizing cost, due to transmission losses, at least one of the power flows in opposite directions in a pair of lines between two nodes will be zero (unless there is a negative price - see [8]).

$$\hat{y}_{lt} = y_{lt} + \tfrac{1}{2}\tilde{y}_{lt}(y_{lt}), \quad \forall l \in \mathcal{L}, t \in \mathcal{T},$$
$$\check{y}_{lt} = y_{lt} - \tfrac{1}{2}\tilde{y}_{lt}(y_{lt}), \quad \forall l \in \mathcal{L}, t \in \mathcal{T},$$
$$\hat{y}_{lt} \le Y_l - \bar{Y}_{lt}, \quad \forall l \in \mathcal{L}, t \in \mathcal{T},$$
$$Z_{nn'}(y_{nn't} - y_{n'nt}) = (z_{n't} - z_{nt}), \quad \forall (n,n') \in \hat{\mathcal{L}}, t \in \mathcal{T},$$
$$\hat{y}_{lt} \le Y_l - \tilde{Y}_{lt}, \quad \forall l \in \check{\mathcal{L}}, t \in \mathcal{T}.$$

Water balance at reservoirs and junctions:

For each lake, the state equals the total of the state in the previous TP, inflow, and net water flow. For each head pond, the state equals the total of the state in the previous TP and net water flow. For a tributary junction, the incoming flows and inflow equal to the outgoing flows. For a non-tributary junction or a hydro station, the incoming flows are equal to the outgoing flows.

$$x_{rt} = x_{r,t-1} + \tilde{T}\bar{T}\lambda_{rt}\omega_{rt} + \tilde{T}\bar{T}(\textstyle\sum_{(i,r)\in\mathcal{A}} \tilde{w}_{irt} - \sum_{(r,i)\in\mathcal{A}} w_{rit}), \quad \forall r \in \hat{\mathcal{R}}, t \in \mathcal{T},$$
$$x_{rt} = x_{r,t-1} + \tilde{T}\bar{T}(\textstyle\sum_{(i,r)\in\mathcal{A}} \tilde{w}_{irt} - \sum_{(r,i)\in\mathcal{A}} w_{rit}), \quad \forall r \in \mathcal{R}, r \notin \hat{\mathcal{R}}, t \in \mathcal{T},$$
$$\textstyle\sum_{(i,j)\in\mathcal{A}} \tilde{w}_{ijt} + \lambda_{jt}\omega_{jt} = \sum_{(j,i)\in\mathcal{A}} w_{jit}, \quad \forall j \in \hat{\mathcal{J}}, t \in \mathcal{T},$$
$$\textstyle\sum_{(i,j)\in\mathcal{A}} \tilde{w}_{ijt} = \sum_{(j,i)\in\mathcal{A}} w_{jit}, \quad \forall j \in \mathcal{H} \cup \mathcal{J}, j \notin \hat{\mathcal{J}}, t \in \mathcal{T}.$$

Flows:

If there is delay for flows in an arc, then the flow leaving an arc is either the flow in transit that is due to leave the arc or the flow entering the arc back in time by the number of TPs in delay, otherwise it is the same as that entering the arc in the same TP. If there is a lower bound or an upper bound for an arc, then the flow needs to satisfy these bounds, with a penalty variable for violation.

$$\tilde{w}_{at} = \bar{w}_{at}, \quad \forall a \in \tilde{\mathcal{A}}, t \in \mathcal{T}, t \leq \bar{t}_a,$$

$$\tilde{w}_{at} = w_{a,t-\bar{t}_a}, \quad \forall a \in \tilde{\mathcal{A}}, t \in \mathcal{T}, t > \bar{t}_a,$$

$$\tilde{w}_{at} = w_{at}, \quad \forall a \in \mathcal{A}, a \notin \tilde{\mathcal{A}}, t \in \mathcal{T},$$

$$w_{at} + \breve{w}_{at} \geq \breve{b}_a, \quad \forall a \in \breve{\mathcal{A}}, a \notin \breve{\mathcal{A}}', t \in \mathcal{T},$$

$$w_{at} + \breve{w}_{at} \geq \breve{b}'_a, \quad \forall a \in \breve{\mathcal{A}}', t \in \tilde{\mathcal{T}},$$

$$w_{at} - \hat{w}_{at} \leq \hat{b}_a, \quad \forall a \in \hat{\mathcal{A}}, t \in \mathcal{T}.$$

Domain for variables:

All variables ≥ 0, except θ.

2 EP model

It is known (see e.g. [11]) that the line flows from any given optimal dispatch maximizes the sum of historical nodal price and net flow at demand nodes. This model seeks to scale historical demand for each node so that the demand estimates obtained are consistent with the historical dispatches and historical prices, subject to historical flows in HVDC lines and bounds on the scales. The demand scales are set to be between 0.9 and 1.1, which are relaxed to 0.8 or 1.2 for a very small number of TPs. Note that dispersed demand in a region is to be scaled but not the concentrated one, which is the demand of a large aluminium smelter in TIW and the demand in BEN. To avoid power flow loops between two nodes to lose power, we use binary variables (taking a value of 0 or 1) for flow directions to restrict the power to flow between two nodes in one way only, which gives a mixed integer problem. The demand scales are used in Central, and the line flows along with historical nodal prices are used to computed line outages and thus the effective line capacities in Central.

The formulation of the model is presented as follows.

Time and location

Sets

$\mathcal{T}$ TPs in decision horizon, actually only one TP.

$\mathcal{O}$ islands.

Demand

Sets

$\mathcal{N}$ demand nodes.

$(\mathcal{N},\mathcal{O})$ nodes in islands.

Parameters

D_{nt} demand at node n in TP t.

$\bar{D}_n$ demand at node n that is not to be scaled.

$\check{\alpha}_{nt}$ demand scale lower bound at node n in TP t.

$\hat{\alpha}_{nt}$ demand scale upper bound at node n in TP t.

π_{nt} historical nodal price at node n in TP t.

Variables

α_{nt} demand scale at node n in TP t.

Generation

Sets

$\mathcal{G}$ generators.

$(\mathcal{G},\mathcal{N})$ generators supplying power to nodes.

Parameters

$\overline{q}_{gt}$ historical dispatch of generator g in TP t.

Transmission

Sets

$\mathcal{L} = (\mathcal{N},\mathcal{N})$ transmission lines, indexed by l or (n,n').

$\hat{\mathcal{L}} \subset \mathcal{L}$ AC lines.

$\check{\mathcal{L}} \subset \mathcal{L}$ HVDC lines.

Parameters

Y_l	nominal capacity of line l.
Z_l	line reactance of AC line l.
$\bar{y}_{lt}$	historical power flow in HVDC line l in TP t.

Variables

y_{lt}	power flow in line l in TP t.
$\hat{y}_{lt}$	power sent into line l in TP t.
$\check{y}_{lt}$	power received from line l in TP t.
$\tilde{y}_{lt}(y_{lt})$	power loss in transmission computed from y_{lt} in line l in TP t.
z_{nt}	voltage angle at node n in TP t for an AC line.
r_{lt}	binary variable for flow direction in line l in TP t, .

Objective:

The objective is to maximize the sum of historical nodal price and net flow at demand nodes.

$$\max \textstyle\sum_{n \in \mathcal{N}} \pi_{nt} \sum_{(n',n) \in \mathcal{L}} (\check{y}_{n'nt} - \hat{y}_{nn't}).$$

Meet demand:

Dispatches from generators and net flow of power meet scaled demand at each node. Demands scales are bounded.

$$\textstyle\sum_{(g,n) \in (\mathcal{G},\mathcal{N})} \bar{q}_{gt} + \sum_{(n',n) \in \mathcal{L}} (\check{y}_{n'nt} - \hat{y}_{nn't}) = \alpha_{nt}(D_{nt} - \bar{D}_n) + \bar{D}_n, \quad \forall n \in \mathcal{N}, t \in \mathcal{T},$$

$$\check{\alpha}_{nt} \leq \alpha_{nt} \leq \hat{\alpha}_{nt}, \quad \forall n \in \mathcal{N}, t \in \mathcal{T}.$$

Transmission:

Power sent into a line is calculated as the power flow plus half of the loss, with the loss being computed from a piecewise linear function on power flow, and power received from a line is the power flow minus half of the loss. For an AC line, power flow direction is restricted to be one way only, power sent is constrained by the capacity accounting for the flow direction, and power flow and reactance of the line and voltage angles at its ends satisfy the Kirchhoff's Law. For a HVDC line, power sent is fixed at the historical flow.

