Pricing battery storage using SDDP

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Research performed while the authors were participating in the Architecture of Green Energy Systems Program hosted by the Institute for Mathematical and Statistical Innovation (IMSI), and supported by the National Science Foundation
(Grant No. DMS-1929348).

ShapiroFest, Atlanta, March 17, 2025.

Energy & Environment   Energy Transitions   Renewables & Advanced Energy   United States and Canada

New Atlanticist | May 13, 2024

California's battery boom is a case study for the energy transition

By Joseph Webster

California is the country's largest and most mature solar market, but it's also changing in important ways. On April 25, California marked a major milestone, as it became the first state to deploy 10 gigawatts (GW) of battery storage capacity. This large-scale deployment of lithium-ion storage batteries is leading to lower solar "curtailment," or when electricity generation is suppressed due to price signals or physical oversupply. Curtailment is a problem because it means solar power stations, for example, are producing less electricity than they could, contributing less to the overall energy mix than they otherwise might.

Figure: CAISO battery boom [New Atlanticist, May 2024]

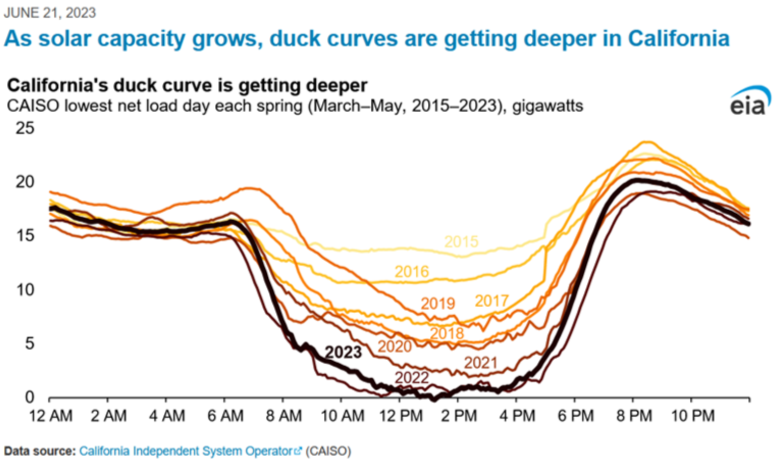

Figure: CAISO Duck curves [California Independent System Operator]

Electricity dispatch and pricing

- System operators solve a multiperiod dispatch problem to schedule generators and batteries and compute prices.
- Needs forecasts of future renewable generation (wind and solar).
- Better to use a scenario tree? [Wong & Fuller, 2007; Pritchard et al, 2010]
- Requires market participants to agree on scenarios ...
- ...and gives intractable problems [Shapiro & Nemirovski, 2005] and potentially inconsistent prices. [Hogan, 2020]
- What about using SDDP?

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SDDP example solution

Agent decision rules

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Economic dispatch example

$x_i(t)$ = dispatch of generator $i \in \mathcal{G}$ in period t;

$\bar{x}_i$ = dispatch of generator i in period $t-1$;

$y_j(t)$ = storage in battery $j \in \mathcal{B}$ at end of period t;

$\bar{y}_j$ = storage in battery j at end of period $t-1$;

u_j = discharge from battery j in period t;

v_j = charge input to battery j in period t;

$$\mathcal{X}_i(\bar{x}) = \{x \mid 0 \le x \le q_i, x - \bar{x}_i \le \rho_i, \bar{x}_i - x \le \sigma_i\},$$

$$\begin{aligned}\mathcal{Y}_j(\bar{y}) = \{&(y,u,v) | 0 \le y \le E_j, 0 \le u \le r_j, 0 \le v \le s_j, \\ &y = \bar{y}_j - u + \eta_j v\}.\end{aligned}$$

Economic dispatch and pricing: period t

$$\text{EP}(t):\quad \min \sum_{i \in \mathcal{G}} c_i(x_i(t)) + L z(t)$$

$$\text{s.t.}\quad \sum_{i \in \mathcal{G}} x_i(t) + \sum_{j \in \mathcal{B}} u_j(t) - \sum_{j \in \mathcal{B}} v_j(t) + z(t) = d(t) + w(t), \quad [\pi(t)]$$

$$x_i(t) \in \mathcal{X}_i(x(t-1)), \quad i \in \mathcal{G},$$

$$(y_j(t), u_j(t), v_j(t)) \in \mathcal{Y}_j(y(t-1)), \quad j \in \mathcal{B},$$

$$w(t) \geq 0, z(t) \in [0, d(t)].$$

[Here $c_i(x)$ is a convex increasing function of x; L is VOLL.]

