Investment and Operational Planning for Electricity Markets with Massive Entry of Renewable Energy

Nicolás Hernández-Santibáñez **Alejandro Jofré** Matías Vera

Universidad de Chile & Center for Mathematical Modeling (CNRS IRL 2807)
UTFSM ETH Zürich

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Equilibria and Optimization for Energy: An Appreciation of Andy Philpott

Outline

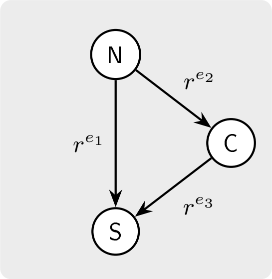
Why renewables change the planning problem

- Chile committed at COP25 to 70% clean and renewable electricity by 2030.
- Renewables are increasingly the *cheapest* source, and the territory (Atacama solar, southern wind) is ideal.
- But renewable capacity is uncertain — intermittency propagates into every long-term decision.

The question

How should a system operator *invest* in renewable capacity *and operate* the network, jointly, under uncertainty and over a continuous horizon?

Chile, three-node abstraction



North / Center / South, linked by lossy transmission lines.

Where this sits in the literature

- **Capacity expansion** is usually mixed-integer stochastic programming, in static or discrete multi-period form (Conejo–Carrión–Morales; Sagastizábal; Philpott).
- **Continuous-time stochastic control** has been used for operation, storage, real options — rarely for *investment with a network*. Aïd et al. drop the network; Hernández–Jofré–Possamaï keep it but with *fixed* capacities.

Our contribution

1. A *continuous-time* model coupling investment *and* operation on a *lossy network*.
2. Capacity is a *controlled stochastic process*, not a constant.
3. A clean reduction to a *state-constrained* stochastic control problem, characterized by an HJB equation.

Network, capacity and demand

Network. Graph (V,E): nodes $i \in V$ are producers with capacity/demand (Q^i, D^i); edges carry flows $\phi^e \in [\underline{\phi}^e, \overline{\phi}^e]$ with resistance $r^e \geq 0$.

Uncertainty. On $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$, a control $\nu \in \mathcal{U}$ (U-valued, U compact) drives an Itô capacity–demand pair $(Q^\nu_s, D^\nu_s) \in \mathbb{R}^N_+ \times \mathbb{R}^N_+$.

Network operation. A production/flow pair (q,ϕ) is admissible if, for every node and almost every path,

$$q^i_s + \sum_{e \in K_i} \mathrm{sgn}(e,i)\, \phi^e_s \;\geq\; D^i_s + \sum_{e \in K_i} \frac{r^e}{2}(\phi^e_s)^2, \qquad q_s \in [0, Q_s],\;\; \phi_s \in [\underline{\phi}, \overline{\phi}].$$

- Supply + net inflow $\geq$ demand + quadratic transmission losses.
- Demand is met *pathwise* ($p = 1$): rationing is unacceptable for regulators.

The ISO's problem (P)

The operator minimizes joint *operation* + *investment* cost:

$$\underset{(\nu,q,\phi)\in\mathcal{U}\times\mathcal{A}}{\text{minimize}}\ \mathbb{E}\left[\int_0^T \sum_{i=1}^N c^i(q_s^i, Q_s^{i,\nu}) + h^i(\nu_s, Q_s^{i,\nu}, D_s^{i,\nu})\,\mathrm{d}s\right]$$

$$\text{s.t. } q_s^i + \textstyle\sum_{e\in K_i} \operatorname{sgn}(e,i)\phi_s^e \geq D_s^{i,\nu} + \sum_{e\in K_i} \frac{r^e}{2}(\phi_s^e)^2, \quad \forall s,\ \mathbb{P}\text{-a.s.}$$

$$q_s \in [0, Q_s^\nu], \quad \phi_s \in [\underline{\phi}, \overline{\phi}].$$

Production cost c^i

Convex, increasing; $c^i(0,\cdot) = 0$. Models a cheap renewable tier and more expensive coal / gas tiers.

Investment cost h^i

Split later as $\nu = (\mu, \alpha)$: μ buys renewable capacity, α stabilizes volatility.

Centralized ISO: investment decisions read as recommendations to producers under full information.

