Submission on

Consultation Paper:

High Spring Washer Pricing

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Executive Summary

The Electricity Commission has been presented with an undesirable trading situation in which a spring-washer effect led to high spot prices. This has resulted in a Consultation Paper that seeks to address the fact that in spring-washer situations small errors in line impedance or security constraint right-hand sides often lead to large errors in spot prices. It is desirable for purchasers to have confidence in the pricing process, and so the Proposal in the Consultation Paper aims to make corrections to ex-post prices to account for any measurement errors in input parameters. The corrections will be made by decreasing the security constraint right-hand side and re-solving the dispatch model (SPD). This will be done only in circumstances in which a spring-washer situation is claimed to be occurring.

The Electric Power Optimization Centre (EPOC) makes the following observations on this document:

1. The Proposal should not be restricted to solely spring-washer events.
2. The “downward pressure on prices” argument is insufficient to justify relaxing constraints arbitrarily.
3. A perturbation to all loads in every trading period before prices are computed can account for errors in load measurement.
4. Any decrease in parameters should be chosen to be consistent with the size of the errors. Ideally any change in parameters should result in prices that amount (in expectation) to the same exact payment for loads with zero error.
5. Choosing arbitrary ratios to trigger an ex-post analysis is likely to lead to unforeseen participant behaviour, and should be avoided if possible.

Introduction

The Electricity Commission has been presented with an undesirable trading situation in which a spring-washer effect led to high spot prices. This has resulted in a Consultation Paper that seeks to address the fact that in spring-washer situations small errors in line impedance or security constraint right-hand sides often lead to large errors in spot prices. It is desirable for purchasers to have confidence in the pricing process, and so the proposal in the Consultation Paper aims to make corrections to ex-post prices to account for any measurement errors in input parameters. The corrections will be made by decreasing the security constraint right-hand side and re-solving the dispatch model (SPD). This will be done only in circumstances in which a spring-washer situation is claimed to be occurring.

A curious feature of this proposal is its focus on spring-washer effects as a pathology that requires special treatment. These effects, though sometimes producing high prices, represent real electrical conditions, and should not be treated as pathological artifacts of a poor model. Moreover, price errors will occur in situations that are not of the spring-washer variety. For example small perturbations in load can increase dispatch across an offer-tranche boundary leading to a (possibly very large) change in spot price. It should be a matter of some concern in this situation that the “price of the last MW of consumption” and the “price of the next MW of consumption” differ markedly even though these terms are sometimes used interchangeably in explanations of marginal prices to the industry. Indeed SPD returns the linear programming dual value which might be either of these limits or some price in between.

So the real issue to be dealt with by the Commission proposal should be sensitivity of prices to data errors*, irrespective of whether this has occurred from a spring-washer effect*. This makes the consultation paper’s efforts to discriminate between high prices caused by spring-washer effects and other high-price outcomes unnecessary. What is important is to identify sensitive solutions and then to immunize them somehow against errors being magnified.

Robust solutions

There is a lot of discussion in the Consultation Paper about *robustness* of prices. This is interpreted as insensitivity of prices to small inaccuracies in the input data.

The subject of making optimization problems robust to data perturbations has received much attention in the mathematical programming community recently under the name *robust optimization* [1]. Researchers have sought fast algorithms to compute solutions to linear programs that are feasible for all choices of parameters in some defined “uncertainty set”, while optimizing a given objective function. Engineering design is an obvious application area.

Robust optimization is difficult to apply in this area since it is likely that there is no single dispatch that will be feasible for all realizations of the parameters. For example any small perturbation in load will necessitate a new dispatch.

It is also not clear whether robustness of the form sought by the Consultation Paper is achievable in this setting. In the situations in which the price is very sensitive to input data perturbations, the proposal in the Consultation Paper is to relax the right-hand side of a security constraint by a small amount in the hope that the resulting prices are no longer sensitive. This will only be possible if the LP basis matrix changes by virtue of this relaxation to one in which the inverse is insensitive to data perturbations. It is not clear that this will happen in all circumstances. (Even if this constraint has a large "shadow price" its relaxation might not result in any change if another constraint becomes binding.)

Of course it is possible to change enough of the input data to SPD to make prices insensitive, but this might substantially alter the dispatch and price signals that we wish SPD to deliver to market participants.

If robustness is not always achievable without losing price signals then what can be done? It is not desirable for market participants to pay for electricity at prices computed using erroneous data. However data errors are inevitable, so this will happen. Nevertheless it is possible to immunize market participants to some extent from the negative effects of these errors. This means that the dispatch models should arrive at prices that give payments by purchasers (and payments to generators) which would have occurred with exact data. This is discussed in the next section.

