Supply Function Equilibria with Pivotal Electricity Suppliers*

Talat S. Genc[a] & Stanley S. Reynolds[b]

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Abstract. The concept of a supply function equilibrium (SFE) has been widely used to study generators' bidding behavior and market power issues in wholesale electricity markets. Observers of electricity markets have noted the important role that *pivotal suppliers*, those who can substantially raise the market price by unilaterally withholding generation output, sometimes play. However the literature on SFE has not considered the potential impact of pivotal suppliers on equilibrium predictions. We formulate a model in which generation capacity constraints can cause some suppliers to be pivotal. In symmetric and asymmetric versions of the model we show that the presence of pivotal suppliers reduces the set of supply function equilibria. We show that the size of the equilibrium set depends on observable market characteristics such as the amount of industry excess capacity and the load ratio (ratio of minimum demand to maximum demand). As the amount of excess capacity falls and/or the load ratio rises, the set of supply function equilibria becomes smaller; the equilibria that are eliminated are the lowest-priced, most competitive equilibria.

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[a] Department of Economics, University of Guelph, Ontario, Canada N1G 2W1, email: tgenc@uoguelph.ca
[b] Department of Economics, Eller College of Management, University of Arizona, Tucson, AZ 85721, email: reynolds@eller.arizona.edu

1. Introduction

The supply function equilibrium (SFE) concept has become a widely used tool to study the exercise of market power by sellers in multi-unit auction environments. SFE models assume that each seller submits a supply function for divisible output to the auctioneer, who sets a uniform market clearing price. Klemperer and Meyer (1989) (hereafter KM) characterize supply function equilibria in environments for which product demand is uncertain. They show that there are multiple equilibria when the range of demand variation is bounded. Roughly speaking, these equilibria are contained in a range of prices between the Cournot price and the competitive price.

The SFE concept has found its widest application in the analysis of wholesale electricity auctions. Many of these auctions are run as uniform price, multi-unit auctions in which power sellers submit offer schedules indicating their willingness to supply. Examples of applications of the SFE concept to wholesale electricity markets include Green and Newbery (1992), Newbery (1998), Rudkevich, et al. (1998), Green (1999), Baldick and Hogan (2002), and Baldick, Grant and Kahn (2004). These papers consider a variety of extensions of the KM model, including production capacity constraints, cost asymmetries, potential entry, multi-step cost functions, and forward contracting.

Recent assessments of wholesale electricity market performance have emphasized the role of the extent of excess production capacity in the market and the ability of a single supplier to influence the market price by withholding production (see Bushnell, Knittel and Wolak (1999), Joskow and Kahn (2001), Lave and Perekhodtsev (2001), Borenstein, Bushnell and Wolak (2002), and Perekhodtsev, et al. (2002)). The term "pivotal supplier" has been used to describe an electricity supplier that is able to dictate the price in the auction by withholding some portion of its production from the auction. One or more pivotal suppliers are most likely to be present when demand (or, load) is near its peak, when available production capacity in the market is limited relative to the peak load, and when suppliers' capacities are asymmetrically distributed.

While prior applications of the SFE approach to electricity markets have considered a variety of extensions of the basic SFE model, these applications have not adequately addressed the impact of production capacity constraints nor have they examined the potential role of pivotal suppliers. In this paper we formulate a simple model of a wholesale electricity auction for which the notion of a pivotal supplier has a natural interpretation. We assume that demand varies over time (during the trading period), and is perfectly inelastic up to a maximum price. We focus on the case in which firms' marginal costs are identical and constant up to production capacity. In the symmetric model, firms have identical capacities as well as costs. In the asymmetric model, firms have differing capacities. As in other SFE models with bounded demand variation, there is a continuum of equilibria.

The introduction of production capacity constraints into a SFE model changes the analysis in a fundamental manner. The SFE model without capacity constraints utilizes global concavity of suppliers' profit functions and first-order necessary conditions to determine optimal price-quantity pairs along with the curvature of the equilibrium supply function. When a firm's rivals have capacity constraints so that the firm is a pivotal supplier, the firm's profit function is not globally concave in price. In addition to a locally optimal price-quantity pair associated with the SFE necessary condition, a pivotal supplier may have a global profit optimum at the maximum price, or price cap. By withholding output, a pivotal supplier can unilaterally move the market price to the price cap. We examine the connection between pivotal suppliers and the set of supply function equilibria. In symmetric and asymmetric versions of the model we show that when pivotal suppliers are present, the set of equilibria is reduced relative to when no suppliers are pivotal. The size of the equilibrium set depends on observable market characteristics such as the amount of industry excess capacity, the load ratio (ratio of minimum demand to maximum demand), the number of suppliers, and the amount of low-cost base load production capacity. For example, as the amount of industry excess capacity falls and/or the load ratio rises, the set of supply function equilibria becomes smaller; the equilibria that are eliminated are the lowest-priced, most competitive equilibria.

The significance of these results turns on a question of equilibrium selection. Our SFE model has multiple equilibria. The introduction of capacity constraints that cause some suppliers to be pivotal will eliminate some Pareto dominated equilibria. If suppliers always select the Pareto dominant SFE then the introduction of capacity constraints will have no impact on observed behavior. We discuss several types of evidence that suggest that suppliers would not select the Pareto dominant SFE.

In section 2 the literature on pivotal electricity suppliers and on supply function equilibrium analysis is reviewed. Section 3 describes the formulation used in this paper. In section 4 we consider the incentives of firms to bid units in at the price cap, and how these incentives limit the equilibrium set for the symmetric firms model. We also allow for a broader class of supply strategies and describe an optimal supply response to rivals' supply strategies. These more general strategies may further limit the equilibrium set. Section 5 considers extensions of the basic model and provides a discussion of our results. In Section 5 we show that the qualitative nature of our results carries over to a model in which market demand is not perfectly inelastic. Section 6 concludes.

2. Background

2.1 *Pivotal Electricity Suppliers*

Wholesale electricity markets are particularly vulnerable to supplier market power. These markets are often organized as auctions in which suppliers submit offer schedules. Buyers in these auctions typically have little or no price sensitivity, so that the price elasticity of demand is close to zero. For example, buyers may be electricity distributors that are obligated to serve whatever quantity (load) their customers request at a fixed retail price. Generation suppliers have production capacity constraints, with marginal generation costs that rise sharply as production approaches the capacity limit. In many parts of the world, wholesale markets are linked together via a transmission grid. However, the ability of buyers to acquire power from distant generation suppliers may be limited by transmission capacity constraints. Taken together, these characteristics of wholesale electricity markets can yield significant market power for individual suppliers when demand is near a peak level. By unilaterally withholding a modest amount of production, a supplier may be able to achieve a large price increase for its output, because of the combination of zero or low demand elasticity and limited production capacity of rival suppliers. Borenstein, Bushnell and Wolak (2002) estimate that 50 percent of electricity expenditures in California in summer 2000 could be attributed to exercise of market power by generation suppliers.

The concept of a pivotal supplier embodies an extreme type of market power that may be present in wholesale electricity markets. Suppose that market demand is perfectly inelastic up to a maximum price, or price cap, and suppliers have capacity constraints. If the market demand quantity exceeds the sum of production capacities of all of a firm's rivals then that firm is said to be a *pivotal supplier*. The firm is pivotal in the sense that it can move the market price to the price cap by withholding a sufficient amount of its capacity from the market. We will develop a model in which the concept of a pivotal supplier plays a crucial role in limiting the set of supply function equilibria. However, it is important to note that production capacity constraints can also contribute to supplier market power and limit the set of equilibria when demand is downward sloping. This point is developed in section 5.4.

Bushnell, et al. (1999) use the concept of a pivotal supplier to develop a binary indicator variable to measure a supplier's pivotal status. [1] The Pivotal Supplier Index (PSI) for a supplier at a point in time is set equal to one if the supplier is pivotal, and is set to zero if the supplier is not pivotal. Bushnell, et al. (1999) find that suppliers in the Wisconsin/Upper Michigan (WUMS) region exercised market power by raising prices well above the competitive level when one or more suppliers were pivotal.

The Residual Supply Index (RSI) is a generalized form of PSI that was devised by the California Independent System Operator (see Sheffrin (2001, 2002)). RSI is calculated as the ratio of residual supply (total supply minus largest seller's capacity) to the total demand. Using summer 2000 peak hourly data from the California Power Exchange, Sheffrin (2001) shows that there is negative correlation between the Lerner Index and RSI. She finds that when RSI is about 1.2, the average price-cost markup is zero.

Perekhodtsev, et al. (2002) formulate and analyze a game theoretic model in which symmetric, capacity constrained firms submit offers to supply into a uniform price auction. They assume a fixed, inelastic demand. They restrict attention to simple bidding strategies in which each firm submits a single offer of either a low price equal to marginal cost or a high price equal to the price cap. The equilibrium is in mixed strategies, with the probability of a high price depending on the supply margin, the difference between industry capacity and the fixed demand (load). As the supply margin decreases (excess capacity decreases) the expected price in equilibrium rises, due to a higher likelihood that a pivotal supplier chooses a high price.

2.2 *Supply Function Equilibrium Models*

Wilson (1979) analyzes an auction model in which each agent submits a bid in the form of a demand function for continuously divisible shares of a good. Wilson analyzed versions of the

[1] This is an unpublished paper. It was incorrectly cited as a published paper in the *J. of Industrial*

model both with and without private information held by bidders. Klemperer and Meyer (1989) formulate a model in which each seller submits a supply function for divisible output as a function of the market price. There is no private information in their model, but the level of demand is uncertain at the time sellers submit supply functions. A uniform market price is determined by the intersection of the realization of the demand function and the aggregate supply function. The necessary conditions for equilibrium yield a system of differential equations that equilibrium supply functions must satisfy. If the range of demand variation is bounded then there is a continuum of equilibria, with the highest equilibrium price ranging from the competitive price to the Cournot price.

Green and Newbery (1992) applied the SFE model to analyze competition in the British wholesale electricity spot market. This market was run as a uniform price auction in which power sellers submit offer schedules. Green and Newbery argue that in a symmetric model, suppliers should select the symmetric equilibrium that yields the highest profit. Using demand and cost parameters to reflect conditions in the early years of the England and Wales wholesale electricity market, they show that at the most profitable symmetric SFE, the two dominant bulk electricity suppliers were predicted to choose supply functions that yield prices far above marginal costs and cause substantial deadweight losses**.** Their SFE predicted prices were well above observed prices. Wolfram (1999) performs a more detailed analysis of pool outcomes in the England and Wales market. She also finds that the most profitable symmetric SFE yields predicted prices that are substantially above actual pool prices.

Green and Newbery (1992), Newbery (1998), and Baldick and Hogan (2002) formulate SFE models that include production capacity constraints for suppliers. They argue, quite correctly, that if a solution of the SFE differential equation system violates any firm's capacity constraint, then that

Economics in earlier versions of our paper.

solution cannot be a SFE (see Green and Newbery (1992, pp. 938-939) and Baldick and Hogan (2002, p. 12)). However, these papers neglect another effect of capacity constraints. Even when capacity constraints are non-binding for all firms at market clearing prices for a proposed vector of supply function strategies, these constraints have an impact on a firm's profitability of changing its supply function. A firm may find it profitable to change its supply by withholding output and bidding up the price, given that its rivals are capacity constrained. This incentive to withhold output can limit the set of supply functions that survive as equilibrium strategies when demand is high and rivals' ability to increase supply is limited by capacity constraints.[2]

Using a formulation similar to ours, Holmberg (2004) assumes that load exceeds total capacity with positive probability (or, for our model, that peak load exceeds total capacity) and shows that there is a unique SFE with supply functions that reach the price cap when output is equal to capacity. Our analysis assumes that peak load never exceeds total industry capacity. However, Proposition 2 and Corollary 1 of this paper imply that as the load distribution becomes concentrated near total industry capacity, the set of symmetric equilibria shrinks to the least competitive symmetric SFE. This is similar to Holmberg's result.

