Randomness, Risk, and Electricity Prices

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Motivation: understanding electricity markets

EX-ANTE

1. Define a welfare-maximizing outcome for society (optimization);
2. Design a price mechanism that incentivizes this under assumptions about participant behaviour: perfect competition, complete markets, increasing marginal cost (complementarity);
3. Test and improve the mechanism ex-ante in small-scale models as assumptions are relaxed to reflect reality (game theory);

EX-POST

1. Implement the mechanism in practice and observe historical market outcomes (statistics);
2. Benchmark historical outcomes against a theoretical competitive counterfactual.
   - Need large-scale models to replicate real system constraints.
   - Better when equilibrium determined by optimization.
   - When is this appropriate in presence of uncertainty?

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Summary

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2. Design a pricing mechanism for two-stage stochastic model
   - deterministic example
   - two-stage stochastic example
   - generalization of example
3. Benchmark a multistage hydro system

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Example: battery, solar panels, and generator

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2. At stage 2:
   - *solar* supplies ξ at zero cost;
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- Maximize total system welfare

$$\text{P:} \quad \max_{x,y,z \geq 0} \quad 16z - z^2 - \frac{1}{2}x^2 - y^2$$

$$\text{s.t.} \quad x + y + \xi \geq z$$

- The solution (x, y, z) to P maximizes the Lagrangian

$$16z - z^2 - \frac{1}{2}x^2 - y^2 + \pi\,(x + y + \xi - z)$$

where x, y, z and π satisfy

$$0 \leq x + y + \xi - z \perp \pi \geq 0.$$

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Decomposition gives market equilbrium

- Each agent $a \in \{1, 2, 3, 4\}$ maximizes profit at prices π,

$$\begin{aligned}
\text{P}(1)&: \max_{x \geq 0} \quad \pi x - \tfrac{1}{2}x^2 \\
\text{P}(2)&: \max_{y \geq 0} \quad \pi y - y^2 \\
\text{P}(3)&: \max_{z \geq 0} \quad 16z - z^2 - \pi z \\
\text{P}(4)&: \quad \pi \xi
\end{aligned}$$

  where x, y, z and π satisfy equilibrium constraints

$$0 \leq x + y + \xi - z \perp \pi \geq 0.$$

- This is an example of a MOPEC (Multiple Optimization Problem with Equilibrium Constraints).
- Assume that agents treat π as a fixed parameter when they optimize, so this is not a model for imperfect competition.

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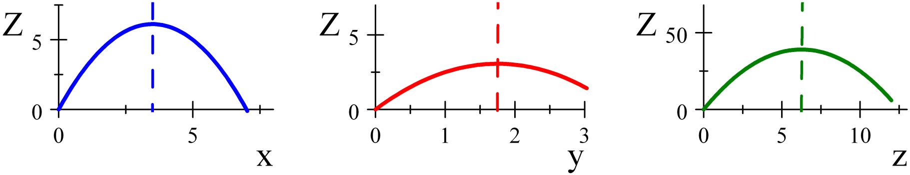
Deterministic equilibrium

Suppose $\xi = 1$. Then $\pi = \$3.5$. Agent welfare at this price is graphed.



- The battery collects $\pi = \$3.5$ per unit for 3.5 units.
- The generator collects $\pi = \$3.5$ per unit for 1.75 units.
- The solar generator collects $\pi = \$3.5$ per unit for 1 unit.
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- Total system welfare is 51.75.

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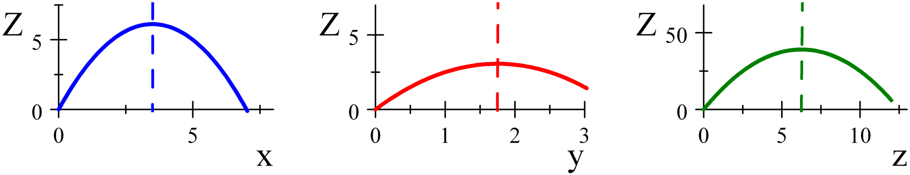

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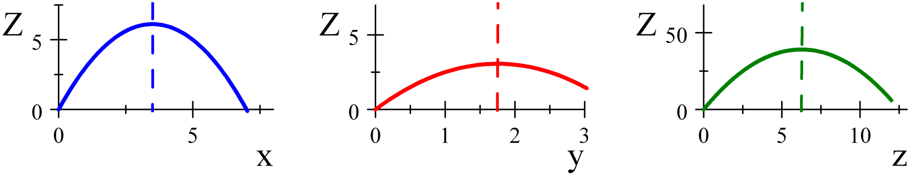

