

# Application of sensitivity analysis for energy power management

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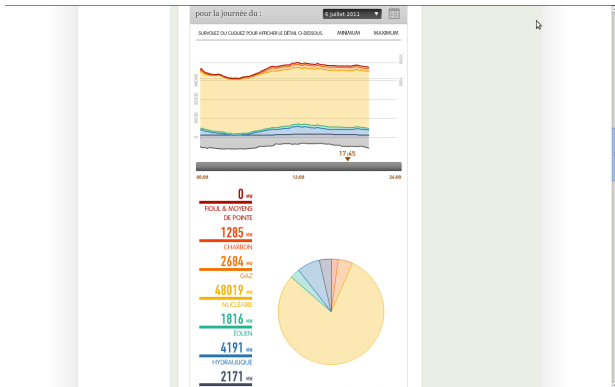
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4. Flexible power stations (Hydro) are used for supplying peak demand and un-flexible power stations (Nuclear) are booked for supplying off-peak demand and for base load;
5. EDF electricity management consists in minimizing the production cost safely;

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Figure: Electricity load: source <http://www.rte-france.com/fr/>

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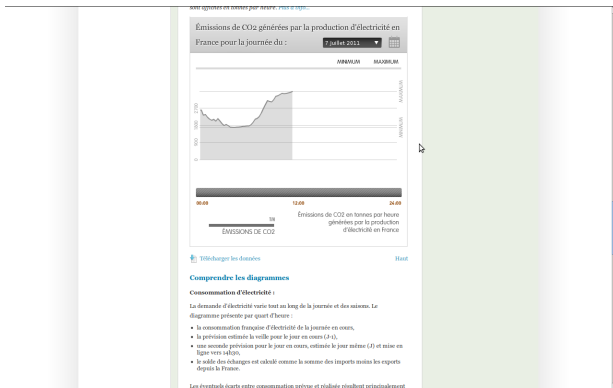


Figure: CO<sub>2</sub> Emission: source <http://www.rte-france.com/fr/>



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- Anticipate long-term investment (stochastic, multi-year);
- Schedule power-stations shut-down for maintenance and refuelling (stochastic, multi-year);
- Objectify the values of stocks of reserves (stochastic, multi-year);

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- Take market positions;
- Objectify appeals of all power stations for supplying demand (sample based, daily);
- Perform automatic and/or manual re-declarations, for matching more tightly the load curve with generation curve (real-time, intra-day);



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## Main focus of this talk

Studying some different formulations for the problem of scheduling power station shut-down.



# Outline

## Problem formulation

Current formulation of the problem

Weakness of the current formulation

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## First proposal

- Continuous time re-formulation
- Advantages and limits

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# Current formulation of the problem

## Problem description

- 58 reservoirs representing 80% of the generation capacity must be stopped periodically about 4 – 5 weeks in order to perform maintenance operations and to refuel;
- The number of parallel shut-downs is limited and a delay must be observed between two consecutive operations of a maintenance team;
- Of course EDF must still supply demand of its clients;

# Current formulation of the problem

## Decision variables

- $s(t)$  energy in the stock;
- $u(t)$  power generated during one time-step;
- $a(t)$  0 if stop and 1 otherwise;

## Parameters

- $c(x, t)$  cost of amount  $x$  of energy generated (convex, increasing);
- $d(t)$  demand at time  $t$ ;
- $\psi(x, T)$  value of the stock at the end of the study period;
- $\mathbb{S}$  set of power plants which need maintenance;

## Current formulation of the problem

Brief sketch of the current formulation :

$$\min \mathbb{E} \left[ \sum_{t=1}^{T-1} c(d(t) - \sum_{i \in \mathcal{S}} u^i(t) a^i(t), t) + \psi(s(T), T) \right]$$

$$s^i(t+1) = s^i(t) - u^i(t)$$

$$s^i(t) \in [0, \bar{s}^i(t)]$$

$$u^i(t) \in [0, \bar{u}^i(t)]$$

$$a^i(t) \in \{0, 1\}$$

$$a \in \{\text{constraints of the planning}\}$$

$u^i(t)$  random variable that observes the noise trajectory causally

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Uncertainty arises from demand and from the duration of the maintenance which could vary between more or less 2 weeks



## Weaknesses of the current formulation

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In practice even if the first campaign (period between the end of a stop and the end of subsequent stop) is fixed the second campaign may vary a lot according to noise trajectory.

