Supply function equilibrium in electricity markets

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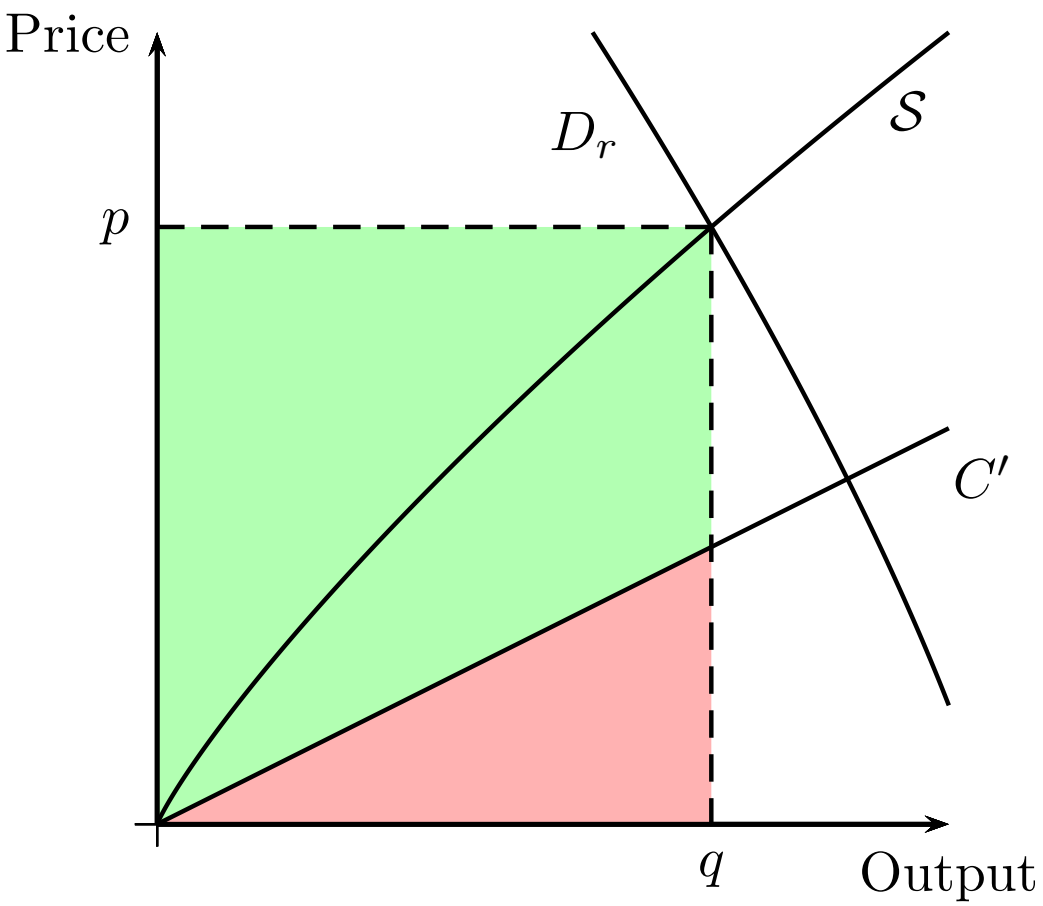
SFE and electricity markets

- Uncertainty in demand and renewable output
- Concentrated industry with high barriers to entry
- Generators are strategic profit-maximizers
- Offer stacks represented by curves
- SFE models have implications for market design and regulation

Optimal Offers



Optimal Offers

For each possible residual demand curve D_r,

$$\max_{q,p} \mathbb{E}\left[pq - C\left(q\right)\right]$$
$$\text{subject to } q = D_r\left(p\right).$$

Optimality condition

$$\frac{q}{p - C'\left(q\right)} = -D_r'.$$

Stochastic residual demand

Generators optimize supply functions for all possible realizations of a stochastic demand shock ε.
Residual demand is then

$$D_{r,i}(p;\varepsilon) = \varepsilon - \sum_{j \neq i} q_j(p).$$

Equilibrium

- Supply function equilbrium is a Nash equilibrium in supply functions
- First-order conditions are a system of differential equations
- Under standard assumptions, a system of *ordinary* differential equations

State of SFE

- Survey by Holmberg and Newbery (2010):
  - Forward contracts reduce market power
  - Equilibrium selection by capacity constraints, price cap
  - Empirical support for theory
- recent progress on:
  - uniqueness: Genc and Reynolds (2011)
  - existence: Anderson (2013)
  - pay-as-bid pricing: Anderson, Holmberg and Philpott (2013)
  - effect of transmission constraints: Wilson (2008), Holmberg and Philpott (2015)

Thesis contributions

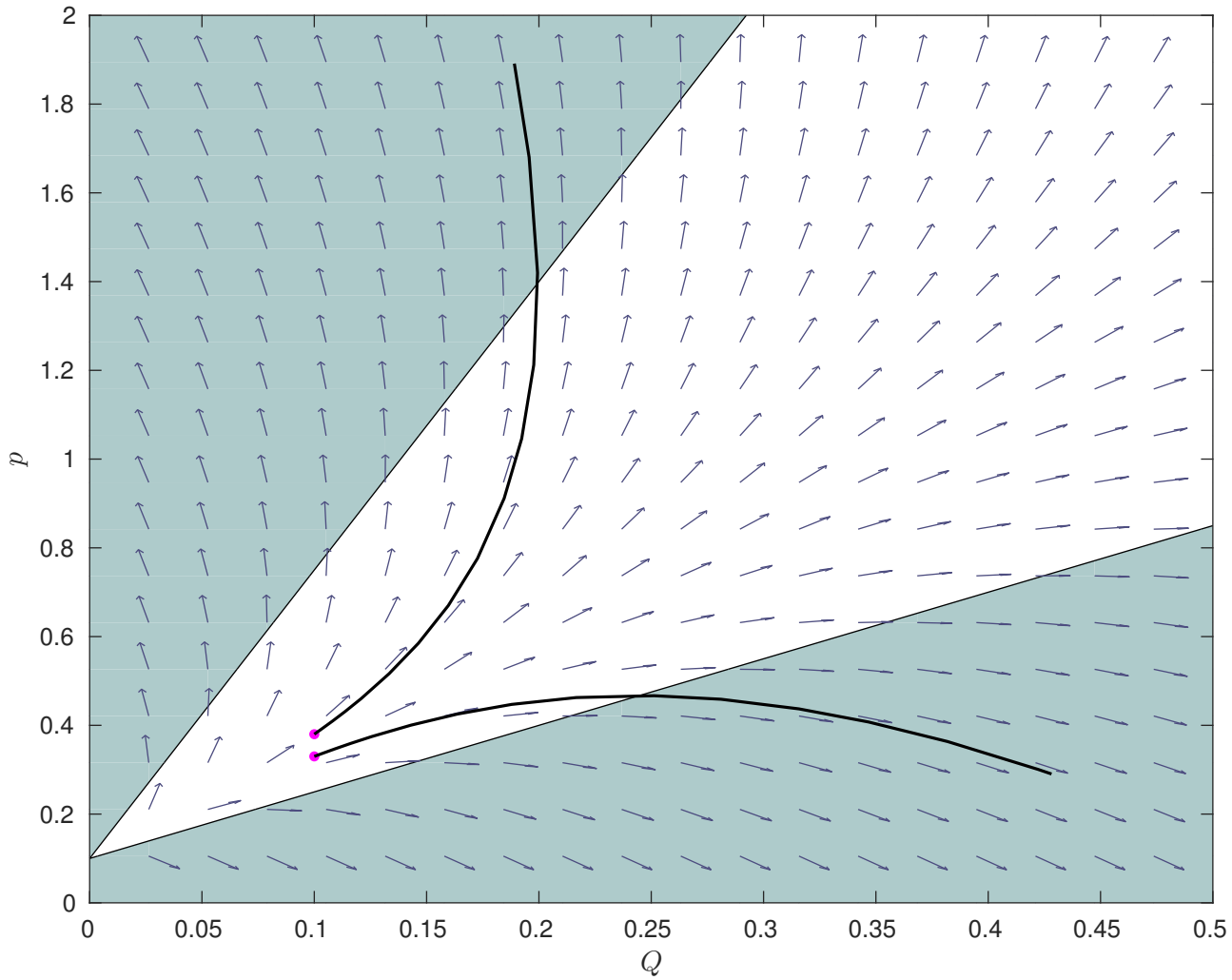
- Numerical methods
- More types of side payment
- Calculation of SFE over transmission constraints

Numerical challenges

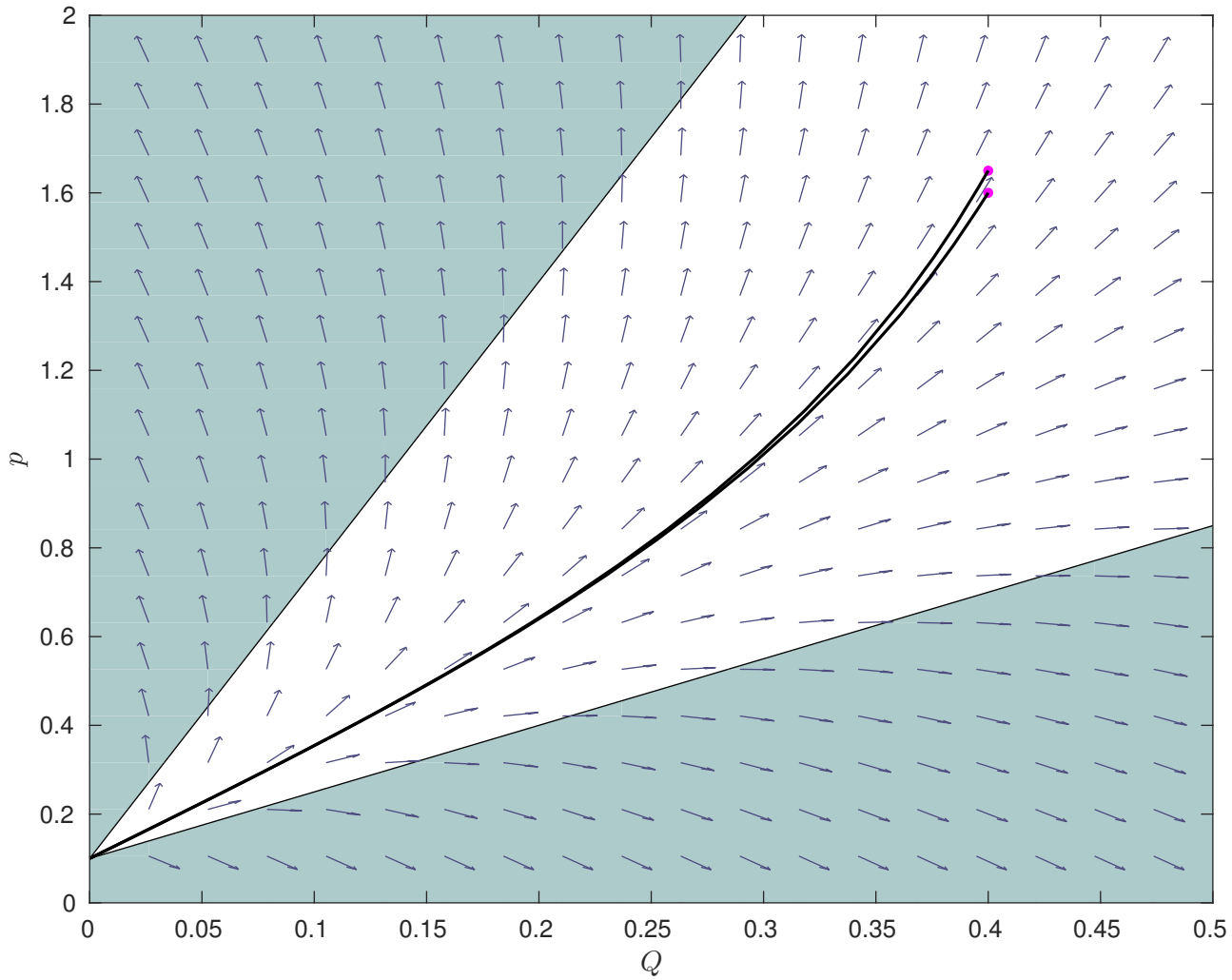
System of ordinary differential equations can be difficult to solve:

- High-order
- Non-linear
- Unstable

Instability



Instability



Contracts

- Forward sales (contracts for differences)
- Option contracts
- Retail sales (fixed-price variable volume)
- Income from non-dispatchable renewables (wind, geothermal)
- Financial transmission rights

Q: How do contract arrangements affect spot market offers and prices?

Financial derivatives

For financial derivatives, obligation from forward sales and options is a function of price

$$B(p),$$

the **portfolio obligation function**.