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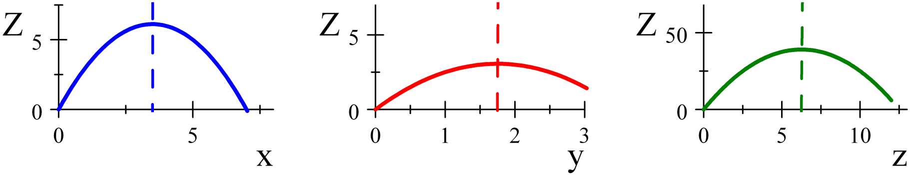

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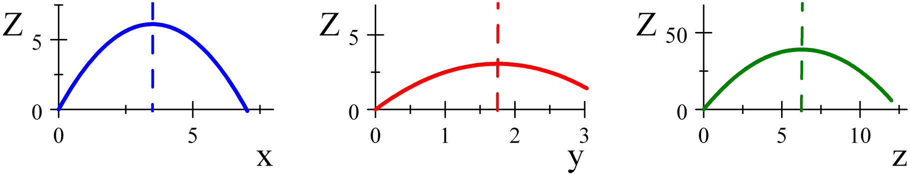

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Welfare theorems of partial equilibrium

Subject to conditions of convexity, completeness, and perfect competition we get:

- First welfare theorem: Suppose for some π, and each $a \in \mathcal{A}$, that x_a solves the agent problem P(a). If π and x satisfy the market clearing condition then x solves the system planning problem P.

- Second welfare theorem: If x solves the system planning problem P then there is some π so that each component x_a solves the agent problem P(a), and π and x satisfy the market clearing condition.

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Example: random sunshine

- At stage 1: battery stores energy x at cost $\frac{1}{2}x^2$.
- At stage 2:
  - solar supplies random $\xi(\omega)$ at zero cost;
  - *generator* supplies $y(\omega)$ at cost $y(\omega)^2$;
  - consumer consumes $z(\omega) = x + y(\omega) + \xi(\omega)$;
  - consumer utility for z is $16z - z^2$.
- Maximize expected welfare $\mathbb{E}[Z(x, \omega)]$ where

$$Z(x, \omega) = 16z(\omega) - z(\omega)^2 - y(\omega)^2 - \frac{1}{2}x^2$$

System optimization is a two-stage stochastic program

Suppose $\xi(\omega_1)= 1$ and $\xi(\omega_2)= 3$ with equal probability.

Optimal solution is

- $x=3$.
- $y(\omega_1)=2 \quad y(\omega_2)=1$;
- $z(\omega_1)=6 \quad z(\omega_2)=7$;
- $\pi(\omega_1)=4 \quad \pi(\omega_2)=2$;

Solution value $= \mathbb{E}[Z(x, \omega)] = 54.5$.

Risk-neutral market equilbrium

A set of prices $\pi(\omega)$ and x^*, $y^*(\omega)$, $z^*(\omega)$ satisfying:

$$x^* \in \arg\max_{x \geq 0} \quad \mathbb{E}[\pi(\omega)x - \tfrac{1}{2}x^2]$$

$$y^*(\omega) \in \arg\max_{y(\omega) \geq 0} \quad \mathbb{E}[\pi(\omega)y(\omega) - y(\omega)^2],$$

$$z^*(\omega) \in \arg\max_{z(\omega) \geq 0} \quad \mathbb{E}[16z(\omega) - z(\omega)^2 - \pi(\omega)z(\omega)],$$

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Payments and revenue adequacy
[Pritchard, Zakeri, P., 2010]

$$\pi(\omega_1) = \$4, \quad \pi(\omega_2) = \$2.$$

- Battery paid $\mathbb{E}[\pi] = \$3$ per unit for 3 units.
- Solar paid $\pi = \$4$ per unit for 1, $\pi = \$2$ per unit for 3.
- Generator paid $\pi = \$4$ per unit for 2, $\pi = \$2$ per unit for 1.
- Consumer pays $\pi = \$4$ per unit for 6, $\pi = \$2$ per unit for 7.
- Half the time there is a shortfall of \$3 and half the time a surplus of \$3. The market is not revenue adequate in every outcome. The market clearing agent bears the risk.

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Alternative payment scheme

[Zakeri et al, 2018], [Cory-Wright et al, 2018]

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- Half the time the battery is producing at a loss. Risk is transferred from the auctioneer to battery.
- Battery bears the cost of nonanticipativity [c.f. de Maere d'Aertrycke, Shapiro, Smeers, 2013].