$$
\begin{aligned}
\hat{y}_{lt} &= y_{lt} + \tfrac{1}{2}\tilde{y}_{lt}(\bar{y}_{lt}), \quad \forall l \in \mathcal{L}, t \in \mathcal{T}, \\
\check{y}_{lt} &= y_{lt} - \tfrac{1}{2}\tilde{y}_{lt}(\bar{y}_{lt}), \quad \forall l \in \mathcal{L}, t \in \mathcal{T}, \\
r_{nn't} + r_{n'nt} &\leq 1, \quad \forall (n,n') \in \hat{\mathcal{L}}, t \in \mathcal{T}, \\
\hat{y}_{lt} &\leq r_{lt} Y_l, \quad \forall l \in \hat{\mathcal{L}}, t \in \mathcal{T}, \\
Z_{nn'}(y_{nn't} - y_{n'nt}) &= (z_{n't} - z_{nt}), \quad \forall (n,n') \in \hat{\mathcal{L}}, t \in \mathcal{T}, \\
\hat{y}_{lt} &= \bar{y}_l, \quad \forall l \in \check{\mathcal{L}}, t \in \mathcal{T}.
\end{aligned}
$$

Domain for variables:

$$\text{All variables} \geq 0.$$

3 Inter model

This model is to give boundary conditions for daily and weekly Central in order to compare the dispatch of Central with that of the market. These boundary conditions are the states of reservoirs including lakes and head ponds and the inflows in transit. In doing this, this model also estimates scaling factors for nominal conversion factors of offering hydro generators and for inflows, which are also used in Central.

The CDS contains only daily averages of tributary inflows, many of which vary over the course of a day. Even the averages are sometimes approximations of the true values, and reservoir levels at the beginning and end of a day fail to capture water flows that may be in transit between days. Moreover the conversion factors can vary with reservoir head level, and so nominal values of these might not correspond with the water releases associated with a given dispatch level.

We are using historical dispatches for offering hydro generators to meet hydrology constraints and boundary conditions. We assume scaling factors for conversion factors to be the same over a day for simplicity. The objective of Inter is to minimize spill of water and absolute deviations from historical states of lake reservoirs. To minimize these deviations the conversion factor for each generator is allowed to vary with bounds. In the daily Inter model, the absolute deviations from historical states of lake reservoirs at the end of each day is penalized, and the initial states of lake reservoirs of each day are set to be the historical states at the end of the previous day. This is run for a decision horizon of two weeks for each week, so that we get a solution for a week each time. Note that we allow the states of head ponds and flows in transit to be passed over days as we do not have historical data for these. The states of reservoirs, including lakes and head ponds, and the flows in transit at the end of each day give the boundary conditions for daily Central, and the scaling factors for nominal conversion factors and for inflows are used in Central.

Then we solve the weekly Inter model to obtain the states of reservoirs and inflows in transit at the end of each week for the boundary conditions of weekly Central. The weekly Inter has the same structure as the daily Inter, except using the scaling factors estimated in daily Inter, penalizing the absolute deviations from historical states of lake reservoirs at the end of each week, and setting the initial states of lake reservoirs of each week to be the historical states at the end of the previous week.

It is important here to make some observations about the use of Inter. If we consider it more important to match historical reservoir levels then we should penalize deviations from these levels more severely than spill. The difficulty in doing this is that the historical solution might appear to spill large volumes of water to match what might be erroneous daily storage or inflow observations. Recall that the lake inflow values are daily averages, and tributary inflows are estimates, and so variations in historical generation might be accommodating changes in these inflow values that we do not have recorded.

Since spill appears to be relatively rare in practice, our approach is to penalize spill more heavily than matching historical states. This means that the market solution in each TP is not forced to spill past stations so as to match historical states. Thus, as a comparison, we solve Central using synthetic boundary conditions, which are a set of states of reservoirs and flows in transit that the market would have attained with average inflows, minimal spill and historical dispatches.

Hence, we use the penalty cost on flow bound violation for water spilled and deviation from historical states of lake reservoirs with scaling by 10 and 5 respectively. We also scaled the cost in the next week by 0.9, so that the solution gives a minimal penalty cost in the current week at the possible expense of the next week.

The formulation of the model is presented as follows.

Time

Sets

$\mathcal{T}$	TPs in decision horizon.
$\bar{\mathcal{T}}$	TPs at night time.
$\hat{\mathcal{T}}$	the first TPs of days (weeks) for daily (weekly) Inter.
$\check{\mathcal{T}}$	the last TPs of days (weeks) for daily (weekly) Inter.
$\mathcal{P}$	days.
$\tilde{\mathcal{T}}_p$	TPs in day p.

Parameters

$\check{t}$	the last TP in $\mathcal{T}$.
$\bar{T}$	the number of hours in a TP.
$\tilde{T}$	the number of seconds in an hour.
$\hat{T}$	the number of TPs in one day.

Generation

Sets

$\mathcal{H}$	offering hydro generators.
$\mathcal{S}$	hydro stations.
$(\mathcal{H}, \mathcal{S})$	offering hydro generators in stations.

Parameters

$\overline{q}_{ht}$	historical dispatch of offering hydro generator h in TP t.
γ_h	nominal conversion factor for offering hydro generator h.
$\check{\delta}$	lower bound for scaling factors.
$\hat{\delta}$	upper bound for scaling factors.

Variables

δ_{st}	reciprocal of scaling factor for station s in TP t.

Reservoir and junction

Sets

$\mathcal{R}$	reservoirs.
$\hat{\mathcal{R}} \subseteq \mathcal{R}$	reservoirs that are lakes.
$\check{\mathcal{R}} \subseteq \mathcal{R}$	reservoirs that are head ponds.
$\mathcal{J}$	junctions.
$\hat{\mathcal{J}} \subseteq \mathcal{J}$	junctions with tributary inflows.

Parameters

X_r	capacity of reservoir r.
γ_r	specific energy of reservoir r.
$\bar{x}_{rt}$	historical state of reservoir r in TP t.

Variables

x_{rt}	state of reservoir r in TP t.
$\check{x}_{rt}$	deviation from $\bar{x}_{rt}$ from below.
$\hat{x}_{rt}$	deviation from $\bar{x}_{rt}$ from above.

Inflow

Parameters

ω_{it}	inflow of lake reservoir or tributary junction i in TP t.
$\hat{\lambda}$	upper bound for scaling factors on inflows.

Variables

λ_{it}	scaling factor on inflow of reservoir or tributary junction i in TP t.

Flow

Sets

$\mathcal{I} = \{\mathcal{R}, \mathcal{J}, \mathcal{H}, Sea\}$	nodes in the water network.
$\mathcal{A} = (\mathcal{I}, \mathcal{I})$	arcs in the water network.
$\check{\mathcal{A}}$	arcs with lower bound.
$\check{\mathcal{A}}' \subset \check{\mathcal{A}}$	arcs with lower bounds at night time.
$\hat{\mathcal{A}}$	arcs with upper bound.
$\bar{\mathcal{A}}$	arcs through which water spill around stations.
$\tilde{\mathcal{A}}$	arcs with delay for flows.

Parameters

$\check{b}_a$	lower bound of arc a.
$\check{b}'_a$	lower bound of arc a at night time.
$\hat{b}_a$	upper bound of arc a.
ρ	penalty cost on flow bound violation.
$\bar{t}_a$	the number of TPs for which flow is in transit in arc a.
$\bar{w}_{at}$	flow in transit that will leave arc a in TP t.
$\tilde{\gamma}_a$	conversion factor for water spilled for arc a.
$\check{\gamma}$	the minimum conversion factor for water spilled.

Variables

w_{at}	flow entering arc a in TP t.
$\tilde{w}_{at}$	flow leaving arc a in TP t.
$\check{w}_{at}$	penalty variable for lower bound violation on arc a in TP t.
$\hat{w}_{at}$	penalty variable for upper bound violation on arc a in TP t.

Cost scaling

Parameters

c_t cost scaling in TP t.

$\bar{c}$ cost scaling for spill.

$\tilde{c}$ cost scaling for deviation from historical state.