A hierarchy: operation reacts to investment

Key structural idea. Once ν (hence (Q^ν, D^ν)) is fixed, the operational pair (q, ϕ) is chosen *last* — it just keeps the network feasible at minimum production cost.

Feasibility function. Define

$$G(Q, D) := \max_{(q,\phi)\in[0,Q]\times[\underline{\phi},\overline{\phi}]} \min_{i=1,\dots,N} \Big(q^i - D^i + \sum_{e\in K_i} \operatorname{sgn}(e,i)\phi^e - \tfrac{r^e}{2}(\phi^e)^2 \Big).$$

$G(Q, D) \geq 0 \iff (Q, D)$ is operationally feasible. Set
$\mathcal{U}_G = \{\nu : G(Q^\nu_s, D^\nu_s) \geq 0 \ \forall s,\ \mathbb{P}\text{-a.s.}\}$.

Cost decomposition

$$V = \inf_{\nu\in\mathcal{U}_G} \Big\{ V_\nu + \mathbb{E}\int_0^T \sum_i h^i(\nu_s, Q^{i,\nu}_s, D^{i,\nu}_s)\,\mathrm{d}s \Big\}, \qquad V_\nu = \operatorname{val}(P_\nu).$$

The operational variables (q, ϕ) are replaced by a state constraint plus the value V_ν.

The Day-Ahead Problem (DAP) gives V_ν explicitly

Fix a path and time: the inner problem is the static

$$\underset{(q,\phi)}{\text{minimize}} \sum_{i=1}^{N} c^i(q^i, Q^i) \quad \text{s.t. network operation holds.}$$

- **Kirchhoff's law**: at any optimum the supply constraints are *tight*, $T^i(q,\phi;D) = 0$.
- Under convex, strictly increasing c^i and $r^e > 0$: the DAP has a unique solution $(q^\star,\phi^\star)(Q,D)$.
- $K := G^{-1}(\mathbb{R}_+)$ is *closed and convex*; $(q^\star,\phi^\star)$ is measurable on K.

Pathwise optimality

$(\tilde{q},\tilde{\phi})$ solves (P_ν) iff

$$\lambda \otimes \mathbb{P}\big[(\tilde{q},\tilde{\phi}) = (q^\star,\phi^\star)(Q^\nu, D^\nu)\big] = 1,$$

hence

$$V_\nu = \mathbb{E}\int_0^T \sum_i c^i\big(q^{\star,i}(Q_s^\nu, D_s^\nu), Q_s^{i,\nu}\big)\mathrm{d}s.$$

Equivalent state-constrained problem $(\tilde{P})$

Substituting the explicit operation removes (q, ϕ) entirely:

$$\boxed{\begin{aligned} &\underset{\nu \in \mathcal{U}}{\text{minimize}}\ \mathbb{E}\left[\int_0^T \sum_{i=1}^N c^i\big(q^{\star,i}(Q_s^\nu, D_s^\nu), Q_s^\nu\big) + h^i(\nu_s, Q_s^{i,\nu}, D_s^{i,\nu})\,\mathrm{d}s\right] \\ &\text{s.t. } G(Q_s^\nu, D_s^\nu) \geq 0, \quad \forall s \in [0,T],\ \mathbb{P}\text{-a.s.} \end{aligned}}$$

- A state-constrained stochastic control problem on the closed, convex set K.
- K is in general *non-smooth* and *unbounded*; controls are *bounded*.
- No off-the-shelf HJB characterization covers this combination — this is the technical heart of the talk.

Generic state-constrained control problem

Work with $X^{\nu,t,x}$ solving $\mathrm{d}X_s = b(X_s,\nu_s)\mathrm{d}s + \sigma(X_s,\nu_s)\mathrm{d}W_s$, cost $J(\nu;t,x) = \mathbb{E}\int_t^T f(X_s,\nu_s)\mathrm{d}s$, and two value functions on a set C with $\overline{C^\circ} = C$:

$$V(t,x) = \inf_{\nu\in\mathcal{U}(t,x)} J(\nu;t,x), \qquad V^\circ(t,x) = \inf_{\nu\in\mathcal{U}^\circ(t,x)} J(\nu;t,x),$$

constraining the state to C and to C°, respectively.

