Minisymposium: Equilibria and optimization for energy systems:

An appreciation of the work of Andy Philpott

- Energy systems raise optimization questions across multiple time scales:
  - real-time dispatch and pricing;
  - hydro and storage operation;
  - market equilibrium and investment;
  - long-term decarbonization planning.
- A recurring theme is the interaction between:
  - uncertainty,
  - market design,
  - optimization models,
  - and practical energy policy.
- This minisymposium brings together collaborators and colleagues of Andy Philpott and highlights recent contributions to these topics.

A long collaboration!

- This paper submitted December 22, 1980

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A CLASS OF CONTINUOUS NETWORK FLOW PROBLEMS*

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The solution in discrete time of the problem of maximizing the flow in a network with time-varying arc capacities and storage at the nodes is a straightforward extension of the static case. In this paper the problem is formulated and solved in continuous time. A continuous version of the Ford–Fulkerson theorem is proved, and an analogue of the labelling algorithm developed. An example is given to clarify some of the ideas of the paper and the duality theory for this problem is discussed.

How Long-Term Demand Forecasts Shape Optimal Decarbonization Paths

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The setting

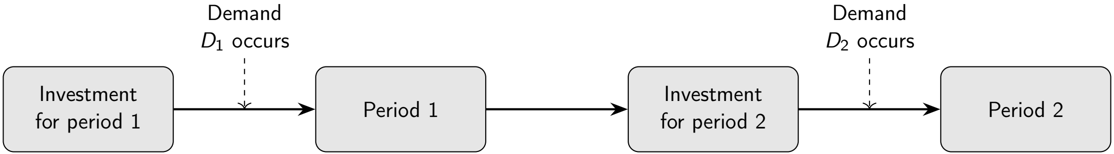
- Many countries have committed to challenging carbon reduction targets.
- Electricity demand is expected to grow and renewables are expected to become cheaper, but by how much is uncertain.
- How should we balance renewable and non-renewable investment to meet carbon targets under substantial demand and cost uncertainty?
- How do we decide between early large scale investment in renewables, and a "wait and see" strategy?
- How does the amount of demand uncertainty influence the optimal renewable investment path and the total expected costs?
- And what happens with uncertainty in renewable costs?

Outline of the talk

- Set up the (three-stage) stochastic model.
- Explain the connections to a double critical fractile solution.
- Give the main result characterising an optimal solution with no cost uncertainty.
- What does this look like with realistic parameter choices?
- Conclusions

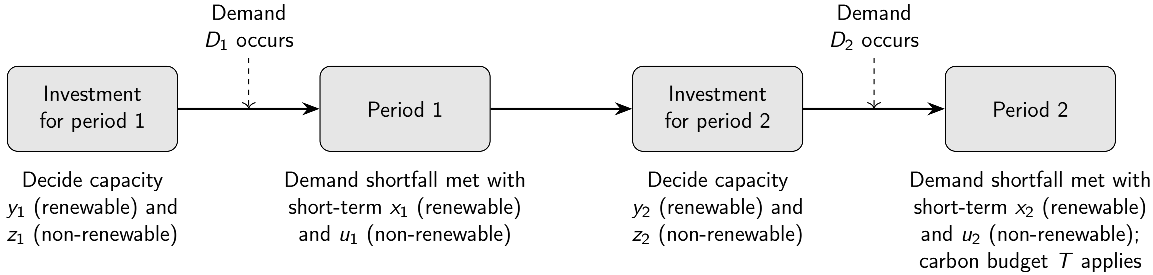
The model

- Two periods of equal length, with demands D_1 and D_2.
- Long term capacity is built in advance, and if built for period 1 is still available in period 2.
- Short term capacity can be adjusted according to the demand that occurs.
- Both renewable and non-renewable capacity can be built. The fixed carbon budget T applies at the end of period 2.