## Weaknesses of the current formulation

Independently of the computational burden, this problem has some structural weaknesses:

### Weaknesses

1. one weakness of this formulation is due to the decision variable  $a^i(t)$  which is open-loop;
2. second weakness is due to the lack of information allowing to perform local search based sensitivity of objectif function with respect to the date of stop;

In practice even if the first campaign (period between the end of a stop and the end of subsequent stop) is fixed the second campaign may vary a lot according to noise trajectory.

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# Continuous time-reformulation

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## Point of view of the continuous formulation

- For a given planning of stops, we seek how to locally change the planning in order to improve the global utility function.
- The idea is to introduce date of beginning of the stop and the stop duration in order to reformulate the problem;



# Continuous time-reformulation



## Continuous time-reformulation

Given a feasible planning its valorization  $\Psi(\tau)$  then reads:

$$\begin{aligned} \min_{u(\cdot), s(\cdot)} \quad & \int_0^T c \left( d(t) - \sum_{i \in \mathbb{S}} u^i(t) (1 - \mathbb{I}_{[\tau^i, \tau^i + \Delta^i]}(t)), t \right) dt + \psi(s(T), T) \\ \text{u.c.} \quad & \dot{s}^i(t) = -u^i(t) + r^i(t) \mathbb{I}_{[\tau^i, \tau^i + \Delta^i]}(t), \quad \forall i \in \mathbb{S}, \\ & s^i(\tau^i) \geq 0, \quad \forall i \in \mathbb{S} \\ & 0 \leq u^i(t) \leq \bar{u}^i(t) \quad \forall i \in \mathbb{S} \end{aligned}$$

The objective is only to compute directionnal derivative of  $\Psi(\tau)$  in order to locally improve this planning.

## Continuous time-reformulation

Lets consider the following problem :

$$\varphi(x) = \min_{y \in F} \{f(x, y) \mid g(x, y) = 0\}$$

- How to compute  $\varphi'$  :

$$\varphi'(x) = f'_x(x, y(x)) + \langle f'_y(x, y(x)), y'(x) \rangle$$

- Lets  $p$  be the optimal multiplier of the constraint then, under mild assumptions see [BS00] we have :

$$\varphi'(x) = f'_x(x, y^*) + \langle p, g'_x(x, y^*) \rangle$$

## Continuous time-reformulation

- Let  $A(j)$  the set of power stations which are stopped when power station  $j$  starts its stop ;

$$A(j) = \{i \mid \tau^j \in [\tau^i, \tau^i + \Delta^i]\} \setminus \{j\}$$

- Let  $B(j)$  the set of power stations which are stopped when power station  $j$  ends its stop ;

$$B(j) = \{i \mid \tau^j + \Delta^j \in [\tau^i, \tau^i + \Delta^i]\} \setminus \{j\}$$

### Formal result (hand made)

$$\begin{aligned} \Psi'_{\tau^j}(\tau) &\sim c \left( d(\tau^j) - u^j(\tau^j) + \sum_{i \in \mathcal{C}A(j)} u^i(\tau^j), \tau^j \right) + c \left( d(\tau^j + \Delta^j) + \sum_{i \in \mathcal{C}B(j)} u^i(\tau^j + \Delta^j), \tau^j + \Delta^j \right) \\ &- c \left( d(\tau^j) + \sum_{i \in \mathcal{C}A(j)} u^i(\tau^j), \tau^j \right) - c \left( d(\tau^j + \Delta^j) - u^j(\tau^j + \Delta^j) + \sum_{i \in \mathcal{C}B(j)} u^i(\tau^j + \Delta^j), \tau^j + \Delta^j \right) \\ &- p^j(\tau^j + \Delta^j) \times r^j(\tau^j + \Delta^j) + p^j(\tau^j) \times r^j(\tau^j) \end{aligned}$$



## Advantages and limits

1. For theoretical justification of this hand-made interpretation see [PBB11];
2. The problem has been investigated in a the deterministic framework;
3. This analysis gives an idea about how the issue related to the displacement of a stop reads;

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# Dynamic programming re-formulation (discrete-time)

## Point of view of the Dynamic programming re-formulation

- The date of stops are stopping-time for the cost process induced by the energy management of all the reserves;
- We solve the Bellman equation;