- **Dynkin operator** $\mathcal{L}^a(x,p,X) = b(x,a)^\top p + \frac{1}{2}\mathrm{Tr}\big(\sigma\sigma^\top(x,a)X\big)$.
- **Supersolution** on $[0,T)\times C$ (easy direction) and **subsolution** on $[0,T)\times C^\circ$ for

$$-\partial_t\varphi + \sup_{u\in U}\{-\mathcal{L}^u(x,D\varphi,D^2\varphi) - f(x,u)\} = 0.$$

- Both V, V° solve the *same* HJB. Equality needs a comparison principle.

The obstacle, and the way around it

- Classical results (Soner; Katsoulakis; Ishii–Loreti) assume the constraint set is *smooth* (C^1 or C^3) and/or that b, σ, f are *bounded*.
- Bouchard–Nutz drop boundedness but require the value's envelope to be of class $R(O)$ — automatic when C is C^1.
- Our K is not smooth even in simple examples.

Class $R(C^\circ)$ (Bouchard–Nutz)

Near each boundary point there is an interior trajectory $\varepsilon \mapsto x + l(\varepsilon)$, $t + \lambda(\varepsilon)$, along which the value function is continuous. *Convex $C \Rightarrow$ every continuous function is $R(C)$.*

Strategy: replace smoothness by an *inward-pointing condition* (PIC) and *prove* $(V^\circ)^\star \in R(C^\circ)$ directly.

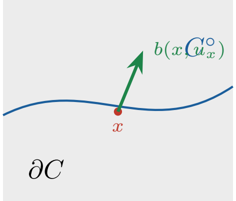
The Pointing-Inward Condition (PIC)

Assumption (Inward pointing)

For each $x \in \partial C$ there exist $u_x \in U$, $r > 0$, $\eta > 0$ with

$$\sigma(z, u_x) = 0 \;\; \forall z \in B(x, \eta) \cap C, \qquad B\big(z + t\, b(z, u_x), rt\big) \subseteq C^\circ \;\; \forall t \in (0, r).$$

- At the boundary there is a control that kills the volatility and drifts strictly inward.
- Under PIC we show $(V^\circ)^\star$ is of class $R(C^\circ)$ (adapting Bouchard–Nutz to the non-smooth set).
- Combined with quadratic growth, this yields the comparison principle.



Main characterization theorem

Theorem (HJB characterization)

Under the PIC (and a Lipschitz admissible-control assumption),

$$V = V^\circ \text{ on } [0,T] \times C^\circ, \quad V \text{ is continuous,}$$

and V is the unique constrained viscosity solution on $[0,T) \times C$ of

$$-\partial_t \varphi + \sup_{u \in U} \{-\mathcal{L}^u(\cdot, D\varphi, D^2\varphi) - f(\cdot, u)\} = 0, \qquad \varphi(T,\cdot) = 0,$$

in the class of polynomial-growth functions whose u.s.c. envelope is of class $R(C^\circ)$.

- "Constrained solution": supersolution on C, subsolution on C°.
- Proof chain: weak DPP $\Rightarrow$ super/sub-solution $\Rightarrow$ $R(C^\circ)$ via PIC $\Rightarrow$ comparison $\Rightarrow$ uniqueness.

Specializing the dynamics

Split the control $\nu = (\mu, \alpha)$: investment μ (capped by a budget) and stabilization $\alpha \in [0,1]^N$.
For $U = U_{\bar\mu} \times [0,1]^N$, $U_{\bar\mu} = \{\mu \geq 0 : \sum_i h^i_\mu \mu^i \leq \bar\mu\}$,

$$\mathrm{d}Q^i_s = \mu^i_s \,\mathrm{d}s + \sigma^i\,(Q^i_s - \hat{Q}^i)(1-\alpha^i_s)\,\mathrm{d}W_s, \qquad \mathrm{d}D^i_s = p^i_D(s)\,\mathrm{d}s.$$

- Drift of Q = *technology acquisition* (renewables); volatility acts only on the renewable excess $Q - \hat{Q}$.
- α *turns off* volatility at a unit cost h^i_α — e.g. buying hydro/storage in emergencies.
- $\bar\mu$ = *annual renewable investment budget*.

PIC reduces to a budget condition

Economic reading of the PIC (Remark)

On ∂K the system produces *at maximum* just to meet demand. A small wrong move violates demand — catastrophic (and illegal). So the operator pays *whatever it costs* ($\alpha \equiv 1$, emergency local generation) to return inside K.