Optimal Offers with financial derivatives

For each possible residual demand curve D_r,

$$\max_{q,p} \mathbb{E}\left[pq - C\left(q\right)\right]$$
$$\text{subject to } q = D_r\left(p\right).$$

Optimality condition

$$\frac{q}{p - C'\left(q\right)} = -D_r'.$$

Optimal offers with financial derivatives

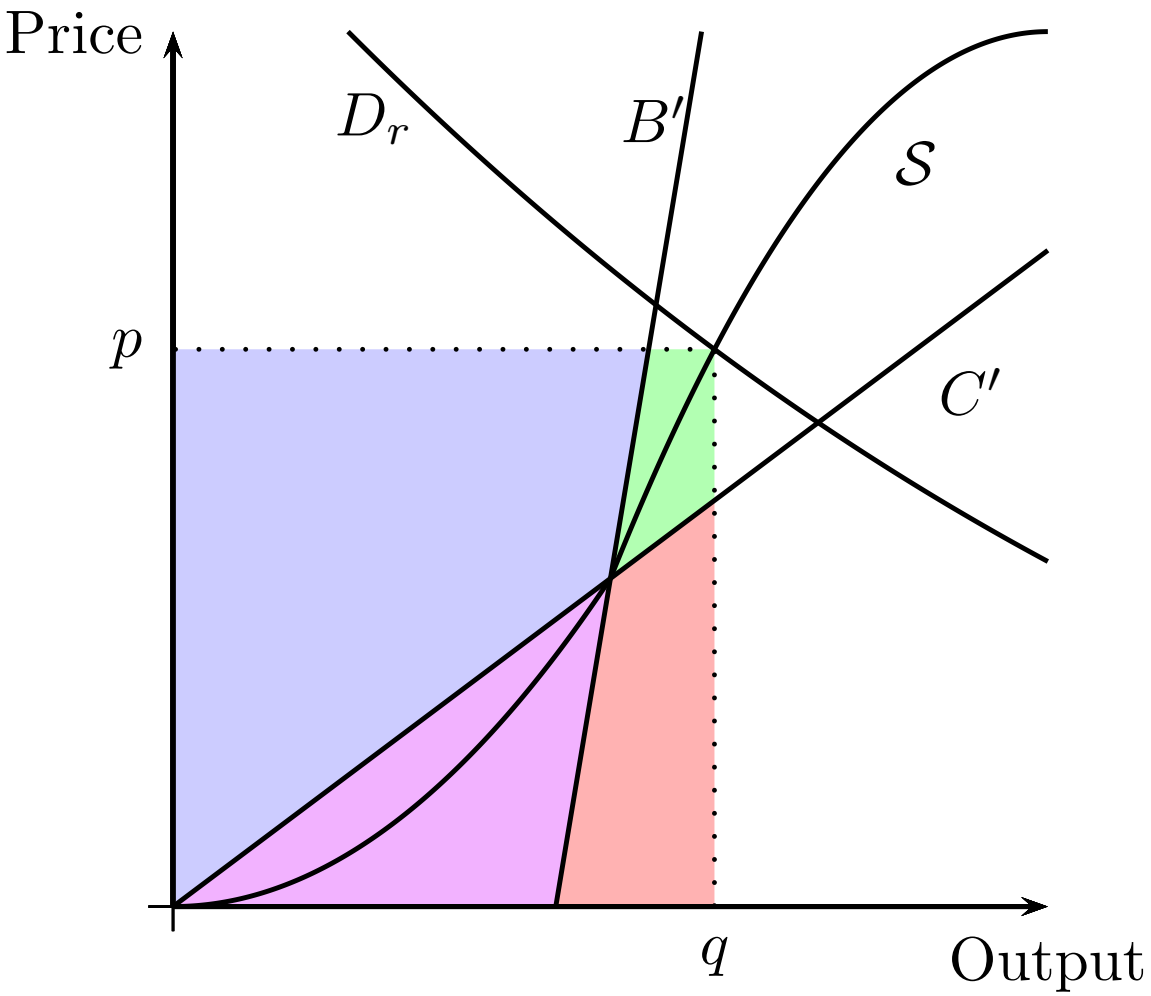
For each possible residual demand curve D_r,

$$\max_{q,p} \mathbb{E}\left[pq - C(q) - \boldsymbol{B}(\boldsymbol{p})\right]$$
$$\text{subject to } q = D_r(p).$$

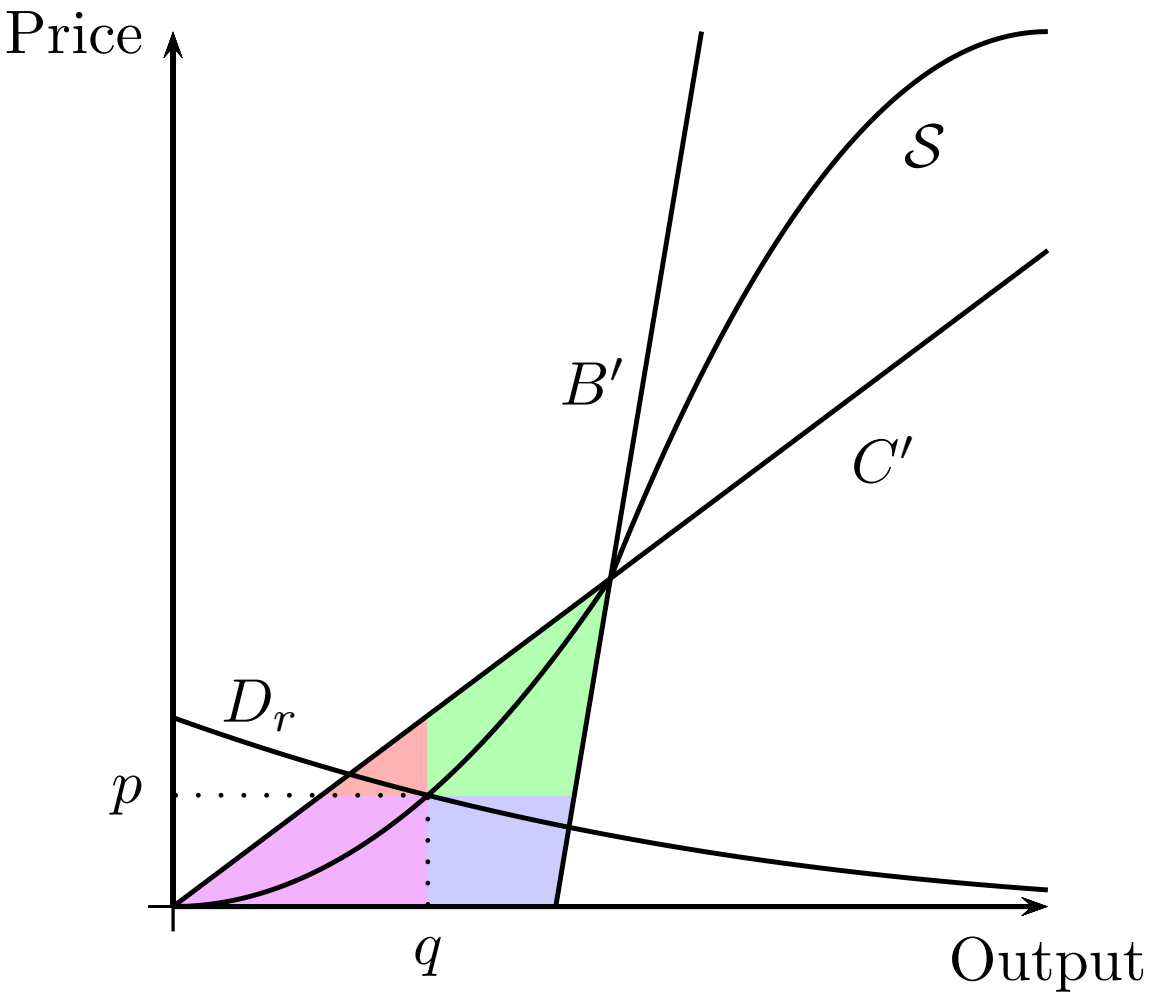
Optimality condition

$$\frac{q - \boldsymbol{B}'(\boldsymbol{p})}{p - C'(q)} = -D_r'.$$

Optimal offers with financial derivatives

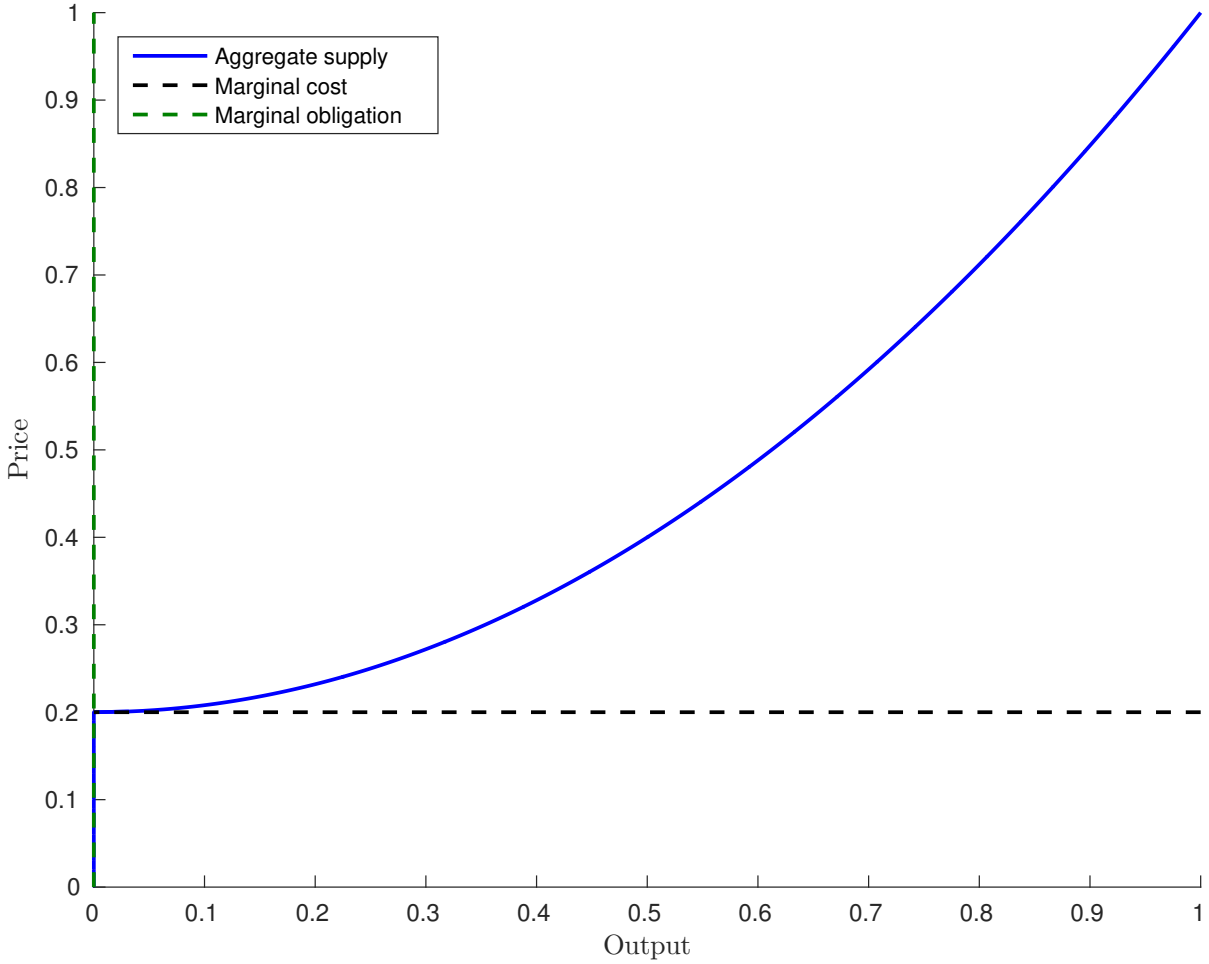


Optimal offers with financial derivatives



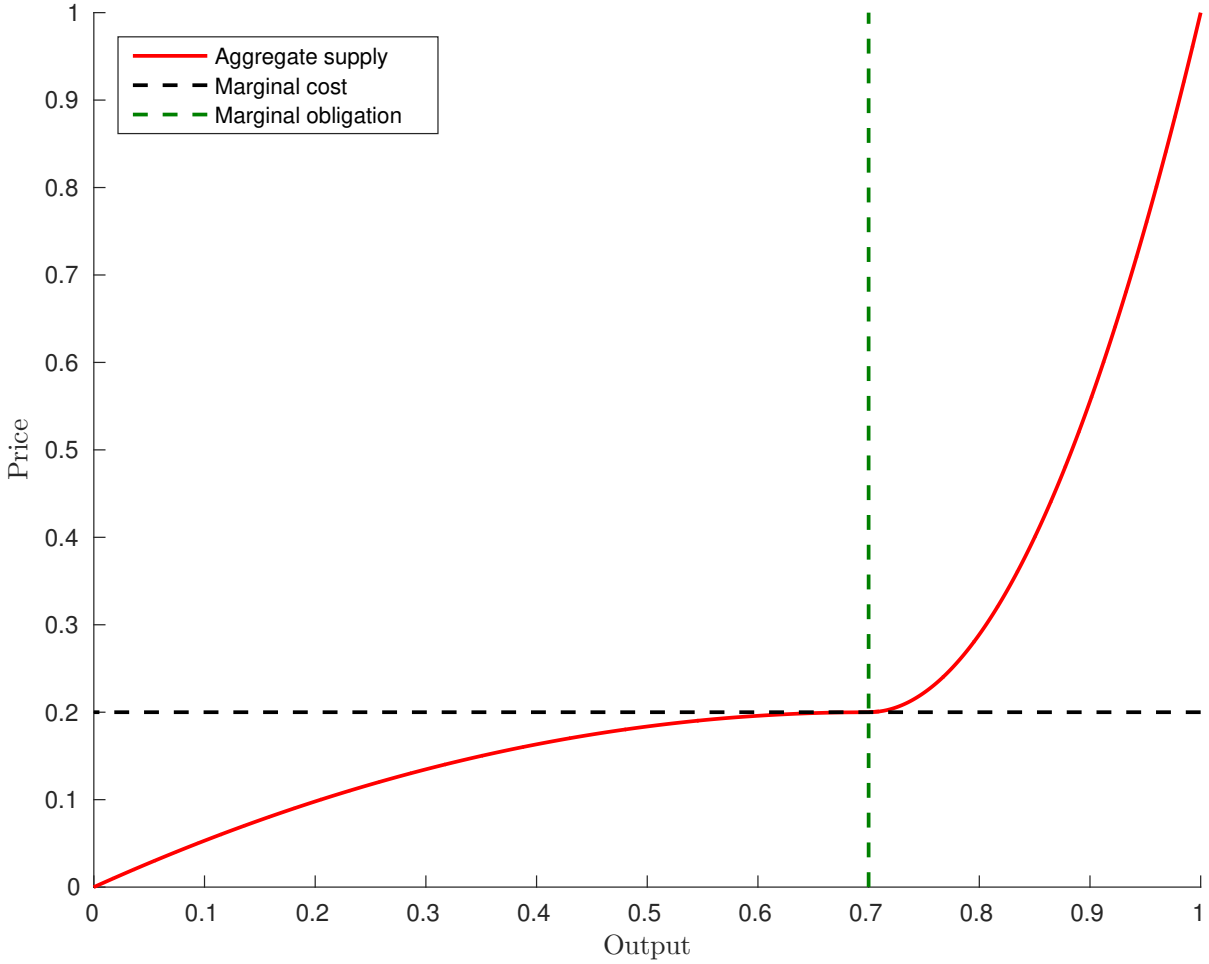
Forward sales (CfD)