3. A Supply Function Equilibrium Model

3.1 *Model Formulation*

We formulate a simple supply function equilibrium model that highlights the role that pivotal suppliers play in determining the range of equilibrium outcomes. We assume that the quantity demanded is perfectly inelastic for prices up to some exogenous market reserve price, $\overline{p}$. This reserve price could represent either buyers' maximum willingness to pay for wholesale

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2 SFE models have focused on local optimality conditions for a firm's supply response to rivals' supply strategies. Local optimality conditions yield the system of differential equations that a SFE must satisfy. However, when there are capacity constraints it is important to check for global optimality of a firm's supply response. Rivals' capacity constraints can yield a non-concave payoff function for which a local optimum need not be globally optimal.

electricity or, a price cap imposed by regulators.[3] The load, or quantity demanded, in the market for prices up to $\overline{p}$ varies during the course of a market trading period (e.g., a day) according to a function $N(t)$, called the load function, where $t \in [0,1]$ is an index of time during the trading period. If prices do not exceed $\overline{p}$, then total sales during the trading period are, $\int_{t=0}^{1} N(t)dt$. The assumption that load (or demand) varies deterministically during a trading period has been adopted in several applications of supply function equilibrium models to electricity markets, such as Green and Newbery (1992), Newbery (1998), and Baldick, Grant, and Kahn (2004). Green and Newbery (1992) explain that a time-varying demand formulation is mathematically equivalent to Klemperer and Meyer (1989)'s formulation of uncertain demand with bounded variation.[4] Thus, our model of time-varying load can be interpreted as a model of stochastic load in which bidders submit supply functions before learning the load realization.

We assume a particular form for the load function in order to simplify calculations that we perform later. Our qualitative results do not appear to be sensitive to the functional form. Let $N(t) = N(0)l^t$, where $l \in (0,1)$. The load during a trading period is ordered from its highest level to its lowest level according to the index *t*. The parameter *l* is referred to as the *load ratio*, representing the ratio of the minimum load to the maximum load during the trading period.[5]

There are $n \geq 2$ electricity suppliers. We assume initially a very simple cost structure. Each supplier has constant marginal cost of production, *c*, up to its capacity constraint, where $0 \leq c < \overline{p}$. We employ this "single-step" cost function in order to simplify equilibrium calculations. In Section 5 we discuss the implications of "multi-step" cost functions. The capacity constraint for supplier *i* is

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3 See von der Fehr and Harbord (1993).

4 If $N(t)$ is decreasing in *t*, as we will assume, then the time-varying load formulation is equivalent to a stochastic formulation in which the load quantity *Q* has support $[lN(0), N(0)]$ and cumulative distribution function, $F(Q) = 1 - N^{-1}(Q)$.

5 A related measure is the *load factor*, which is the ratio of the time averaged demand to the maximum demand. For our specification of the load function, the load factor is increasing in *l*.

K_i. Total capacity for all suppliers is, $K \equiv \sum_{i=1}^{n} K_i$. We assume throughout our analysis that $K \geq N(0)$; i.e., we assume that suppliers have sufficient capacity to meet the maximum demand.

Each supplier *i* is assumed to choose a supply function, $s_i(p)$, which is non-decreasing and right continuous in *p*, satisfies $0 \leq s_i(p) \leq K_i$, and is piecewise-differentiable on $[c, \overline{p}]$. Suppliers are assumed to select supply functions simultaneously prior to the beginning of the trading period. The auctioneer (or, system operator) constructs an aggregate supply function and determines a market clearing price for each time *t* of the trading period. If $p(t)$ is the market clearing price at *t* and supplier *i* submitted $s_i(p)$, then *i* is obligated to produce up to $s_i(p(t))$ units at *t* and will be paid $p(t)$ per unit for this production. Note that a supplier is not permitted to condition their supply function on the time *t* or the particular load at *t* within a trading period; instead supply functions are held fixed for the duration of the trading period.[6]

Right continuity of supply functions is more general than the continuous supply function employed in most SFE analyses.[7] Right continuity allows for two types of supply strategies that are important to consider when pivotal suppliers are present. First, it allows a strategy of offering all units at the price cap and no units for prices below the cap. Second, it allows for a discontinuous jump in quantity supplied up to full capacity for prices above candidate SFE prices. Each supplier's supply function is held fixed during the trading period. Let $S(p) \equiv \sum_j s_j(p)$ and $S_{-i}(p) \equiv S(p) - s_i(p)$.

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[6] Under the stochastic load interpretation of the model, supply functions are submitted before the realization of the load. The auctioneer selects some clearing price p^* after the realization of the load, and supplier *i* produces up to $s_i(p^*)$ units. For instance, our model could be applied to electricity auctions in which suppliers submit supply schedules for each ½ hour period of the day, and there is some uncertainty on the part of suppliers about the load for each of these ½ hour periods.

[7] Right continuity is the analog of left continuity of a bidder's demand function in a sales auction. Kremer and Nyborg (2004) allow bidders to use left continuous demand functions in their analysis of the effect of alternative allocation rules. Our interest is in the interaction of capacity constraints and supply strategies, rather than the impact of alternative allocation rules.

If $N(t) > S(p)$ for all p then we assume there is no trade at time t. Otherwise, a uniform market clearing price, $p^m(t)$, is established at time t that satisfies

$$(3.1) \qquad p^m(t) = \min\{p \in [0, \overline{p}] : N(t) \le S(p)\} .$$

It is possible that there is excess supply at the market clearing price. We assume a pro-rata on the margin (PRM) rule for allocating excess supply. This rule is commonly used in electricity auctions. Under PRM demand is first allocated to offers below the clearing price. Next, the remaining demand (quantity demanded at the clearing price less supply quantity offered at prices below the clearing price) is allocated to suppliers according to a pro-rata rule. If p' is the clearing price at time t then the PRM rule yields sales for firm i as follows:

$$(3.2) \qquad q_i = s_{i-}(p') + (N(t) - S_-(p')) \frac{s_i(p') - s_{i-}(p')}{S(p') - S_-(p')}$$

where $s_{i-}(p') = \lim_{p \uparrow p'} s_i(p)$ and $S_-(p') = \lim_{p \uparrow p'} S(p)$.

3.2 *Supply Function Equilibrium*

A supply function equilibrium (SFE) is a Nash equilibrium in supply function strategies. Consider the profit for firm i at time t, given that its rivals have chosen continuous supply functions, $\{s_j(p)\}_{j \ne i}$. If the clearing price is p and firm i supplies the residual demand, $N(t) - S_{-i}(p)$, at t then its profit is:

$$(3.3) \qquad \pi_i(p,t) = (p - c)[N(t) - S_{-i}(p)]$$

We seek a supply function $s_i(p)$ for firm i that has the property that the clearing price p maximizes $\pi_i(p,t)$ in (3.3) and $s_i(p) = N(t) - S_{-i}(p)$, for each time t. If its rivals' supply functions are differentiable then the necessary condition for an (interior) optimal price for time t for each firm i yields a system of ordinary differential equations for the n supply functions:

(3.4) $$\sum_{j\neq i} s_j'(p) = \frac{s_i(p)}{(p-c)},\ i = 1,\dots,n$$

This is a simple version of the differential equation system that has been intensively studied in the SFE literature.[8]

A SFE derived from (3.4) has the characteristic that a firm's supply function maximizes its profit at each time *t* during the trading period, given the supply functions chosen by its rivals. This implies that the way in which the load is distributed between its minimum and maximum levels does not change the set of SFE. In addition, if the maximum load is held fixed, changing the minimum load does not change the set of SFE; however it does change the predicted interval of equilibrium prices.

3.3 *Pivotal Suppliers*

Supplier *i* is pivotal at time *t* during a trading period if $\sum_{j\neq i} K_j < N(t)$. When this inequality holds, supplier *i* would be able to raise the market price at *t* to $\overline{p}$ either by withholding some of its capacity (i.e., physical withholding) or by submitting a supply function with some portion of its capacity bid in at price $\overline{p}$ (i.e., economic withholding), and still sell at least $N(t) - \sum_{j\neq i} K_j$ units. Note that a pivotal supplier can move the market price to $\overline{p}$ unilaterally. If $\sum_{j\neq i} K_j < N(1)$ then supplier *i* is pivotal for the entire trading period.

We emphasize that there are three important parameters in our analysis, which we treat as exogenous. One parameter is the number of suppliers, $n \geq 2$. A second is the load ratio, *l*. A third parameter is the capacity index, *k*. The capacity index is the ratio of the total amount of production capacity, *K*, held by all *n* suppliers, to the maximum load, *N*(0); i.e., $k \equiv K / N(0)$. Our assumption regarding *K* implies that $k \geq 1$. Larger values of *k* indicate greater excess capacity held by suppliers.

8 This version is simple because there is no demand function term, due to perfectly inelastic demand, and because marginal cost is constant.

4. Symmetric Firms Model

In the previous section it was assumed that all suppliers have a common marginal cost *c* for production up to capacity. In this section we also suppose that total capacity is equally divided among suppliers; $K_i = K/n,\ i = 1,...,n$. For this symmetric suppliers formulation, equation (3.4) may be used to derive a continuum of candidate symmetric supply function equilibria. We use the term "candidate" because some symmetric supply functions derived from (3.4) may fail to be equilibria due to the presence of pivotal suppliers.

If all suppliers utilize a common supply function, $s(p)$, then the system of equations in (3.4) simplify to the following single differential equation:

(4.1) $$s'(p) = \frac{s(p)}{(n-1)(p-c)}$$

There is a continuum of solutions to (4.1) of the form:

(4.2) $$s(p) = \frac{N(0)}{n}\left[\frac{p-c}{p(0)-c}\right]^{1/(n-1)}$$

The supply function solutions in (4.2) are a special case of results in Rudkevich, et al. (1998). They derive SFE solutions for a symmetric model with zero demand elasticity and general cost functions (including multi-step marginal cost functions).

The *candidate* supply function equilibrium strategies in (4.2) are indexed by the initial price, $p(0)$, which can take on any value in the interval, $(c, \overline{p}]$. If $n = 2$ then the supply functions in (4.2) are linear. If $n > 2$ then the supply functions are non-linear, as in Figure 1. This figure illustrates three of these solutions, including the limiting cases of the most competitive solution (a horizontal supply function at $p = c$) and the least competitive solution (with $p(0) = \overline{p}$).

Equilibrium prices and profit per firm are as follows:

(4.3) $$p^*(t) = c + (p(0)-c)l^{t(n-1)}$$

(4.4) $$\Pi^{SFE} = \int_{t=0}^{1} (p^*(t)-c)(N(t)/n)dt = \frac{N(0)(1-l^n)(p(0)-c)}{-n^2 \ln l}$$

Π^{SFE} is profit per firm associated with a candidate SFE, where it is understood that this profit depends on the initial price $p(0)$ that indexes the candidate SFE. Π^{SFE} is independent of the capacity index, *k*.

Proposition 1. Fix the number of suppliers, $n \geq 2$. Then $s(p) = \frac{N(0)}{n}\left[\frac{p-c}{\overline{p}-c}\right]^{1/(n-1)}$ is a symmetric Nash equilibrium supply function strategy for any capacity index and load ratio parameters satisfying $k \geq 1$ and $l \in (0,1)$.

Proof. See the Appendix.