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- Two-stage stochastic program gives a market design assuming risk neutrality for all players.
- Maximizing expected social welfare will give shortfalls when participants and market clearing agent are risk averse.
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Example: risk-averse agents

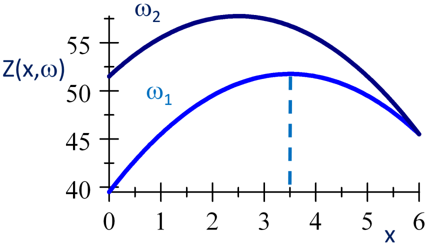
- At stage 1: battery stores energy x at cost $\frac{1}{2}x^2$.
- At stage 2:
  - *generator* supplies y at cost y^2;
  - solar supplies random $\xi(\omega)$ at zero cost;
  - consumer consumes $z(\omega) = x + y(\omega) + \xi(\omega)$;
  - consumer utility for z is $16z - z^2$.
- Maximize risk-adjusted welfare $\mathbb{F}[Z(x, \omega)]$ where $\mathbb{F}$ is defined by a risk measure.

Example: $\mathbb{F}$ is worst-case ($\mathbb{W}$)

maximize worst-case social welfare:

$$\max_{x \geq 0, y(\omega) \geq 0} \min\{Z(x, \omega_1), Z(x, \omega_2)\}$$

$$Z(x, \omega) = 16(x + y(\omega) + \xi(\omega)) - (x + y(\omega) + \xi(\omega))^2 - \frac{1}{2}x^2 - y(\omega)^2$$



Solution $x = 3.5$, worst-case is ω_1, risk adjusted value is 51.75.

Risk-averse equilibrium

A set of prices $\pi(\omega)$ and x^*, $y^*(\omega)$, $z^*(\omega)$ satisfying:

$$x^* \in \arg\max_{x \geq 0} \quad \mathbb{F}[\pi(\omega)x - \tfrac{1}{2}x^2]$$

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Risked equilibria: $\mathbb{F}$ is worst-case ($\mathbb{W}$)

A	ξ	π	x^*	y^*	z^*	Z	Z	Z	Z
ω_1	1	4.5	2.5	2.25	5.75	8.13	5.06	33.06	4.5
ω_2	3	2.5	2.5	1.25	6.75	3.13	1.56	45.56	7.5

Unique competitive equilibrium A if $y^*(\omega)$ and $z^*(\omega)$ are constrained to be optimal for each ω (risk-adjusted system welfare=50.75).

B	ξ	π	x^*	y^*	z^*	Z	Z	Z	Z
ω_1	1	3.31	3.09	2.25	6.34	5.46	2.39	40.24	3.31
ω_2	3	3.09	3.09	1.55	7.64	4.78	2.39	40.24	9.27

Best alternative equilibrium B: $y^*(\omega_1)$ is not optimal for ω_1. $z^*(\omega_2)$ is not optimal for ω_2 (risk-adjusted system welfare=51.41).

Monotonicity

[Shapiro, 2017]

- $\mathbb{F}$ is monotone if $Z_1 \leq Z_2$ implies $\mathbb{F}(Z_1) \leq \mathbb{F}(Z_2)$.
- $\mathbb{F}$ is strictly monotone if $\mathbb{F}$ is monotone, and $Z_1 \leq Z_2$ and $Z_1 \neq Z_2$ implies $\mathbb{F}(Z_1) < \mathbb{F}(Z_2)$.
- $\mathbb{F} = \mathbb{W}$ (worst case) is monotone but not strictly monotone.

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Good and bad news about this example

1. GOOD: Risked social plan gives unique x =3.5 for this example, and $y(\omega_1) = 1.75$.
   BAD: $y(\omega_2)$ is not uniquely determined.
2. BAD: Risk-averse competitive equilibrium is not unique.
3. GOOD: The social plan is unique if $\mathbb{F}$ is strictly monotone, for example $\mathbb{F} = \lambda\mathbb{E} + (1-\lambda)\mathbb{W}$ where $\lambda > 0$.
4. GOOD: The risk-averse competitive equilibrium is unique when $\mathbb{F}$ is strictly monotone.
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1. GOOD: Risked social plan gives unique x =3.5 for this example, and $y(\omega_1) = 1.75$.
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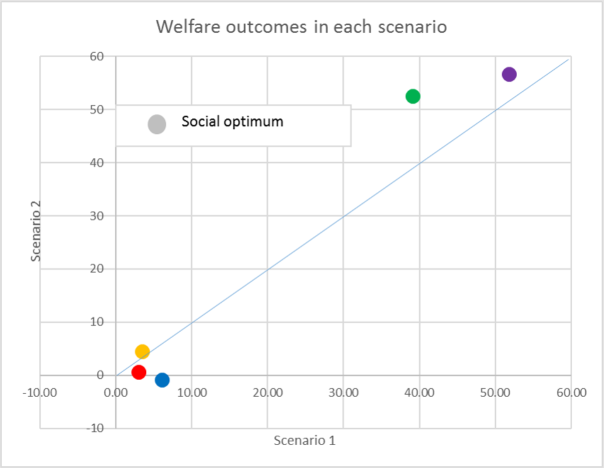
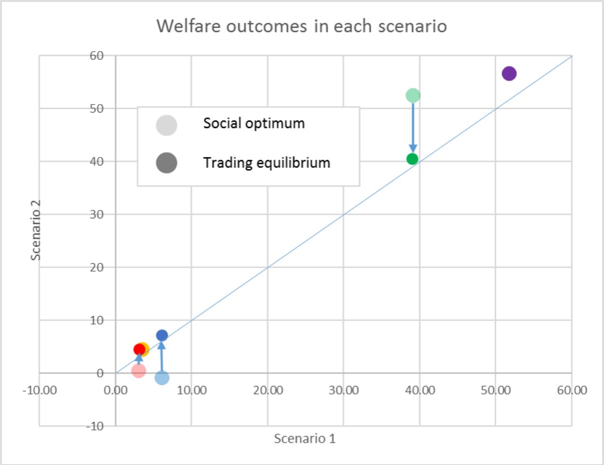
Social plan welfare outcomes: worst-case measure