Without contracts



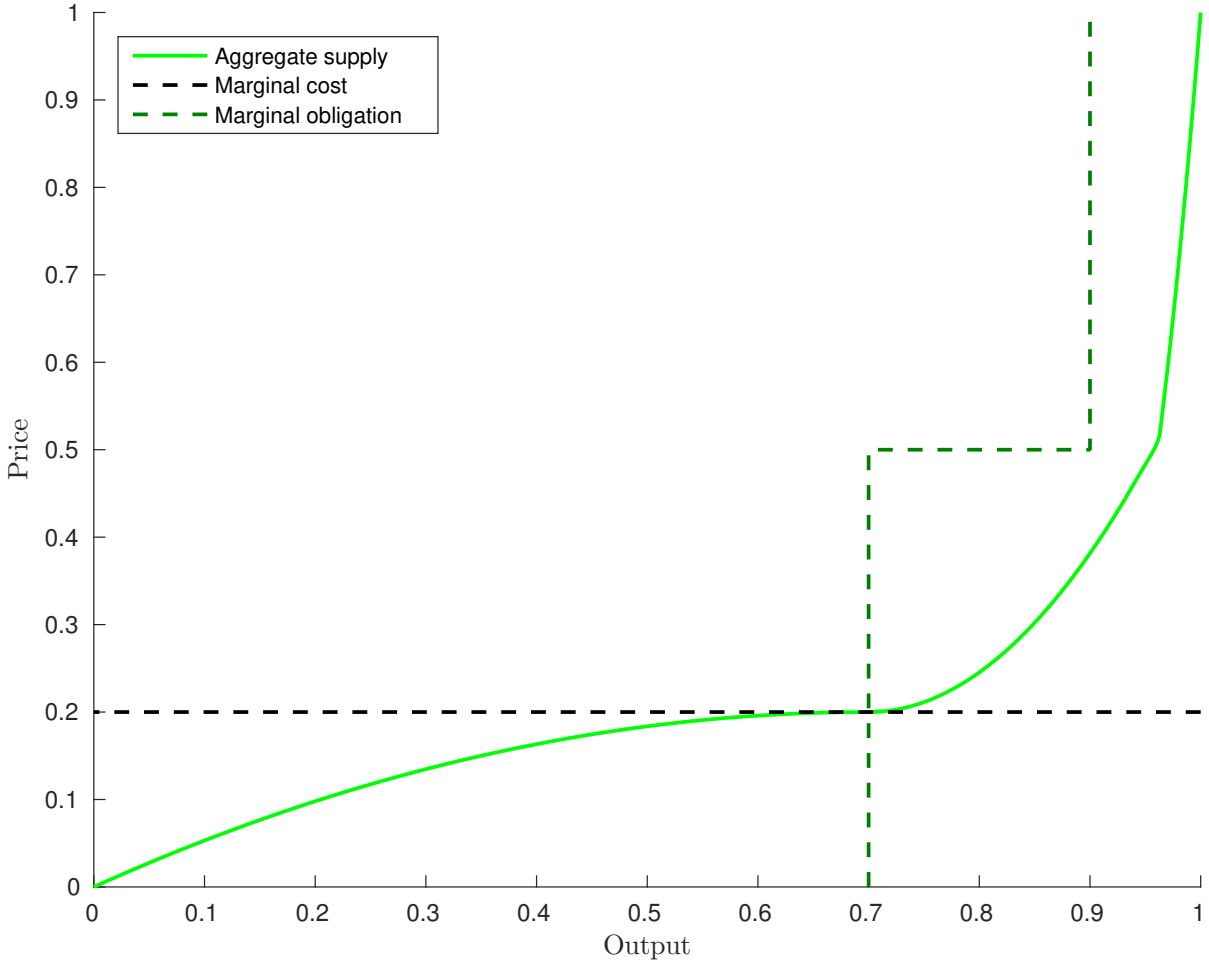
Forward sales (CfD)

With contracts



Cap options

With CfDs and some call options



Linear regression model of retail load

- Oligopolist's retail load x is correlated with market demand shock ε.
- Integrated gentailer must buy at spot to cover retail obligation

$$p \cdot x$$

- Multivariate normal distribution has nice property (linear regression)

$$\mathbb{E}\left[x|\varepsilon\right] = \mu + \lambda\varepsilon.$$

Optimal offers with retail obligations

For each possible residual demand curve D_r,

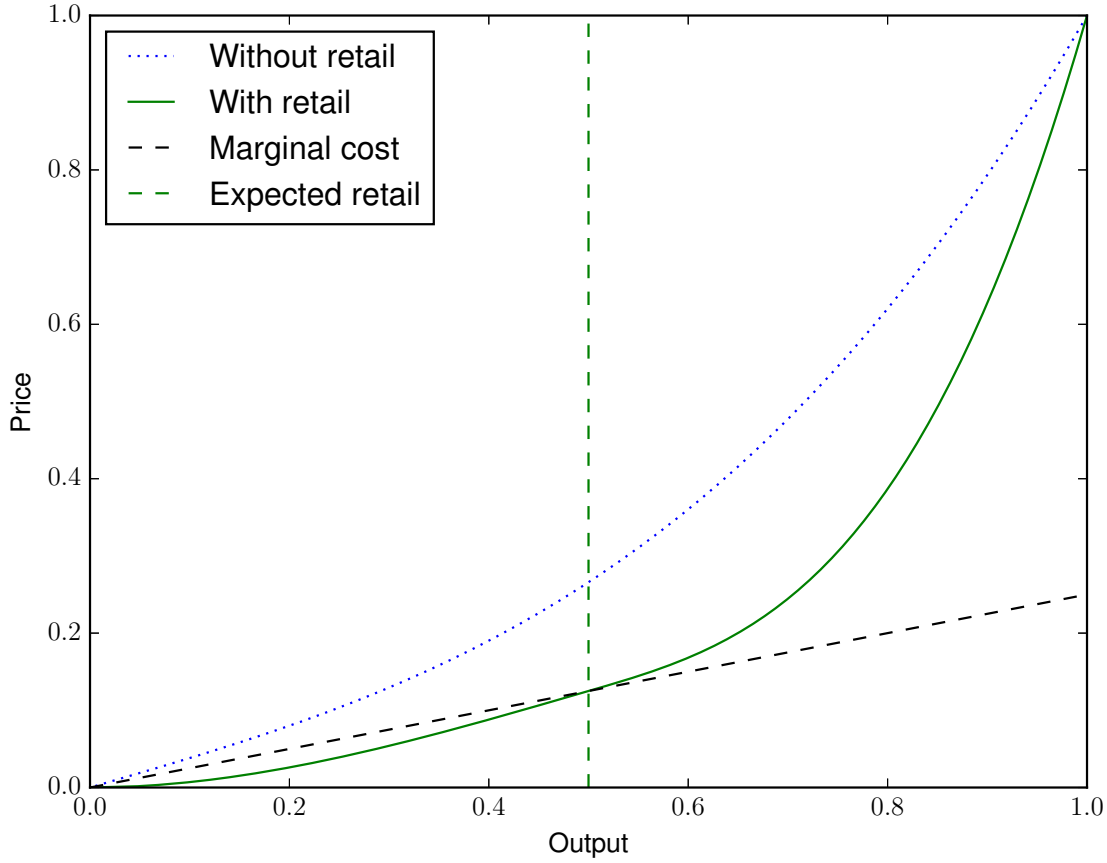
$$\max_{q,p} \mathbb{E}\left[pq - C\left(q\right) - \boldsymbol{p} \cdot \boldsymbol{x}\right]$$
$$\text{subject to } q = D_r\left(p\right).$$

Optimality condition

$$\frac{q - \mathbb{E}\left[\boldsymbol{x} | \varepsilon\left(\boldsymbol{q}, \boldsymbol{p}\right)\right]}{p - C'\left(q\right)} = -D_r'.$$

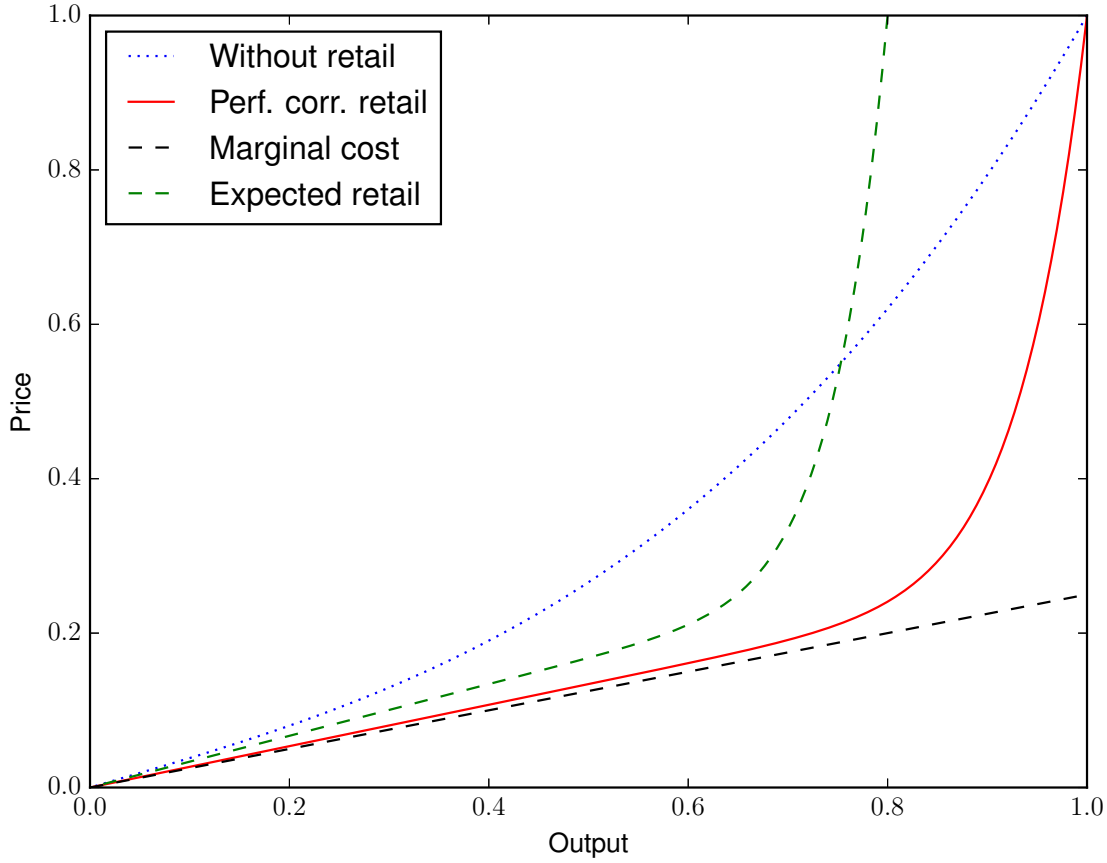
Retail obligations

Independent retail loads are like CfDs: $\rho = 0$



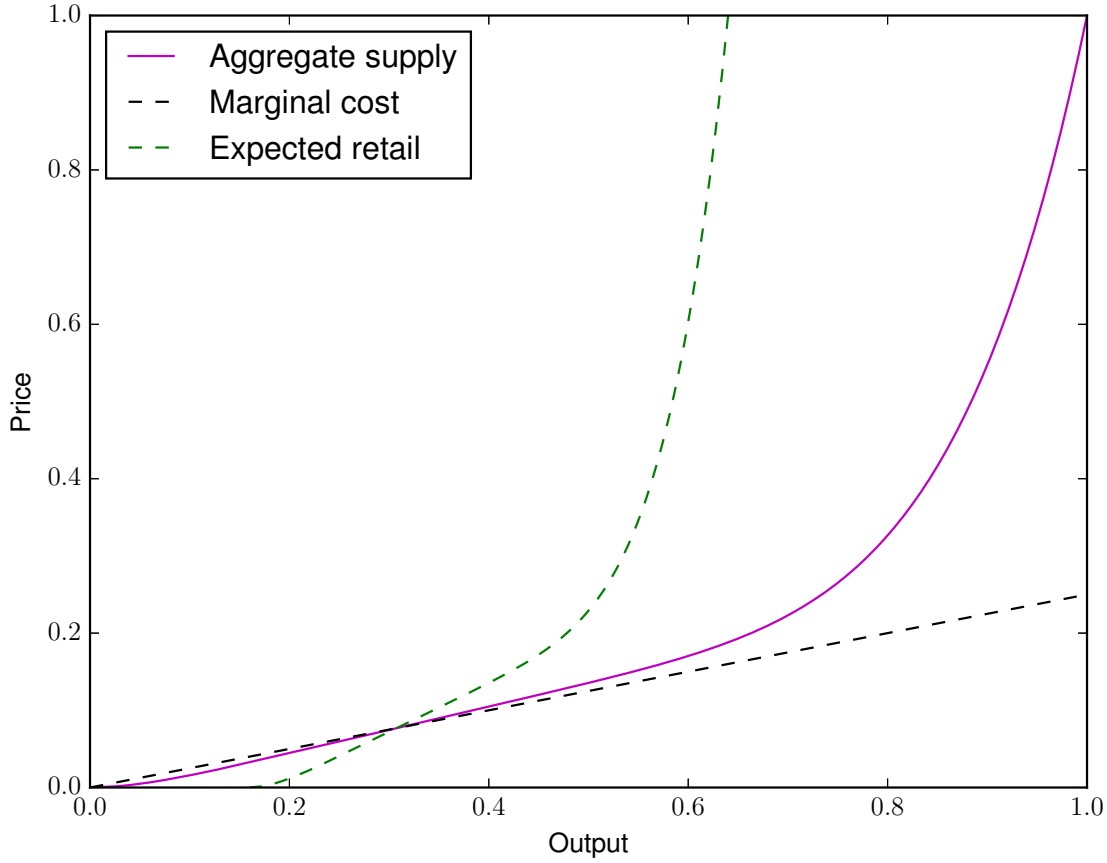
Retail obligations

Perfectly correlated retail loads: $\rho = 1$

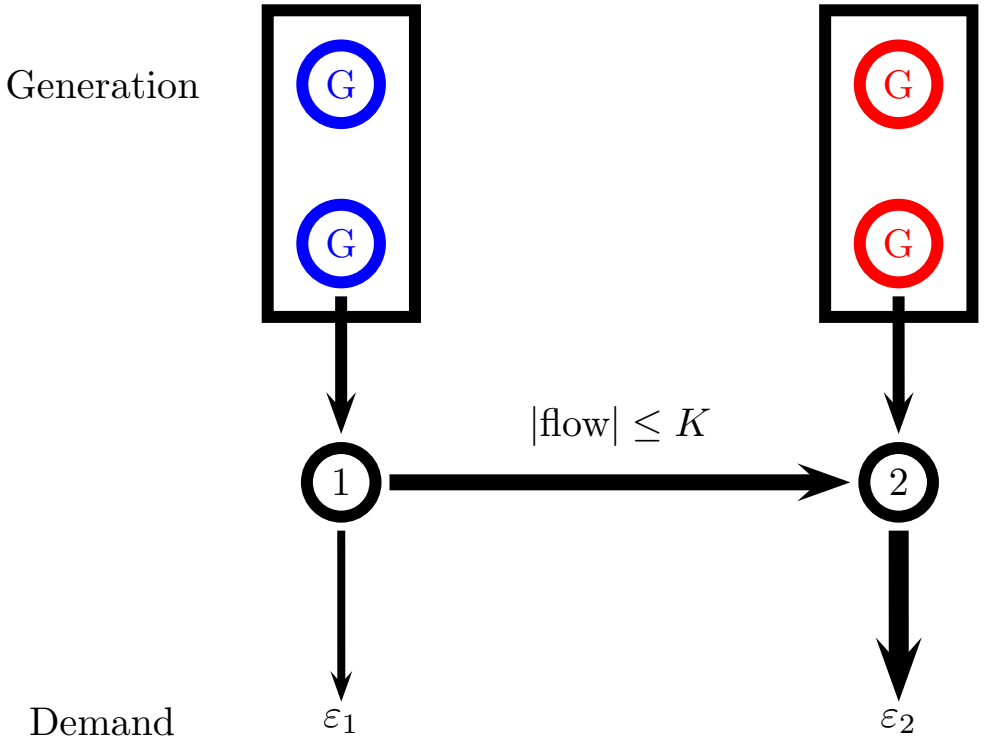


Retail obligations

Linear regression model with correlation: $\rho = 0.5$



Competition over a network



Q: How is competition between generators affected by transmission constraints?