Proposition 1 establishes that the least competitive SFE survives as an equilibrium regardless of the load ratio and the amount of excess capacity. Next we examine how the presence of pivotal suppliers and the extent of excess capacity influence the set of equilibria. Figure 2 illustrates relevant parameter ranges for the capacity index and the load ratio, for a fixed number of suppliers, *n*. The load ratio, *l*, lies between zero and one. The capacity index, *k*, is greater than or equal to one. If $l \in [k(n-1)/n, 1)$ then each firm is pivotal at all times *t* during the trading period. We refer to this as the totally pivotal (TP) case. Let $TP \equiv \{(k,l,n): n \geq 2, n/(n-1) \geq k \geq 1, l \in [(n-1)k/n, 1)\}$. This set is illustrated by the shaded area in Figure 2. A second situation arises when each firm is pivotal for some times during the trading period. We refer to this as the partially pivotal (PP) case. Set *PP* lies below set *TP* in Figure 2;

$PP \equiv \{(k,l,n): n \geq 2, n/(n-1) \geq k \geq 1, l \in (0,(n-1)k/n)\}$.

If parameters are in set *PP* then each firm is pivotal for times between zero and a time τ satisfying, $N(\tau) = (n-1)K/n$. A third case occurs when no firm is pivotal at any time; we refer to this as the never pivotal (NP) case. This occurs when $k \geq n/(n-1)$, and is illustrated by the area *NP* in Figure 2. In this case, any collection of $n-1$ suppliers has enough capacity to serve the peak load.

If parameters are in the *NP* (never pivotal) region then supply functions in (4.2) are supply function equilibria for all initial prices, $p(0) \in (c, \overline{p}]$. These supply functions can be shown to be equilibrium strategies by using the approach outlined by Klemperer and Meyer (1989, p. 1255).[9] Klemperer and Meyer restrict supply functions to be twice continuously differentiable. However, if parameters are in the *NP* region then, under the PRM allocation rule, one can show that equilibria obtained when restricting firms to twice continuously differentiable strategies remain equilibria when firms are permitted to use right continuous strategies.[10] Extending the strategy space from differentiable to right continuous supply functions may yield additional Nash equilibria. One such equilibrium is the competitive equilibrium in which each firm uses the right continuous strategy of providing zero units at prices less than *c* and *K/n* units for prices greater than or equal to *c*.[11] It is possible that there are additional equilibria in discontinuous supply strategies, but we have not identified them.

4.1 *Simple Supply Deviation*

If parameters are in the *TP* or *PP* set then an individual firm has an incentive to deviate from $s(p)$ defined in (4.2) for some initial prices in the interval $(c, \overline{p}]$. We examine a simple type of deviation involving a supply function with no units offered for prices below the maximum price,

[9] For any candidate SFE with initial price $p(0) \in (c, \overline{p}]$, extend the supply function linearly for prices above $p(0)$ with slope, $s'(p(0))$. Then at each time *t*, $p(t)$ and $s(p(t))$ are the globally optimal price and quantity, given rivals' (extended) supply functions.

[10] Suppose that each of firm *i*'s rivals play the twice continuously differentiable supply strategy $s(p)$ given by (4.2). By choosing $s(p)$ as its own supply strategy, firm *i* induces a clearing price $p(t)$ for time *t* that maximizes its profit in (3.3) for *t*, with output of $s(p(t))$. Moreover, since the clearing prices induced by choosing $s(p)$ maximize profit for each time *t*, the choice of $s(p)$ maximizes the total payoff for firm *i*. Firm *i* could not increase its total payoff by selecting a discontinuous supply function.

[11] This supply function may be viewed as a limiting case of the supply function in (4.2), as $p(0)$ approaches *c*.

$\overline{p}$, and all units up to capacity offered at price $\overline{p}$. A deviating firm *i* is assumed to use the right continuous function,

$$(4.5) \qquad s_i(p) = \begin{cases} 0 & \text{if } p < \overline{p} \\ K/n & \text{otherwise.} \end{cases}$$

Each rival firm $j \neq i$ is assumed to play its candidate SFE strategy for prices up to the initial clearing price, and to produce at full capacity for prices at or above the initial price. This yields the right continuous function,

$$(4.6) \qquad s_j(p) = \begin{cases} \dfrac{N(0)}{n}\left[\dfrac{p-c}{p(0)-c}\right]^{1/(n-1)} & \text{if } p < p(0) \\ K/n & \text{otherwise.} \end{cases}$$

This strategy specifies the most aggressive supply behavior possible for prices above candidate SFE prices. It is also possible to use linearly extended supply functions above the initial price as proposed by Klemperer and Meyer. However, the linear extension proposed by Klemperer and Meyer is inconsistent with some symmetric supply function equilibria that can be supported by strategies of the form of (4.6).

We now compare the profit associated with the simple deviation specified by (4.5) to profit from a candidate SFE. If parameters are in the totally pivotal set then the clearing price is $\overline{p}$ at each time *t*, $S_{-i}(\overline{p}) = (n-1)K/n \leq N(t)$ at each time *t*, and all of the remaining demand at $\overline{p}$ is allocated to the deviating firm *i*, so that $q_i = N(t) - (n-1)K/n$ at *t*. Total profit for the deviating firm is,

$$\Pi^D = \int_{t=0}^{1} [N(t) - (n-1)K/n](\overline{p} - c)dt = \frac{N(0)[n^2(1-l) + n(n-1)k \ln l](\overline{p} - c)}{-n^2 \ln l}.$$

Deviation profit exceeds SFE profit if $\Pi^{SFE} < \Pi^D$, or equivalently if,

$$(4.7) \qquad p(0) - c < \phi(k,l,n)(\overline{p} - c)$$

where,

(4.8) $\phi(k,l,n) \equiv [n^2(1-l) + n(n-1)k \ln l]/[1-l^n]$.

The function ϕ produces a fraction for each triple (k,l,n) of parameters in the set *TP*. If the markup at the initial price for a candidate SFE is less than the fraction ϕ of the maximum markup, $\overline{p} - c$, then the candidate SFE is not an equilibrium.

Example 1. Suppose that the capacity index is k =1.4, the load ratio is l = 0.8, and the number of firms is n = 2. This implies that $\phi(k,l,n) \approx 0.486$. Any candidate SFE that does not involve a markup at the initial price that is at least 48.6% of the maximum markup is ruled out as an equilibrium.

Proposition 2. The ϕ function that characterizes the lower bound for the SFE initial markup is decreasing in the capacity index *k*, increasing in the load ratio *l*, and decreasing in the number of suppliers *n* for parameters in the totally pivotal set.

Proof. See the Appendix.

Equation (4.7) provides a sufficient condition for eliminating certain candidate supply function equilibria as equilibria. Proposition 2 implies that the set of candidate supply functions that are ruled out becomes larger as the amount of excess capacity decreases, as the load ratio increases, and as the number of firms (holding total capacity constant) decreases.

Corollary 1. Fix $n \geq 2$ and choose an initial price, $p(0) < \overline{p}$. Then there is a load ratio $l' \in (0,1)$ and a capacity index $k' > 1$ such that supply function strategies given by (4.2) with initial price $p(0)$ are not a Nash equilibrium.

Proof. See the Appendix.

Proposition 1 established that the least competitive symmetric SFE (the SFE with initial price $p(0) = \overline{p}$) survives as an equilibrium regardless of the values of the capacity index and the load ratio. Proposition 2 and Corollary 1 imply that as the amount of excess capacity declines to zero and the load ratio approaches 100%, the set of symmetric equilibria shrinks to the least competitive symmetric SFE. Corollary 1 implies that for any candidate initial price $p(0) < \bar{p}$ there exist a load ratio and a capacity index such that an equilibrium initial price must be greater than $p(0)$.[12] This is reminiscent of a result in Holmberg (2004). For a model similar to ours, he shows that if the load exceeds total capacity with positive probability (or for our model, if peak load $N(0)$ exceeds total capacity *K*) then there is a unique SFE with supply functions that reach the price cap when output is equal to capacity.

Now suppose that parameters are in the partially pivotal set; $(k,l,n) \in PP$. As before we assume a deviating firm i uses the strategy in (4.5) and its rivals $j \neq i$ use the strategy in (4.6). Residual demand at time t for the deviating firm at price $\bar{p}$ is, $[N(t)-(n-1)K/n] > 0$, if $t < \tau \equiv \ln[(n-1)k/n]/\ln(l)$ and residual demand is zero for $t \geq \tau$. Total profit for the deviating firm is,

$$\text{(4.9)} \qquad \hat{\Pi}^D = \int_{t=0}^{\tau} [N(t)-(n-1)K/n](\bar{p}-c)dt = \frac{N(0)[n^2(1-l^\tau)+n(n-1)k\tau \ln l](\bar{p}-c)}{-n^2 \ln l},$$

where τ defined above depends on (k,l,n).

Deviation profit exceeds SFE profit if $\Pi^{SFE} < \hat{\Pi}^D$, or equivalently if,

$$\text{(4.10)} \qquad p(0)-c < \lambda(k,l,n)(\bar{p}-c)$$

where,

$$\text{(4.11)} \qquad \lambda(k,l,n) \equiv [n^2(1-l^\tau)+n(n-1)k\tau \ln l]/[1-l^n], \text{ with } \tau \equiv \ln[(n-1)k/n]/\ln(l).$$

Equations (4.10) and (4.11) provide a sufficient condition that eliminates certain candidate equilibria for the partially pivotal case. If the markup at the initial price for a candidate SFE is less

[12] Note that it is *not* an equilibrium to bid all units at the price cap; i.e., it is not an equilibrium for each firm i to use the strategy defined by (4.5). If all rival firms submit supply schedules at the price cap, then a firm's best response will be to undercut its rivals' supply functions and produce at full capacity for the entire trading period.

than the fraction λ of the maximum markup, $\overline{p} - c$, then the candidate SFE is not an equilibrium. The following is analogous to Proposition 2.

Proposition 3. The λ function that characterizes the lower bound for the SFE initial markup is decreasing in the capacity index *k*, increasing in the load ratio *l*, and decreasing in the number of suppliers *n* for parameters in the partially pivotal set.

Proof. See the Appendix.

Example 2. Consider conditions for a day of peak summer demand. Assume a 20% reserve margin of capacity over typical peak summer demand, so that k =1.2. Suppose there are 3 strategic suppliers, so that n = 3. A representative value for the load ratio on a peak summer day is l = 0.6 (see, for example, data from the New York ISO for summer 2003, at http://nyiso.com/markets/graphs/mkt_toolbox.html). These parameters are in the partially pivotal set; they yield $\lambda(k,l,n) \approx 0.25$. A SFE must have an initial markup of at least 25% of the maximum possible markup for the parameters of Example 2.

4.2 *Optimal Supply Response*

In Section 4.1 we showed how some candidate supply function equilibria could be ruled out as equilibria via a simple supply deviation that involves bidding all units in at the maximum acceptable price, $\overline{p}$. However, this kind of deviation may not be an optimal response to rivals' supply functions. In this section we describe an optimal supply response to rivals' candidate equilibrium strategies. We consider the response of a firm *i* to $n-1$ rival firms *j* who each use the supply strategy in equation (4.6).

An optimal supply response cannot necessarily be constructed by piecing together the optimal (q, p) pairs at each time *t*. The difficulty with this approach is illustrated in Figure 3A for a particular problem based on Example 2 (the problem is described in more detail below). The candidate symmetric SFE strategy for this problem is illustrated by the dashed curve that begins at

$c = 20$ and rises to $p(0) = 42$ at a quantity of 33 units. The arrows in Figure 3A trace out optimal (q, p) pairs as the time index moves from zero to one for this problem. There is an initial time interval during which $\bar{p} = 100$ is the most profitable price and sales for *i* are $q = N(t) - (n-1)K/n$ at time *t*. Following the initial interval the optimal price drops discontinuously to the candidate SFE price $p^*(t)$, defined in (4.3), and the optimal quantity *rises* to $q = N(t)/n$. Firm *i*'s optimal (q, p) pairs follow its candidate SFE supply strategy for the remainder of the time interval. The optimal (q, p) pairs traced out in Figure 3A cannot be an optimal supply response because the resulting supply function would violate the non-decreasing constraint.