Trading makes social plan a risk-averse equilibrium



Recap the battery example

- Risk neutral social optimum (54.5) cannot be achieved by prices that give revenue adequacy and cost recovery in every scenario.
- Risk-averse social optimal solution (51.75) has higher risk-adjusted system welfare than the best risk-averse competitive equilibrium (51.4), so social optimal solution is not an equilibrium.
- Risk-averse social optimal solution turns into an equilibrium with risk-adjusted social welfare 51.75 if agents can trade risk.
- Conversely, if agents can trade risk as well as energy then the resulting risk-averse equilibrium will maximize risk-adjusted social welfare.
- With trading, all agents ignore scenario 2. Prices from scenario 1 system optimal solution ($x = 3.5$) give risk-averse equilibrium. Revenue adequacy and cost recovery is achieved in risk-adjusted expectation.

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Generalization: coherent risk measures

(Artzner et al, 1999)

- Assume a finite sample space for simplicity.
- Worst-case in a minimization setting is an example of a coherent risk measure. Other examples are expectation and average value at risk (AVaR).
- Coherent risk measures satisfy well-known axioms, monotonicity, subadditivity, positive homogeneity, and translation equivariance.
- When minimizing Z, coherent risk measure $\rho(Z)$ has a dual representation

$$\rho(Z) = \sup_{\mathbb{P} \in \mathcal{D}} \mathbb{E}_{\mathbb{P}}[Z]$$

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Coherent risk measures and risk-adjusted expectation $\mathbb{F}$

- Recall we are maximizing risk-adjusted Z.
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Trading risk: Arrow-Debreu securities

- An Arrow-Debreu security for outcome $\omega \in \Omega$ in stage 2 is a contract that has a payout of 1 in outcome ω. We denote the price of such a contract in stage 1 by $\mu(\omega)$.
- Suppose that each agent buys $W_a(\omega)$ Arrow-Debreu securities at stage one, costing $\mu^\top W_a = \sum_{\omega\in\Omega} \mu(\omega) W_a(\omega)$, to receive return $W_a(\omega)$ in outcome ω in stage 2.
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Risk-averse equilibrium

A set of prices $\pi(\omega)$, and x^*, y^*, z^*, satisfying:

$$(\quad x^*) \in \arg\max \mathbb{F}[\pi(\omega)x - \tfrac{1}{2}x^2 \quad]$$

$$(\quad y^*(\omega)) \in \arg\max \mathbb{F}[\pi(\omega)y(\omega) - y(\omega)^2 \quad]$$

$$(\quad z^*(\omega)) \in \arg\max \mathbb{F}[16z(\omega) - z(\omega)^2 - \pi(\omega)z(\omega) \quad]$$

$$0 \leq x^* + y^*(\omega) + \xi(\omega) - z^*(\omega) \perp \pi(\omega) \geq 0, \omega \in \Omega.$$

Risk-averse equilibrium with Arrow-Debreu securities

A set of prices $\pi(\omega)$, $\mu(\omega)$ and x^*, y^*, z^*, W^* satisfying:

$$(W_1^*(\omega), x^*) \in \arg\max \mathbb{F}[\pi(\omega)x - \tfrac{1}{2}x^2 + W_1(\omega)] - \mu^\top W_1$$

$$(W_2^*(\omega), y^*(\omega)) \in \arg\max \mathbb{F}[\pi(\omega)y(\omega) - y(\omega)^2 + W_2(\omega)] - \mu^\top W_2,$$

$$(W_3^*(\omega), z^*(\omega)) \in \arg\max \mathbb{F}[16z(\omega) - z(\omega)^2 - \pi(\omega)z(\omega) + W_3(\omega)] - \mu^\top W_3,$$

$$(W_4^*(\omega)) \in \arg\max \mathbb{F}[\pi(\omega)\xi(\omega) + W_4(\omega)] - \mu^\top W_4,$$