Objective:

The objective is to minimize the penalty cost on flow bound violation, spill and deviation from historical states. The cost for each term is scaled by c_t, that for spill is scaled up by $\bar{c}$, and that for deviation from historical end storage is up by $\tilde{c}$, with $\bar{c} > \tilde{c} > c_t$.

$$\begin{aligned} \min \quad & c_t\rho\check{\gamma}\bar{T}\sum_{a\in\{\check{\mathcal{A}},\check{\mathcal{A}}'\},t\in\mathcal{T}}\check{w}_{at} + c_t\rho\check{\gamma}\bar{T}\sum_{a\in\hat{\mathcal{A}},t\in\mathcal{T}}\hat{w}_{at} + \\ & c_t\bar{c}\rho\bar{T}\sum_{a\in\bar{\mathcal{A}},t\in\mathcal{T}}\tilde{\gamma}_a w_{at} + \frac{1}{\bar{T}}c_t\tilde{c}\rho\check{\gamma}\sum_{r\in\mathcal{R},t\in\check{\mathcal{T}}}(\check{x}_{rt} + \hat{x}_{rt}). \end{aligned}$$

Dispatch:

Offering hydro generators are set to be dispatched at historical levels. The product of reciprocal of scaling factor and hydro dispatch equals the flow through the generator and nominal conversion factor. Given that flows are variables, we use reciprocals of scaling factors at the left hand side of the equation instead of scaling factors at the right hand side to maintain the linearity of the problem. The reciprocal of scaling factors are the same in each day, and constrained by bounds.

$$\begin{aligned} q_{ht} &= \overline{q}_{ht}, \quad \forall h \in \mathcal{H}, t \in \mathcal{T}, \\ \delta_{st}q_{ht} &= \gamma_h\sum_{(h,i)\in\mathcal{A}} w_{hit}, \quad \forall (h,s) \in (\mathcal{H},\mathcal{S}), t \in \mathcal{T}, \\ \delta_{st} &= \delta_{st'}, \quad \forall (h,s) \in (\mathcal{H},\mathcal{S}), t \in \mathcal{T}_p, t' \in \mathcal{T}_p, p \in \mathcal{P}, \\ \frac{1}{\bar{\delta}} &\le \delta_{st} \le \frac{1}{\underline{\delta}}, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}. \end{aligned}$$

Water balance at reservoirs and junctions:

For each lake, in the first TP of each day (week) in daily (week) Inter, the state equals the total of the historical state at the end of the previous day (week), inflow and net flow, in any other TPs, the state equals the total of the state in the previous TP, inflow and net flow. For each head pond, the state equals to the total of the state in the previous TP and net flow. For a tributary junction, the incoming flows and inflow equal the outgoing flows. For a non-tributary junction or hydro station, the incoming flows equal the outgoing flows. States match historical states at the end of each day (week) in daily (weekly) Inter, with penalty variables for violation. The average of scaled inflows in each day matches the daily average.

$$x_{rt} = \bar{x}_{r,t-1} + \tilde{T}\bar{T}\lambda_{rt}\omega_{rt} + \tilde{T}\bar{T}\sum_{(i,r)\in\mathcal{A}} \tilde{w}_{irt} - \tilde{T}\bar{T}\sum_{(r,i)\in\mathcal{A}} w_{rit}, \quad \forall r \in \hat{\mathcal{R}}, t \in \hat{\mathcal{T}},$$

$$x_{rt} = x_{r,t-1} + \tilde{T}\bar{T}\lambda_{rt}\omega_{rt} + \tilde{T}\bar{T}\sum_{(i,r)\in\mathcal{A}} \tilde{w}_{irt} - \tilde{T}\bar{T}\sum_{(r,i)\in\mathcal{A}} w_{rit}, \quad \forall r \in \hat{\mathcal{R}}, t \in \mathcal{T}, t \notin \hat{\mathcal{T}},$$

$$x_{rt} = x_{r,t-1} + \tilde{T}\bar{T}\sum_{(i,r)\in\mathcal{A}} \tilde{w}_{irt} - \tilde{T}\bar{T}\sum_{(r,i)\in\mathcal{A}} w_{rit}, \quad \forall r \in \check{\mathcal{R}}, t \in \mathcal{T},$$

$$\sum_{(i,j)\in\mathcal{A}} \tilde{w}_{ijt} + \lambda_{jt}\omega_{jt} = \sum_{(j,i)\in\mathcal{A}} w_{jit}, \quad \forall j \in \hat{\mathcal{J}}, t \in \mathcal{T},$$

$$\sum_{(i,j)\in\mathcal{A}} \tilde{w}_{ijt} = \sum_{(j,i)\in\mathcal{A}} w_{jit}, \quad \forall j \in \mathcal{H} \cup \mathcal{J}, j \notin \hat{\mathcal{J}}, t \in \mathcal{T},$$

$$x_{rt} - \hat{x}_{rt} + \check{x}_{rt} = \bar{x}_{rt}, \quad \forall r \in \hat{\mathcal{R}}, t \in \check{\mathcal{T}},$$

$$\frac{1}{\hat{T}}\sum_{t\in\tilde{\mathcal{T}}_p} \lambda_{it}\omega_{rt} = \omega_{rt}, \quad \forall j \in \hat{\mathcal{R}} \cup \hat{\mathcal{J}}, p \in \mathcal{P}.$$

Flows:

If there is delay for flows in an arc, then the flow leaving an arc is either the flow in transit that is due to leave the arc or the flow entering the arc back in time by the number of TPs in delay, otherwise it is the same as that entering the arc in the same TP. If there is a lower bound all day round or at night time or an upper bound for an arc, then the flow needs to satisfy these bounds, with a penalty variable for violation.

$$\tilde{w}_{at} = \bar{w}_{at}, \quad \forall a \in \tilde{\mathcal{A}}, t \in \mathcal{T}, t \leq \bar{t}_a,$$

$$\tilde{w}_{at} = w_{a,t-\bar{t}_a}, \quad \forall a \in \tilde{\mathcal{A}}, t \in \mathcal{T}, t > \bar{t}_a,$$

$$\tilde{w}_{at} = w_{at}, \quad \forall a \in \mathcal{A}, a \notin \tilde{\mathcal{A}}, t \in \mathcal{T},$$

$$w_{at} + \check{w}_{at} \geq \check{b}_a, \quad \forall a \in \check{\mathcal{A}}, t \in \mathcal{T},$$

$$w_{at} + \check{w}_{at} \geq \check{b}'_a, \quad \forall a \in \check{\mathcal{A}}', t \in \tilde{\mathcal{T}},$$

$$w_{at} - \hat{w}_{at} \leq \hat{b}_a, \quad \forall a \in \hat{\mathcal{A}}, t \in \mathcal{T}.$$

Domain for variables:

$$\text{All variables} \geq 0.$$

4 DOASA

The DOASA model generates a policy for the release of water from reservoirs, by solving stage problems for dispatch of offering hydro and thermal generators and large fixed hydro generators being treated as offering to meet block demands with inflows sampled from a set of historical inflows in each stage in a plan year of 52 weekly stages. The policy is used in simulation in yearly Central, and is updated at a rolling horizon by 13 weeks with updated reservoir states from yearly Central.

4.1 Modelling assumptions and data estimation

DOASA is an approximation of yearly Central in a yearly planning horizon with weekly decisions, with a three-node transmission network. The representation of the network with major generators is in Figure 3. The data are weekly averages of those by TPs in yearly Central. The units of data and variables are the same as those in yearly Central, except that for block demand and load shedding for which the units are MWh instead of MW.

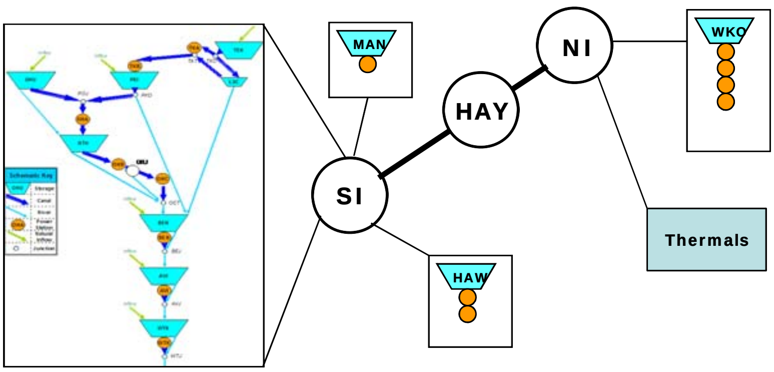

Figure 3: The 3 node transmission network and major generators in DOASA.

Time

The model uses weekly stages. A calendar year is divided into 52 weeks. A plan year is a year of 52 weeks with the starting week chosen to be a particular week. We use some past years for sampling historical inflows. We have chosen 1970 to be the start year for sampling, so that dry years in the 1970s are included, and the year in which the week just before the plan year is to be the end year. More details on sampling inflows from past years are in the later description on inflows.

Location

We model New Zealand as two islands, the North Island and the South Island.