Competition over a network

Generators submit offers *ex ante*, before demand levels are known. Residual demand slope

$$D'_{r,i}\left(q_i, p\right) = -\sum_{j \neq i} P_{i,j}\left(q_i, q_j\left(p\right), \ldots\right) \cdot q'_j\left(p\right)$$

weights $P_{i,j}$ are probability of free flow, conditional on quantities offered at local price.

A tractable model

- Constant and equal marginal costs
- At most two *types* of generator
- Inelastic demand
- Bivariate normal demand shocks

Together these give a planar autonomous system of ODE, under change of variable $t(p)$

Optimal offers in a network

Equilibrium condition: all generators of types $i = 1, 2$ offer $Q_i(t)$ that solves

$$\boldsymbol{A}(Q_1, Q_2) \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}.$$

Determinant and eigenvectors of $\boldsymbol{A}$ determine stable direction of integration

Example

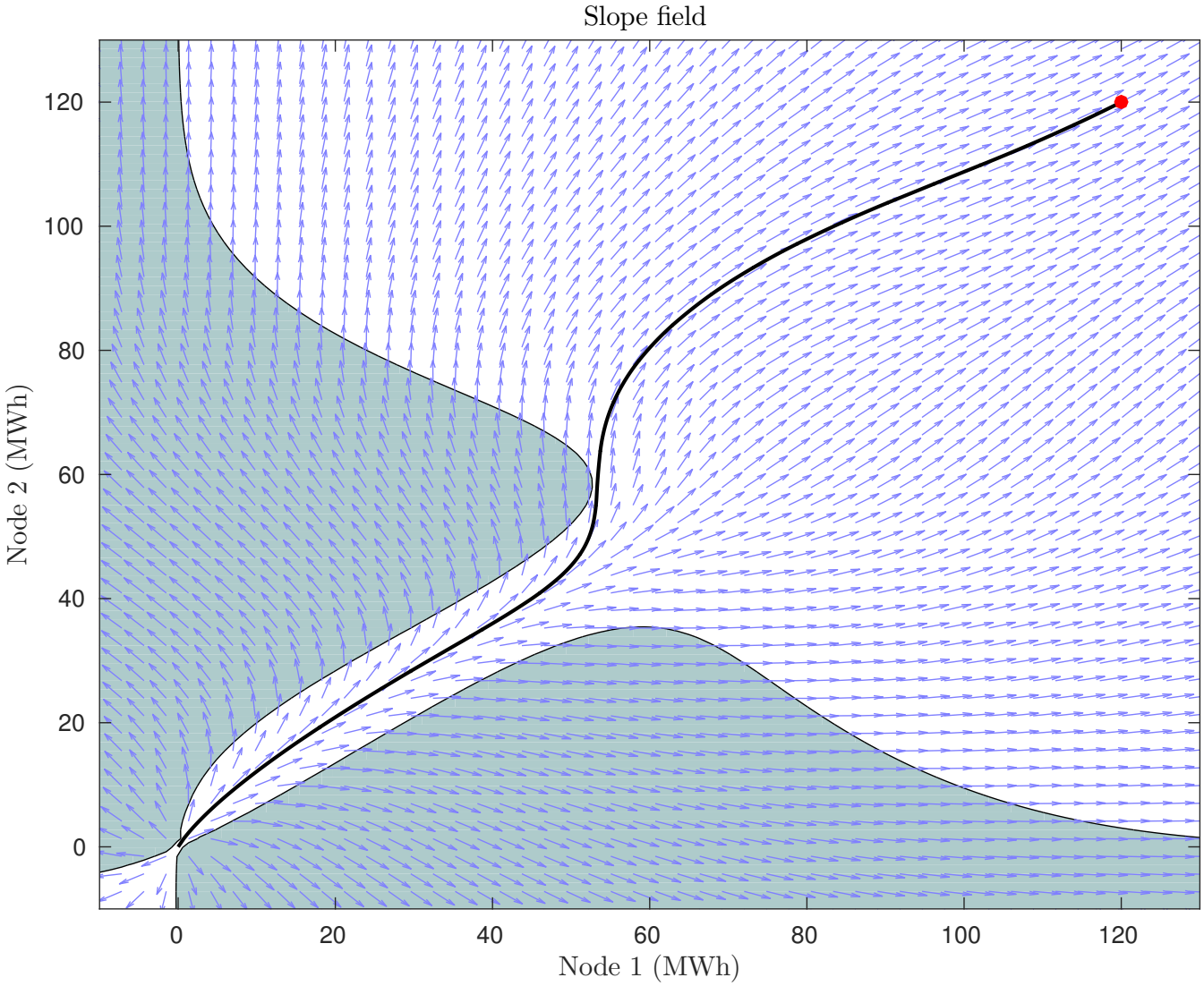
Demand shock distribution independent normals with

$$\mu_1 = 100\text{MWh}$$
$$\mu_2 = 100\text{MWh}$$
$$\sigma_1 = 60\text{MWh}$$
$$\sigma_2 = 40\text{MWh}.$$

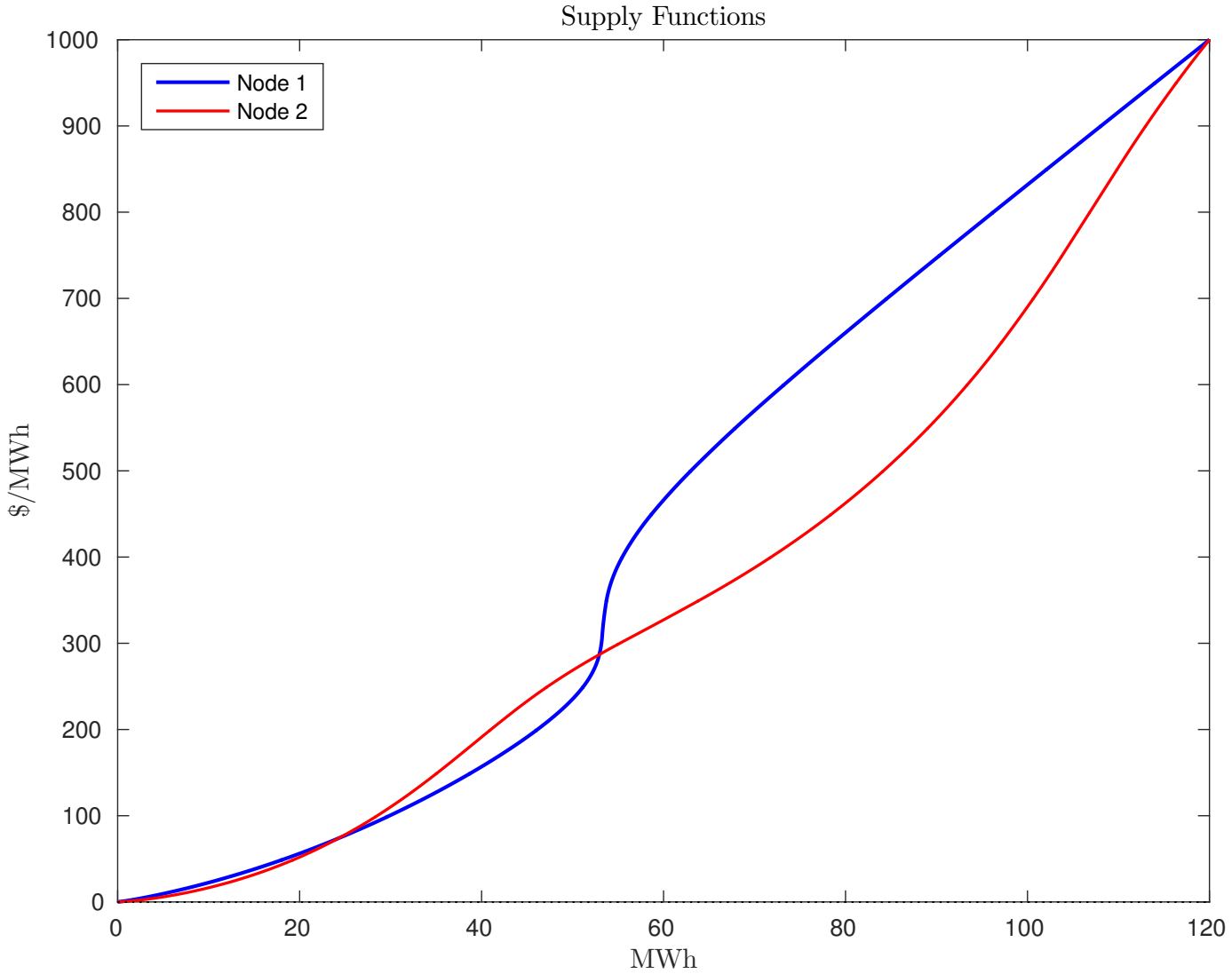
Transmission line capacity 25MWh.
Two firms in each node with production capacities

$$\overline{Q}_1 = 120$$
$$\overline{Q}_2 = 120.$$

Direction Fields



SFE


Supply Functions
Node 1
Node 2
$/MWh
MWh
0
100
200
300
400
500
600
700
800
900
1000
0
20
40
60
80
100
120

Example

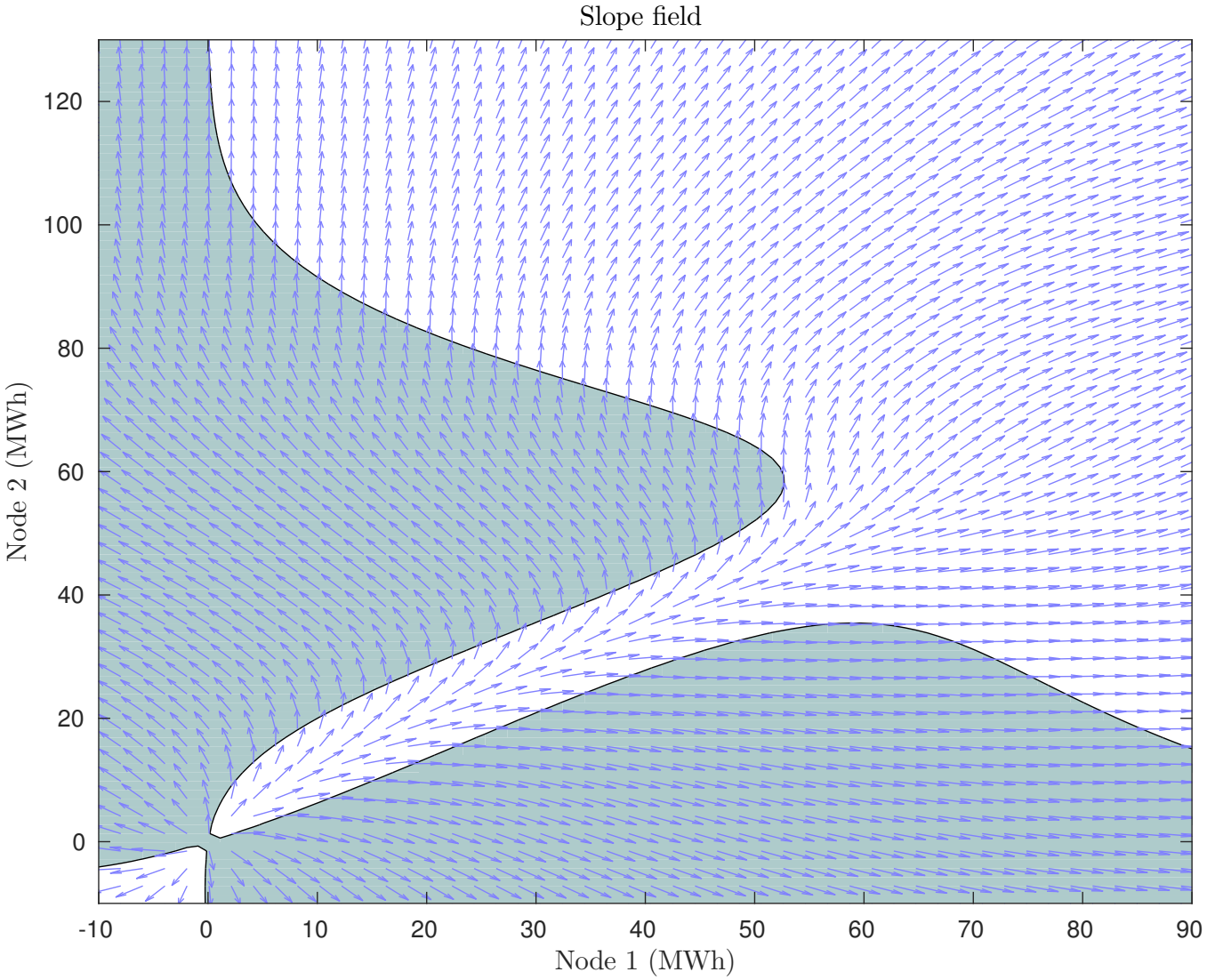
Demand shock distribution independent normals with

$$\mu_1 = 100\text{MWh}$$
$$\mu_2 = 100\text{MWh}$$
$$\sigma_1 = 60\text{MWh}$$
$$\sigma_2 = 40\text{MWh}.$$