Finding the right price

In its proposals for dealing with sensitivity of prices to data errors, the Electricity Commission has espoused the following principle.

If errors in data lead to a distribution of (high) electricity prices then the data should be perturbed in the direction that lowers prices.

This principle is based on the Electricity Commission's specific outcome "..sustained downward pressure on prices". In applying this principle the consultation paper proposes relaxing security constraints by an amount of the maximum of 1MW or 1%, and re-solving the dispatch-pricing model.

Although in general this will result in lower prices, it is not clear that this is an appropriate application of the Electricity Act. Downward pressure on prices can be encouraged by many means, e.g. demand-side response, increased competition, more transmission capacity, but changing the input data for SPD would seem to be outside the intention of this guiding principle.

To understand how to address this issue it is useful to see in principle what the "correct" spot price should be when there are errors in the data that are input to SPD. If we accept that the SPD model with correct data (*D*, say) is a true representation of the dispatch-pricing system, then with this correct data there will be a correct set of prices $\pi(D)$. In practice SPD will be solved with inaccurate data (*D'*, say) which may lead to incorrect prices $\pi(D')$.

The problem of estimating the correct prices $\pi(D)$ from $\pi(D')$ is a statistical inference problem. One might use a least squares or a maximum likelihood approach to this estimation. It is important to understand in this estimation some possible biases which would result in an unfair outcome.

First, the estimation of $\pi(D)$ may be biased. This would happen, for example, if the industry offer stack has a step from price π_1 to $\pi_2 > \pi_1$ just below load *D*. Even if the errors in *D* are unbiased, positive errors in *D* would give the correct price π_2, but some negative errors would give the low price π_1, with the result that the price on average (some combination of π_1 and π_2) will be lower than it would have been with no errors (i.e. π_2).

A second more subtle issue is that the expected payments of consumers at a node may be biased because of errors in load measurement, even though the average prices at this node are not biased. This is because prices and load are positively correlated, so the average of their product will be greater than the product of their averages. We illustrate this with two examples in the Appendix.

Example 1 assumes that prices are a linear increasing function of load in a single node model. Here errors in load measurement lead to unbiased prices, but the average payment made by loads is greater than it should be with perfectly accurate data.

Example 2 assumes that prices come from a step function industry stack. Here the errors are chosen so that the prices are biased up half the time and biased down half the time. However this does not even out the payments which are biased high.

For other parameters in a nodal market (e.g. line reactances) the story is also not so clear cut since the nodal prices are not necessarily positively correlated in the parameter. Nevertheless, these examples illustrate the point that errors in data may entail surplus payments by loads, even though the data errors average out to zero. Thus in some circumstances it is not necessary to invoke the "downward pressure on prices" argument to justify a smaller marginal price than that obtained by taking expectations over an error distribution.

One way of invoking Principle 1 correctly would be to choose a fixed perturbation of data that would result in the same payments by loads as those obtained from correct data. In other words we seek a load reduction δ_i at node *i* and a resulting new price $\pi_i(\delta_i)$ so that the payment $\pi_i(\delta_i)D'_i$ is the same in expectation as the value $\pi_i D_i$ obtained from correct values of these data.

This can be illustrated for a single node in case of Example 1 as follows.

Suppose that SPD is supplied with erroneous metered data $d+\varepsilon$, where E[ε] =0. This results in a spot price $\pi(d+\varepsilon)$. Suppose that

$$\pi(d+\varepsilon) = a\,\varepsilon + b$$

where a>0. Subtracting δ from the metered load before solving SPD will give prices

$$\pi(d+\varepsilon-\delta) = a\,\varepsilon - a\delta + b$$

If the purchaser pays for metered load $d+\varepsilon$, at these corrected prices, then the expected payment is

$$\begin{aligned} E[(d+\varepsilon)\,\pi(d+\varepsilon-\delta)] &= E[(d+\varepsilon)(a\,\varepsilon - a\delta + b)] \\ &= db - da\delta + a\,E[\varepsilon^2] \end{aligned}$$

To make this equal to the exact payment *db*, we set

$$\delta = E[\varepsilon^2]/d$$

Subtracting δ from the metered load before solving SPD will result in prices that give the same payment by purchasers as if SPD had used the correct load *d*. It is easy to see that if $\varepsilon=0$ (i.e. no errors in load) then $\delta = 0$. If $a=0$, then any value of δ will give the same price *b*, so no adjustment in price will occur.