Demand

In the 18-node model, we observe that power is transmitted between the upper North Island and the South Island via a single node Haywards at the lower North Island, and the lines connecting Haywards and the north and south are critical in meeting demands. Thus we choose to aggregate the 18 nodes into 3 nodes, which are NI, HAY and SI.

The aggregation ignores the transmission loss that occurs in the 18 node model. Thus the demand for each of the three demand nodes is not the total of demands in each aggregated region, but the power supplied to meet the demands. This is computed as the power supplied by offering generators and large fixed hydro generators feeding into the aggregated region plus the net power received from the other regions, which are estimated in the EP model. Large fixed hydro generators are taken into account due to their intra-week flexibility in storage and thus their generation.

The demands in each TP are used to construct a load duration curve for the weekly demand with three blocks, which are peak, shoulder and off-peak. The national demands by TPs are arranged from the highest to the lowest and plotted against the TPs to give an empirical distribution curve. We then approximate this curve by the load duration curve, which is a decreasing three-piece constant curve, as shown in Figure 4. Each piece defines a demand block, with the bounds and height defining the hour and average demand of the block. Note that each block contains an even number of TPs since each block size is an integer number of hours. The bounds and heights are chosen to minimize the absolute deviation of the approximate curve from the empirical distribution curve for the week and in each block. Once the block hours are estimated, the block demand for each node is the total of demands by TPs in that block. The block hours are estimated as follows.

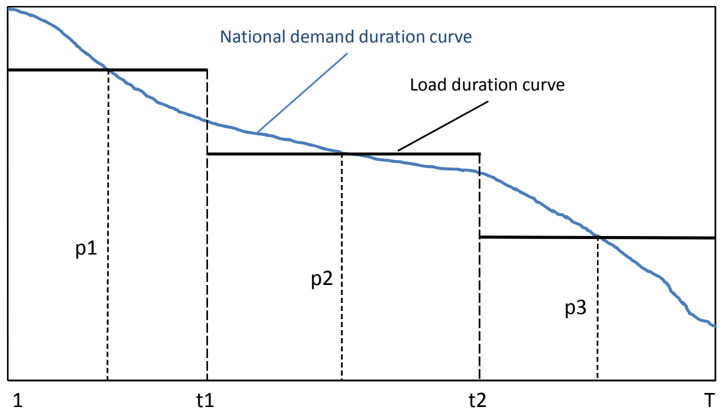

Figure 4: National demand duration curve and load duration curve.

Let T be the maximum TP. Let t be a TP between 1 and T, $f(t)$ be the empirical distribution curve, and $p(t)$ be the approximate curve. Let t_1 and t_2 be the bounds of the second piece, and p_1, p_2 and p_3 be the heights. Then let

$$\begin{aligned} z &= \min \int_0^T |f(t) - p(t)| \, dt \\ &= \min \int_0^{t_1} |f(t) - p_1| \, dt + \int_{t_1}^{t_2} |f(t) - p_2| \, dt + \int_{t_2}^{T} |f(t) - p_3| \, dt. \end{aligned}$$

Setting $\frac{\partial z}{\partial t_1}$ and $\frac{\partial z}{\partial t_2}$ to zero gives

$$\begin{aligned} |f(t_1) - p_1| &= |f(t_1) - p_2| \\ |f(t_2) - p_2| &= |f(t_2) - p_3| \,. \end{aligned} \tag{1}$$

Let t_{01}, t_{02} and t_{03} be such that

$$f(t_{01}) = p_1, \quad f(t_{02}) = p_2, \quad f(t_{03}) = p_3, \tag{2}$$

and let

$$
\begin{aligned}
z_1 &= \min \int_0^{t_{01}} (f(t) - p_1)\, dt + \int_{t_{01}}^{t_1} (p_1 - f(t))\, dt, \\
z_2 &= \min \int_{t_1}^{t_{02}} (f(t) - p_2)\, dt + \int_{t_{02}}^{t_2} (p_2 - f(t))\, dt, \\
z_3 &= \min \int_{t_2}^{t_{03}} (f(t) - p_T)\, dt + \int_{t_{03}}^{T} (p_3 - f(t))\, dt.
\end{aligned}
$$

Setting $\frac{\partial z_1}{\partial p_1}$, $\frac{\partial z_2}{\partial p_2}$ and $\frac{\partial z_3}{\partial p_3}$ to zero gives

$$
t_{01} = \frac{t_1}{2}, \quad t_{02} = \frac{t_1 + t_2}{2}, \quad t_{03} = \frac{t_2 + T}{2}. \tag{3}
$$

Substituting (3) into (2) and then (2) into (1) gives

$$
\begin{aligned}
2f(t_1) &= f(\frac{t_1}{2}) + f(\frac{t_1 + t_2}{2}), \\
2f(t_2) &= f(\frac{t_1 + t_2}{2}) + f(\frac{t_2 + T}{2}),
\end{aligned} \tag{4}
$$

which implies that t_{01}, t_{02} and t_{03} are the midpoints in each block.

Since it is unlikely that the optimal t_1 and t_2 are even numbers so that the optimal t_{01}, t_{02} and t_{03} are integers, the conditions in (4) are unlikely to be satisfied. However, we use a different approach to obtain the block hours.

One approach is to use demand brackets around the demands at midpoint for the conditions in (4). The lower bounds are the demands at the TPs just before the midpoints, denoted by $f(t^-)$, and the upper bounds are those just after the midpoints, denoted by $f(t^+)$. Then the following conditions are used,

$$
\begin{aligned}
f(\frac{t_1}{2}^-) + f(\frac{t_1 + t_2}{2}^-) &\leq 2f(t_1) \leq f(\frac{t_1}{2}^+) + f(\frac{t_1 + t_2}{2}^+), \\
f(\frac{t_1 + t_2}{2}^-) + f(\frac{t_2 + T}{2}^-) &\leq 2f(t_2) \leq f(\frac{t_1 + t_2}{2}^+) + f(\frac{t_2 + T}{2}^+).
\end{aligned} \tag{5}
$$

The conditions are tested for each possible set of t_1 and t_2. However, this approach cannot find a solution for some weeks.

The approach we have used is to use the conditions in (2) and (3) and search for an optimal solution among all possible sets of t_1 and t_2 which gives the lowest absolute deviation of the approximate curve from the empirical distribution curve.

The solutions for all weeks are obtained, which give us the block hours, and then the block demands are computed.

Load shedding is allowed and the setting is the same as that in Central. The cost along with the sectors and segments of demand for load shedding is copied over and displayed in Table 9.

	Up to 5%	Up to 10%	VOLL	North Is	South Is
Industrial	$1,000	$2,000	$10,000	0.34	0.58
Commercial	$2,000	$4,000	$10,000	0.27	0.15
Residential	$2,000	$4,000	$10,000	0.39	0.27

Table 9: Load reduction costs ($/MWh) and proportions of each load that is industrial, commercial, and residential load.

Generation

There are 33 hydro generators feeding energy into NI and SI in the model. These include the 21 offering hydro generators and 12 large fixed hydro generators in Central. The 12 fixed generators have limited intra-week flexibility in storage, which gives flexibility in their generation. The 21 offering generators and 7 of the fixed generators have inflow data available, and thus their dispatches are computed from flows through the generators with their nominal conversion factors. The remaining 5 fixed generators do not have inflow data available and we restrict their average dispatch in each week to be at most the historical average. Each generator has a nominal capacity. The offering generators and three fixed generators (Tokaanu, Rangipo and Waikaremoana as mentioned in Central) have de-rating due to maintenance outage, which are the weekly averages of de-rating in Central. Hydro generators are assumed to be dispatched throughout a demand block at a particular rate. The hydro generators are presented in Table 10.