Sufficient budget assumption

$$\bar{\mu} > \sup_{s \in [0,T]} \left(\sum_{i=1}^{N} h_{\mu}^{i} \, (p_D^i(s))^{+} \right).$$

If the annual budget exceeds the cost of covering demand growth, then the PIC and the admissibility assumptions hold — so the HJB characterization applies.

Verification theorem: V is the unique constrained viscosity solution of the associated HJB with Hamiltonian H_{CE}.

Calibrated three-node Chilean market

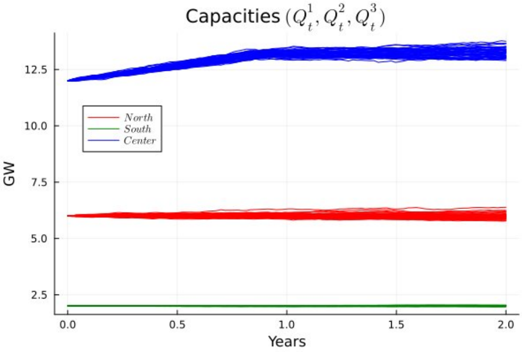
- Per node: renewable + coal + gas, piecewise-linear cost with $0 < c_R < c_{\text{coal}} < c_{\text{gas}}$.
- Time unit: one hour; volatility $\sigma^i = 5\%$; flows in $[-6, 6]$ GW.
- Three horizons: short (2 yr), mid (10 yr), long (20 yr).

Initial conditions (MW)

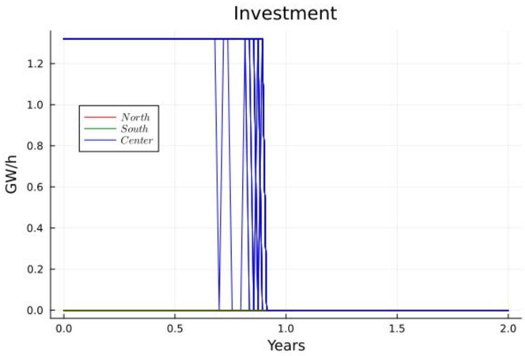
	Q_0	Q_{coal}	Q_{gas}	D_0
North	6000	1800	2400	3000
South	2000	800	1000	1000
Center	12000	1200	8400	6000

Solve HJB by finite differences with a *penalization* heuristic for the state-constrained boundary.

Short-term planning ($T = 2$ yr, constant demand)



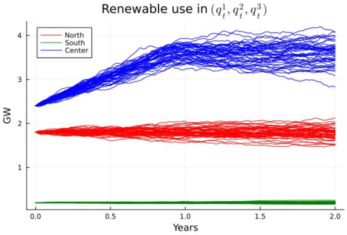
Capacities — 50 trajectories



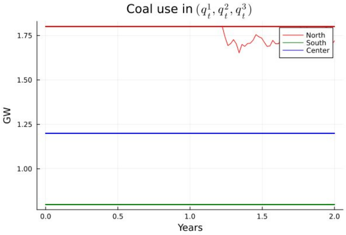
Investment control μ

- Only the Center (blue) grows; the burst of investment lands in a narrow window around year 0.7–0.9, then stops once h_μ^i exceeds the marginal value of energy.
- North and South capacities are essentially flat — only the inherent 5% volatility shows.
- No stabilization ($\alpha \equiv 0$): each node starts with $\sim 2\times$ its demand, so the system is far from the boundary of K.

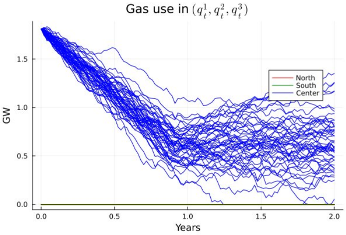
Short-term planning: a local gas → renewable switch