Genc and Reynolds (2004) use an optimal control approach to characterize an optimal supply response of a firm *i* to $n-1$ rival firms.[13] The non-decreasing supply function constraint is captured by constraints that quantity for firm *i* and price are both non-increasing over time. The price is also constrained to be less than or equal to $\bar{p}$. Firm *i* is free to choose the initial price; the initial price for the solution to the control problem may differ from $p(0)$, the initial price associated with the candidate SFE. The necessary conditions allow us to piece together the features of an optimal response.

Free interval: Over a supply segment for which both *q* and *p* are increasing, the necessary conditions imply that,

(4.12) $$(p-c)(n-1)s_j'(p) - q = 0.$$

If the optimal response has a free interval then the price must be less than $p(0)$; if price is greater than $p(0)$ then the residual demand for *i* is perfectly inelastic (with $s_j'(p) = 0$) so that (4.12) could

[13] A continuously differentiable approximation of a rival's supply function in (4.6) is utilized. The approximation is identical to $s_j(\cdot)$ except for a interval of prices above *p*(0). The approximation replaces the discontinuity of $s_j(\cdot)$ at *p*(0) with a rapidly rising quantity supplied over a small interval of prices above *p*(0). By choosing the length of this price interval sufficiently small, the approximation of $s_j(\cdot)$ can be made as close as desired.

not be satisfied for $q > 0$. A (q, p) pair that satisfies (4.12) for $p \le p(0)$ is consistent with the candidate SFE supply strategy.

Binding Price Constraint: If the constraint that price is non-increasing over time is binding then price is constant over a time interval, which corresponds to a horizontal supply segment. Such a segment cannot lie below the candidate SFE; the firm would earn greater profit by raising the price associated with each quantity up to the candidate SFE, and this would not violate the non-decreasing supply function constraint. If there is a horizontal supply segment above the candidate SFE but below the price cap, then the firm could earn greater profit by deviating either with a price reduction down to the candidate SFE, or with a price increase up to the price cap. A horizontal supply segment can only occur at a price equal to the price cap.

Binding Quantity Constraint: Suppose that the constraint that quantity is non-increasing over time is binding. This corresponds to a vertical supply segment. First, note that a (q, p) pair on a vertical supply segment cannot lie below the candidate SFE supply function. If (q, p) lies below the firm's candidate SFE supply function then greater profit could be earned by reducing quantities for each price during this time interval; this change would not violate any constraints. Second, if quantity is constant then price must be falling at a rate such that the quantity supplied by rival firms is falling at the same rate that the load (demand) is falling.

An optimal supply response to $(n - 1)$ rival firms j who use supply strategy (4.6) will be one of two types. (1) The first type has all of time [0,1] as a free interval, so that the candidate SFE is in fact an equilibrium. (2) The second type has a horizontal segment at price $\overline{p}$, possibly a vertical segment extending up to price $\overline{p}$, and possibly an increasing segment (over a free time interval) that corresponds to the bottom portion of the candidate SFE and that connects to the bottom a vertical segment.

Note that the second type of optimal supply response may or may not be a simple deviation; a simple deviation contains only a horizontal segment at the price cap. The following proposition characterizes the set of symmetric SFE for parameters in a subset of the totally pivotal set. For parameters in this subset, an optimal supply response that differs from a candidate SFE must be a simple deviation.

Proposition 4. Suppose that (k,l,n) is in the totally pivotal (*TP*) set and that $l \geq 1-(k-1)(n-1)/n$. Then symmetric SFE are given by (4.2) with $p(0) \in [\phi(k,l,n)(\overline{p}-c)+c, \overline{p}]$.

Proof. See the Appendix.

The inequality $l \geq 1-(k-1)(n-1)/n$ corresponds to the area above the dashed line in Figure 2. When parameters are in set *TP* and this inequality is satisfied, an optimal supply response that differs from the candidate SFE must be a simple deviation. As a consequence, the function $\phi(k,l,n)$ specified in (4.8) defines a tight lower bound on the set of initial SFE prices.

Example 1, revisited. Recall that a simple deviation ruled out candidate supply function equilibria with initial prices, $p(0)-c < \phi(k,l,n)(\overline{p}-c) \approx 0.486(\overline{p}-c)$. The parameters of Example 1 satisfy the conditions of Proposition 4. Therefore, the set of symmetric SFE for Example 1 is given by (4.2) with initial prices satisfying $p(0)-c \geq 0.486(\overline{p}-c)$.

Example 2, revisited. Simple supply deviations rule out candidate supply function equilibria such that $p(0)-c < \lambda(k,l,n)(\overline{p}-c) \approx 0.25(\overline{p}-c)$. If $c=20$ and $\overline{p}=100$ then simple deviations rule out equilibrium initial prices less than 40. Suppose that $p(0)=42$; the candidate symmetric SFE supply function for this initial price is illustrated by the dashed curves in Figures 3A and 3B. Figure 3A illustrates the unconstrained optimal (q,p) pairs for a firm facing rivals playing this candidate SFE strategy. The constrained optimal supply response is illustrated in Figure 3B. The optimal supply response has a vertical segment at $q=9.37$ and a horizontal segment at the price cap,

$\overline{p} = 100$. The arrows in Figure 3B illustrate how prices and quantities move over time for the constrained optimal supply response. So, $p(0) = 42$ cannot be an initial equilibrium price. Consideration of optimal supply responses for example 2 yields an equilibrium set with initial prices satisfying, $p(0) - c \geq 0.3(\overline{p} - c)$.

5. Extensions and Discussion

5.1 *Asymmetric Capacities*

Concerns about market power of pivotal suppliers in wholesale electricity markets have often focused on the largest suppliers (e.g., see Lave and Perekhodtsev (2001)). Suppliers with the greatest generation capacity appear to be most likely to be able to force up the market price by withholding production. However there is relatively little analysis of models in which firms have asymmetric capacities in the SFE literature.[14]

We focus on duopoly markets in which firms differ only in capacities. We assume that, without loss of generality, firm 2 has less capacity than its competitor. We also assume throughout our analysis that $K_1 + K_2 \geq N(0)$. Because of the relative simplicity of our formulation, we can solve for the supply functions that solve the differential equations in closed form and provide a precise condition under which supply function solutions satisfy the non-decreasing property.

If $K_1 > K_2 \geq \frac{1}{2}N(0)$ then the symmetric supply function equilibria that were characterized in Section 4 are feasible, since each firm has enough capacity to meet at least one-half of the peak demand. In this case the incentives to defect from any candidate symmetric SFE are driven by the amount of capacity of the smaller firm. The larger firm's incentive to deviate are identical to the

[14] Green and Newbery (1992) discuss equilibria for asymmetric duopoly. They write that it is, "… more difficult to solve for the pair of (differential) equations for the asymmetric equilibrium than the single equation for the symmetric equilibrium, and the rest of the paper will restrict attention to the symmetric case." Baldick and Hogan (2002) also consider asymmetries in capacities and cost functions. However they note that the differential equation approach of solving for supply functions may not be effective because the resulting supply functions often fail the non-decreasing property.

incentives of firms in a symmetric duopoly market with capacity index $2K_2 / N(0)$. Using the load ratio, l, $k = 2K_2 / N(0)$, and $n = 2$, the ϕ and λ functions derived in Section 4 define the sets of equilibria ruled out by simple deviations.

If $K_1 > \frac{1}{2}N(0) > K_2$ then symmetric equilibria are not feasible because they violate the capacity constraint for firm 2. Following analysis of subsection 3.2, we obtain the following differential equations for the supply functions:

(5.1) $$s_2'(p) = \frac{s_1(p)}{(p-c)}, \quad s_1'(p) = \frac{s_2(p)}{(p-c)}.$$

We use the boundary conditions $s_2(p(0)) = K_2$ and $s_1(p(0)) = N(0) - K_2$.[15] The solution of this pair of differential equations is as follows:

(5.2) $$s_1(p) = \frac{N(0)}{2}\left[\frac{p-c}{p(0)-c} + \frac{p(0)-c}{p-c}\right] - K_2\left[\frac{p(0)-c}{p-c}\right],$$

(5.3) $$s_2(p) = \frac{N(0)}{2}\left[\frac{p-c}{p(0)-c} - \frac{p(0)-c}{p-c}\right] + K_2\left[\frac{p(0)-c}{p-c}\right].$$

These supply functions are non-negative and non-decreasing as long as the load ratio is greater than or equal to $(1-2k_2)^{1/2}$; an asymmetric SFE of this type does not exist for load ratios less than this value. One can show the existence of an asymmetric SFE of the type (5.2)-(5.3). For example, analogous to Proposition 1, it can be shown that at $p(0) = \overline{p}$, (5.2)-(5.3) are asymmetric supply function equilibrium strategies.

Next we calculate equilibrium prices from the market clearing condition:

$$N(t) = s_1\left(p(t)\right) + s_2\left(p(t)\right) = N(0)\frac{p(t)-c}{p(0)-c},$$

[15] It is possible that firm 2 could have an even smaller initial output, coupled with greater initial output for the larger firm. Here we suppose initial conditions consistent with aggressive play by the smaller firm so that he supplies his all available capacity at time zero.

which implies that $p(t) = c + \frac{N(t)}{N(0)}(p(0) - c)$. Let $k_2 \equiv \frac{K_2}{N(0)}$, then profit for firm 1 becomes

$$\Pi_1^{SFE} = \int_{t=0}^{1} (p(t) - c)s_1(p(t))dt = \frac{(p(0) - c)N(0)}{2}\left[\frac{l^2 - 1}{2\ln l} + 1 - 2k_2\right].$$

Note that this profit is dependent on the small firm's capacity index k_2, as well as the initial price, maximum load, load ratio, and marginal cost.

When capacities are asymmetric, the totally pivotal (TP) set is defined as $TP_{asym} \equiv \{(k_2, l) \mid 0 < k_2 < 0.5, \max\{k_2, \sqrt{1 - 2k_2}\} \le l < 1\}$. If $(k_2, l) \in TP_{asym}$ then residual demand at time *t* for firm 1 at price $\overline{p}$ is, $(N(t) - K_2) > 0$. Profit for firm 1 is,

$$\Pi_1^{D} = \int_{t=0}^{1} (\overline{p} - c)(N(t) - K_2)dt = N(0)(\overline{p} - c)\left[\frac{l - 1}{\ln l} - k_2\right].$$

It is also necessary to consider the deviation incentives for the smaller firm. However, Genc (2003) shows that the deviation incentive for the small firm is less than the deviation incentive for the large firm. In the sequel we focus on the deviation incentive for the large firm.

For firm 1, deviation profit exceeds profit associated with the candidate SFE if $\Pi_1^D > \Pi_1^{SFE}$, or equivalently if,

(5.4) $$p(0) - c < \psi(k_2, l)(\overline{p} - c)$$

where, $$\psi(k_2, l) \equiv \frac{4(l - 1 - k_2 \ln l)}{(l^2 - 1 + 2\ln l - 4k_2 \ln l)}.$$

The function, $\psi(\cdot)$ generates a fraction for each pair (k_2, l) of parameters in the set TP_{asym}. If the markup at the initial price for a candidate SFE is less than the fraction ψ of the maximum markup, $\overline{p} - c$, then the candidate SFE is not an equilibrium.

Proposition 5. The $\psi(\cdot)$ function that characterizes the lower bound for the SFE initial markup in the asymmetric case is increasing in the load ratio l and decreasing in the smaller firm's capacity index k_2 for $(k_2, l) \in TP_{asym}$.