Generator	Capacity	Average	Conversion factor	Node	Inflow	De-rating
Arapuni	196.7		0.4348	NI	Yes	Yes
Aratiatia	78		0.2703	NI	Yes	Yes
Atiamuri	84		0.2041	NI	Yes	Yes
Aviemore	230.2		0.3175	SI	Yes	Yes
Benmore_162	270.2		0.8214	SI	Yes	Yes
Benmore_163	270.2		0.8214	SI	Yes	Yes
Clyde_220KV	464		0.5128	SI	Yes	Yes
Karapiro	100		0.2632	NI	Yes	Yes
Manapouri	885		1.5180	SI	Yes	Yes
Maraetai	360		0.5000	NI	Yes	Yes
Ohakuri	112		0.2778	NI	Yes	Yes
Ohau_A	264.2		0.4790	SI	Yes	Yes
Ohau_B	212.2		0.4251	SI	Yes	Yes
Ohau_C	212.2		0.4251	SI	Yes	Yes
Pukaki	160.1		1.2970	SI	Yes	Yes
Roxburgh_110KV	124		0.3876	SI	Yes	Yes
Roxburgh_220KV	210		0.3876	SI	Yes	Yes
Tekapo	25.1		0.2436	SI	Yes	Yes
Waipapa	55		0.1429	NI	Yes	Yes
Waitaki	105		0.1751	SI	Yes	Yes
Whakamaru	100		0.3125	NI	Yes	Yes
Cobb	32		4.4050	SI	Yes	
Coleridge	45		1.0090	SI	Yes	
Mangahao	42		2.5300	NI	Yes	
Matahina	72		0.5950	NI	Yes	
Rangipo	120		1.9600	NI	Yes	Yes
Tokaanu	240		1.7500	NI	Yes	Yes
Waikaremoana	140		3.5400	NI	Yes	Yes
Aniwhenua	25	15		NI		
Patea	30.7	14		NI		
Wheao	24	13		NI		
Highbank	25.2	11		SI		
Waipori	84	22		SI		

Table 10: Hydro generators and their capacities, historical averages, conversion factors, supplying demand nodes and whether have inflows data or de-rating.

There are 12 thermal generators feeding energy into NI, which are the offering thermal generators in Central. Each generator has a nominal capacity, and has de-rating due to maintenance outage, which is the weekly average of de-rating in Central. They run on different fuels and heat rates, and the wholesale costs for fuels vary over time. Thermal generators are assumed to be dispatched at capacities for a particular number of hours in each demand block. The thermal generators are presented in Table 11.

Generator	Capacity	Fuel	Heat rate	Node	Start year	Start week	Endyear	End week
Stratford_220KV	387	gas	7.3	NI				
Huntly_gas	430	gas	6.8	NI	2007	23		
Huntly_main_g1	260	coal	10.5	NI				
Huntly_main_g2	260	coal	10.5	NI				
Huntly_main_g3	260	coal	10.5	NI				
Huntly_main_g4	265	coal	10.5	NI				
Huntly_Peak	50	gas	9.5	NI	2004	23		
NewPlymouth_110KV_g1	120	gas	11	NI			2007	38
NewPlymouth_110KV_g2	120	gas	11	NI			2007	38
NewPlymouth_220KV_g3	120	gas	11	NI			2007	38
Otahuhu_B	396	gas	7.05	NI				
NI_Whirinaki_220KV	159	diesel	11	NI	2004	22		

Table 11: Thermal generators and their capacities, fuel types, heat rates, supply demand nodes and start and end of available time periods.

As we mentioned in Central, the Huntly unit 5 was dispatched at zero price before its official commission date which is day 6 in week 22 in 2007. Since the official commissioning could be anticipated, the generator is set to be available from week 23 in this model. On the other hand, the New Plymouth station was decommissioned on day 2 in week 39 in 2007 due to the sudden discovery of harmful chemicals. Since this was not anticipated, we should have set the generator to be unavailable from week 39 only when making policies after this had become realized, which would be policy generation from quarter 4 of year 2007. However, we have inadvertently assumed this decommissioning for policy generation for all three years, and thus the policies have reserved more water throughout 2007 than one would expect them to without prior knowledge of New Plymouth decommissioning.

By our observations, the maintenance schedules are available at [12] before the start of a year, and we assume that the schedule is known one year ahead at any time of a year. Incorporating de-rating is important for DOASA in generating a policy to control water storage properly for Central. For example, Huntly 3 and 4 are out for maintenance in week 13 in 2006. An experiment shows that ignoring de-rating in DOASA results in low water storage at the beginning of this week in Central, and thus load shedding due to lack of water for hydro generation to offset the outage of Huntly 3 and 4, which does not happen in our result with de-rating being accounted for.

Transmission

For the three demand nodes, there is a pair of lines connecting NI and HAY and an other pair connecting HAY and SI, with power flow in opposite directions. Each line has a nominal capacity subject to de-rating due to outage, which is the weekly average of de-rating in Central. There is no loss in transmission. Power is assumed to be transmitted at a particular rate in each demand block. Note that the assumption of no transmission loss implies that the position of generators is not important if the lines are not constrained, which is not the case in Central. The transmission lines are presented in Table 12.

Start	End	Capacity
NI	HAY	1000
HAY	NI	1000
HAY	SI	1040
SI	HAY	1040

Table 12: Transmission lines and their capacities.

Reservoir and junction

The nine lakes in Central are set to be the only reservoirs in DOASA, and the head ponds, which have small capacities and limited flexibility in storage, are set to be junctions along with the junctions in Central[1]. For simplicity, we do not take into account delays in flows. A policy in this setting may result in extreme low states in Benmore and Aviemore at the end of a week in Central, owing to delays for upstream flows to arrive, and thus possible water shortage for generation at the beginning of the next week in Central. To overcome this, we set lower bounds for the states of these two reservoirs at each week in DOASA. These bounds are the amount of water used for the generation of Benmore and Aviemore stations at

[1]In the paper we did not mention that the three small lakes in the lower Waitaki system are treated as reservoirs, as they are far less important than the six large lakes due to their storage capacities and generation capacities of associated hydro stations. This, however, does prompt us to ignore them in computing future cost in DOASA.

capacity in the delayed TPs. Violation of these bounds is penalized at \$50/MWh, which is an estimate of average water value in New Zealand. The reservoirs are presented in Table 13.

Reservoir	Type	System	Capacity	Lower bound	Specific energy
AVI	Small lake	Waitaki	89,194,289	1,279,897	0.4556
BEN	Small lake	Waitaki	423,451,076	4,819,966	1.2522
HAW	Large lake	Clutha	1,378,764,328	0	0.9004
MAN	Large lake	Manapouri	1,501,878,016	0	1.5180
OHU	Large lake	Waitaki	57,245,219	0	2.5203
PKI	Large lake	Waitaki	2,425,440,000	0	2.5203
TAUPO	Large lake	Waikato	848,624,230	0	2.4056
TEK	Large lake	Waitaki	823,190,000	0	3.9927
WTK	Small lake	Waitaki	19,466,601	0	0.1626

Table 13: Reservoirs and their types, locations, capacities, lower bounds and specific energy.

Inflow

The inflows into reservoirs and junctions are weekly averages of daily inflows used in Central. Data for historical inflows from 1931 to 2008 are available, but only those from 1970 are used. This reduces computational time in policy generation, and still enables the impact of different outcomes of inflows to be assessed, particularly dry years which occurred several times during the 1970s. The set of historical inflows for sampling are those from the starting week in a particular year to the week before the plan year starts, and the years for the set of historical inflows are *sample years*. This gives 35, 36 and 37 historical inflows for sampling for a plan year starting in 2005, 2006 and 2007 respectively. In policy generation and simulation, the inflows in the first stage are fixed, and in each of the other stages, a year is sampled from the sample years and the historical inflows in the corresponding week are used.

Note that we have assumed that inflows are stagewise independent, although we have observed persistence in inflows over weeks, such as dry weeks over a long period. A scenario tree constructed using independent sampling would not capture such persistence, and thus, say, a dry week is followed equally likely by a dry or wet week. To bring the scenario tree closer to the one accounting for persistence, particularly in the early stages of the tree, we use an autoregressive model for the logs of inflows to adjust historical inflows, so that the sampled inflows in early stages of the tree are closer to those that would be given the inflows in the first stage. The model is as follows.

Given the observed inflows in the first week of the plan year, inflows in each week in a particular year are adjusted by

$$I_t = \left(\frac{I_0}{h_0}\right)^{\alpha^t} h_t,$$

where I_t and h_t are the adjusted and historical inflows in week t, I_0 and h_0 are the totals of inflows in a particular set of seven lakes (the six large lakes and Lake Waikaremoana which is treated as a junction in our model) in the first week in the plan year and the corresponding week in the particular year, and α is the power to raise which is estimated as 0.44 in our model. This means that every sequence of inflows I_t starts with I_0, and eventually becomes h_t as the term $(\frac{I_0}{h_0})^{\alpha^t} \to 1$. For example, $\frac{I_0}{h_0}$ for the year 2005 over 1973 is 1.56, and Figure 5 shows that the adjusted inflows for Taupo in the early stages in 1973 have moved closer to the ones in 2005.

Note that to capture persistence of inflows over weeks, a common practice is to use Markov chain in sampling inflows, which samples inflows dependent on the outcome in the previous stage, say low inflows being more likely sampled than high inflows given low inflows in the previous stage. However, we found that in experiments, using a Markov chain of even only 4 states for inflows makes DOASA slow to converge, and it does not give a much better result in using the generated policy in Central. Thus we choose to use independent sampling with the autoregressive model.