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Welfare Theorems

[Ralph and Smeers, 2015], [Gerard et al, 2018]

Suppose agents have coherent risk measures with risk sets $\mathcal{D}_a$ with $\bigcap_{a\in\mathcal{A}} \mathcal{D}_a \neq \emptyset$, and there is a complete market for A-D securities.

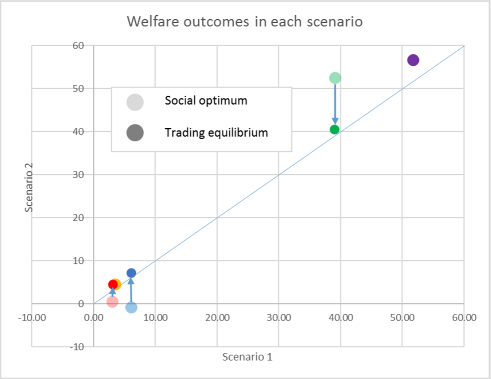
Theorem

If $\{\pi(\omega), \omega \in \Omega\}$, *and* $\{\mu(\omega), \omega \in \Omega\}$ *give a risk-averse equilibrium* $\{(W_1^*(\omega), x^*),(W_2^*(\omega), y^*(\omega)),(W_3^*(\omega), z^*(\omega)), W_4^*(\omega)\}$ *then* $\{x^*, y^*(\omega), z^*(\omega)\}$ *solves the risk-averse social planning problem with risk measure having risk set* $\mathcal{D}_s = \bigcap_{a\in\mathcal{A}} \mathcal{D}_a$.

Theorem

If $\{x^*, y^*(\omega), z^*(\omega)\}$ *solves the risk-averse social planning problem SP with risk set* $\mathcal{D}_s = \bigcap_{a\in\mathcal{A}} \mathcal{D}_a$ *then there exists prices* $\{\pi(\omega), \omega \in \Omega\}$, *and* $\{\mu(\omega), \omega \in \Omega\}$ *and trades in A-D securities so that* $\{(W_1^*(\omega), x^*),(W_2^*(\omega), y^*(\omega)),(W_3^*(\omega), z^*(\omega)), W_4^*(\omega)\}$ *is a risk-averse equilibrium.*

Example with worst case $\mathbb{F} = \mathbb{W}$



Equilibrium prices of A-D Securities are $\mu(\omega_1) = 1$, $\mu(\omega_2) = 0$. Agent a can acquire $W_a(\omega_2)$ at zero cost as long as $0 \leq -W_1^*(\omega_2) - W_2^*(\omega_2) - W_3^*(\omega_2) - W_4^*(\omega_2)$.

Summary

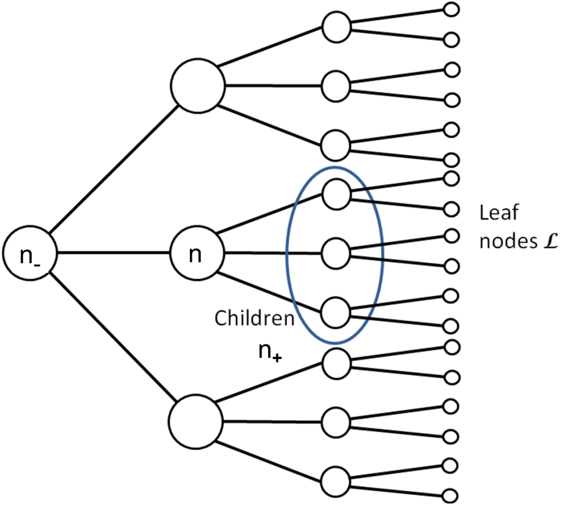
1. Introduction
2. Design a pricing mechanism for two-stage stochastic model
   - deterministic example
   - two-stage stochastic example
   - generalization of example
3. Benchmark a multistage hydro system

Hydroelectricity reservoir optimization

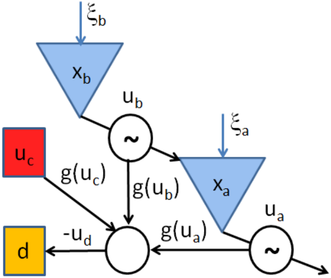
Ohau A power station in New Zealand's South Island