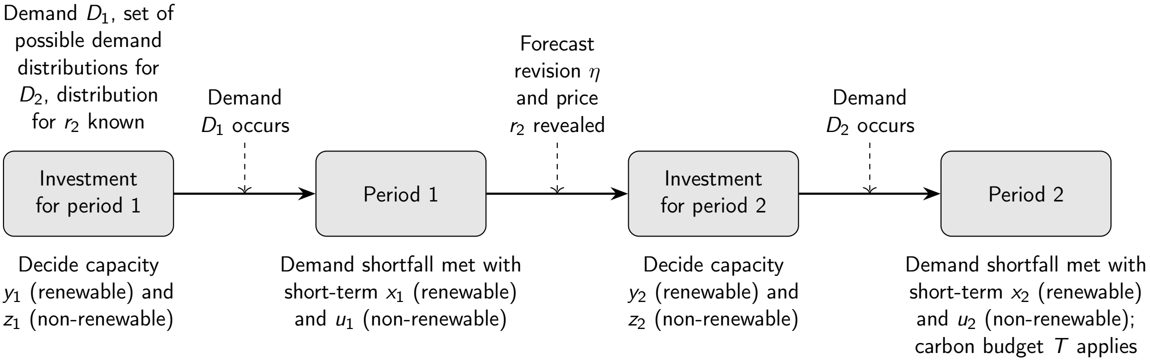
The model: notation

- Long term renewable capacity y_1 and y_2, cost r_1 and r_2 (renewables get cheaper)
- Long term non-renewable capacity z_1 and z_2, cost p_z
- Short term renewable capacity x_1 and x_2, cost p_x
- Short term non-renewable capacity u_1 and u_2, cost p_u



The model: structure of uncertainty

- No uncertainty in D_1. The distribution for D_2 varies according to a shift parameter η, so the cdf for D_2 is $F_\eta(x) = F_0(x - \eta)$ where η is revealed after period 1.
- The second-period renewable investment cost r_2 is uncertain, drawn from a known distribution, and its value is revealed after period 1.



A three-stage stochastic program

$$
\begin{aligned}
\min_{y_1, z_1, u_1, x_1 \geq 0} \quad & 2r_1 y_1 + 2p_z z_1 + p_x x_1 + p_u u_1 + \mathbb{E}_{\eta, r_2}[Q_1(y_1, z_1, u_1, \eta, r_2)] \\
\text{s. t.} \quad & u_1 + x_1 + y_1 + z_1 \geq D_1
\end{aligned}
$$

$$
\begin{aligned}
Q_1(y_1, z_1, u_1, \eta, r_2) = \min_{y_2, z_2 \geq 0} \quad & r_2 y_2 + p_z z_2 + \mathbb{E}_{D_2}[Q_2(y_1, y_2, z_1, z_2, u_1, D_2 + \eta)] \\
\text{s. t.} \quad & z_1 + z_2 \leq T
\end{aligned}
$$

$$
\begin{aligned}
Q_2(y_1, y_2, z_1, z_2, u_1, D_2) = \min_{u_2, x_2 \geq 0} \quad & p_u u_2 + p_x x_2 \\
\text{s. t.} \quad & u_2 + x_2 + y_1 + z_1 + y_2 + z_2 \geq D_2 \\
& u_2 + z_1 + z_2 \leq T \\
& u_2 - u_1 \leq 0
\end{aligned}
$$

Simplifications

Assumption: price ordering

$p_x > p_u > p_z$ and $p_x > r_1 > r_2$.

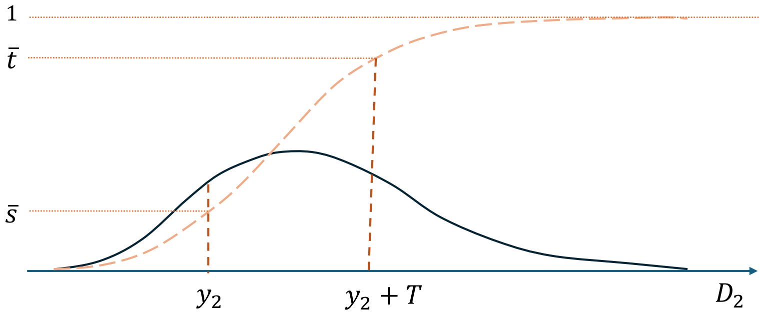
- Short term renewables are not used in period 1: $x_1 = 0$ and $u_1 = D_1 - y_1 - z_1$.
- Long term non-renewables are only built in first period: $z_2 = 0$.
- Second-period short-term generation amounts are $u_2 = \min(u_1, T - z_1, (D_2 - y_1 - y_2 - z_1)_+)$ and $x_2 = (D_2 - y_1 - y_2 - z_1 - u_2)_+$.

Proposition: condition for delaying renewable investment

In an optimal solution, if $2r_1 > p_u + \mathbb{E}[r_2]$ then $y_1 = 0$; if $2r_1 < p_u + \mathbb{E}[r_2]$ then $y_1 > 0$.