Flow

The water network is the same as Central. The bounds for arcs are the same as in Central, except for the lower bound for flows leaving Lake Dunstan which now

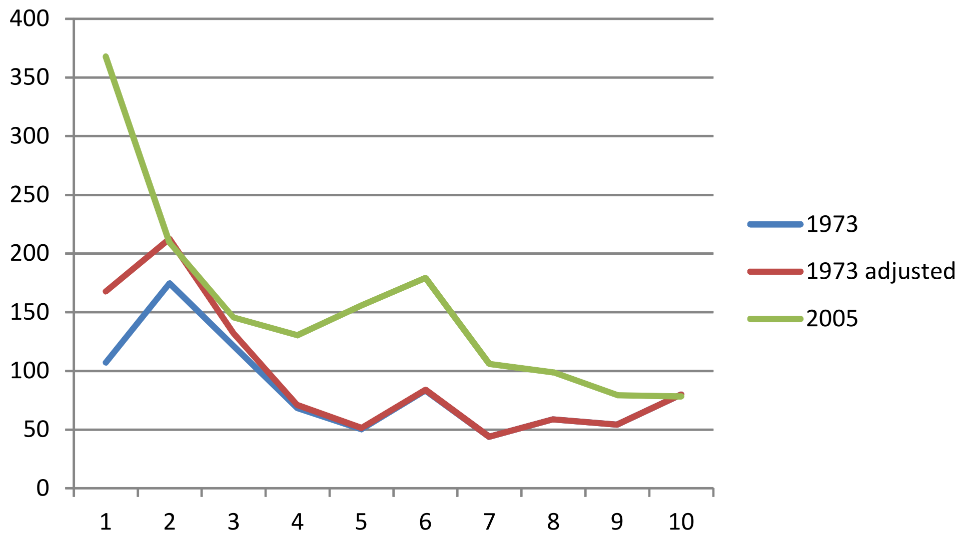

Figure 5: Taupo inflows and adjusted inflows in 1973 and inflows in 2005.

is 50 cumecs that is the weekly average of 120 cumecs at night time in Central. Penalty cost for violation of lower bounds is \$500/MWh. In week 50 in 1995 and week 46 and 47 in 1999, the inflows downstream of the only reservoir Hawea in the Clutha system exceed the upper bound leaving Roxburgh. These inflows, when sampled, incur a penalty cost, and thus lower the value of full storage of Hawea. To reduce this effect, we set the penalty cost for violation of upper bounds to \$50/MWh. For simplicity, there is no delay in the model. For spill, the penalty cost is \$50/MWh.

Future cost (water value) and cuts

The future cost in each stage is computed using cuts, except in the last stage. For all but the last stage, a set of cuts is generated by solving the next stage problems over all possible inflows outcomes. Each cut is a linear function of states of the six large lakes, and the set of cuts give a lower bound to the future cost, as shown in Figure 6. A policy is a collection of these cuts over 52 stages. Note that we use the six large lakes in computing the future cost, as they are important in the hydro systems, and this also reduces the dimension of state variables in the cuts and thus speeds up convergence in generating a policy.

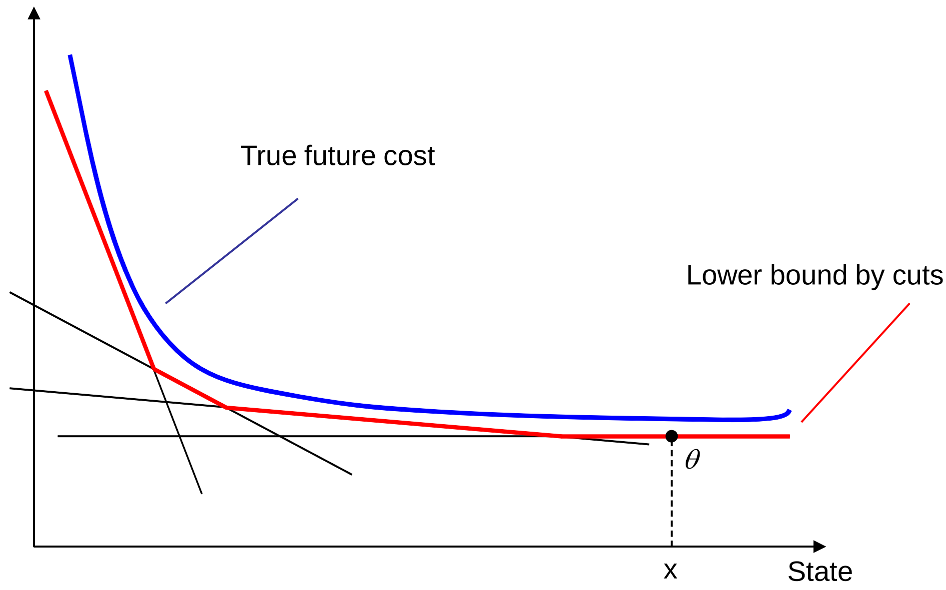

Figure 6: The cuts give the future cost θ at a particular state x.

For the last stage, the difference of the state and the initial state of the plan year for each reservoir is converted into energy, and then the future cost is computed as the total of the energy at \$50/MWh plus a base value. The base value is the total energy of the reservoirs at full capacity, which gives a lower bound of zero for the future cost in this stage, and thus those in all previous stages. The future cost is lower if the states are higher and thus this setting encourages reserving water.

Note that we may use cuts to compute the future cost for the last stage stage. For example, we may use DOASA to generate a policy using the current approach, and use the cuts in the first stage as the cuts for the last stage, and then re-solve DOASA to generate a policy. We may also solve DOASA to generate cuts for the next plan year, and then use the cuts for the first stage of the next plan year as the ones for the last stage of the current plan year. However, both approaches are time-consuming, and they may not give a better policy as they use the current approach to compute cuts for the last stage.

Minzone

The DOASA model minimizes expected fuel cost and so it is risk neutral. This means that shortages might be more frequent than is considered desirable. In practice the Electricity Commission in New Zealand has a mandate to intervene in the electricity market when reservoir levels become very low. This intervention usually involves a public electricity savings campaign. It can be triggered when total reservoir levels fall below what is called the national *minzone* (in MWh), which we obtain from [6]. We have implemented this in DOASA and any violation of the minzone incurs a penalty of $9000/MWh. This means that up to 10% of load reduction (that has penalty costs of at most $4000/MWh) will occur in preference to minzone violation[2]. We have also added two minzone constraints that reflect reservoir levels in each island. The South Island minzone is 250,000 MWh less than the national minzone as we observe in [7] and has the same violation penalty. The North Island minzone is computed to enable environmental minimum flow constraints at Lake Karapiro to be met with high probability. Violations of this minzone are penalized at a lower value of $500/MWh, as breaches in these flow constraints would typically be allowed in preference to load shedding. The minzones are presented in Figure 7.

For the computation of the North Island minzone, since Taupo is the only reservoir in the North Island, we are actually computing a minzone for Taupo. We first search for the year between 1970 and 2004 with the lowest total inflow in the Taupo system for the first half year. This gives the year 1978. Then we use the inflows in 1978 to obtain the minimum state of Taupo in each week that satisfies the Karapiro lower bound along with the inflows in the next week by solving the

[2]In solving the model, we accidentally set the penalty cost for minzone violation to $4000/MWh. Since violation of minzone means lower states which will incur higher future cost, the total cost for violation of minzone is actually always higher than 4000$/MWh. Thus the preference in load shedding and minzone violation is unchanged.