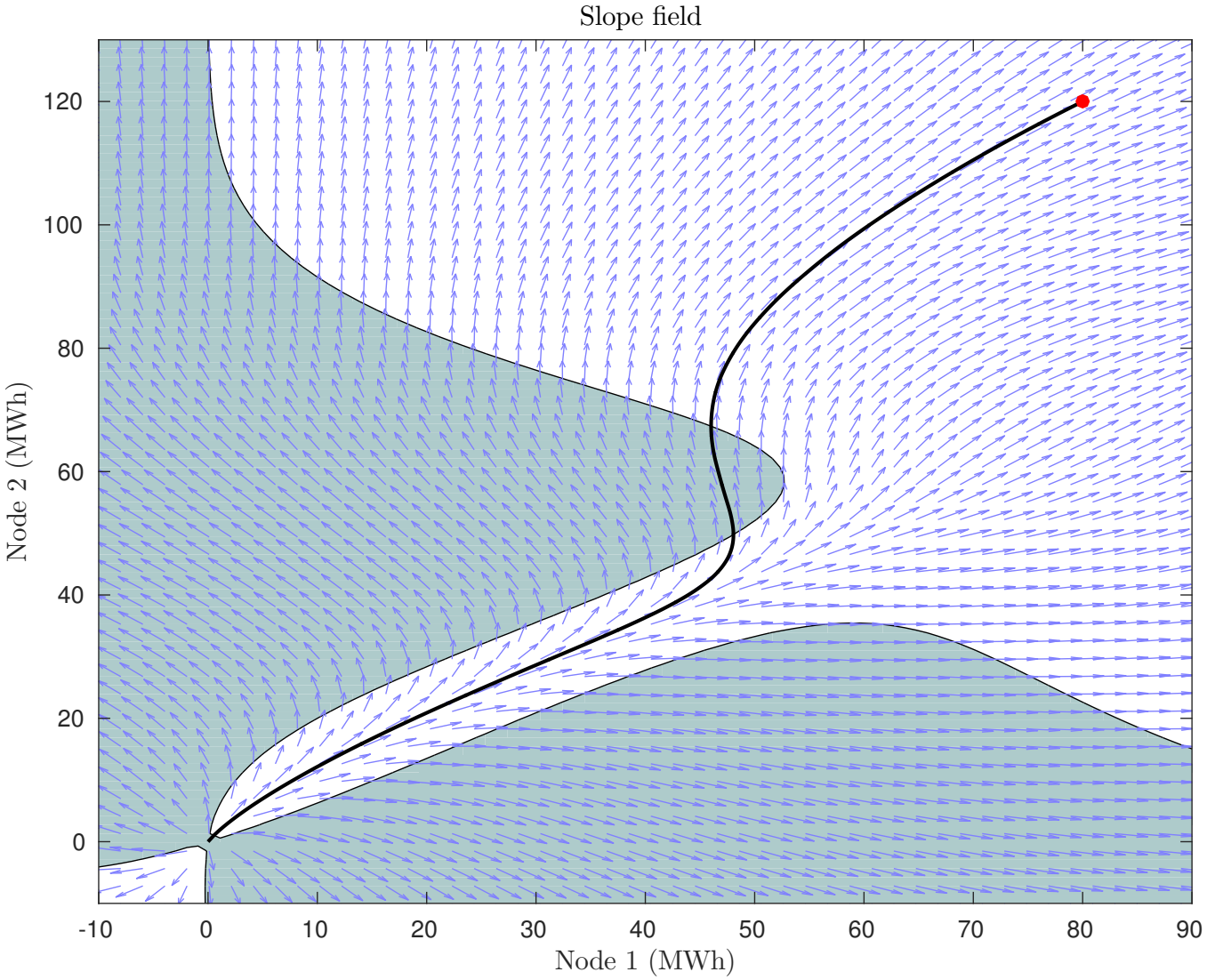
Transmission line capacity 25MWh.
Two firms in each node with production capacities

$$\overline{Q}_1 = 120$$
$$\boldsymbol{\overline{Q}_2 = 80.}$$

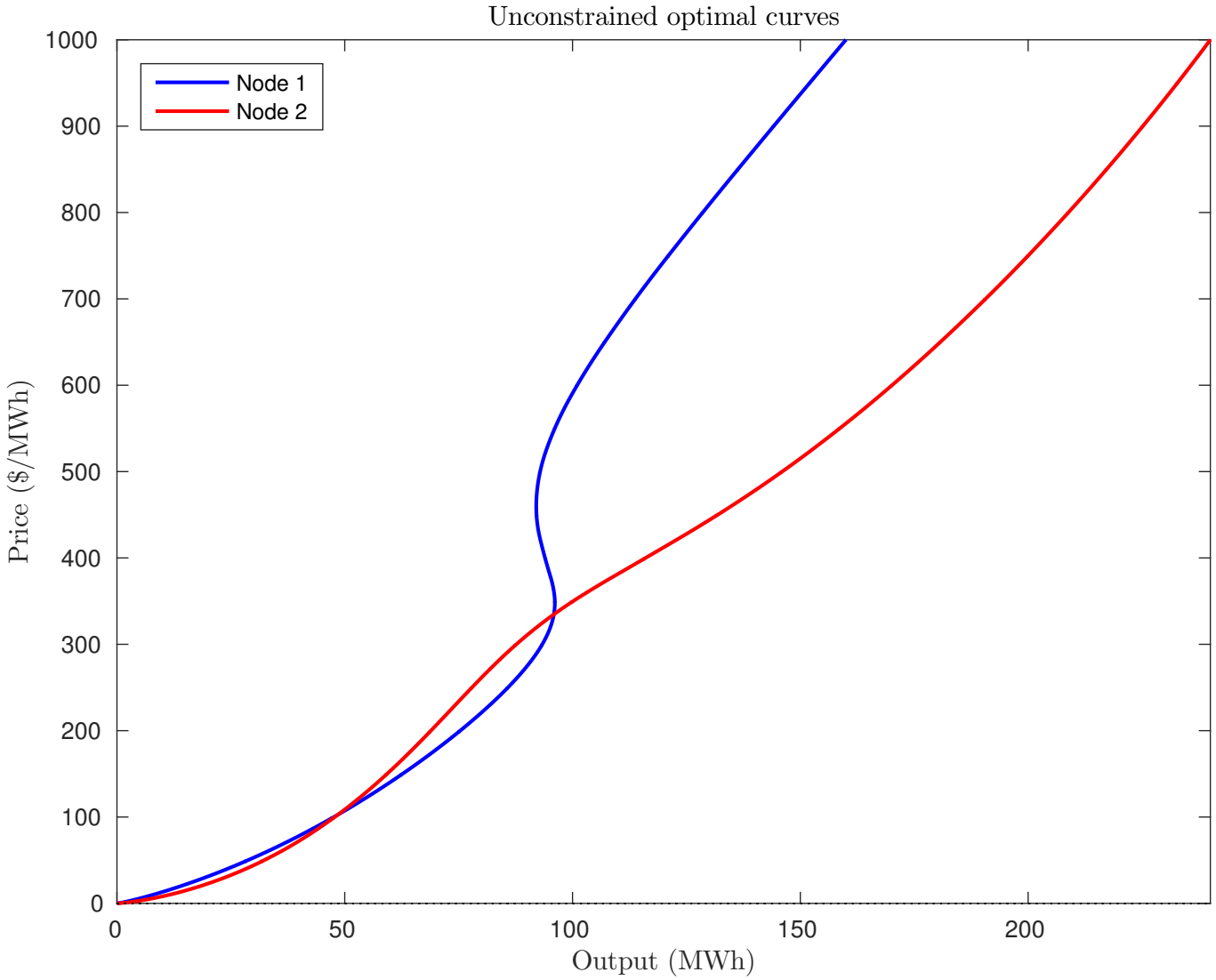
Direction Fields



Direction Fields



Non-monotone supply functions



Ironing

Unconstrained equilibrium condition

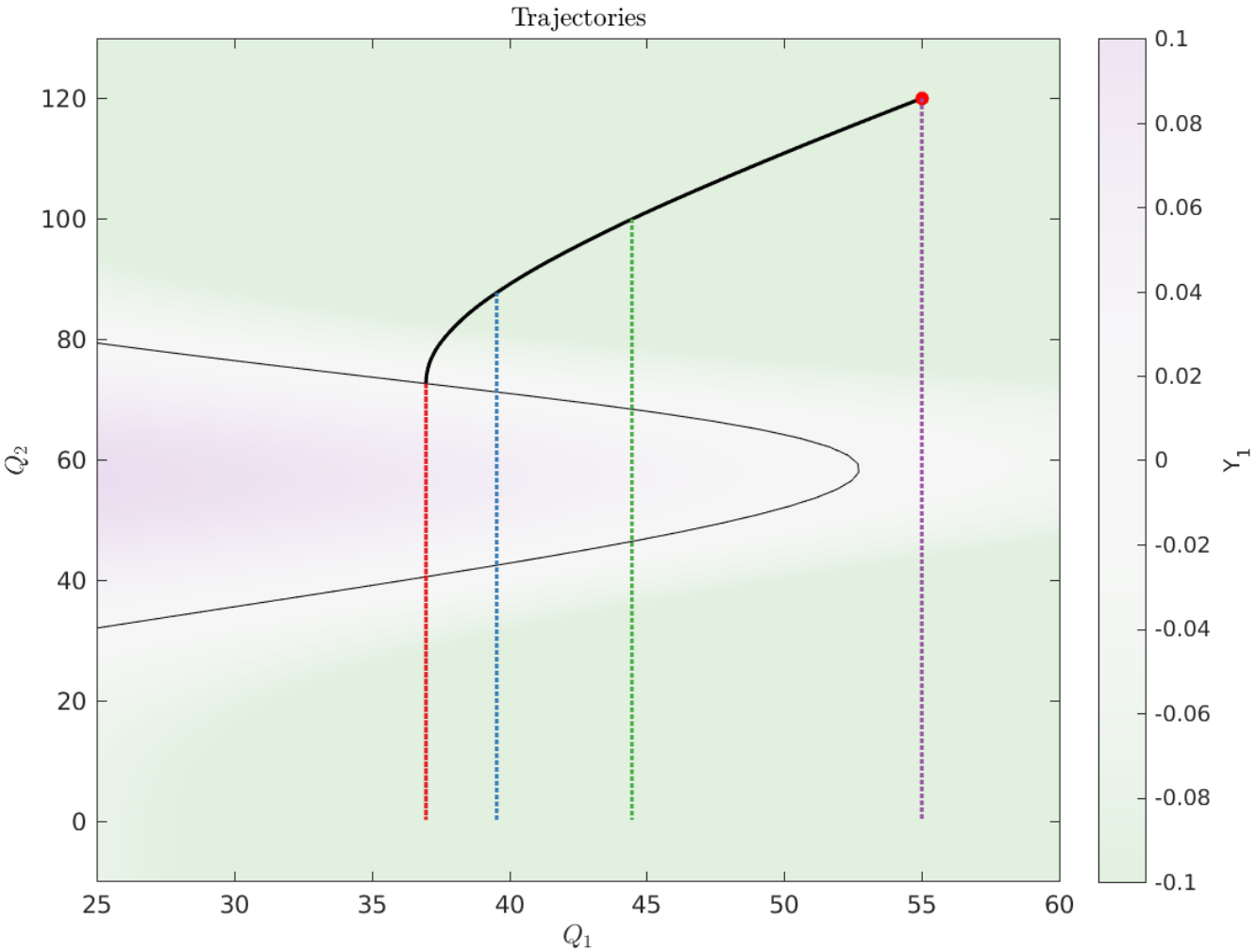
$$Z\left(Q\left(t\right)\right) = \boldsymbol{A} \cdot \dot{Q} - Q = 0 \qquad \forall t.$$

Ironing condition, if $\dot{Q}_m = 0$ on $[t_0, t_1]$, then

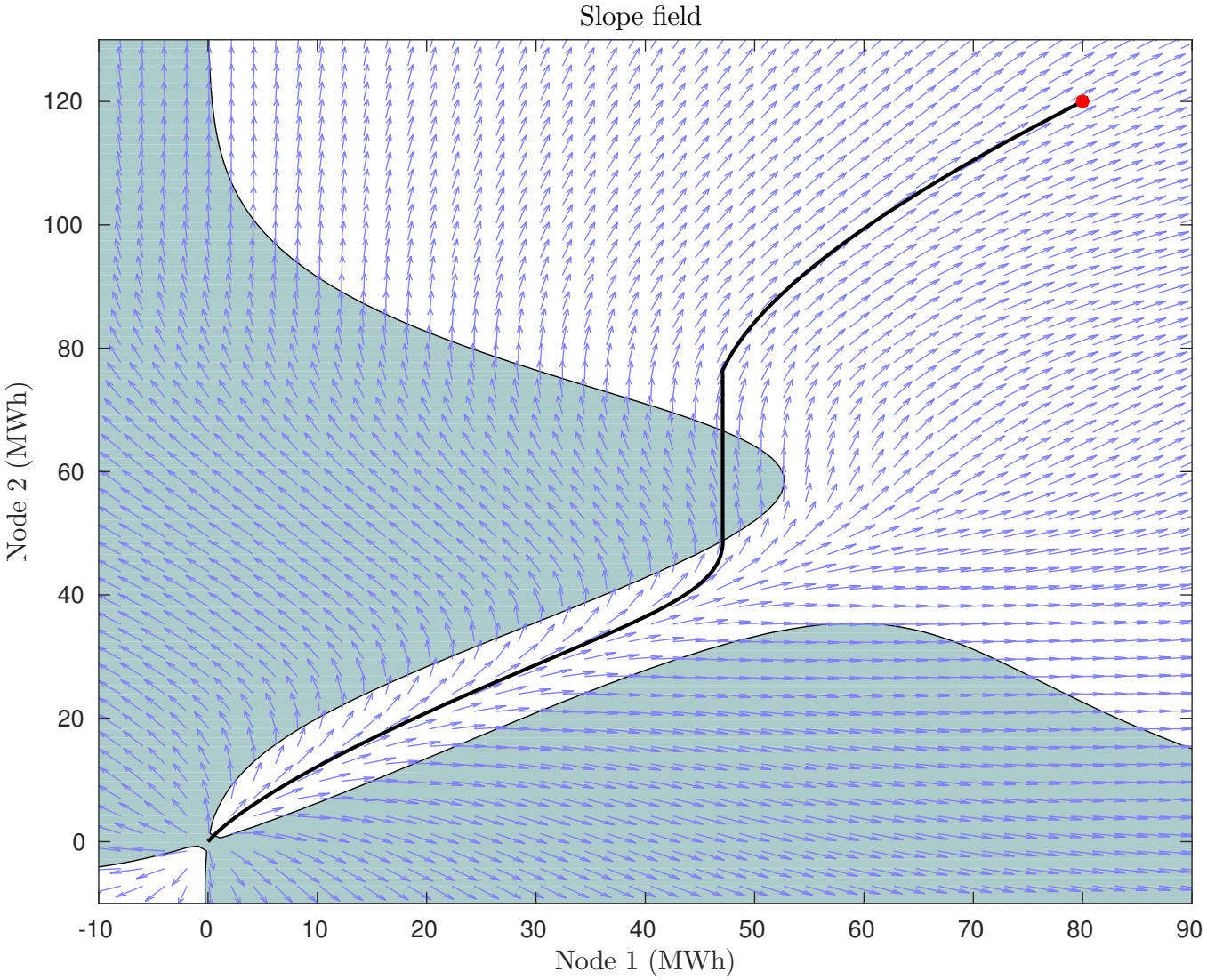
$$\int_{t_0}^{t_1} Z_i\left(Q\left(t\right)\right)\, dt = 0$$