Renewable



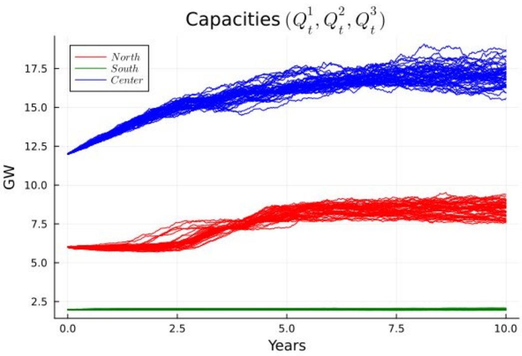
Coal



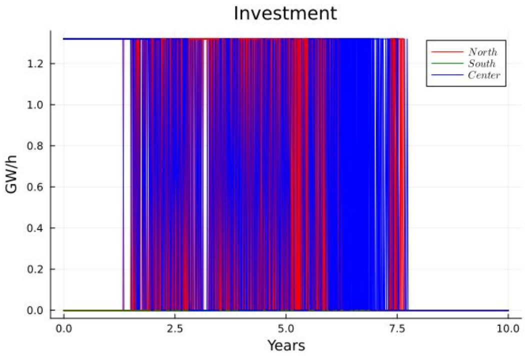
Gas

- The whole adjustment is *within the Center*: gas (blue) falls sharply while renewable (blue) climbs — coal is barely touched.
- Flows stay essentially constant: over two years there is no time for a *structural* change in transmission, only a change in the *type* of production.
- System-wide renewable share rises only modestly, from ∼44% to ∼55% — the short horizon understates the model's value.

Mid-term planning ($T = 10$ yr, $+2\%$/yr demand): investment



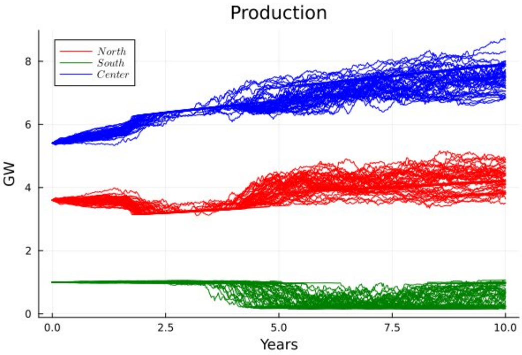
Capacities (Q^1, Q^2, Q^3) — 50 trajectories



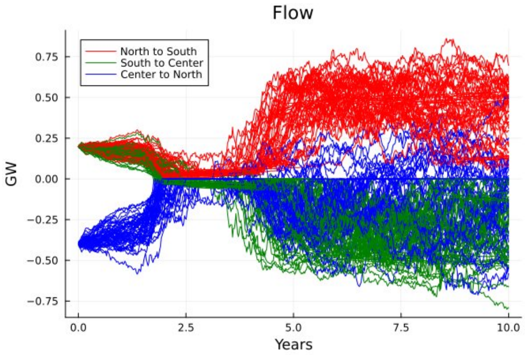
Investment control μ

- Sustained growth at North and Center; the South (green) stays flat — building there is far more expensive.
- Investment is shared between North and Center until $\approx$ year 8, with an early preference for the Center.
- No stabilization ($\alpha \equiv 0$): initial capacity is large relative to demand, so the system stays

Mid-term planning: operation and flows



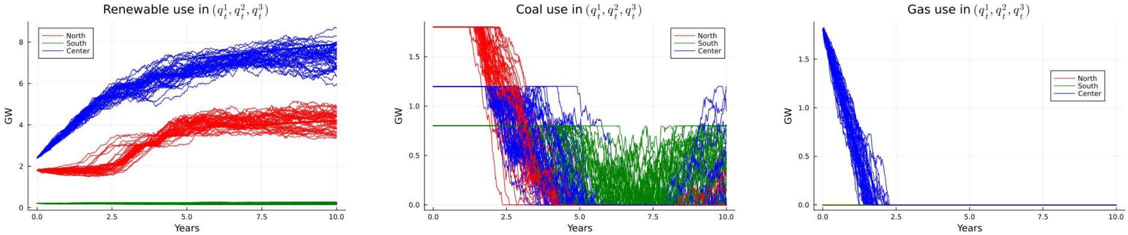
Production (q^1, q^2, q^3)



Line flows (signed)

- Operation reacts to investment through the Day-Ahead Problem — renewable uncertainty propagates into (q, ϕ).
- Flows reverse around year 2.5: once local transitions finish, the North begins *exporting to the South* (red curve climbs).
- Geography pays off: the South is served by *transmission*, not local capacity.