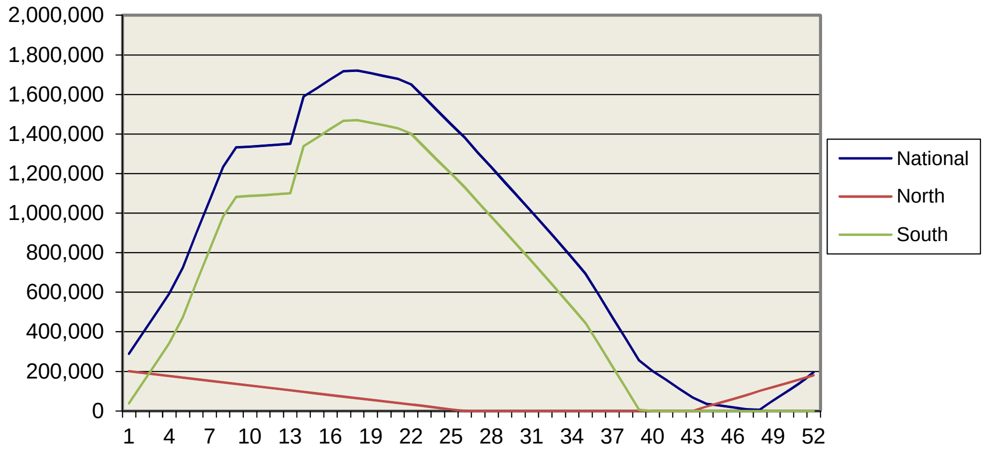

Figure 7: National, North Island and South Island minzones.

following problem,

$$\begin{aligned} \min \quad & \textstyle\sum_{t\in\mathcal{T}} v_t, \\ s.t. \quad & v_{t-1} = \max(T_t(b - \textstyle\sum_{i\in\mathcal{I}} \omega_{it}) + v_t, 0), \quad \forall t \in \mathcal{T}, \\ & v_0 = v_{\bar{t}}. \end{aligned}$$

Here v_t is the minimum state of Taupo in week t, T_t is the number of seconds in week t, b is the Karapiro lower bound, ω_{it} is the inflow in a reservoir or tributary junction $i \in \mathcal{I}$, where $\mathcal{I}$ is a set of reservoir and tributary junctions above Karapiro, and $\bar{t}$ is the last week. Finally, we convert these states into energy in MWh, plot them against the weeks and fit straight lines that are just above them to obtain the minzone. Note that since inflows in each week are independently sampled, lower inflows than 1978 may be sampled and thus minzone satisfaction might still give violation of the Karapiro lower bound, and minzone violation does not necessarily mean the Karapiro lower bound will be violated in simulation.

DOASA generation and simulation

The policy is generated by the DOASA algorithm (see [10]). The algorithm has a forward pass and a backward pass in each iteration. In the forward pass, the inflows in the first stage are fixed, and the inflows from the second stage to the second to last stage are sampled, and the states solved in each stage are the initial states for the next stage. The stage cost at the first stage gives a lower bound to the cost in the plan year. In the backward pass, from the last stage to the second stage, stage problems for inflows in all sample years are solved and a cut is computed for the previous stage. We set the algorithm to terminate after 150 iterations. This process takes approximately 2 hours on an operating system of Microsoft Windows Server 2003 R2, Standard Edition, Service Pack 2 on Intel Core 2 Quad CPU Q9550 @ 2.83GHz, 2.93 GB of RAM.

In sampling inflows in the forward pass, we used expected inflows for the first five iterations to generate expectation cuts. These cuts give a quick lower bound to the future cost, which is valid by Jensen's inequality. We also use importance sampling so that low inflows, which have large adverse impact, are sampled more frequently in each stage. From iteration forty-one, in each of the first two iterations of every ten iterations, a year is sampled from the three years with the lowest national inflows in each stage, and the inflows in the sampled year are used.

For the convergence of the algorithm, the lower bound is tested against an estimated upper bound of the cost in the plan year in simulation. In simulation, the inflows in the first stage are fixed, and the inflows in each of the remaining stages are sampled. For each sampled inflows sequence throughout the year, after solving each stage problem, the running cost in each stage is computed by subtracting out the future cost from the stage cost. The total of the running costs in the plan year and the future cost at the last stage gives the cost in the plan year for this inflows sequence. Then the average of the total costs over all sampled inflows sequences gives the estimated upper bound, and the standard error and a 95% confidence interval are calculated. When the 95% confidence interval contains the lower bound that is generated in 150 iterations, we deem that the algorithm has converged. Note that we have used a sample of 100 inflows sequence in simulation. A larger sample may result in a narrower confidence interval and thus more iterations for the algorithm to converge. This may result in a better policy but also require more computational time.

In sampling inflows in simulation, we have used antithetic sampling to reduce the variance of outcomes. In each stage, we arrange the sample years with respect to their national inflows, and for every two inflow sequences, if the year with the ith lowest inflow is sampled in the first sequence, then the year with the ith highest inflow will be used in the second sequence.

4.2 Formulation

The formulation of the DOASA model is presented as follows.

Time and location

Sets

$\mathcal{T}$	stages in the plan year.
$\mathcal{O}$	islands.

Parameters

$\check{t}$	the last week in $\mathcal{T}$.
$\tilde{T}$	the number of seconds in an hour.

Demand

Sets

$\mathcal{N}$	demand nodes.
$(\mathcal{N}, \mathcal{O})$	nodes in islands.
$\mathcal{P}$	blocks of demand.
$\mathcal{U}$	sectors of demand.
$\mathcal{V}$	segments of demand.

Parameters

D_{npt}	block demand at node n in block p at stage t.
T_{pt}	block hour in block p at stage t.

ψ_{uv} cost for load shedding in segment v in sector u.

π_{uo} proportion of sector u in island o.

$\bar{\pi}_v$ proportion of segment v.

Variables

d_{npuv} load shedding in segment v, sector u, block p and node n.

Generation

Sets

$\mathcal{H}$ hydro generators.

$\bar{\mathcal{H}} \subset \mathcal{H}$ hydro generators with no inflow data.

$\tilde{\mathcal{H}} \subset \mathcal{H}$ hydro generators with de-rating.

$(\mathcal{H}, \mathcal{N})$ hydro generators supplying power to nodes.

$\mathcal{S}$ hydro stations.

$(\mathcal{H}, \mathcal{S})$ hydro generators in stations.

$\mathcal{M}$ thermal generators.

$\tilde{\mathcal{M}} \subset \mathcal{M}$ unavailable thermal generators.

$(\mathcal{M}, \mathcal{N})$ thermal generators supplying power to nodes.

$\mathcal{F}$ fuels.

$(\mathcal{M}, \mathcal{F})$ thermal generators using fuels.

Parameters

Q_g capacity of hydro or thermal generator g.

$\tilde{Q}_{gt}$ de-rating of hydro or thermal generator g at stage t.

$\bar{q}_h$ historical average dispatch of hydro generator $h \in \bar{\mathcal{H}}$.

γ_h nominal conversion factor for hydro generator h.

δ_{ht} scaling factor for hydro generator h at stage t.

κ_m heat rate of thermal generator m.

ϕ_{ft} wholesale cost of fuel f at stage t.

Variables

q_{hp} dispatch of hydro generator h in block p.

$\hat{q}_{mp}$ dispatching hours of thermal generator m in block p.

Transmission

Sets

$\mathcal{L} = (\mathcal{N}, \mathcal{N})$	transmission lines, indexed by l or (n, n').

Parameters

Y_l	nominal capacity of line l.
$\tilde{Y}_{lt}$	de-rating of line l at stage t.

Variables

y_{lp}	power flow in line l in block p.

Reservoir and junction

Sets

$\mathcal{R}$	reservoirs.
$\hat{\mathcal{R}}$	reservoirs that are large lakes.
$\bar{\mathcal{R}}$	reservoirs that have lower bounds for states.
$(\mathcal{R}, \mathcal{O})$	reservoirs in islands.
$\mathcal{J}$	junctions.
$\hat{\mathcal{J}} \subseteq \mathcal{J}$	junctions with tributary inflows.

Parameters

X_r	capacity of reservoir r.
$\bar{\gamma}_r$	specific energy for reservoir r.
$\hat{\gamma}$	the maximum specific energy of reservoirs.
$x_{r,0}$	initial state of reservoir r of the plan year.
σ	end value for states at the last stage.
$\check{X}_r$	lower bound for state of reservoir r.
ς	penalty cost for state lower bound violation.
Z_t	national minzone at stage t.
$\bar{Z}_{ot}$	minzone for island o at stage t.
ι	penalty cost for violation of national minzone.
$\bar{\iota}_o$	penalty cost for violation of minzone for island o.

Variables

x_{rt}	state of reservoir r at stage t.
$\check{x}_r$	penalty variable for state lower bound violation for reservoir r.
z	penalty variable for violation of national minzone.
$\bar{z}_o$	penalty variable for violation of minzone for island o.

Inflow

Sets

$\bar{\mathcal{I}} \subseteq \{\mathcal{R}, \hat{\mathcal{J}}\}$	reservoirs and tributary junctions for scaling inflows.

Parameters

ω_{it}	inflow of reservoir or tributary junction i at stage t.
λ_{it}	scaling factor for inflow of reservoir or tributary junction i at stage t.