The principle of perturbing load downwards to allow for errors in load data is simple and theoretically justified under the assumption that errors in load measurement do not bias spot prices. It means that purchasers might pay less than they did without such a perturbation, and generators be paid less, but it cannot be seen as an unfair wealth transfer as each pays or is paid the correct amount as computed with perfectly accurate data. The perturbation can be applied in all trading periods before a final pricing run of SPD. In many cases it will not result in any changes in price. When it does these changes will on average compensate purchasers for any extra payments they have made because of metering errors. We discuss how to determine these perturbations in Appendix 2.

Trigger ratios

The Commission recommends (as one option) that SPD is re-solved only when the spot price exceeds the highest offer price by some trigger ratio (set to be 5), which is intended to indicate a spring-washer event. Although the motivation for the proposal in the Consultation Document was a spring-washer event, there does not seem to be any reason that these should be singled out as being the only times that prices need to be made robust. If not, then there is no need to try and identify these using trigger ratios.

The advantage of a trigger is to restrict the application of the Proposal to a few trading periods. In the normal course of events, it is supposed that prices are robust to data inaccuracies, and so there is no need to relax security constraints in these circumstances. However the effects of non-robustness come from several sources:

1. the LP basis matrix inverse is very sensitive to small changes in its coefficients (possibly from a spring-washer effect)
2. the industry offer stack is steep at the dispatch point
3. the measurement errors are large

Any of these circumstances should give purchasers some grounds to seek a correction to ex-post prices, even though a trigger condition had not been met. A drawback of a fixed trigger strategy is that choosing any such value might lead to unforeseen participant behaviour. For example marginal generators might have an incentive to increase the price of their marginal offer to keep the trigger inactive. In this case, ex-post re-solves of the dispatch can be manipulated to some extent by the marginal generator.

References

[1] Bertsimas, D. and Thiele, A. “Robust and data-driven optimization: modern decision-making under uncertainty”, www.optimization-online.org.

Appendix 1: Examples

Example 1

Consider a single-node market with fixed load d. With this load and fixed offers that spot price is π. Now suppose that SPD is supplied with erroneous data $d+\varepsilon$. This results in a spot price $\pi(d+\varepsilon)$. Since the dispatch problem is convex, it is clear that the spot price $\pi(d+\varepsilon)$ at the node is nondecreasing in ε. In practice $\pi(d+\varepsilon)$ will be a step function (since offer curves are step functions), but let us suppose for the moment that it is linear and strictly increasing in ε.

In various trading periods we will observe different values of ε leading to different values of $\pi(d+\varepsilon)$. To estimate π we need to make some assumptions about the distribution of ε. We would expect that measurement errors would not be systematically biased, and so it is reasonable to assume that ε has mean 0. Let us assume that it is also normally distributed. It follows that $\pi(d+\varepsilon)$ is also normally distributed with mean π, so a maximum likelihood estimate of π is given by a sample average of $\pi(d+\varepsilon)$ observations.

If the purchaser consumes d in each trading period then the correct payment to be made by the load is πd. If they are required to pay $\pi(d+\varepsilon)$ in each trading period then on average they will pay the correct amount.

If on the other hand the purchaser is charged for $d+\varepsilon$, then on average they pay

$$E[(d+\varepsilon)\,\pi(d+\varepsilon)] = dE[\pi(d+\varepsilon)] + E[\varepsilon\,\pi(d+\varepsilon)] > dE[\pi(d+\varepsilon)] = \pi d.$$

The inequality comes about because $\pi(d+\varepsilon)$ at the node is strictly increasing in ε so

$$E[\varepsilon\,\pi(d+\varepsilon)] > 0.$$

Example 2

Consider a single-node market in which half the trading periods have demand d=4 and half have demand d=6. Suppose that the generator offer stack has a step from price π_1 to $\pi_2 > \pi_1$ at load d=5. Now suppose that when d=4, there is an error of 2 in the load, so the metered load is d=2 and d=6 with equal probability. Similarly suppose when d=6, there is an error of 2 in the load, so the metered load is d=4 and d=8 with equal probability. Thus when d=4 is the true load, the expected payment of the consumer is $0.5(2\pi_1 + 6\pi_2)$ and when d=6 is the true load, the expected payment of the consumer is $0.5(4\pi_1 + 8\pi_2)$. The average payment over all trading periods is then equal to

$$0.25\ (2\pi_1 + 6\pi_2) + 0.25\ (4\pi_1 + 8\pi_2) = 1.5\pi_1 + 3.5\pi_2$$

The payment that should be made (with accurate load measurements) is

$$0.5\ (4\pi_1) + 0.5\ (6\ \pi_2) = 2\pi_1 + 3\pi_2$$

which is less than $1.5\pi_1 + 3.5\pi_2$ because $\pi_2 > \pi_1$.