An example: one battery, one generator

Assume $T = 24$, $c_i(x) = 70.0x$, $\sigma = \infty$. Other parameters are as follows.

$q = 70.0$	$E = 30.0$	$\eta = 0.8$
$r = 10.0$	$s = 10.0$	$\rho = 10.0$
$L = 500.0$	$x^0 = 35.0$	$y^0 = 4.0$

Table: Parameter values for example

Example demand

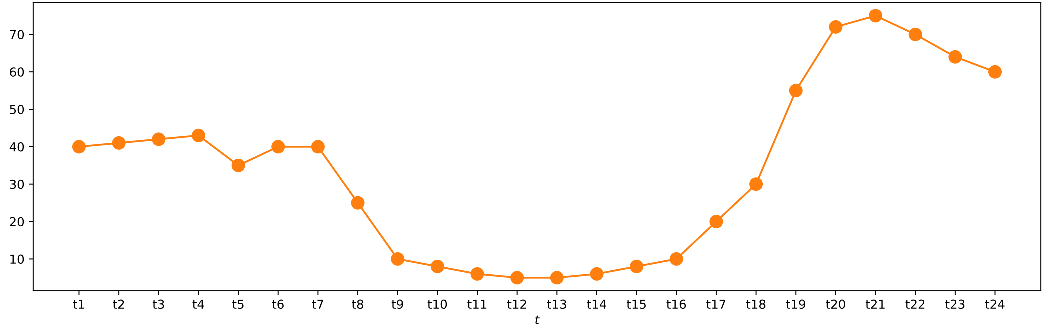

Figure: Example values of $d(t)$ for $t = 1, 2, \ldots, 24$. We add stagewise independent random noise chosen from -4.0, -2.0, 0.0, 2.0, 4.0 with equal probabilty

Uncertain net demand modeled by a scenario tree.

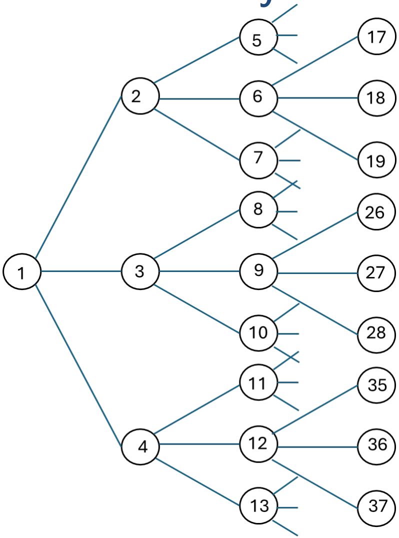

Figure: A scenario tree. We write n_- for the parent of node n, for example, $8_- = 3$. SDDP requires (some form of) stagewise independence.

Eeconomic dispatch and pricing in a scenario tree

$$\text{SP:} \quad \min \sum_{n \in \mathcal{N}} P(n) \left(\sum_{i \in \mathcal{G}} c_i(x_i(n)) + Lz(n) \right)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{G}} x_i(n) + \sum_{j \in \mathcal{B}} u_j(n) - \sum_{j \in \mathcal{B}} v_j(n) + z(n) = d(n) + w(n),$$

$$[P(n)\pi(n)], \quad n \in \mathcal{N},$$

$$x_i(1) = x_0, \quad x_i(n) \in \mathcal{X}_i(x(n_-)), \quad \forall i, n \in \mathcal{N} \setminus \{1\},$$

$$y_j(1) = y_0, \quad (y_j(n), u_j(n), v_j(n)) \in \mathcal{Y}_j(y(n_-)), \forall j, n \in \mathcal{N} \setminus \{1\},$$

$$w(n) \geq 0, z(n) \in [0, d(n)], \quad n \in \mathcal{N}.$$

Optimal dispatch gives energy prices π

- Dual variables on demand constraints are $P(n)\pi(n)$ that decouple SP into agent problems. [Ferris & P., 2022]