(Photo by By Ulrich Lange, Bochum, Germany - Own work)

A scenario tree represents uncertain inflow outcomes



Hydroelectric optimization and equilibrium with storage



Storage (i.e. batteries, hydroelectric reservoirs, pumped storage) adds dynamics. We have a storage state variable x_a for agent a affected by controls (e.g. reservoir releases, u, possibly from other agents) and random disturbances ξ (e.g. inflows).

$$x_a(n) \leq x_a(n_-) + \sum_{b \in \mathcal{A}} T_{ab} u_b(n) + \xi_a(n), \quad a \in \mathcal{A}, n \in \mathcal{N}.$$

In each node $n \in \mathcal{N}$, agents produce electricity to meet demand (that is also treated as an agent).

Nested risk measure

- Controls $u(m) \in \mathcal{N}\backslash\{0\}$ give benefits $Z(m)$, $m \in \mathcal{N}\backslash\{0\}$.
- Assume known future risk-adjusted benefit $\theta(n)$ in node $n \in \mathcal{L}$.
- polyhedral risk sets $\mathcal{D}(n)$, $n \in \mathcal{N}$ with known extreme points $\left\{\mathbb{P}^k(m), m \in n_+, k \in \mathcal{K}(n)\right\}$.
- Risk adjustment is recursive:

$$
\begin{aligned}
\theta(n) &= \min_{\mathbb{P}\in\mathcal{D}(n)} \sum_{m\in n_+} \mathbb{P}(m)(Z(m)+\theta(m)) \\
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A multistage risk-averse optimization problem with storage

(P., Ferris, Wets, 2016, Ferris and P., 2018)

$$\text{SO}(\mathcal{D}): \max_{u,x,\theta} \theta(0) - \sum_{a\in\mathcal{A}} C_a(u_a(0))$$

$$\text{s.t. } \theta(n) = \min_{\mathbb{P}\in\mathcal{D}(n)} \sum_{m\in n_+} \mathbb{P}(m)(-\sum_{a\in\mathcal{A}} C_a(u_a(m)) + \theta(m)),$$

$$x_a(n) \leq x_a(n_-) + \sum_{b\in\mathcal{A}} T_{ab} u_b(n) + \xi_a(n), \quad \text{[water]}$$

$$\sum_{a\in\mathcal{A}} g_a(u_a(n)) \geq 0 \quad \text{[energy]}$$

$$\theta(n) = \sum_{a\in\mathcal{A}} V_a(x_a(n)), \quad n \in \mathcal{L},$$

$$u_a(n) \in \mathcal{U}_a, \quad x_a(n) \in \mathcal{X}_a, \quad n \in \mathcal{N}, \quad a \in \mathcal{A}.$$

New definitions for multistage

Definition

For $n \in \mathcal{N} \setminus \mathcal{L}$ the social planning risk set is

$$\mathcal{D}_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n).$$

Definition

Given any node $n \in \mathcal{N} \setminus \mathcal{L}$, an Arrow-Debreu security for node $m \in n_+$ is a contract that charges a price $\mu(m)$ in node n to receive a payment of 1 in node $m \in n_+$.

Some assumptions

- Assumption 1: All risk sets $\mathcal{D}_a(n)$ lie strictly inside the positive orthant, implying strictly monotone $\mathbb{F}_a$.
- Assumption 2: (Complete risk markets) At every node $n \in \mathcal{N} \setminus \mathcal{L}$, there is an Arrow-Debreu security for each child node $m \in n_+$.
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Multistage risk-averse agent optimization

$$\mathrm{AO}_a(\pi, \alpha, \mu, \mathcal{D}_a): \quad \max_{u_a, x_a, W_a, \theta_a} \; Z_a^0(u, x, W) + \theta_a(0)$$

s.t.

$$\theta_a(n) = \min_{\mathbb{P}\in\mathcal{D}(n)} \sum_{m\in n_+} \mathbb{P}(m)(Z_a^m(u_a, x_a, W_a) + W_a(m) + \theta_a(m)),$$

$$\theta_a(n) = V_a(x_a(n)), \quad n \in \mathcal{L},$$

$$u_a(n) \in \mathcal{U}_a, \; x_a(n) \in \mathcal{X}_a, \quad n \in \mathcal{N},$$

$$\begin{aligned} Z_a^n(u, x, W) = \;& \pi(n) g_a(u_a(n)) - C_a(u_a(n)) \text{ [energy profit]} \\ & + \alpha_a(n)\left(x_a(n_-) - x_a(n) + \xi_a(n)\right) \text{ [stored water]} \\ & + \textstyle\sum_{b\in\mathcal{A}} \alpha_b(n) T_{ba} u_a(n) \text{ [transferred water]} \\ & - \textstyle\sum_{m\in n_+} \mu(m) W_a(m). \quad \text{[cost of A-D purchases]} \end{aligned}$$