## Dynamic programming re-formulation (discrete-time)

For sake of simplicity we consider the following deterministic discrete time model with only one stop to manage:

$$\min \sum_{t=0}^{T-1} c(d(t) - u(t), t) \mathbb{I}_{[0, \tau] \cup [\tau + \Delta, T]}(t) + c(d(t), t) \mathbb{I}_{[\tau, \tau + \Delta]}(t) + \psi(s(T), T)$$

$$u.s. \quad s(t+1) = s(t) - u(t)$$

$$s(\tau + \Delta) = s(\tau) + R$$

$$s(\tau) \in I$$

$$u(t) \in [0, \bar{u}]$$

## Dynamic programming re-formulation (discrete-time)

The continuation cost  $\Gamma(s, \theta)$  is the value of the following problem:

$$\min \sum_{t=0}^{T-1} c(d(t) - u(t), t) \mathbb{I}_{[\theta+\Delta, T]}(t) + c(d(t), t) \mathbb{I}_{[\theta, \theta+\Delta]}(t) + \psi(s(T), T)$$

$$u.c. \quad s(t+1) = s(t) - u(t)$$

$$s(\theta + \Delta) = s$$

Lets  $W(s, \theta)$  denotes the optimal value starting from state  $s$  at time  $\theta$ :

$$W(s, \theta) = \min_{u, \tau} \sum_{t=\theta}^{\tau} c(d(t) - u(t), t) + \Gamma(s(\tau) + R, \tau)$$

$$s(t+1) = s(t) - u(t)$$

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$$s(\theta) = s$$



## Dynamic programming re-formulation (discrete-time)

$\Gamma$  and  $W$  are solutions of the backward recursions:

$$\Gamma(s, \theta) = \min_u \{c(d(\theta + \Delta) - u, \theta + \Delta) + \Gamma(s - u, \theta + 1)\} + c(d(\theta), \theta) - c(d(\theta + \Delta), \theta + \Delta)$$

$$\Gamma(s, T) = \psi(s, T)$$

$$W(s, \theta) = \min \left\{ \min_u c(d(\theta) - u, \theta) + \Gamma(s + R, \theta), \min_u c(d(\theta) - u, \theta) + W(s - u, \theta + 1) \right\}$$

And  $\tau^*(s, \theta)$  is the first time  $\beta$  such:

$$\Gamma(s^*(\beta) + R, \beta) + \min_u c(d(\beta) - u, \beta) = W(s^*(\beta), \beta) \text{ with } s^*(\theta) = s$$



# Advantages and limits of dynamic programming

- Dynamic programming provides feedback decisions;
- It is relatively easy to consider the markovian case;
- Dimensionality is still the main weakness of dynamic programming;



# Dynamic programming re-formulation (continuous time)

## Point of view of the Dynamic programming re-formulation

- The date of stops are stopping-time for the cost process induced by the energy management of all the reserves;
- We solve the HJB equation;

## Dynamic programming re-formulation (continuous time)

For sake of simplicity we consider the following model with only one stop to manage:

$$\min_{u, \tau} \int_0^{\tau} c(d(t) - u(t), t) dt + \int_{\tau}^{\tau+\Delta} c(d(t), t) dt + \int_{\tau+\Delta}^T c(d(t) - u(t), t) dt + \psi(s(T), T)$$

$$u.c. \quad \dot{s}(t) = -u(t)$$

$$s(\tau) \in I$$

$$s(\tau + \Delta) = s(\tau) + R$$

$$u(t) \in [0, \bar{u}(t)]$$

## Dynamic programming re-formulation (continuous time)

$\Gamma$  denotes the continuation cost after  $\tau$ :

$$\forall s \in I + R, \quad \Gamma(s, \theta) = \min_{u(t)} \left\{ \int_{\theta+\Delta}^T c(d(t) - u(t), t) dt + \psi(s(T), T) \right. \\ \left. + \int_{\theta}^{\theta+\Delta} c(d(t), t) dt + \psi(s(T), T) \right\}$$

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$$u(t) \in [0, \bar{u}(t)]$$

## Dynamic programming re-formulation (continuous time)

The Bellman function  $W(x, z, \beta)$  satisfies the HJB equation:

$$\frac{\partial W}{\partial \beta}(x, z, \beta) + \min_v \left\{ c(d(\beta z) - v, \beta z) - \frac{\partial W}{\partial x}(x, z, \beta) v z \right\} = 0$$

$$W(x, z, 1) = \Gamma(x + R, z)$$

Then the optimal time is to stop is:

$$\tau^*(s, t) \in \arg \min_z W(s, z, t) \quad (1)$$

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Each approach has raised a difficulty. Next step will consist in combining these two methodologies as in [BCG10].





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