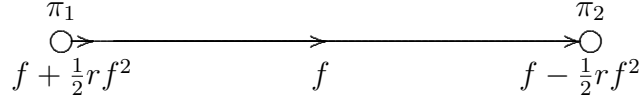


Consider a single-line network. We assume quadratic losses for simplicity, although the same results will hold for other convex loss models, e.g. piecewise linear as in SPD.



Let the loss coefficient be r , so that with physical flow f the lost power is $r f^2$. The losses are apportioned equally between the two ends of the line, so that the power sent from node 1 is $f_1 = f + \frac{1}{2} r f^2$ and the power received by node 2 is $f_2 = f - \frac{1}{2} r f^2$. (We take the positive sense of power flow to be from node 1 to node 2; throughout this analysis, negative flow values can be used to represent flow in the reverse direction, without changing any of the algebraic expressions.)

If the line is not capacity-constrained, the effect of the losses on nodal pricing is determined by marginal quantities:

$$\frac{\pi_2}{\pi_1} = \frac{df_1}{df_2} = \frac{df_1}{df} \bigg/ \frac{df_2}{df} = \frac{1 + r f}{1 - r f}.$$

The loss rental (i.e. the spot market revenue surplus) for this situation is

$$\pi_2 f_2 - \pi_1 f_1 = \pi_1 \left(\frac{1 + r f}{1 - r f} \right) \left(f - \frac{1}{2} r f^2 \right) - \pi_1 \left(f + \frac{1}{2} r f^2 \right) = \frac{\pi_1 r f^2}{1 - r f}.$$

Now suppose that balanced FTRs with total net quantity F have previously been allocated. Their total coupon payment is $(\pi_2 - \pi_1)F$. After the loss rental has been used to fund this payment, the remaining surplus is

$$\left(\frac{\pi_1 r f}{1 - r f} \right) (f - 2F).$$

This quantity could be negative, i.e. revenue inadequacy. Moreover, there is no simple limit that can be placed on F (absent assumptions about f) to ensure revenue adequacy.

It is well-known that revenue adequacy can be achieved by ensuring that the flow represented by the totality of the FTRs is feasible for the network used in the spot market. For the single-line example, this would require (at least some) unbalanced FTRs, such that the total coupon payment is

$$\pi_2 \left(F - \frac{1}{2} r F^2 \right) - \pi_1 \left(F + \frac{1}{2} r F^2 \right) = \frac{\pi_1 r F}{1 - r f} (2f - F),$$

corresponding to lossy flow. After the loss rental has been used to fund this payment, the remaining surplus is

$$\frac{\pi_1 r (f - F)^2}{1 - r f},$$

which is always non-negative.