Multistage risk-trading equilibrium

A multistage risk-trading equilibrium $\text{RTE}(\mathcal{D}_{\mathcal{A}})$ is a stochastic process of prices $\{\pi(n)\}$, $\{\alpha_a(n)\}$, $\{\mu(n)\}$, and a corresponding collection of actions, $\{u_a^*(n)\}, \{x_a^*(n)\}, \{W_a^*(n)\}, \{\theta_a^*(n)\}$ with the property that $(u_a^*, x_a^*, W_a^*, \theta_a^*)$ solves the problem $\text{AO}_a(\pi, \alpha, \mu, \mathcal{D}_a)$, and at every node $n \in \mathcal{N}$

$$0 \leq \pi(n) \quad \perp \quad \sum_{a \in \mathcal{A}} g_a(u_a^*(n)) \geq 0, \quad \text{[energy market]}$$

$$0 \leq \alpha_a(n) \quad \perp \quad -x_a^*(n) + x_a^*(n_-) + \sum_{b \in \mathcal{A}} T_{ab} u_b^*(n) + \xi_a(n) \geq 0, \quad \text{[water market]}$$

$$0 \leq \mu(n) \quad \perp \quad -\sum_{a \in \mathcal{A}} W_a^*(n) \geq 0, \quad \text{[risk market]}.$$

First welfare theorem

[Ferris and P., 2018]

Suppose Assumptions 1 and 2 hold, and consider a set of agents $a \in \mathcal{A}$, each endowed with a polyhedral node-dependent risk set $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$ satisfying Assumption 3.

Theorem

If $\{\pi(n)\}$, $\{\alpha_a(n)\}$, and $\{\mu(n)\}$ form a multistage risk-trading equilibrium with $\{u_a^*(n)\}, \{x_a^*(n)\}, \{W_a^*(n)\}, \{\theta_a^*(n)\}$, then (u^*, x^*, θ_s^*) is a solution to $SO(\mathcal{D}_s)$ where $\mathcal{D}_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$ and $\theta_s^*(n) = \sum_{a \in \mathcal{A}} \theta_a^*(n)$.

Second welfare theorem

[P.,Ferris,Wets, 2016]

Suppose Assumptions 1 and 2 hold, and consider a set of agents $a \in \mathcal{A}$, each endowed with a polyhedral node-dependent risk set $\mathcal{D}_a(n)$ satisfying Assumption 3.

Theorem

Let (u^*, x^*, θ_s^*) be a solution to $SO(\mathcal{D}_s)$ with risk sets $\mathcal{D}_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$, giving rise to Lagrange multipliers $\alpha_a(n)$ (for storage) and $\pi(n)$ (for energy). Then there exists $\mu(n)$ so that the prices $\{\pi(n)\}$, $\{\alpha_a(n)\}$, $\{\mu(n)\}$ and actions $\{u_a^*(n)\}, \{x_a^*(n)\}, \{W_a^*(n)\}, \{\theta_a^*(n)\}$ form a multistage risk-trading equilibrium.

Benchmarking the NZ wholesale electricity market
(Joint work with Ziming Guan)

- NZ uses a standard nodal pricing market design with 250 nodes.
- Wholesale market is dispatched every 30 minutes using optimization software called SPD.
  - energy (and spinning reserve) offers submitted by generators;
  - generation levels and reserve levels allocated to every generator;
  - 250 locational marginal (nodal) prices;
- SPD inputs (offers, network constraints and demand) are made public two weeks after the day of dispatch.
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Benchmarking the NZ wholesale electricity market

- In market, thermal generators offer supply curves that deviate from marginal cost.
- Hydrogenerators use a market value of water to define the marginal cost of their offer.
- In counterfactual, thermal generators offer at marginal cost.
- We use a counterfactual value of water based on a system optimization using risk-averse SDDP [Shapiro et al, 2013, P., de Matos, Finardi, 2013].
- We use 35 historical inflow sequences to train SDDP policy and a nested coherent risk measure based on a one-step conditional risk measure equal to $0.5\mathbb{E} + 0.5\mathbb{W}$.
- If market is perfectly competitive, and risk markets are complete, and we are using the correct risk measure for agents, then we expect outcomes to be be similar.