Proof. See the Appendix.

Equation (5.4) provides a sufficient condition for eliminating some candidate equilibria, just as (4.7) did for the symmetric case. Proposition 5 indicates that the set of equilibria being ruled out becomes larger as the capacity index decreases and the load ratio increases. The example that follows illustrates how the minimum SFE initial markup varies as the small firm's share of total capacity decreases.

Example 3. Let the load ratio be $l = 0.8$, and fix the amount of total capacity such that $k = 1.5$ (that is, $K_1 + K_2 \equiv K = 1.5N(0)$). Define the capacity share of the small firm as, $g \equiv K_2 / K$. Even with asymmetric capacities, symmetric supply function equilibria remain possible as long as $K_2 \geq \frac{1}{2}N(0)$; in terms of the small firm's share of capacity, this is equivalent to $g \in [\frac{1}{3}, \frac{1}{2}]$. For shares in this interval, the minimum SFE markup is given by $\phi(2kg, l, n) = \phi(3g, 0.8, 2)$. For shares below 1/3, a SFE must be asymmetric; the minimum SFE markup is given by $\psi(kg, l) = \psi(1.5g, 0.8)$. Figure 4 illustrates how the minimum SFE markup varies with the small firm's share of capacity. As the smaller firm's share of capacity drops from 50% to 33%, the minimum SFE markup increases from 36% to 98% of the maximum possible markup, $(\overline{p} - c)$. Thus almost all of the impact of a falling capacity share for the small firm occurs when the equilibria are symmetric, but capacities are asymmetric. As the share falls below 33%, the minimum SFE markup is already very high, and rises slowly. If the small firm's share of capacity falls below 12% then a SFE does not exist.

5.2 *Symmetric Firms with Step Marginal Costs*

In preceding sections we assumed that all suppliers have a common, constant marginal cost c for production up to capacity. Our approach can be extended to the more realistic case in which electricity generation firms have multi-step marginal cost functions. Rudkevich, et al. (1998) derive SFE results for a multi-step marginal cost model with identical firms. Genc (2003) examines the impact of pivotal suppliers on the set of supply function equilibria when each supplier has a two-step marginal cost schedule comprised of low-cost base load generation units and higher cost peak load units. We summarize two key findings from Genc (2003). First, with rising marginal cost steps, even the most competitive symmetric SFE can yield positive profits since market clearing prices may be above marginal cost for base load units. As a consequence, the presence of pivotal suppliers may not have any effect on the set of supply function equilibria. This is in contrast to the constant marginal cost case, in which the presence of pivotal suppliers always eliminates the most competitive supply function equilibria (the lower bound on equilibrium markups is always positive if there are pivotal suppliers). Second, the fraction of total capacity that is comprised of base load units has an impact on the set of supply function equilibria. By analyzing a series of numerical examples, Genc (2003) finds that the lower bound for the SFE initial price is non-increasing in the proportion of total capacity that is comprised of base load units.

5.3 *Equilibrium Selection*

In prior analyses based on the SFE approach, neither a change in the load distribution nor in the amount of excess capacity has any impact on the predicted set of equilibrium supply functions. In this paper we have shown that both of these factors do influence the set of equilibria. Relatively competitive supply function strategies are eliminated as equilibrium strategies when pivotal suppliers are present.

The symmetric model has a continuum of Pareto-ranked equilibria. Proposition 1 indicates that the most profitable (least competitive) SFE (the SFE with initial price equal to the price cap) is an equilibrium for the symmetric model for all possible parameter values. If suppliers are able to coordinate on the most profitable supply function equilibrium then the fact that some less profitable equilibria are eliminated when the load ratio and the capacity index change would be irrelevant for observed supply behavior. There are several reasons to question the ability of suppliers to coordinate on the most profitable SFE. First, as noted in Section 2.2, both Green and Newbery (1992) and Wolfram (1999) found that the most profitable symmetric SFE predicted prices were substantially above actual pool prices for the England and Wales wholesale electricity market. Second, evidence from laboratory experiments on coordination games shows that human subjects often fail to coordinate on the most profitable Nash equilibrium (see Van Huyck, et al. (1990)). Failure becomes more likely as the number of players rises and the complexity of the environment increases. Third, empirical evidence on the impact of pivotal suppliers cited in Section 2.1 suggests that the presence of pivotal suppliers does have an impact on wholesale electricity market prices. In the context of our analysis, if suppliers were consistently able to coordinate on the most profitable SFE then the presence of pivotal suppliers would have no impact on market prices.

5.4 *Downward Sloping Demand*

A downward sloping demand curve can be introduced in order to capture price-responsive buyers in the wholesale market and/or price-taking competitive fringe suppliers. One might conjecture that the results in previous sections depend crucially on our assumption of perfectly inelastic demand, up to a price cap. In this section we show that capacity constraints can also reduce the size of the equilibrium set in a model with downward sloping demand, even when the capacity constraints would not be binding in any candidate supply function equilibrium.

We utilize a model from Newbery (1998). Demand is assumed to be linear in price and decreasing over time during a trading period;

(5.5) $$D(p,t) = a(t) - bp \ ,$$

where $a'(t) < 0$ for $t \in [0,1]$ and $b > 0$. Marginal cost is constant and identical for each of $n \geq 2$ suppliers and is normalized to zero. In the absence of capacity constraints, Newbery shows that symmetric equilibrium supply functions are of the form:

$$(5.6) \qquad s(p) = \begin{cases} Ap - bp\ln(p), & \text{for } n = 2 \\ Ap^{1/(n-1)} - \dfrac{bp}{n-2}, & \text{for } n > 2. \end{cases}$$

A is a constant of integration that depends on the initial price, $p(0)$. The initial price may take on any value between the competitive price of zero and the Cournot price, $a(0)/b(n+1)$. Note that our equilibrium supply function for perfectly inelastic demand in (4.2) is a special case of (5.6) with b = 0. Several equilibrium supply functions are illustrated in Figure 5.

Now consider capacity constraints. As before, we assume that total capacity is K and that each supplier has capacity K/n. Suppose first that total capacity is less than the competitive quantity for peak demand, $a(0)$. This situation is illustrated by the dashed vertical line in Figure 5 at capacity per firm of K'/n. Green and Newbery (1992) note that capacity constraints like this rule out a subset of supply function equilibria; namely those equilibria involving supply functions that intersect $D(p,0)/n$ to the right of K'/n. While we do not provide a proof, it can be shown that capacity constraints per firm of K'/n may also eliminate additional supply functions as equilibria that intersect $D(p,0)/n$ to the left of K'/n.

Now suppose that total capacity K is greater than the competitive quantity for peak demand, $a(0)$. This situation is illustrated by the dashed vertical line in Figure 5 at capacity per firm of K''/n. Capacity constraints of this type would not directly rule out any of the symmetric supply function equilibria in (5.6) with initial prices between the competitive and Cournot prices. Each firm would have more than enough capacity to produce the maximum competitive quantity per firm of $a(0)/n$. However, capacity constraints of this type limit the ability of suppliers to expand production in response to high prices induced by a supply restriction initiated by any single firm. This in turn impacts the shape of the residual demand a firm faces at each time in the trading period. For prices below $p(0)$ residual demand is relatively price responsive, with price-derivative, $-(b+(n-1)s'(p))$. Residual demand is less price responsive for prices above $p(0)$, with price-derivative, $-b$. The difference in residual demand slopes above and below the initial SFE price causes profit to be non-concave in price. There may be a global profit optimum for a price above $p(0)$, in addition to a local profit optimum at the "SFE price" below $p(0)$, for some times in the trading period. Therefore, a supplier may have an incentive to defect from a candidate SFE involving a relatively low initial price, by restricting output and pushing price into the less price-responsive part of residual demand. The following proposition characterizes the maximal extent to which non-binding capacity constraints limit the set of symmetric supply function equilibria.

Proposition 6. Suppose that demand is given by (5.5) and that marginal cost is constant (normalized to zero) for each of $n \geq 2$ suppliers. Define a threshold price, $p^T \equiv \frac{a(0)}{2b}\left[1-\sqrt{\frac{n-1}{n}}\right]$.

i) If $K > a(0)$ then the supply function in (5.6) is a symmetric SFE for any initial price satisfying $p^T \leq p(0) \leq a(0)/b(n+1)$.

ii) Given an initial price $p(0) < p^T$, there exist total capacity $K > a(0)$ and smallest demand quantity intercept $a(1) < a(0)$ such that there is not a symmetric SFE with initial price $p(0)$.

Proof. See the Appendix.

The result is illustrated in Figure 5. There is a symmetric SFE associated with each initial price between the threshold price p^T and the Cournot price, as long as total capacity exceeds the competitive demand quantity, $a(0)$. However, for any initial price below p^T, there will not be a symmetric SFE if the amount of excess capacity $(K-a(0))$ and the extent of demand variation

$(a(0)-a(1))$ are both sufficiently small. For $n=2$ the threshold price is approximately 44 % of the Cournot price.

Proposition 6 is analogous to the results in Proposition 2 and Corollary 1 for perfectly inelastic demand. We showed that if the amount of excess capacity is sufficiently small and the load ratio is sufficiently close to one then there is a single supply function equilibrium; the least competitive SFE. When demand is downward sloping, an interval of the most competitive supply function equilibria is eliminated if the amount of excess capacity and the extent of demand variation are sufficiently small. However, an interval of the least competitive equilibria survives as long as there is enough capacity to produce the competitive output.

6. Conclusions

The supply function equilibrium (SFE) model has been applied to numerous analyses of generators' bidding behavior and electricity market power issues. The important role that pivotal suppliers play in wholesale electricity markets has been documented empirically. However SFE models in which pivotal suppliers can play a role have not been considered in the literature.

In this paper we examine the connection between pivotal suppliers and the set of supply function equilibria. We include production capacity constraints in our model to allow for the possibility that a single supplier is pivotal; that is, the supplier can unilaterally move the market price by a large amount by withholding a small amount of capacity. While other SFE analyses have considered capacity constraints, we argue that these analyses failed to properly account for the impact of these constraints due to a focus on local, rather than global, optimality conditions.

In our constant marginal cost, symmetric firms model, we show that when pivotal players are present, the set of supply function equilibria shrinks. It is the most competitive equilibria that are eliminated when there are pivotal suppliers. The equilibrium set shrinks as the number of players and the capacity index decrease and the load ratio increases. In the asymmetric firms model we show that the larger firm's deviation incentives determine which strategies are ruled out as equilibrium strategies.

Production capacity constraints introduce a non-concavity in profit functions. In addition to a local profit optimum at the "SFE price" there may be a global profit optimum at a higher price. This can provide an incentive for defection from relatively competitive supply strategies that satisfy local optimality conditions. We show that this effect of capacity constraints is present in a model with downward sloping demand, as well as for the model with perfectly inelastic demand. We also show how our approach may be extended to the empirically more realistic case of multi-step marginal costs. We derive results that show how the amount of base-load (low cost) capacity alters the minimal exercise of market power.

Most of this analysis is based on a kind of "simple deviation" from proposed equilibrium strategies, in which all of a supplier's capacity is bid in at the maximum price. However, we also consider more complex optimal supply responses, based on an optimal control formulation. We show that for a subset of parameter values, it is sufficient to consider simple deviations. For parameters outside of this subset, we illustrate how an optimal supply response can rule out strategies as equilibria that could not be ruled out by a simple deviation.

There are a variety of possible extensions of our analysis. More general cost functions that better approximate costs of electricity generation from various plant types could be introduced. We could perform a more general analysis of a model with a downward sloping demand. We conjecture that the spirit of our results would carry through to more general models. That is, production capacity constraints shape the set of equilibrium supply functions via their role in limiting rivals' ability to respond to a firm's high price-low output supply strategy.