$$
\begin{array}{lll}
\mathrm{GP}(i): & \max & \displaystyle\sum_{n \in \mathcal{N}} P(n)(\pi(n) x_i(n) - c_i(x_i(n))) \\
& \text{s.t.} & x_i(1) = x_0, \quad x_i(n) \in \mathcal{X}_i(x(n_-)), \forall i, n,
\end{array}
$$

$$
\begin{array}{lll}
\mathrm{CO}: & \max & \sum_{n \in \mathcal{N}} P(n)(\pi(n) - L) z(n) \\
& \text{s.t.} & 0 \le z(n) \le d(n), \forall n.
\end{array}
$$

$$
\begin{array}{lll}
\mathrm{BP}(j): & \max & \displaystyle\sum_{n \in \mathcal{N}} P(n)\pi(n)(u_j(n) - v_j(n)) \\
& \text{s.t.} & y_j(1) = y_0, \quad (y_j(n), u_j(n), v_j(n)) \in \mathcal{Y}_j(y(n_-)), \forall j, n.
\end{array}
$$

Drawbacks of scenario trees

- The scenario tree reflects the system operator view of the future and is not a consensus of market participant views, who prefer to "put their money where their mouths are";
- Even with a shared view, the future will (almost surely) not be a scenario in the tree;
- Solving scenario-based problems is impossible at scale;
- With stagewise independent or Markov noise we can use SDDP [SDDP.jl: Dowson and Kapelevich, 2021];
- Prices π from SDDP models are not stagewise independent, so agent problems require scenario trees. [Barty et al, 2010]

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Example problem with stagewise independent demand

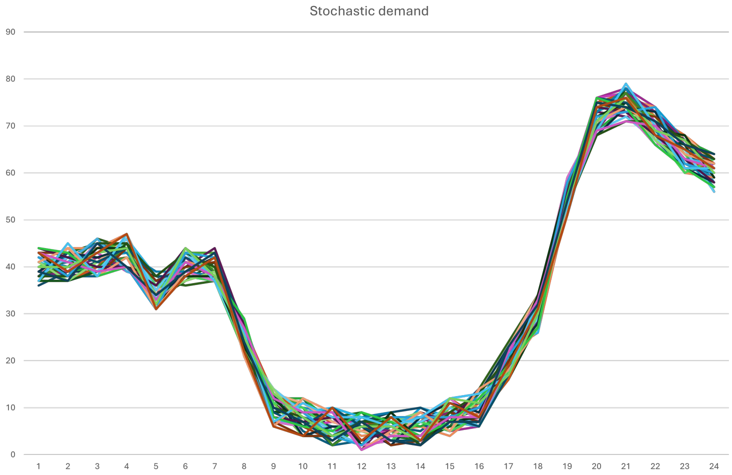

Figure: Example of simulated demand realizations (5 equiprobable outcomes per stage).

SDDP.jl solution

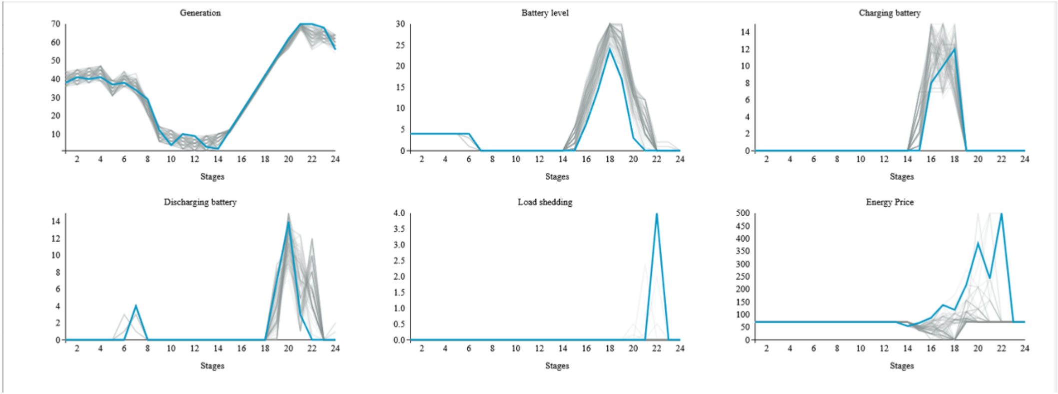

Figure: 100 simulations of optimal SDDP policy (100 cuts). LB=57126 UB=57148 $\pm$ 21