An example case showing the double critical fractile (r_2 fixed)

- Suppose that $y_1 = z_1 = 0$. We need to determine y_2.
- For $D_2 > y_2$ we need to use some short term non-renewables, i.e. $u_2 > 0$. But if $D_2 > y_2 + T$ we also need to use more expensive short term renewable x_2.
- Let $\bar{s} = F^{-1}(y_2)$ and $\bar{t} = F^{-1}(y_2 + T)$.
- The optimal choices $\bar{t}$ and $\bar{s}$ satisfies $\gamma \bar{s} + (1-\gamma)\bar{t} = B = \frac{p_x - r_2}{p_x}$ where $\gamma = p_u / p_x$



The possible solution structures: First stage decisions plus y_2 (r_2 fixed)

(R,N)	y_1	z_1	y_2
$(0,M)$	0	T	$F_\eta^{-1}(B)-T$
$(0,P)$	0	$T-F_0^{-1}(C)+F_0^{-1}(A)$	$F_\eta^{-1}(C)-T$
$(0,0)$	0	0	$F_\eta^{-1}(\bar{s})$
(P,M)	D_1-T	T	$F_\eta^{-1}(B)-D_1$
(P,P)	D_1-T	$T-F_0^{-1}(C)+F_0^{-1}(A)$	$F_\eta^{-1}(C)-D_1$
$(P,0)$	D_1-T	0	$F_\eta^{-1}(\bar{s})-D_1+T$
$(M,0)^*$	D_1	0	$F_\eta^{-1}(B)-D_1$
$(P,0)^*$	$D_1-F_0^{-1}(C^*)+F_0^{-1}(A^*)$	0	$F_\eta^{-1}(C^*)-D_1$

- $A=\frac{2(p_u-p_z)}{p_u}$, $B=\frac{p_x-r_2}{p_x}$, $C=\frac{p_x-r_2+2(p_z-p_u)}{p_x-p_u}$, $A^*=\frac{2(p_u-r_1)}{p_u}$, $C^*=\frac{p_x-r_2+2(r_1-p_u)}{p_x-p_u}$

Which solution structure is optimal? (r_2 fixed)

- It turns out that the first-period investment decisions are the same even if η is known and F_0 is replaced by F_η.
- The optimal choice of structure is independent of the distribution of the demand signal η.

Proposition: Delayed renewable investment

Suppose that $2r_1 \geq p_u + r_2$. Then the optimal solution follows one of the structures listed above with $y_1 = 0$ and $z_2 = 0$:

(i) If $C \leq A$, then solution structure $(0, M)$ is optimal.

(ii) If $A < C < 1$ and $F_0^{-1}(C) - F_0^{-1}(A) < T$, then solution structure $(0, P)$ is optimal.

(iii) Otherwise, solution structure $(0, 0)$ is optimal, and there is a unique $(\bar{s}, \bar{t})$ pair satisfying the simultaneous equations: $\bar{t}(1-\gamma) + \bar{s}\gamma = B$, together with either $F_0^{-1}(\bar{t}) - F_0^{-1}(\bar{s}) = T$ or $\bar{t} = 1$ (and $F_0^{-1}(\bar{t}) - F_0^{-1}(\bar{s}) < T$).

Which solution structure is optimal? (r_2 fixed)

Proposition: Early renewable investment

Suppose that $2r_1 < p_u + r_2$. Then the optimal solution follows one of the structures listed above, with $y_1 > 0$ and $z_2 = 0$:

(i) If $r_1 < p_z$ and $C^* < A^*$, then solution structure $(M, 0)^*$ is optimal.

(ii) If $r_1 < p_z$, $A^* < C^*$ and $F_0^{-1}(C^*) - F_0^{-1}(A^*) < T$, then solution structure $(P, 0)^*$ is optimal.

(iii) If $r_1 \geq p_z$ and $C \leq A$, then solution structure (P, M) is optimal.

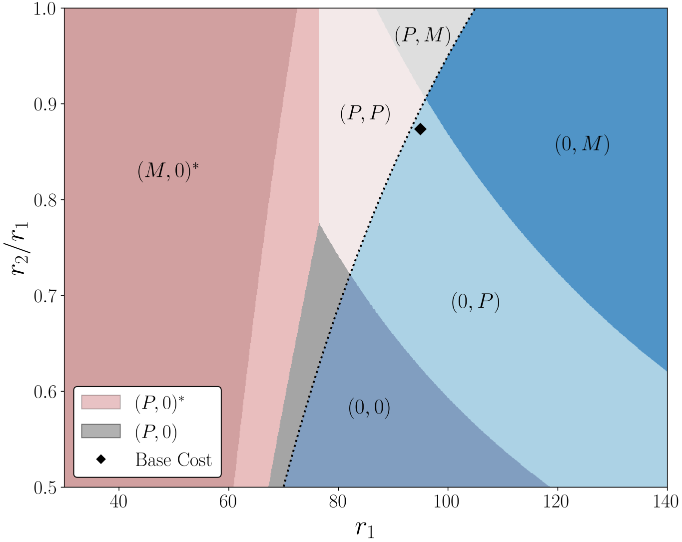
(iv) If $r_1 \geq p_z$, $A < C$ and $F_0^{-1}(C) - F_0^{-1}(A) < T$, then solution structure (P, P) is optimal.