+ some rules for corners.jgj

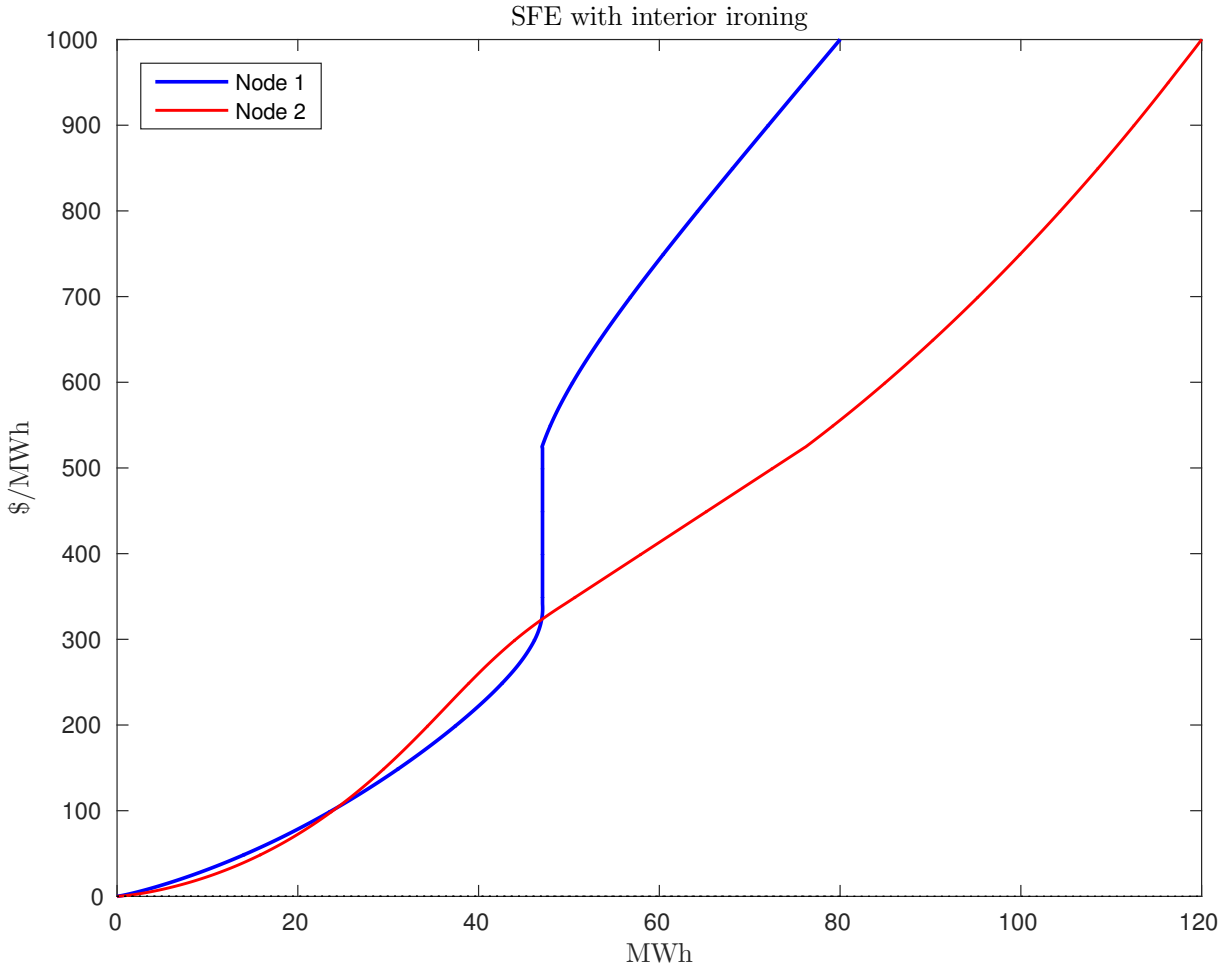
Ironing


Trajectories
120
100
80
60
40
20
0
Q_2
25
30
35
40
45
50
55
60
Q_1
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
Y_1

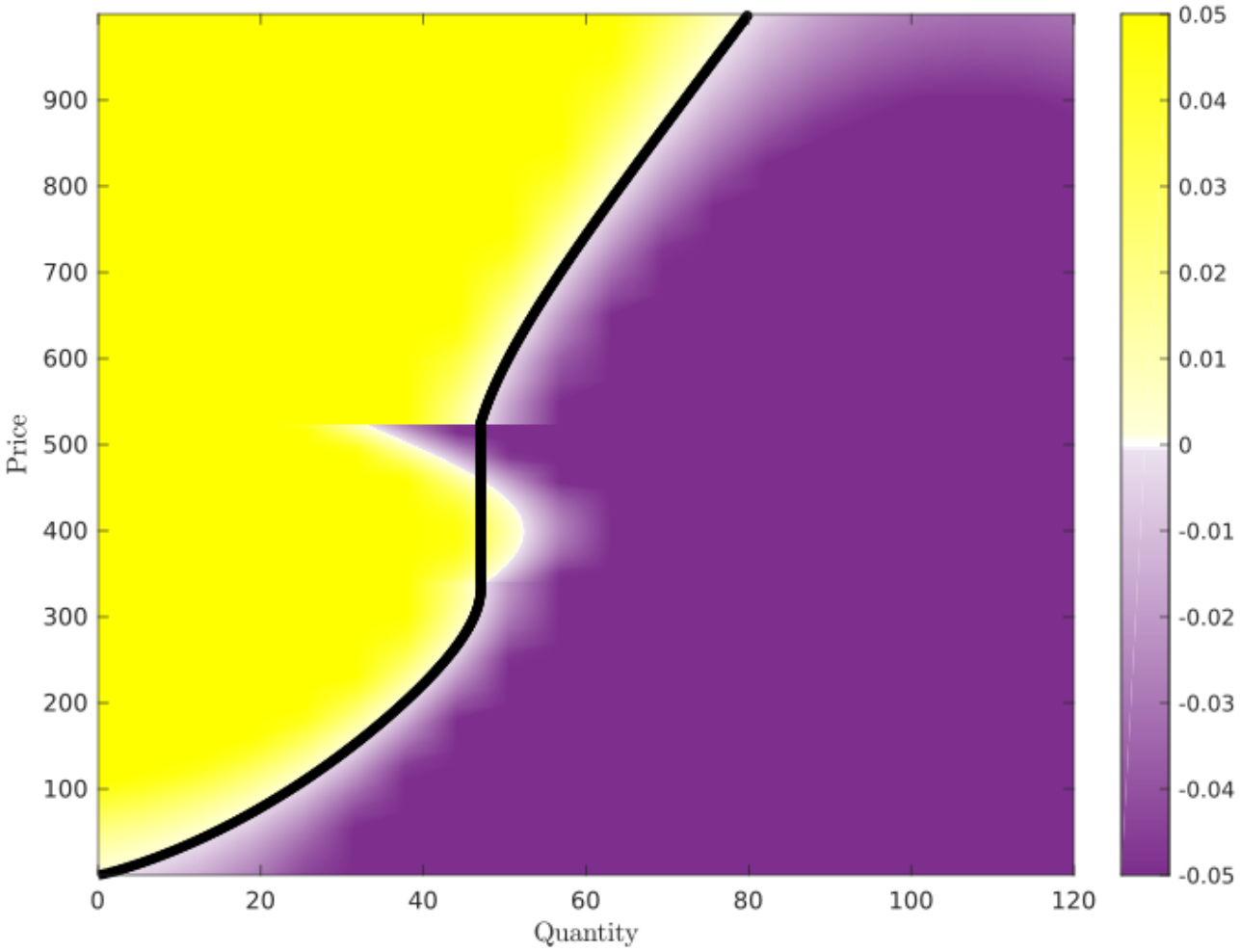
Ironed trajectory



Ironed supply functions



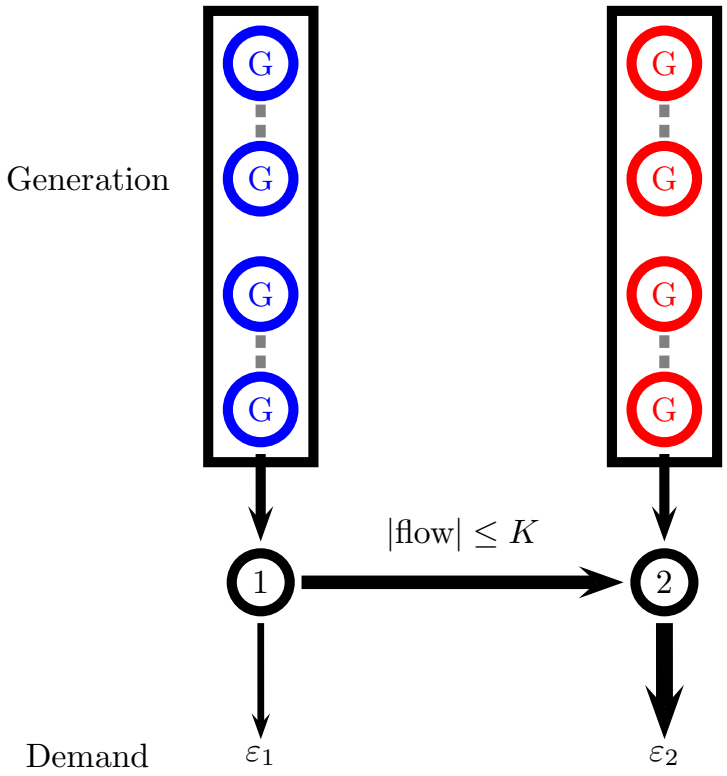
Verify second-order conditions



Applications of network model

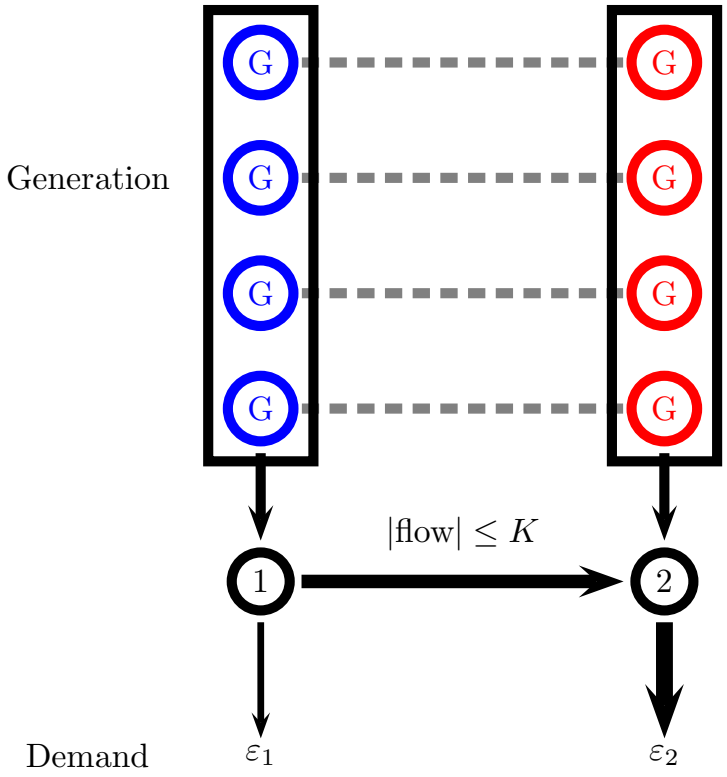
- Plants at other nodes
- Locational CfDs
- Financial transmission rights
- Virtual asset swaps

Local duopolies



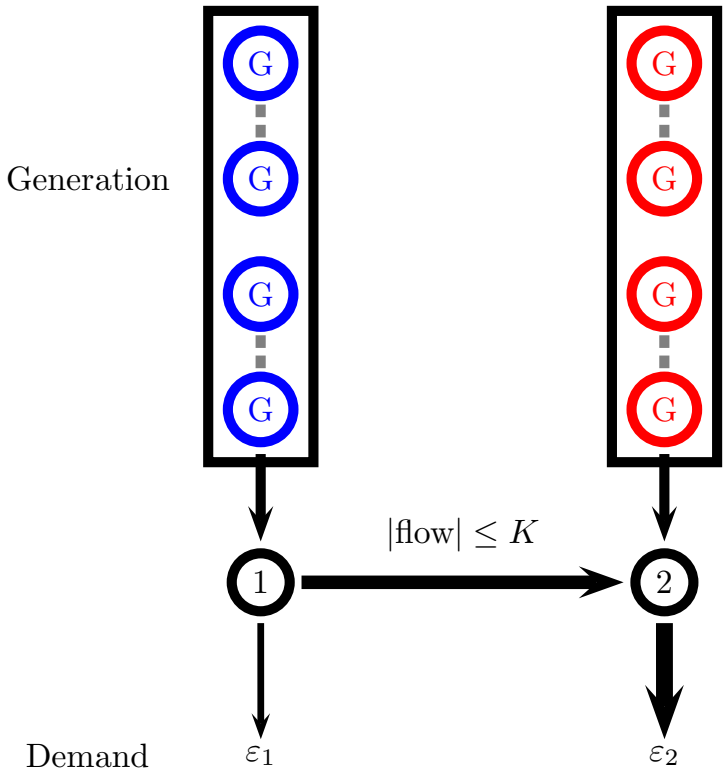
Node 1 plants: 60MW
Node 2 plants: 50MW
Two firms each node