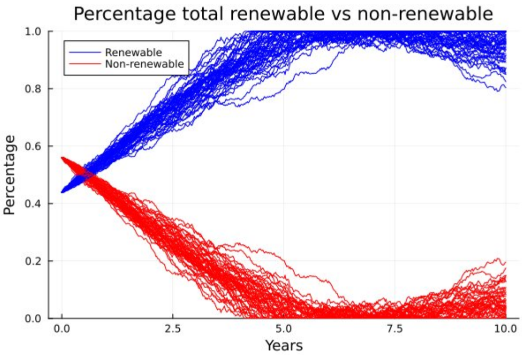
Mid-term planning: the gas → coal → renewable shift



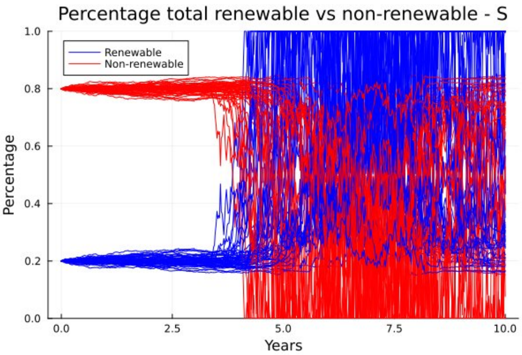
Renewable Coal Gas

- **Phase 1** ($\lesssim$ yr 1): gas is retired at the Center (blue collapses to zero by year ∼2.5).
- **Phase 2**: coal phased out at North and Center as renewables ramp up.
- **Phase 3** (from yr ∼4): the South leans on imported renewables; coal lingers only as a residual buffer.

Mid-term planning: renewable share of the mix



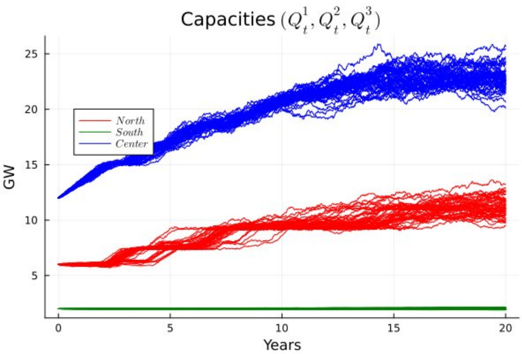
System-wide

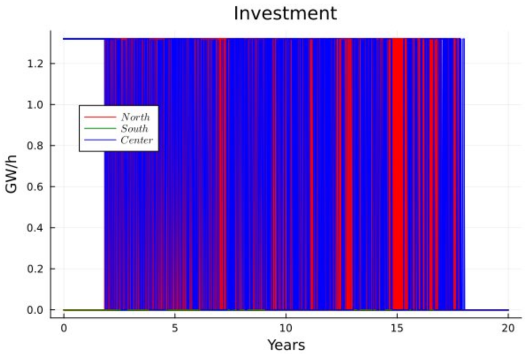


South only

- System-wide renewable share climbs from ~55% to near 100% by year 7, then eases slightly as demand keeps growing.
- The South is the laggard and the noisy one: its own share stays volatile because it is balanced through *flows*, not local build-out — the network constraint at work.

Long-term planning ($T = 20$ yr): investment



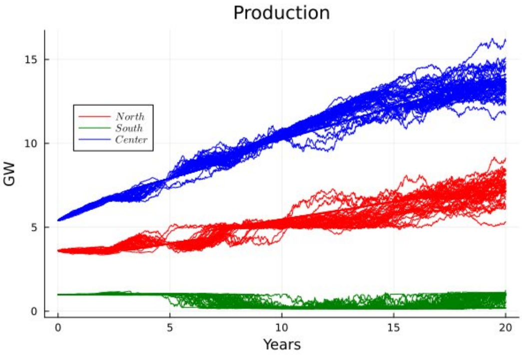


Capacities (Q^1, Q^2, Q^3)