(v) Otherwise, solution structure $(P, 0)$ is optimal where $\bar{s}$ is defined from the unique $(\bar{s}, \bar{t})$ pair satisfying the simultaneous equations: $\bar{t}(1-\gamma) + \bar{s}\gamma = B$, together with either $F_0^{-1}(\bar{t}) - F_0^{-1}(\bar{s}) = T$ or $\bar{t} = 1$ (and $F_0^{-1}(\bar{t}) - F_0^{-1}(\bar{s}) < T$).

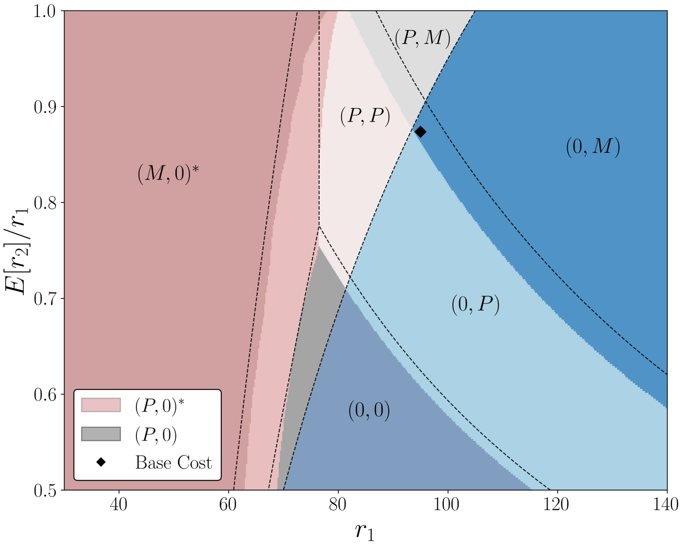
What numbers would be appropriate for the UK?

- Consider two 5-year periods.
- Use levelised costs from Lazard $p_z = \$76.5$ (mid-point estimate CCGT), $p_u = \$105$ (top end estimate for CCGT corresponding to delaying retirement of an expensive existing generator).
- Costs for long term renewable taken for offshore wind - but allow for variations.
- For short term renewable costs we make an estimate from demand response $p_x = \$190$.
- We net out existing generation (allowing for some retirement) from the demand to be met. We get a mean value for D_2 of 157 TWh of additional generation over 10 years.
- Uncertainty in D_2 is high. Our best estimate of a 5-year uncertainty gives $\sigma = 15$.
- We use an 80% renewable energy target (assuming significant use of CCS) giving $T = 14$ in additional carbon emissions.

Solution structures for varying renewable costs



When r_2 is uncertain (uniform distribution over range ±20% around mean)



Overall costs

- Look at base costs: $r_1 = \$95$, $r_2 = \$83$

		z_1	$y_1 + \mathbb{E}[y_2]$	Expected cost
(a)	$\sigma = 15,\ T = 14$	12.182	146.255	21412.75
(b)	$\sigma = 15,\ T = 7$	5.182	153.255	21657.75
(c)	$\sigma = 10,\ T = 14$	12.788	145.170	21080.50
(d)	$\sigma = 10,\ T = 7$	5.788	152.170	21325.50

- Changes in σ have a larger impact on expected cost than halving the carbon budget.
- When $\sigma = 15,\ T = 14$ and there is an uncertain r_2 ($\pm 20\%$ around mean) we get a solution with $z_1 = 14$, $y_1 + \mathbb{E}[y_2] = 145.17$ and expected cost 21255.10.
- Compared with the fixed r_2 case (top line of the table) costs are reduced because of a real option effect.

Summary

- The problem is a three-stage stochastic optimization problem where the underlying linearity enables a complete solution with uncertain demand revealed over time.
- The proof in the paper is through taking a limit of the solution for a finite set of demand values - letting the number of values approach ∞.
- We fix the demand distribution only after the first stage decisions, but a mixture of shifted distributions means the first period decision is independent of the shift.
- The paper discusses the shadow price of the carbon budget, and links this to the effect of changes in σ.
- High demand uncertainty is expensive (and can involve higher costs than from decreasing the carbon budget).
- Cost uncertainty requires a numerical solution. We find that cost uncertainty implies that overall costs are reduced (through a real options effect).