Real Asset Swap



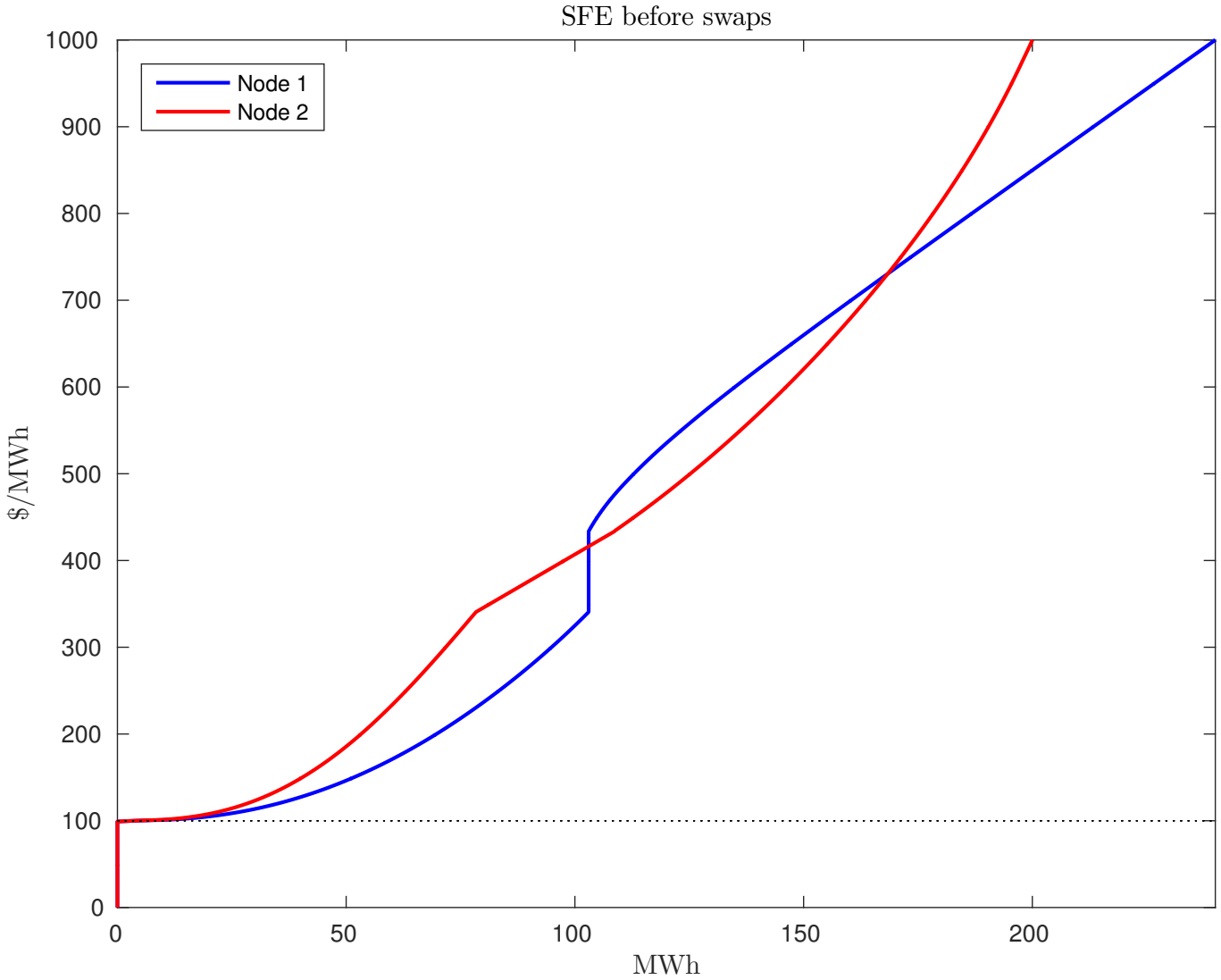
Node 1 plants: 60MW
Node 2 plants: 50MW
Four firms, each with plant at both nodes

Virtual asset swap

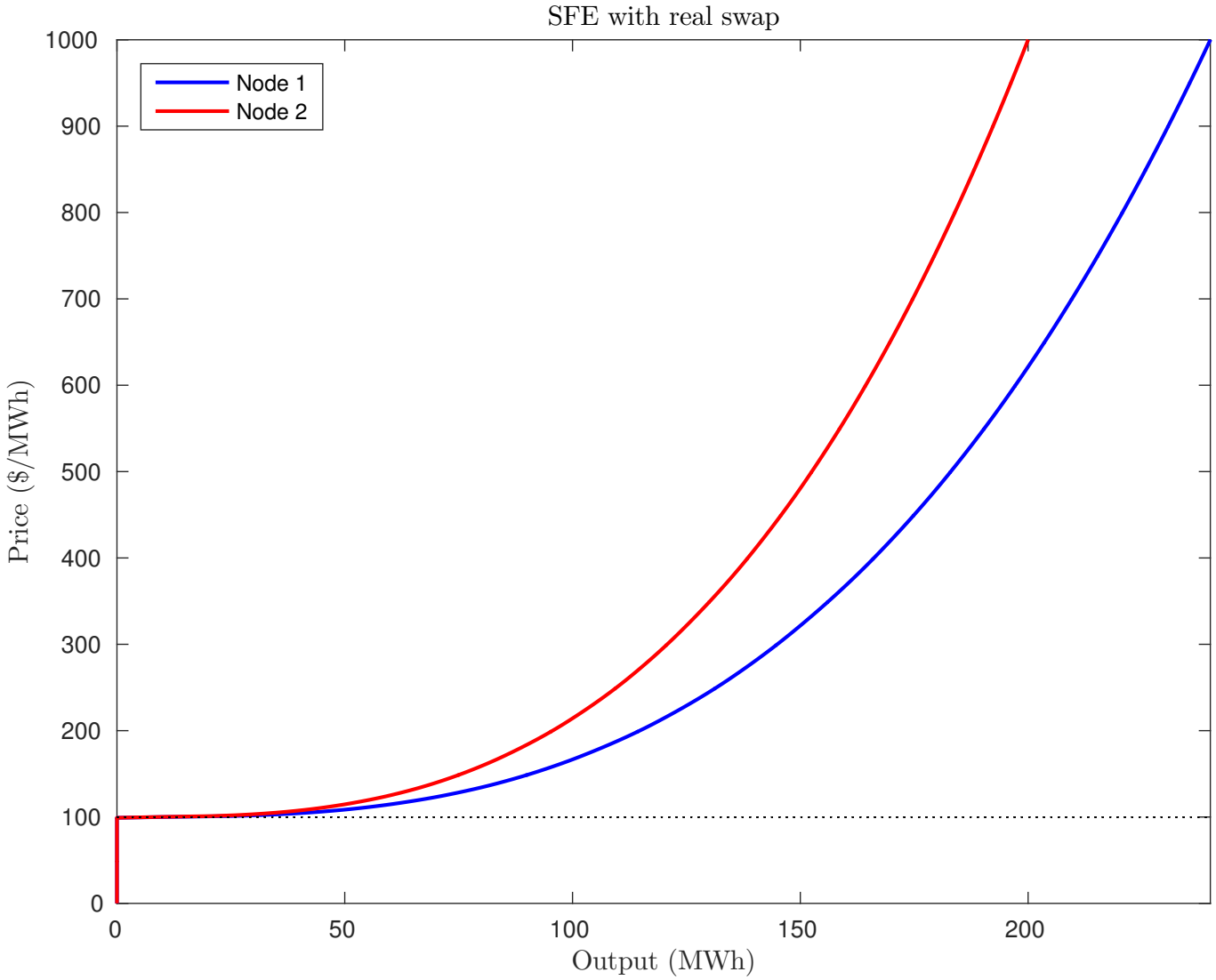


Two firms each node
50MW financial swap contract

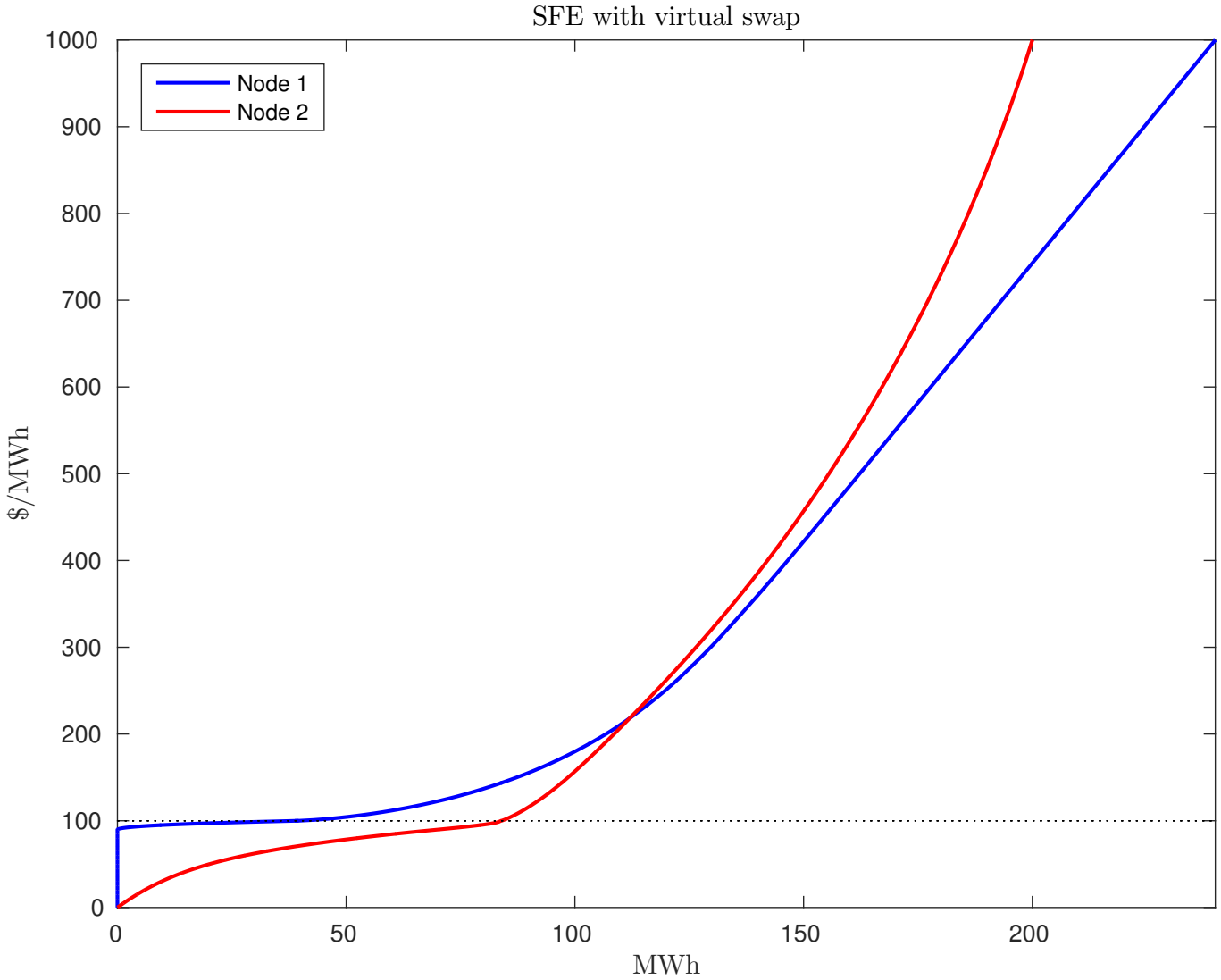
Local duopolies



Real asset swap



Virtual asset swap



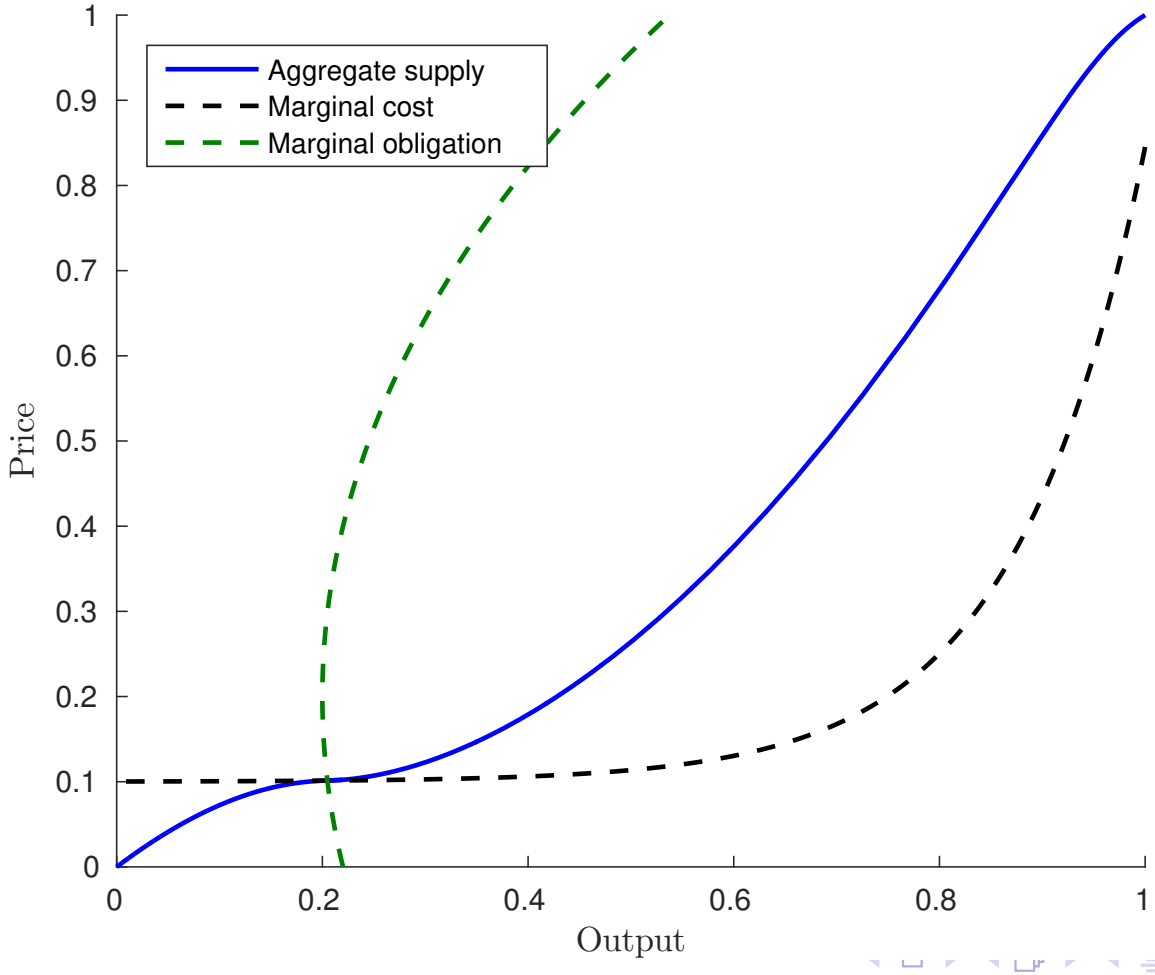
Summary

- With numerical methods we can solve far more SFE models than could be solved analytically
- Obligations from financial contracts are as important as cost of production, in determining spot market mark-ups
- SFE models with strategic exploitation of transmission constraints can be solved