APPENDIX

Proof of Proposition 1. Suppose that each of firm *i*'s rivals uses supply function strategy,

(A1) $$s(p)=\frac{N(0)}{n}\left[\frac{p-c}{\overline{p}-c}\right]^{1/(n-1)}.$$

This supply function satisfies the capacity constraint for all $p\in[c,\overline{p}]$ since $k\geq 1$. Profit for firm *i* at time *t* is,

$$\pi_i(p,t)=(p-c)\left[N(t)-\frac{(n-1)N(0)}{n}\left[\frac{p-c}{\overline{p}-c}\right]^{1/(n-1)}\right].$$

$\pi_i(\cdot,t)$ is twice differentiable and strictly concave for $p\in[c,\overline{p}]$. Therefore, for each $t\in[0,1]$ there is a unique price in $[c,\overline{p}]$ that maximizes $\pi_i(\cdot,t)$; this price is defined in (4.3) with $p(0)=\overline{p}$. Firm *i*'s choice of the supply function defined by (A1) implements the optimal prices with a non-decreasing supply function. Since prices are profit-maximizing at each $t\in[0,1]$, these prices maximize the integral of firm *i*'s profits over $t\in[0,1]$. Firm *i*'s choice of the supply function defined by (A1) is a best response to rivals' choices of the same supply function. □

Proof of Proposition 2. We begin by listing several facts about functions of parameters (l,n) that we use in the proof. We omit the proof of a fact when the proof is straightforward.

Fact 1: $1-l^n+nl^n\ln l>0$ for $l<1$.

Fact 2: $1-l+\ln l<0$ for $l<1$.

Fact 3: $b(l,n)\equiv n(1-l)+n\ln l-\ln l\in(0,(1-l)/l)$ for $l\in((n-1)/n,1)$.

Proof of Fact 3: $b(1,n)=0$ and $\partial b(l,n)/\partial l=-n+(n-1)/l<0$ since $l\in((n-1)/n,1)$. As a result, $b(l,n)>0$. Next we consider the proposed upper bound for $b(l,n)$, $a(l)\equiv(1-l)/l$. We note that $a(1)\equiv 0$. One can show that $a'(l)<\partial b(l,n)/\partial l<0$ for $l\in((n-1)/n,1)$. This slope condition, coupled with the condition $b(1,n)=a(1)=0$, establishes that $b(l,n)<a(l)$ for $l\in((n-1)/n,1)$.

Fact 4: $(n-1)\ln l+1\in(0,l)$ for $l\in((n-1)/n,1)$.

Proof of Fact 4: First note that $e^{-1/(n-1)}<\dfrac{n-1}{n}$ for $n\geq 2$.Taking logs of both sides of the inequality yields, $-1/(n-1)<\ln((n-1)/n)<\ln l$, where the second inequality follows from the lower bound on *l*. Rearranging this inequality yields $(n-1)\ln l+1>0$. Next we consider the upper bound for

$(n-1)\ln l+1$. Note that $\ln l < l-1$ for l < 1. Furthermore, $(n-1)\ln l \le \ln l$ for l < 1, since $n \geq 2$. Combining these two inequalities yields, $(n-1)\ln l < l-1$, or $(n-1)\ln l+1 < l$, which is the desired result.

The remainder of the proof is divided into three parts, with each part devoted to the impact of a change in one element of (k,l,n).

(a) *Capacity index k*: Note that $\phi(.)$ is an affine function in k, and

$\partial\phi/\partial k = [n(n-1)\ln l]/[1-l^n] < 0$, since $\ln l < 0$. □

(b) *Load ratio l*: We initially suppose that k = 1. At k = 1 we have,

$$\text{(A2)} \qquad l(1-l^n)^2\frac{\partial\phi}{\partial l} = l^n[(n^3-n^2)\ln l + l(n^2-n^3)+n^3-n^2+n] + [n^2(1-l)-n].$$

Define the function $\mu(l,n)$ as the right-hand-side of equation (A2). Note that $\mu(1,n)=0$. The partial of μ with respect to the load ratio simplifies to,

$\partial\mu/\partial l = n^2 l^n[(n(n-1)\ln l)/l - n^2 + 1 + n^2/l] - n^2$.

Let $q(l) = [n(n-1)\ln l/l - n^2 + 1 + n^2/l] - l^{-n}$. We claim that $q(l) < 0$, since $q(1)=0$, and, $\partial q/\partial l = -n[(n-1)\ln l + 1]/l^2 + n[l^{-n+1}]/l^2 > 0$ because of fact 4. This in turn implies that $\mu_l(l,n) < 0$ for l < 1, so that $\mu(l,n) > 0$ for l < 1. Using (A2), we have $\phi_l(1,l,n) > 0$.

Now we extend this result to k > 1. Differentiating ϕ_l with respect to k yields,

$$\phi_{lk} = \frac{1}{(1-l^n)^2}\left((1-l^n)\left(\frac{n(n-1)}{l}\right) + n(n-1)nl^{n-1}\ln(l)\right) = \frac{n(n-1)}{l(1-l^n)^2}\left[1-l^n+nl^n\ln(l)\right]$$

The term in square brackets is positive by Fact 1, so that, $\phi_{lk} > 0$. This result, coupled with $\phi_l(1,l,n) > 0$, implies that $\phi_l(k,l,n) > 0$ for all $(k,l,n) \in TP$. □

(c) *Number of suppliers n*: We initially suppose that k = 1. At k = 1 we have,

$$\text{(A3)} \qquad (1-l^n)^2\frac{\partial\phi}{\partial n} = \{1-l^n+nl^n\ln l\}[n(1-l)+n\ln l-\ln l] + n(1-l^n)[1-l+\ln l].$$

The sign of $\partial\phi/\partial n$ is not clear, because the first term on the RHS of (A3) is positive (by Facts 1 and 3) and the last term is negative (by Fact 2). Define the function $\omega(l,n)$ as the right-hand-side of equation (A2). Note that $\omega(1,n)=0$. The partial of ω with respect to the load ratio may be expressed as,

$$\partial\omega(l,n)/\partial l = E+F+G+H,$$

where, $E = (n^2 l^{n-1} \ln l)(n(n-1) + (n-1)\ln l)$, $F = (1 - l^n + nl^n \ln l)(-n + (n-1)/l)$, $G = (-n^2 l^{n-1})(1 - l + \ln l)$, and $H = n(1 - l^n)(-1 + 1/l)$.

By using the facts stated at the beginning of the proof we can establish the following inequalities: $E < 0$, $F > 0$, $G > 0$, and $H > 0$. Let $D \equiv E + H$. Note that $D = 0$ at $l = 1$.

Consider the derivative of D with respect to the load ratio,

$$\partial D/\partial l = \partial E / \partial l + \partial H / \partial l,$$

where,

$\partial E/\partial l = (n^2 l^{n-2}(1 + (n-1)\ln l))(n(1-l) + (n-1)\ln l) + (n^2 l^{n-1} \ln l)(-n + (n-1)l^{-1}) > 0$, since the first term on the RHS is the product of two positive terms (using Facts 3 and 4) and the second term is the product of two negative terms, and

$\partial H/\partial l = (-n^2 l^{n-1}(-1 + 1/l)) + (n - nl^n)(-1/l^2) < 0$.

Next we compare the magnitude of $\partial E/\partial l$ and $\partial H/\partial l$. Comparing the first terms in these two derivatives, we have,

(A4) $$\left|-n^2 l^{n-1}(-1 + 1/l)\right| > (n^2 l^{n-2}(1 + (n-1)\ln l))(n(1-l) + (n-1)\ln l),$$

by using Facts 3 and 4. Define the function $m(l,n)$ as the sum of the last term in $\partial E / \partial l$ and the last term in $\partial H / \partial l$, so that, $m(l,n) = (n^2 l^{n-1} \ln l)(-n + (n-1)l^{-1}) + (n - nl^n)(-1/l^2)$. By using $m(1,n) = 0$ combined with the kind of derivative argument that we have used several times in this proof, we can establish that $m(l,n) < 0$. This fact, coupled with the inequality (A4), yields, $\partial D/\partial l = \partial E / \partial l + \partial H / \partial l < 0$, which in turn establishes that $D \equiv E + H > 0$. Signing D was the final step in establishing that $\partial \omega(l,n) / \partial l = E + F + G + H > 0$. This derivative result for $\omega(l,n)$ establishes that $\omega(l,n) < 0$. Since we defined the RHS of (A3) as $\omega(l,n)$, we have shown that $\partial \phi/\partial n$ is negative, evaluated at $k = 1$.

The final step in the proof of part (c) is to extend the derivative result for $\partial \phi/\partial n$ to $k > 1$. We have,

$$(1 - l^n)^2 \frac{\partial \phi}{\partial n} = (1 - l^n + nl^n \ln l)[n(1-l) + kn \ln l - k \ln l] + n(1 - l^n)[1 - l + k \ln l],$$ which is affine in k. Also,

$$(1 - l^n)^2 \frac{\partial^2 \phi}{\partial n \partial k} = [1 - l^n + nl^n \ln l](n-1)\ln l + (n - nl^n)\ln l < 0,$$

where the inequality follows because the term in square brackets is positive by Fact 1. As *k* increases $\frac{\partial \phi}{\partial n}$ becomes more negative, thus the result of part (c) follows. □

Proof of Corollary 1. It is profitable for a firm to deviate from its candidate equilibrium supply strategy (defined by (4.2)) if, $p(0)-c<\phi(k,l,n)(\bar{p}-c)$. The $\phi(\cdot)$ function is not defined at $l=1$. However, note that $\lim_{l\uparrow 1}\phi(k,l,n)=n-k(n-1)$. For $l<1$ we define $\phi(\cdot)$ as in (4.8); for $l=1$ we define $\phi(k,1,n)=n-k(n-1)$. Then $\phi(\cdot)$ is continuous in (k,l) for $(k,l)\in[1,n/(n-1)]\times(0,1]$, for each $n\geq 2$. Note that $\phi(1,1,n)=1$.

Let $d\langle x,x'\rangle$ be the metric for the Euclidean distance between two vectors x and x' in $\mathbb{R}^2$. Define $\varepsilon\equiv(\bar{p}-p(0))/(\bar{p}-c)>0$. By the continuity of $\phi(\cdot)$ in (k,l), there exists $\delta(\varepsilon)>0$ such that $|\phi(1,1,n)-\phi(k,l,n)|<\varepsilon$ if $d\langle(1,1),(k,l)\rangle<\delta(\varepsilon)$. Let $k'=1+\frac{1}{2}\delta(\varepsilon)$ and $l'=1-\frac{1}{2}\delta(\varepsilon)$. Then $d\langle(1,1),(k',l')\rangle<\delta(\varepsilon)$ and $1-\phi(k',l',n)<\varepsilon$. This final inequality is equivalent to, $p(0)-c<\phi(k',l',n)(\bar{p}-c)$. Thus, if the capacity index is $k'>1$ and the load ratio is $l'<1$ then a simple deviation is more profitable than using the candidate SFE strategy. □

Proof of Proposition 3. Let $h(k,l,n,\tau)\equiv[n^2(1-l^\tau)+n(n-1)k\tau\ln l]/[1-l^n]$. Then $\partial h/\partial\tau=n\ln l[(n-1)k-nl^\tau]/(1-l^n)=0$, since the term in square brackets is zero by the definition of τ. We will use this equality for the proof of the following.