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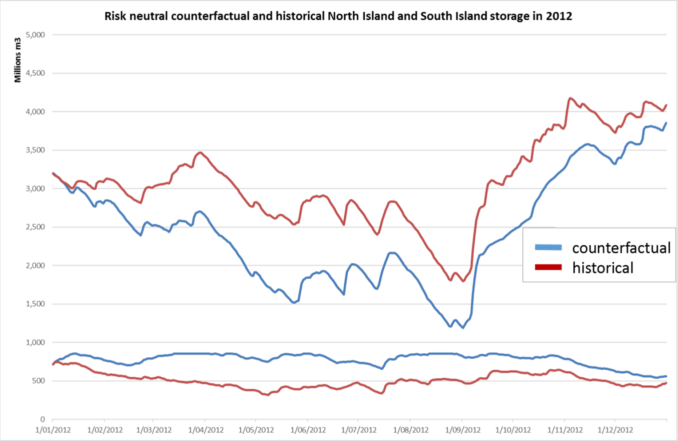
South Island reservoir storage 2012



Storage levels from risk-neutral benchmark simulation compared with historical (market) levels in 2012.

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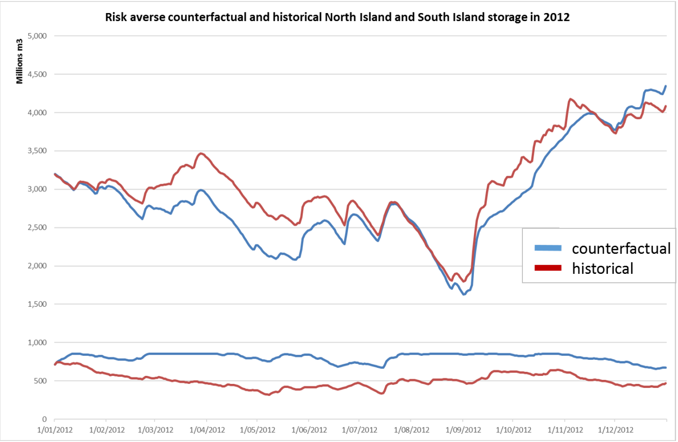

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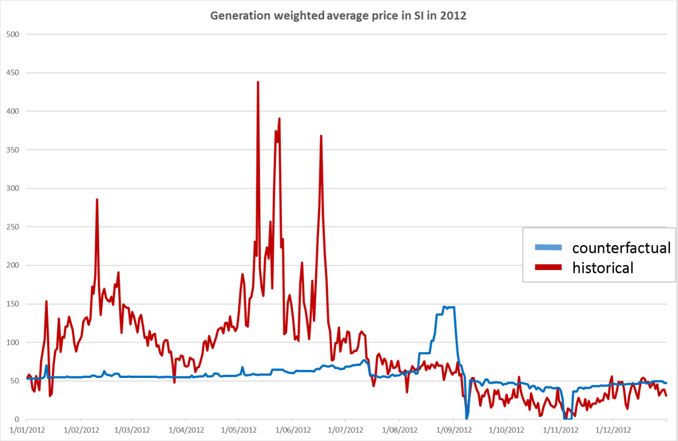
South Island wholesale electricity prices 2012



Generation-weighted average price from risk-neutral benchmark simulation compared with historical (market) prices in 2012.

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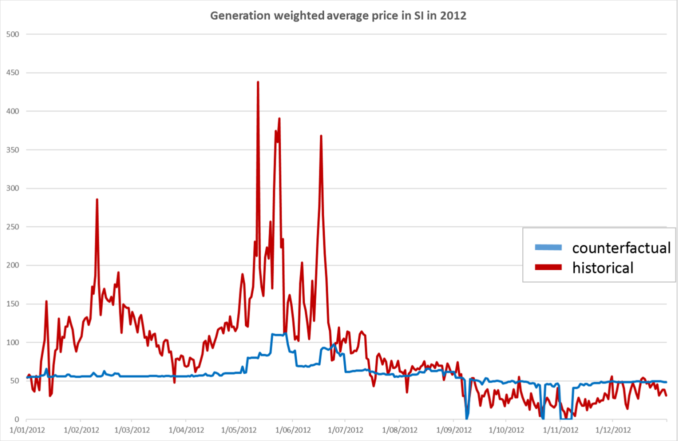

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Some practical questions

- How to solve the MOPEC for multistage risk-averse equilibrium?
  - complete risk market gives a multistage optimization model formulated in scenario tree.
  - most popular algoithm for hydrothermal scheduling is SDDP [Pereira & Pinto, 1991] (see e.g. Julia package sddp.jl by Dowson.)
  - computation of solutions to incomplete risk market problems at scale is challenging (see [Höschle et al, 2017] for a decomposition approach).
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Conclusion

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- Market efficiency in hydro-dominated systems can be measured with benchmark models based on optimization by assuming complete markets for risk.
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