Flow

Sets

$\mathcal{I} = \{\mathcal{R}, \mathcal{J}, \mathcal{H}, Sea\}$	nodes in the water network.
$\mathcal{A} = (\mathcal{I}, \mathcal{I})$	arcs in the water network.
$\check{\mathcal{A}}$	arcs that have lower bounds.
$\hat{\mathcal{A}}$	arcs that have upper bounds.
$\bar{\mathcal{A}}$	arcs through which water spill around stations.

Parameters

$\check{b}_a$	lower bound of arc a.
$\hat{b}_a$	upper bound of arc a.
$\check{\rho}$	penalty cost on violation of flow lower bound.
$\hat{\rho}$	penalty cost on violation of flow upper bound.
$\tilde{\gamma}_a$	conversion factor for water spilled through arc a.
$\tilde{\rho}$	penalty cost on spill of water.

Variables

w_{ap}	flow in arc a in block p.
$\check{w}_{ap}$	penalty variable on violation of flow lower bound in arc a in block p.
$\hat{w}_{ap}$	penalty variable on violation of flow upper bound in arc a in block p.

Cut

Parameters

$\mathcal{K}$	cuts to compute future cost.
α_k	intercept in cut k.
β_{rk}	slope for large lake reservoir r in cut k.
$\bar{\theta}$	base value for future cost at the last stage.

Variables

θ_t	future cost at stage t.

Objective:

The objective is to minimize fuel cost, load shedding cost, penalty cost for violation of minzones, state lower bound and flow bounds, penalty cost for spill, and future cost. Note that the penalty cost for violation of state lower bound and flow bounds are higher than spill.

$$\begin{aligned}\min \quad & \textstyle\sum_{(m,f)\in(\mathcal{M},\mathcal{F}),p\in\mathcal{P}} \phi_{ft}\kappa_m \hat{q}_{mp}(Q_m - \tilde{Q}_{mt}) + \sum_{n\in\mathcal{N},p\in\mathcal{P},u\in\mathcal{U},v\in\mathcal{V}} \psi_{uv} d_{npuv} + \\ & \iota z + \textstyle\sum_{o\in\mathcal{O}} \bar{\iota}_o \bar{z}_o + \frac{1}{\tilde{T}} \varsigma \hat{\gamma} \check{x}_r + \check{\rho}\hat{\gamma}\tilde{T} \sum_{a\in\check{\mathcal{A}},p\in\mathcal{P}} T_{pt} \check{w}_{ap} + \\ & \hat{\rho}\hat{\gamma}\tilde{T} \textstyle\sum_{a\in\hat{\mathcal{A}},p\in\mathcal{P}} T_{pt} \hat{w}_{ap} + \tilde{\rho} \sum_{a\in\bar{\mathcal{A}},p\in\mathcal{P}} \tilde{\gamma}_a T_{pt} w_{at} + \theta_t.\end{aligned}$$

Future Cost:

From each stage before the last stage, the future cost is computed as the point-wise maximum of cuts of a particular set of states for large lake reservoirs. For the last stage, it is computed as the sum of the value for the difference of the states and initial states of the plan year and the base value.

$$\begin{aligned}\theta_t &\geq \alpha_k + \textstyle\sum_{r\in\hat{\mathcal{R}}} \beta_{rk} x_{rt}, \quad \forall k \in \mathcal{K}, t \in \mathcal{T}, t < \check{t}, \\ \theta_t &= \bar{\theta} + \frac{1}{\tilde{T}} \sigma \textstyle\sum_{r\in\mathcal{R}} \bar{\gamma}_r (x_{r,0} - x_{rt}), \quad \forall t = \check{t}.\end{aligned}$$

Dispatch

For a hydro generator that has inflow data, its dispatch in a block is computed from a conversion factor and flows through the generator in this block, while for the one that doesn't, its average dispatch is constrained by the historical average generation. For a hydro generator with de-rating, its dispatch is below capacity minus de-rating, while for the one without de-rating, its dispatch is below capacity. For unavailable thermal generators, the dispatching hours are set to zero.

$$q_{hp} = \delta_{ht}\gamma_h \sum_{(h,i)\in\mathcal{A}} w_{hip}, \quad \forall h \in \mathcal{H}, h \notin \bar{\mathcal{H}}, p \in \mathcal{P},$$

$$\sum_{p\in\mathcal{P}} T_{pt} q_{hp} \leq \bar{q}_h \sum_{p\in\mathcal{P}} T_{pt}, \quad \forall h \in \bar{\mathcal{H}}, p \in \mathcal{P},$$

$$q_{hp} \leq Q_h - \tilde{Q}_{ht}, \quad \forall h \in \tilde{\mathcal{H}}, p \in \mathcal{P},$$

$$q_{hp} \leq Q_h, \quad \forall h \in \mathcal{H}, h \notin \tilde{\mathcal{H}}, p \in \mathcal{P},$$

$$\hat{q}_{mp} = 0, \quad \forall m \in \tilde{\mathcal{M}}, p \in \mathcal{P}.$$

Meet demand:

In each block at each node, generation from hydro and thermal generators and net power flow satisfy demand minus load shedding. Load reduction is bounded.

$$T_{pt} \sum_{(h,n)\in(\mathcal{H},\mathcal{N})} q_{hp} + \sum_{(m,n)\in(\mathcal{M},\mathcal{N})} (Q_m - \tilde{Q}_{mt}) h_{mp} +$$

$$T_{pt} \sum_{(n',n)\in\mathcal{L}} (y_{n'np} - y_{nn'p}) \geq D_{ntp} - \sum_{u\in\mathcal{U},v\in\mathcal{V}} d_{npuv}, \quad \forall n \in \mathcal{N}, p \in \mathcal{P},$$

$$d_{npuv} \leq \pi_{uo}\bar{\pi}_v D_{npt}, \quad \forall (n,o) \in (\mathcal{N},\mathcal{O}), u \in \mathcal{U}, v \in \mathcal{V}, p \in \mathcal{P}.$$

Transmission:

Power flow in a line is constrained by the nominal capacity minus de-rating.

$$\hat{y}_{lp} \leq Y_l - \tilde{Y}_{lt}, \quad \forall l \in \mathcal{L}, p \in \mathcal{P}.$$

Water balance at reservoirs and junctions:

For each reservoir, the state equals to the total of the state in the previous week, inflow and net flow. For a tributary junction, the incoming flows and inflow equal to the outgoing flows. For a non-tributary junction or hydro station, the incoming flows are equal to the outgoing flows. For a reservoir with a lower bound for state, its state needs to meet the lower bound, which has a penalty on violation.

$$x_{rt} = x_{r,t-1} + \tilde{T} \sum_{p \in \mathcal{P}} T_{pt}(\lambda_{rt}\omega_{rt} + \sum_{(i,r)\in\mathcal{A}} w_{irp} - \sum_{(r,i)\in\mathcal{A}} w_{rip}), \quad \forall r \in \mathcal{R},$$
$$\sum_{(i,j)\in\mathcal{A}} w_{ijp} + \lambda_{jt}\omega_{jt} = \sum_{(j,i)\in\mathcal{A}} w_{jip}, \quad \forall j \in \hat{\mathcal{J}}, p \in \mathcal{P},$$
$$\sum_{(i,j)\in\mathcal{A}} w_{ijp} = \sum_{(j,i)\in\mathcal{A}} w_{jip}, \quad \forall j \in \mathcal{H} \cup \mathcal{J}, j \notin \hat{\mathcal{J}}, p \in \mathcal{P},$$
$$x_r + \check{x}_r \geq \check{X}_r, \quad \forall r \in \bar{\mathcal{R}}.$$

Flows:

A flow through an arc satisfies the bounds, with a penalty on violation.

$$\begin{aligned} w_{ap} + \check{w}_{ap} &\geq \check{b}_a, \quad \forall a \in \check{\mathcal{A}}, p \in \mathcal{P}, \\ w_{ap} - \hat{w}_{ap} &\leq \hat{b}_a, \quad \forall a \in \hat{\mathcal{A}}, p \in \mathcal{P}. \end{aligned}$$

Minzone:

The national storage satisfies the national minzone, and the total storage in each island satisfies the minzone in that island, with a penalty on violation.

$$\begin{aligned} \tfrac{1}{\tilde{T}} \sum_{r\in\mathcal{R}} \bar{\gamma}_r x_r + z &\geq Z_t, \\ \tfrac{1}{\tilde{T}} \sum_{(r,o)\in(\mathcal{R},\mathcal{O})} \bar{\gamma}_r x_r + \bar{z}_o &\geq \bar{Z}_{ot}, \quad \forall o \in \mathcal{O}. \end{aligned}$$

Domain for variables:

All variables ≥ 0.

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