Plot of prices from optimal policy

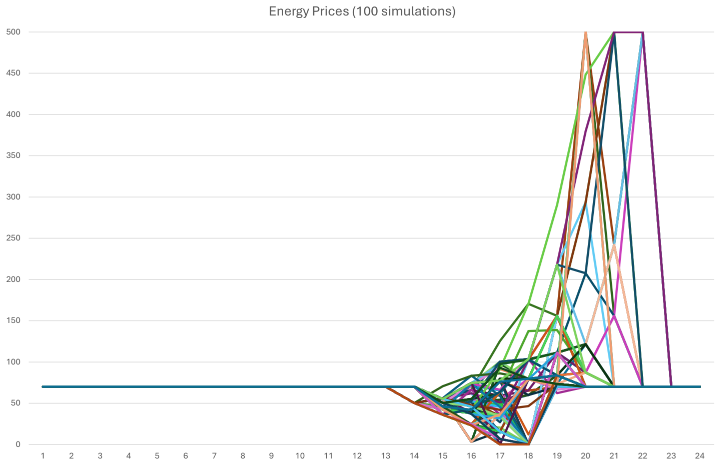

Figure: System marginal prices from 100 simulations of optimal stochastic policy computed using SDDP.jl.

System stage problem and expected future cost

$$
\begin{aligned}
\text{EP}(t)\text{: min} \quad & \sum_{i \in \mathcal{G}} c_i(x_i(t)) + Lz(t) {\color{red} + C^t(x, y)} \\
\text{s.t.} \quad & \sum_{i \in \mathcal{G}} x_i(t) + \sum_{j \in \mathcal{B}} u_j(t) - \sum_{j \in \mathcal{B}} v_j(t) + z(t) = d(t) + w(t), \\
& x_i(t) \in \mathcal{X}_i(x(t-1)), \quad i \in \mathcal{G}, \\
& (y_j(t), u_j(t), v_j(t)) \in \mathcal{Y}_j(y(t-1)), \quad j \in \mathcal{B}, \\
& w(t) \geq 0, z(t) \in [0, d(t)].
\end{aligned}
$$

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Expected future cost provided by agents

$$
\begin{aligned}
\text{ADR}(t)\text{: min}\quad & \sum_{i \in \mathcal{G}} c_i(x_i(t)) + Lz(t) + \sum_{i \in \mathcal{G}} G_i^t(x_i) + \sum_{j \in \mathcal{B}} B_j^t(y_j) \\
\text{s.t.}\quad & \sum_{i \in \mathcal{G}} x_i(t) + \sum_{j \in \mathcal{B}} u_j(t) - \sum_{j \in \mathcal{B}} v_j(t) + z(t) = d(t) + w(t), \\
& x_i(t) \in \mathcal{X}_i(x(t-1)), \quad i \in \mathcal{G}, \\
& (y_j(t), u_j(t), v_j(t)) \in \mathcal{Y}_j(y(t-1)), \quad j \in \mathcal{B}, \\
& w(t) \geq 0, z(t) \in [0, d(t)].
\end{aligned}
$$

Agent decision rules

- System operator collects future cost functions $G_i^t(x_i)$ and $B_j^t(y_j)$ from agents and uses them in place of $C^t(x, y)$.
- This is an example of an agent decision rule (ADR).
- An ADR for agent a in period t is a function of any parameter of the stage t problem, and a's dispatch (storage) at end of t.
- An ADR for agent a expresses the expected future cost to a of being in a given state at the end of each period.

Dispatch process for generators and batteries

- Generator agents $i \in \mathcal{G}$ provide system operator with cost $c_i(x))$.
- Generator agents $i \in \mathcal{G}$ provide system operator with ADR defined by G_i^t.
- Battery agents $j \in \mathcal{B}$ provide system operator with ADR defined by B_j^t.
- System operator solves single-stage problem ADR(t) and computes dispatch and system marginal price $\pi(t)$.
- Generator i is paid $\pi(t)x_i(t)$.
- Battery j is paid $\pi(t)(u_j(t) - v_j(t))$.