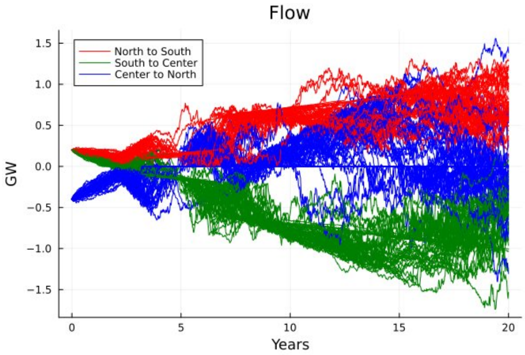
Investment control μ

- Capacity at North and Center grows throughout; the South again never expands. Investment is dense and sustained to year ~18, far longer than in the mid-term case.
- Driver: with +2%/yr demand over 20 years, $\hat{Q}^i < D^i_{20}$ at every node — so $\alpha \equiv 0$ has *positive probability* of leaving K.
- The state constraint is now theoretically active, yet stabilization stays at zero: heavy

Long-term planning: a fully bidirectional network



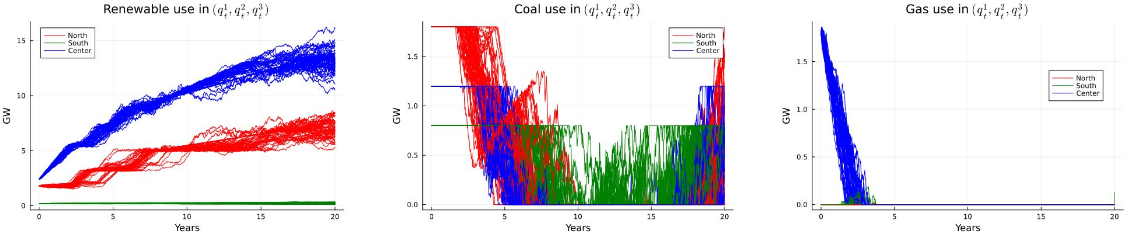
Production (q^1, q^2, q^3)



Line flows (signed)

- Production rises steadily at North and Center to meet growing demand; the South stays near its modest local output.
- Flows now fan out widely in both directions — North→South and Center→North both grow to $\sim$1 GW, with large dispersion.
- The network is worked hard: long-horizon decarbonization is fundamentally a *transmission*

Long-term planning: the source mix over two decades



Renewable Coal Gas

- Gas is retired early (gone by year $\sim$4) and renewable use climbs steadily at North and Center for the full horizon.
- Coal dips deep then returns: as demand approaches available capacity late in the horizon, coal is reactivated as a buffer — the constraint making itself felt.
- Mid- vs. long-term look alike in α but for *opposite* reasons: abundant capacity vs. low exit probability.

Take-aways

- First continuous-time model coupling investment + operation with a lossy network and stochastic capacity.
- Hierarchy of controls + the Day-Ahead Problem reduce (P) to a state-constrained stochastic control problem on a convex, non-smooth K.
- A PIC tailored to electricity markets delivers the unique-viscosity-solution characterization despite non-smoothness and unboundedness.
- Numerics on the Chilean network: geography and transmission matter most over the long horizon — build in the North/Center, serve the South by flow.

Future work

- Drop ISO–producer alignment: a sequential / Nash differential game with a *shared state constraint* on the controlled state.
- A general theory of continuous-time stochastic games with state constraints — of interest beyond electricity markets.
- Rigorous convergence of the penalization scheme in the stochastic setting (known deterministically: Bokanowski–Forcadel–Zidani).
- Richer uncertainty: seasonal/multiplicative capacity, storage and batteries as endogenous stabilization.

Thank you! Questions welcome.

Backup — the equivalent problem $(\tilde{P})$ and H_{CE}

$$H_{CE}(t,x,y,r,w,v,X) = \sup_{u\in U}\left\{-r-\sum_{i=1}^{N}\left[u_\mu^i w^i + p_D^i(s)v^i + \frac{1}{2}\left(\sigma^i(1-u_\alpha^i)(x^i-\hat{Q}^i)\right)^2 X^{ii} + h^i(u,x,y)\right]\right\}.$$

HJB:

$$-\partial_t\varphi + H_{CE}(s,x,y,\partial_s\varphi, D_x\varphi, D_y\varphi, D_x^2\varphi) - \sum_{i=1}^{N} c^i(q^\star(x,y),x) = 0, \quad \varphi(T,\cdot) = 0.$$

Solved numerically by finite differences with penalization $\rho\, d_K(x,y)$ on the right-hand side, ρ large.