(a) *Capacity index k:* We take the total derivative of $\lambda(.)$ with respect to *k*.

$$\frac{d\lambda}{dk}=\frac{\partial h}{\partial\tau}\frac{\partial\tau}{\partial k}+\frac{\partial\lambda}{\partial k}.$$

Here $\partial\lambda/\partial k=n(n-1)\tau\ln l/(1-l^n)<0$, since $\ln l<0$. Thus $\frac{d\lambda}{dk}<0$. □

(b) *Load ratio l:* Take the total derivative of $\lambda(.)$ with respect to *l*: $\frac{d\lambda}{dl}=\frac{\partial h}{\partial\tau}\frac{\partial\tau}{\partial l}+\frac{\partial\lambda}{\partial l}$.

We need to determine the sign of the term, $\partial\lambda/\partial l$:

$$l(1-l^n)^2\frac{\partial\lambda}{\partial l}=l^n[\tau k(n^3-n^2)\ln l+l^\tau(\tau n^2-n^3)+n^3-\tau kn^2+\tau kn]+[\tau n^2(k-l^\tau)-\tau kn].$$

Note that $l(1-l^n)^2 \frac{\partial \lambda}{\partial l}\Big|_{\tau=0} = 0$. By the definition of τ,

(A4) $\tau n^2(k-l^\tau) - \tau kn = \tau n^2[k(n-1)/n - l^\tau] = 0$.

Define

(A5) $Y(.) = [\tau k(n^3-n^2)\ln l + l^\tau(\tau n^2 - n^3) + n^3 - \tau kn^2 + \tau kn]$.

If we show that $\partial Y/\partial \tau < 0$ then the proof of part (b) is complete. We obtain,

$$\partial Y/\partial \tau = k(n^3-n^2)\ln l + nk(1-n) + l^\tau n^2 + l^\tau(\tau n^2 - n^3)\ln l.$$

Denote $y = \partial Y/\partial \tau$. Then $\partial y/\partial k = (n^3-n^2)\ln l + n(1-n) < 0$ implies that y reaches its maximum at k=1. Now re-write y at k=1 as,

$$\bar{y} = (n^3-n^2)\ln l + n(1-n) + l^\tau n^2 + l^\tau(\tau n^2 - n^3)\ln l.$$

We will show that the maximum of $\bar{y}$ is negative. Then,

$$\partial \bar{y}/\partial \tau = l^\tau n^2 \ln l[2 + (\tau - n)\ln l] < 0.$$

Now if we solve $\bar{y} = 0$ for τ, we obtain $\bar{\tau} = \ln(1 + 1/[(n^2-n)\ln l - n])/\ln l$, but $\bar{\tau}$ is less than the constraint $\tau(k=1) = \ln((n-1)/n)/\ln l$. These imply that $\partial Y/\partial \tau < 0$.

Thus by the results of (A4) and (A5), the proof of this proposition is complete. □

(c) *Number of suppliers n:* We take the total derivative of $\lambda(.)$ with respect to n.

We obtain $\frac{d\lambda}{dn} = \frac{\partial h}{\partial \tau}\frac{\partial \tau}{\partial n} + \frac{\partial \lambda}{\partial n}$, where we need to determine the sign of second term. Now if we show that for all τ, the maximum of $\partial \lambda/\partial n$ is negative then the proof is complete. The partial derivative of λ with respect to n becomes

$(1-l^n)^2 \frac{\partial \lambda}{\partial n} = (1 - l^n + nl^n \ln l)[n(1-l^\tau) + \tau k(n-1)\ln l] + n(1-l^n)[1 - l^\tau + \tau k \ln l]$. Note that

$(1-l^n)^2 \frac{\partial \lambda}{\partial n}\Big|_{\tau=0} = 0$. Now consider how the partial derivative of λ with respect to n varies with τ:

$$(1-l^n)^2 \frac{\partial^2 \lambda}{\partial n \partial \tau} = (1 - l^n + nl^n \ln l)[-nl^\tau \ln l + k(n-1)\ln l] + n(1-l^n)(-l^\tau \ln l + k \ln l).$$

The term in square brackets is zero, given the definition of τ. The fact that $k \geq 1 > l^\tau$ implies that $(1-l^n)^2 \frac{\partial^2 \lambda}{\partial n \partial \tau} < 0$. Since $\frac{\partial \lambda}{\partial n}\Big|_{\tau=0} = 0$ and $\frac{\partial^2 \lambda}{\partial n \partial \tau} < 0$, we have $\frac{\partial \lambda}{\partial n} < 0$ for $\tau = \ln[k(n-1)/n]/\ln l > 0$.

As a consequence, the total derivative of $\lambda(.)$ with respect to n becomes negative. □

Proof of Proposition 4. Suppose it is optimal for firm *i* to deviate from its candidate SFE strategy. Then an optimal supply response specifies price $\overline{p}$ and quantity $q(t) = N(t) - (n-1)K/n$ for $t \in [0, \tau)$ for some $\tau > 0$. For $t \in [\tau, 1]$ the quantity is held fixed at $q(\tau)$ and the price is equal to $p(0)$, the initial price for the candidate SFE. The parameter restriction $l \geq 1 - (k-1)(n-1)/n$ implies that the clearing price does not fall below $p(0)$ when the quantity is fixed at $q(\tau)$, for all $\tau \in (0,1]$. The restriction that parameters are in set *TP* implies that $q(\tau) \geq 0$, for all $\tau \in (0,1]$.

If a deviation from the candidate SFE strategy is optimal, the profit associated with the deviation is, $W(\tau) = \int_0^\tau (\overline{p} - c) q(t) dt + (1-\tau) q(\tau)(p(0) - c)$, for $\tau \in (0,1]$. The first and second derivatives are,

$$W'(\tau) = (\overline{p} - p(0)) q(\tau) + N(0) \ln(l)(1-\tau) l^\tau (p(0) - c),$$

$$W''(\tau) = N(0) l^\tau \ln(l)[(\overline{p} + c - 2p(0)) + \ln(l)(1-\tau)(p(0) - c)].$$

An optimal supply response requires that τ is chosen from the interval (0,1] to maximize $W(\cdot)$. Note two features of $W(\cdot)$. First, if $W(\cdot)$ is concave at some $\tau' > 0$ then $W(\cdot)$ is strictly concave for $\tau \in (\tau', 1]$. Second, $W'(1) = (\overline{p} - p(0)) q(1) \geq 0$. These two features imply that there cannot be an interior optimum for τ; the optimal supply response sets $\tau = 1$. This corresponds to maintaining the price at $\overline{p}$ for all times $t \in [0,1]$, which is a simple deviation. □

Proof of Proposition 5.

(a) *Capacity index* k_2: We take the derivative of $\psi(.)$ with respect to k_2. After re- arranging terms we have,

$$sign\left(\frac{\partial \psi}{\partial k_2}\right) = sign\left((-4\ln l)(1-\psi_1)(l^2 - 1 + 2\ln l - 4k_2 \ln l)\right) < 0,$$

since the first two terms are positive, and the final term is negative. □

(b) *Load ratio* l: After taking the derivative of $\psi(.)$ with respect to *l*, we obtain,

$$sign\left(\frac{\partial \psi}{\partial l}\right) = sign\left(4(1 - k_2/l)(l^2 - 1 + 2\ln l - 4k_2 \ln l) - 4(l - 1 - k_2 \ln l)(2l + 2/l - 4k_2/l)\right)$$

The sign is not clear yet, because the first and last terms are positive and other terms are negative. Let $Z(.) = 4(1 - k_2/l)(l^2 - 1 + 2\ln l - 4k_2 \ln l) - 4(l - 1 - k_2 \ln l)(2l + 2/l - 4k_2/l)$.

We will show that the minimum of *Z*(.) is positive. Note that $Z(l = k_2) > 0$, and moreover

$$\frac{\partial Z}{\partial l} = 4(-k_2 / l^2)(l^2 - 1 + 2\ln l - 4k_2 \ln l) - 4(l - 1 - k_2 \ln l)(2 - 2/l^2 + 4k_2 / l^2) > 0,$$ since the first three terms in parentheses are negative, and the last term is positive. Hence the result follows. □

Proof of Proposition 6. Part *i*) Each of a firm's $(n-1)$ rivals use the supply function $s(p)$ in (5.6) for $p \le p(0)$ and supply K/n for $p > p(0)$. Therefore, the firm's profit as a function of price and time is,

$$\pi(p,t) = \begin{cases} p(a(t) - bp - (n-1)s(p)) & \text{for } p \le p(0) \\ p(a(t) - bp - (n-1)K/n) & \text{for } p > p(0) \end{cases}$$

For each $t \in [0,1]$ there is a locally optimal price, $p^*(t) \le p(0)$, and associated quantity, $s(p^*(t))$; the candidate SFE price and supply quantity, respectively.

If $K \ge n(a(0) - 2bp(0))/(n-1)$ then $\pi_p(p,t) < 0$ for $p > p(0)$ and the locally optimal SFE price, $p^*(t)$, is globally optimal at *t*, for each $t \in [0,1]$.

If $K < n(a(0) - 2bp(0))/(n-1)$ then there is a time $\tau \in (0,1]$ such that there is a locally optimal price $\tilde{p}(t) > p(0)$ for each $t \in [0,\tau)$. The inequality $K > a(0)$ implies,

(A6) $$\pi(\tilde{p}(0),0) = \frac{(a(0) - (n-1)K/n)^2}{4b} < \frac{a(0)^2}{4bn^2}.$$

At time zero the locally optimal candidate SFE price is $p(0)$, the initial price associated with the candidate SFE.

(A7) $$\pi(p^*(0),0) = \pi(p(0),0) = p(0)(a(0) - bp(0))/n \ge \frac{a(0)^2}{4bn^2}.$$

The weak inequality is implied by $p(0) \ge p^T$. Combining (A6) and (A7) yields, $\pi(p(0),0) > \pi(\tilde{p}(0),0)$. That is, the local price optimum at $p(0)$ is a global profit optimum for time zero.

Next, differentiate profits with respect to *t*, for $t \in [0,\tau)$.

(A8) $$\frac{d\pi(p^*(t),t)}{dt} = \pi_p(p^*(t),t)\frac{dp^*(t)}{dt} + p^*(t)a'(t) = p^*(t)a'(t)$$

(A9) $$\frac{d\pi(\tilde{p}(t),t)}{dt} = \pi_p(\tilde{p}(t),t)\frac{d\tilde{p}(t)}{dt} + \tilde{p}(t)a'(t) = \tilde{p}(t)a'(t)$$

The π_p terms in (A8) and (A9) are zero because $p^*(t)$ and $\tilde{p}(t)$ are locally optimal prices. The inequalities $\tilde{p}(t) > p^*(t)$ and $a'(t) < 0$ imply that

$$0 > d\pi(p^*(t),t)/dt > d\pi(\tilde{p}(t),t)/dt$$

for $t \in [0,\tau)$. That is, locally optimal profits for prices above $p(0)$ fall faster than locally optimal profits for prices at or below $p(0)$. Since $\pi(p(0),0) > \pi(\tilde{p}(0),0)$, we have shown that $p^*(t)$ is a global profit optimum for $t \in [0,\tau)$. For $t \in [\tau,1]$ profits are strictly decreasing in price for $p > p(0)$, so $p^*(t)$ is a global profit optimum for $t \in [\tau,1]$ as well. This establishes that supply function $s(p)$ in (5.6) is a profit maximizing response to the choice of $s(p)$ by each of $(n-1)$ rivals.