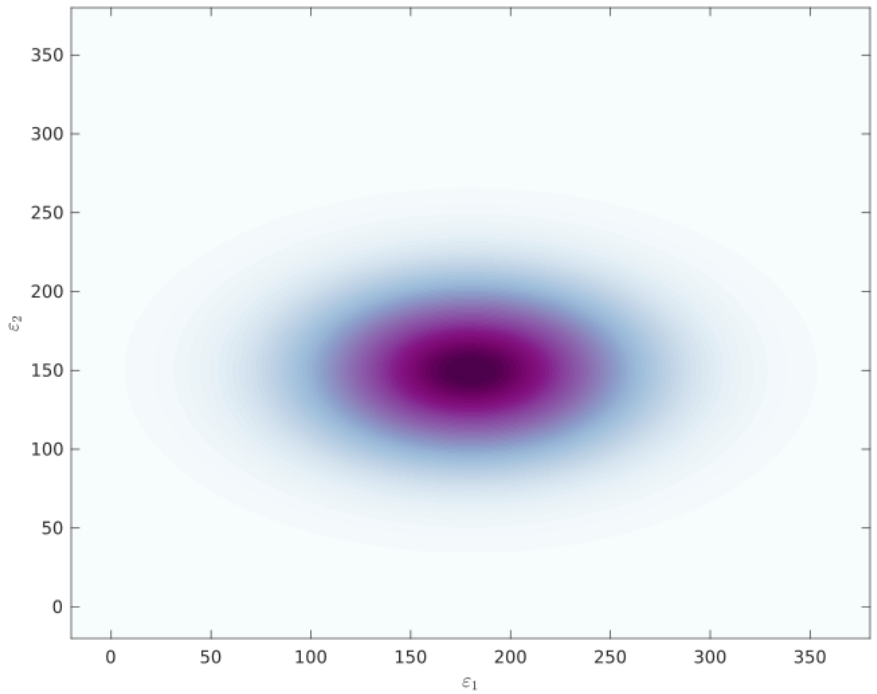
EXTRA SLIDES

Exotic portfolios

Symmetric SFE with non-linear portfolio obligation and production cost functions

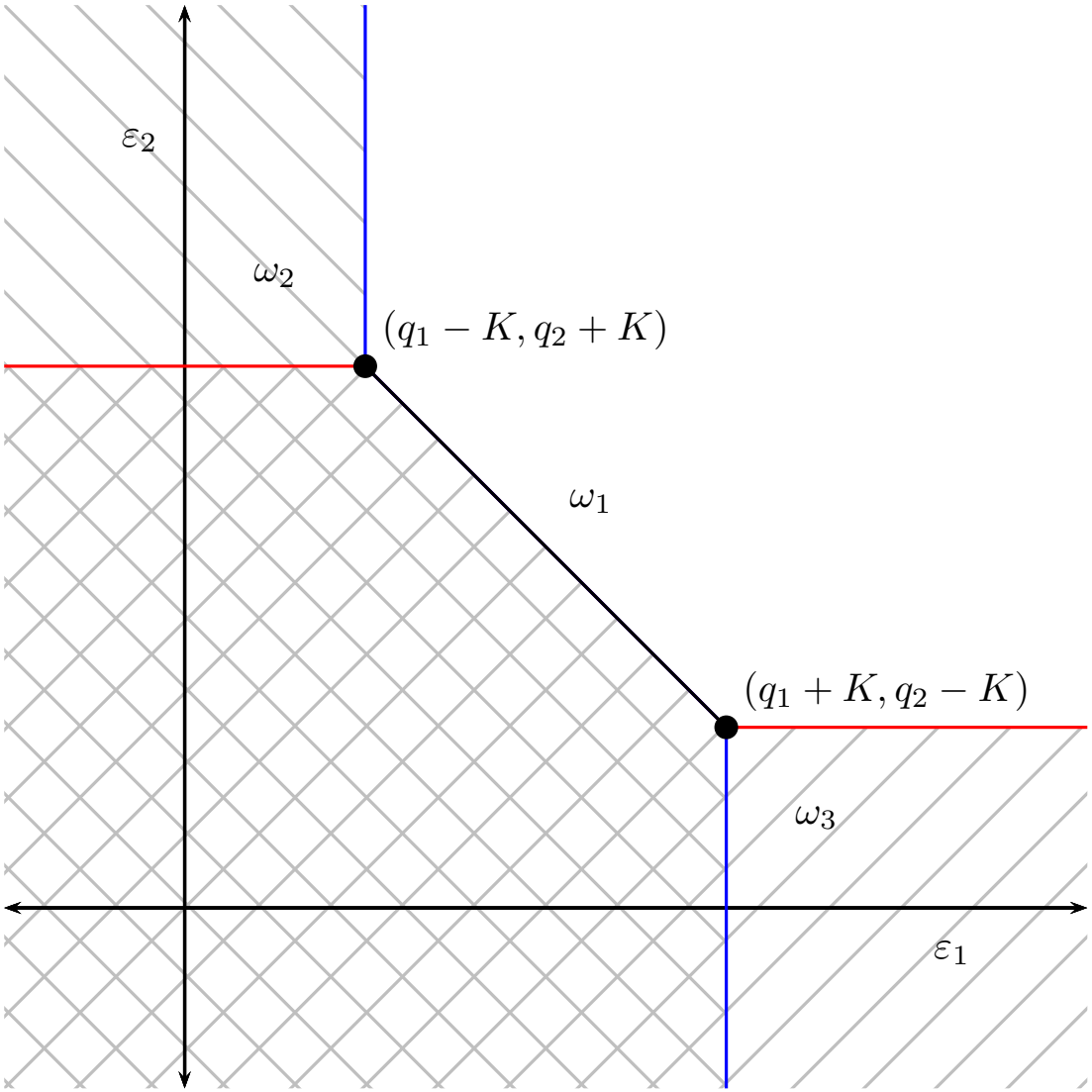


The nicest distribution for shocks

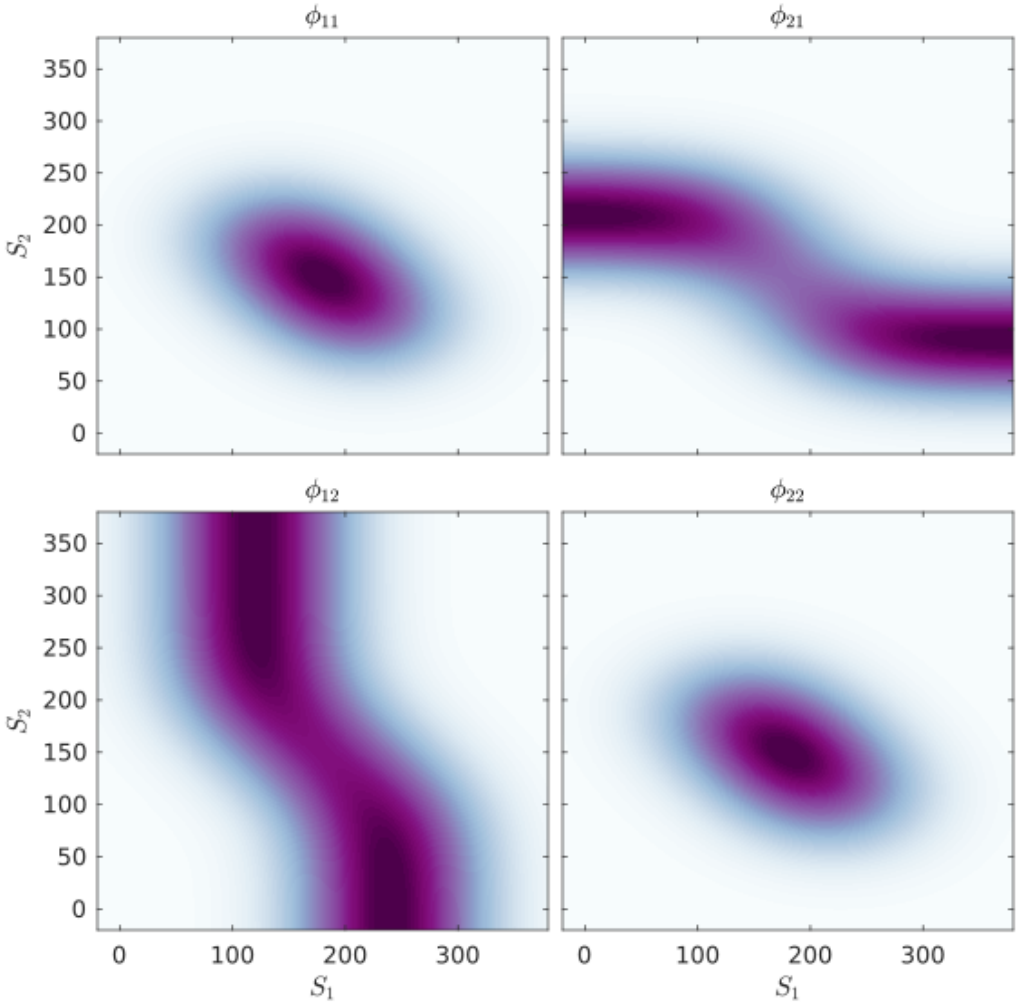
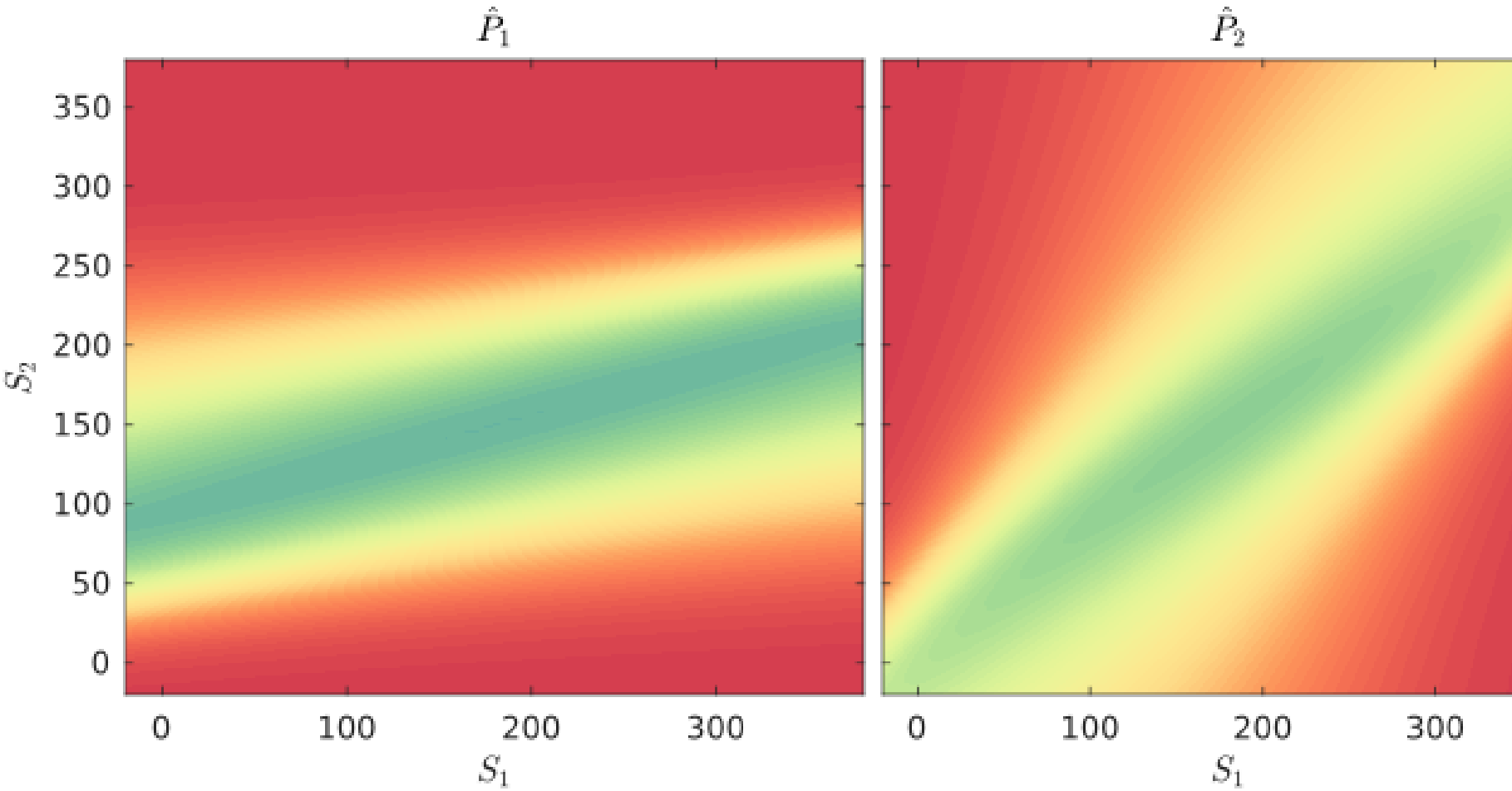


The glorious Gaussian!

Constrained demand distribution



Constrained demand distribution



Conditional probability of free flow