Remarks

- ADR(t) is a deterministic convex optimization problem (assuming no unit commitment).
- This means price $\pi(t)$ gives budget balance for system operator (i.e. revenue adequacy).
- Price $\pi(t)$ defines a perfectly competitive equilibrium for stage t, so agents recover costs.
- Does dispatch problem ADR(t) yield social optimum?
- If all agents and system operator agree on probability distribution of future demand then ADRs can recover social optimum.

How agents might choose an ADR

- SDDP defines (approximate) Bellman function $C^t(x, y)$ at stage t (using cuts).
- Suppose given $(x(t-1), y(t-1))$ the optimal dispatch with $C^t(x, y)$ yields actions $(x^*(t), y^*(t))$.
- Given $(x(t-1), y(t-1))$ agent a makes a forecast $(\tilde{x}^t, \tilde{y}^t)$ of $(x^*(t), y^*(t))$.
- Propose that agent $i \in \mathcal{G}$ and $j \in \mathcal{B}$ offer ADRs:

$$\tilde{G}_i^t(x_i) = C^t(x_i, \tilde{x}_{-i}^t, \tilde{y}^t),$$

$$\tilde{B}_j^t(y_j) = C^t(\tilde{x}^t, y_j, \tilde{y}_{-j}^t).$$

ADRs can be system optimal

Theorem

Suppose given $(x(t-1), y(t-1))$, that each agent a makes a perfect forecast $(\tilde{x}^t, \tilde{y}^t)$ of $(x^*(t), y^*(t))$ (for example they might all solve SDDP model with the same shared data). Then

1. the solution for ADR(t) using $\sum_{i\in\mathcal{G}} \tilde{G}_i^t(x_i) + \sum_{j\in\mathcal{B}} \tilde{B}_j^t(y_j)$ is optimal for EP(t) with $C^t(x, y)$;
2. prices from EP(t) and the solution to ADR(t) defines a perfectly competitive equilibrium where all agents optimize profit in period t at system prices accounting for their ADR.

Experiment with imperfect forecast

- In battery example, suppose we solve SDDP, and simulate over many sample paths. This gives expected cost $=$57,148 $\pm$ 21.
- Let $(\tilde{x}^t, \tilde{y}^t)$ denote average values of generation and average values of battery storage at each stage.
- We then simulate the solution of ADR(t) using the approximation

$$\sum_{i \in \mathcal{G}} \tilde{G}_i^t(x_i) + \sum_{j \in \mathcal{B}} \tilde{B}_j^t(y_j).$$

- Simulated policy gives 58,082 $\pm$ 76. Some social optimality is lost since $(\tilde{x}^t, \tilde{y}^t) \neq (x^*(t), y^*(t))$ (varying with each sample path).

"To do" list

- Unit commitment requires binary variables.
- Dispatch must also meet many side constraints (e.g. reserve)
- Agents can hold a portfolio of technologies.
- Agents have different views of the future.
- Agents have different risk preferences.
- Agents may be strategic: i.e. not reveal their true future costs.

The End

Happy 75th Birthday Alexander!

Agents are strategic

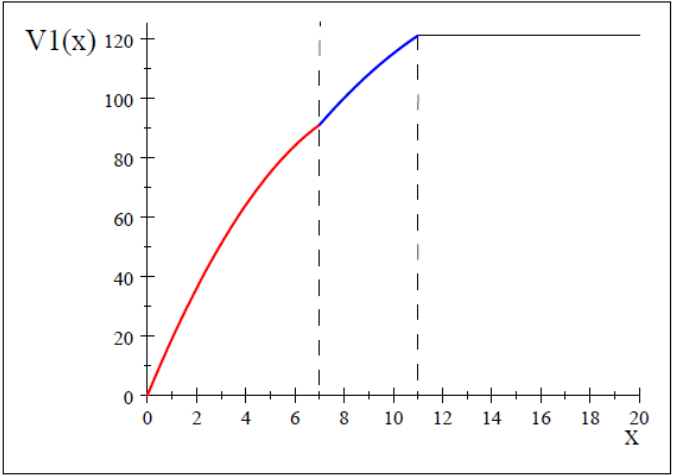

Figure: ADR from Cournot game [Crampes & Moreaux, 2001]