Part *ii*) Define the Cournot profit function at time *t* as, $\pi^C(x,y,t) = x(a(t)-x-y)/b$, where *x* is own output and *y* is rivals' total output. Let,

$$\pi^T \equiv \max_{x\geq 0} \pi^C(x,(n-1)a(0)/n,0) = \frac{a(0)^2}{4bn^2}.$$

π^T is the maximum Cournot profit at time zero if each rival produces its competitive output, $a(0)/n$. The threshold price p^T is implicitly defined as the lowest price satisfying, $\pi^T = p(a(0)-bp)/n$. Profit from the candidate SFE strategy is π^S where,

$$\pi^S \equiv \int_{t=0}^{1} p^*(t)s(p^*(t))dt < p(0)(a(0)-bp(0))/n < \pi^T.$$

The right-most inequality follows from $p(0) < p^T$. Let $\varepsilon \equiv \pi^T - p(0)(a(0)-bp(0))/n > 0$.

Suppose the firm adopts a constant supply strategy with output equal to, $\frac{1}{2}(a(0)-(n-1)K/n)$. The market clearing price at *t* is $(a(t)-\frac{1}{2}a(0)-(n-1)K/2n)/b$ and profit at *t* is, $(a(0)-(n-1)K/n)(2a(t)-a(0)-(n-1)K/n)/4b$. Total profit for the constant supply strategy is,

$$\pi^{CS} \equiv \int_{t=0}^{1} [(a(0)-(n-1)K/n)(2a(t)-a(0)-(n-1)K/n)/4b]dt > (a(1)-(n-1)K/n)^2/4b$$

Define $\delta \equiv a(0)\varepsilon/2\pi^T > 0$ and suppose that $K < a(0)+\delta$ and $a(1) > a(0)-\delta$. Then

$$\pi^{CS} > (a(1)-(n-1)K/n)^2/4b > (a(0)-\delta)^2/4bn^2 > \pi^T - 2\pi^T\delta/a(0) > \pi^T - \varepsilon.$$

This inequality combined with the definition of ε implies that $\pi^{CS} > \pi^S$; that is, profits associated with the constant supply strategy exceed profits from the candidate SFE strategy. This demonstrates that there exist $K > a(0)$ and $a(1) < a(0)$ such that a deviation from the candidate SFE strategy is profitable when $p(0) < p^T$. □

Figure 1
Candidate Symmetric SFE

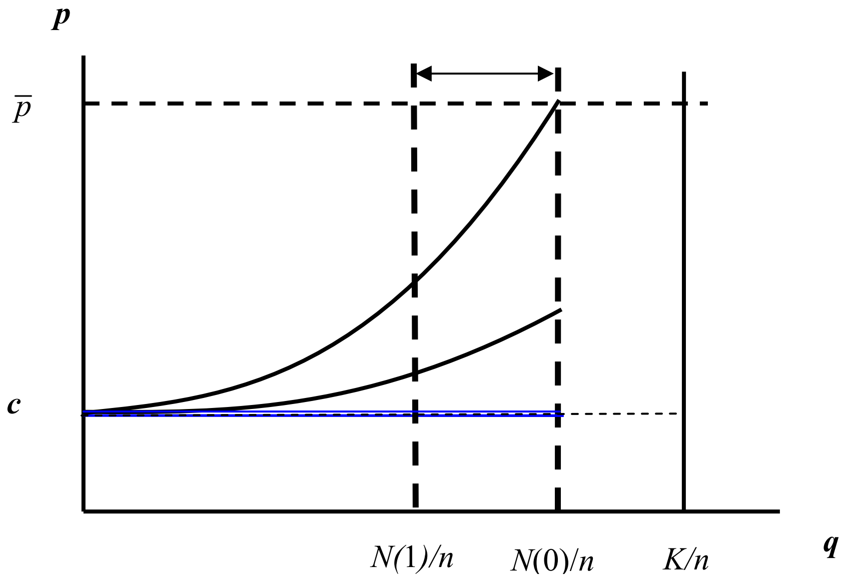
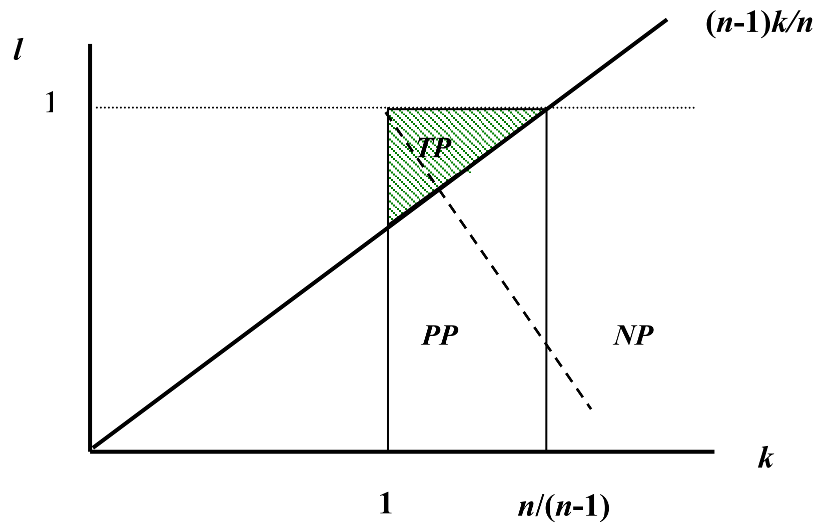


Figure 2
Parameter Ranges for Capacity Index (k)
and Load Ratio (l)*



* Assumes a fixed number of firms (n). Simple deviation is the optimal supply response in the area above the dashed line in TP region.

Figure 3A
Optimal (q , p) Pairs

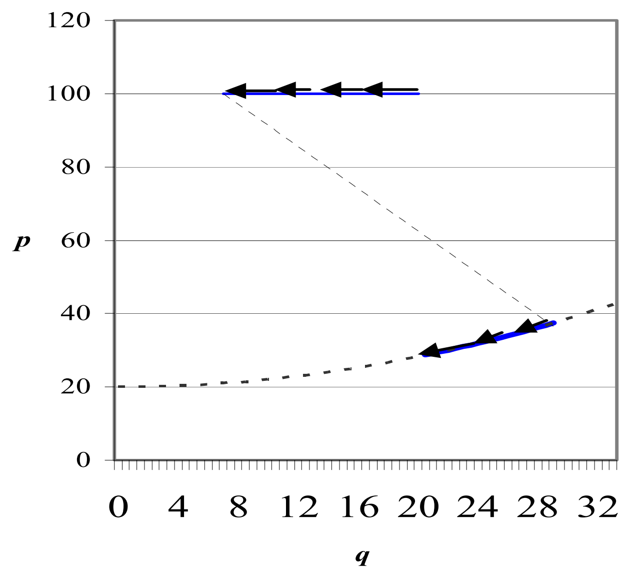


Figure 3B
Constrained Optimal (q , p) Pairs

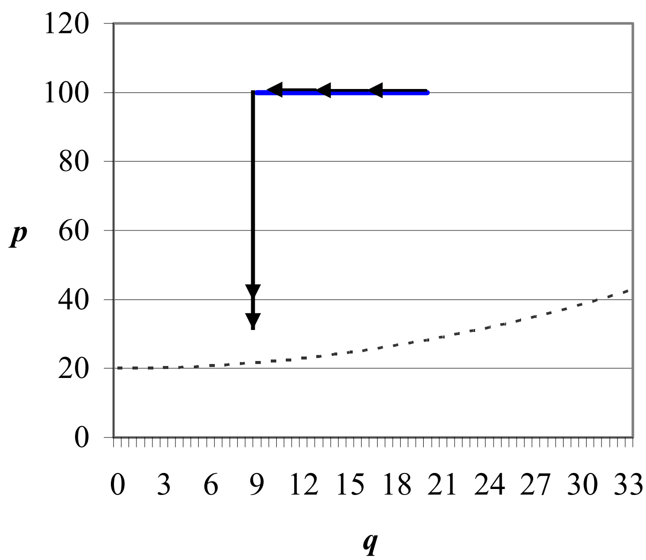


Figure 4
Minimum SFE Markups with Asymmetric Capacities
(As fraction of maximum markup)

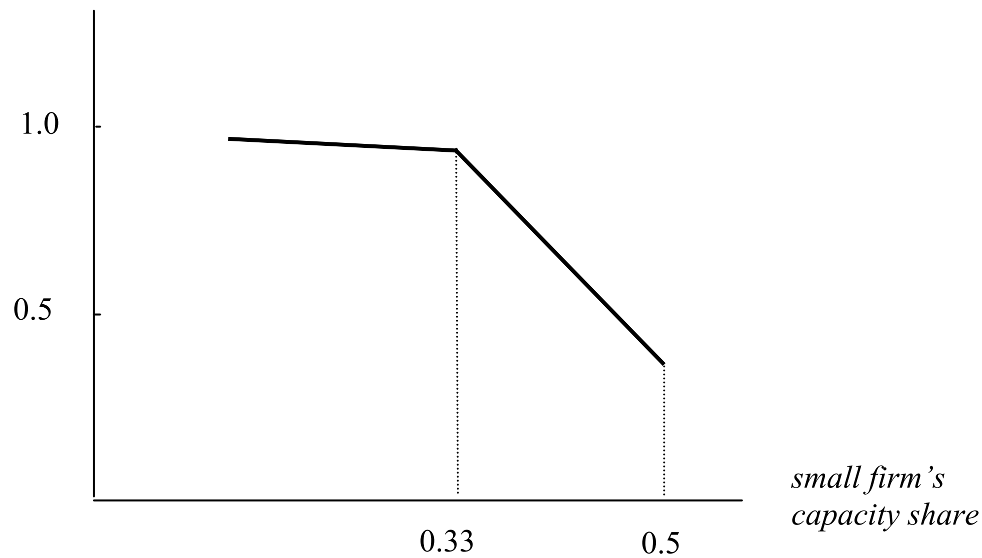
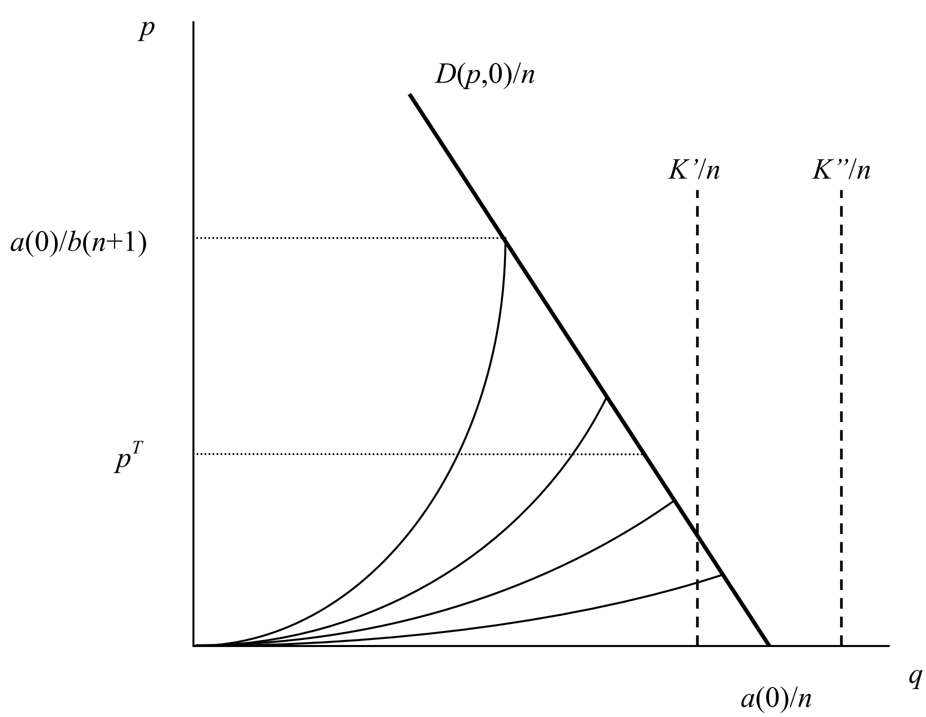


Figure 5
Supply Function Equilibria with Downward Sloping Demand



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