Strategic declaration of risk sets in stochastic market clearing

Vincent Leclère
joint work with Andy Philpott

SIAM Conference on Optimization – Edinburgh

A short story

- I met Andy in 2012, as a PhD student wishing to ~~visit New Zealand~~ learn more about electricity markets. Luckily, I arrived in Auckland when he got reviews on an SDDP paper and I got on board and started a large part of my scientific career this way.
- In 2016, as a young assistant professor wishing to ~~return to New Zealand~~ continue collaborating with Andy, I sent a PhD student that wanted to ~~visit New Zealand~~ learn more about equilibrium to visit Andy, and got on board. We wanted to compute equilibrium efficiently, and showed that there was non-uniqueness and instability.
- In 2021(?), Andy was in Cambridge, and I got the opportunity to work with him again. We took three (4? 5?) years, a visit to Paris and a visit to Chicago to finish the paper.

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Motivation: stochastic market clearing

Electricity markets clear several agents under uncertainty:

- producers, consumers, storage, and portfolios submit offers and face common clearing constraints;
- renewable output and demand are uncertain when some commitments must be made;
- prices are generated by the market-clearing optimization problem.

A useful idealization is a two-stage stochastic market:

commitment x_a $\longrightarrow$ scenario $\omega \in \Omega$ $\longrightarrow$ recourse and settlement.

Arrow–Debreu securities trade scenario risk: the security for scenario ω costs $\mu(\omega)$ now and pays 1 if ω occurs.[1]

Question: what happens when the risk sets needed by the operator are private information?

[1] See Pritchard–Zakeri–Philpott, "A single-settlement, energy-only electric power market for unpredictable and intermittent participants", and Zakeri et al., "Pricing wind".

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Reference works for Section 1

Complete risk trading

Michael Ferris, Andy Philpott and Roger Wets, "Equilibrium, uncertainty and risk in hydro-thermal electricity systems", *Mathematical Programming*, 2016.

Incomplete-market warning

Henri Gérard, Vincent Leclère and Andy Philpott, "On risk averse competitive equilibrium", *Operations Research Letters*, 2018.

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Two-stage model and random disutilities

Let Ω be finite. Agent $a \in \mathcal{A}$ chooses first-stage and recourse decisions,

$$x_a, \qquad y_a(\omega), \quad \omega \in \Omega.$$

The physical market clears scenario by scenario. Here $y_a(\omega)$ denotes a signed net injection/trade:

$$\sum_{a \in \mathcal{A}} y_a(\omega) = 0, \qquad \omega \in \Omega.$$

Agent a's random disutility is the vector

$$Z_a(x_a, y_a) = \big(Z_a(x_a, y_a, \omega)\big)_{\omega \in \Omega}.$$

With AD trades $W_a(\omega)$, the net disutility becomes $Z_a - W_a$, and the AD market clears when

$$\sum_{a \in \mathcal{A}} W_a(\omega) = 0, \qquad \omega \in \Omega.$$

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Coherent risk measures as risk sets

For the finite coherent-risk setting used here,

$$\rho_a(Z) = \sup_{\mathbb{Q} \in \mathcal{P}_a} \mathbb{E}_{\mathbb{Q}}[Z],$$

where $\mathcal{P}_a \subseteq \Delta(\Omega)$ is a closed convex risk set.[2]

Object	Risk set	Evaluation
reference expectation	$\{\mathbb{P}^0\}$	$\mathbb{E}_{\mathbb{P}^0}[Z]$
polyhedral coherent risk	polytope in $\Delta(\Omega)$	worst supporting probability
worst-case risk	$\Delta(\Omega)$	$\max_\omega Z(\omega)$

A larger risk set gives a more conservative evaluation of random disutilities.

[2] Artzner–Delbaen–Eber–Heath, "Coherent Measures of Risk".

Complete-market equivalence

In the finite/polyhedral coherent-risk setting, with a nonempty intersection and complete Arrow–Debreu risk trading, risked equilibrium can be represented by a social planner problem.[3]

Write $\mathcal{P}^{\cap} = \bigcap_{a \in \mathcal{A}} \mathcal{P}_a$. The striking point is that the social planner does not use the union, an average, or a representative risk measure, but the common risk set:

$$\text{risked competitive equilibrium} \iff \min_{x} \sup_{\mathbb{Q} \in \mathcal{P}^{\cap}} \mathbb{E}_{\mathbb{Q}} \left[\sum_{a \in \mathcal{A}} Z_a(x, \omega) \right].$$

Why this matters here

If a market-clearing mechanism needs $\mathcal{P}^{\cap}$, then the operator needs information about each agent's risk set.

Gérard–Leclère–Philpott (2018) use the complete-market benchmark before showing that incomplete risk markets can produce multiple and unstable equilibria.

[3] Ferris–Philpott–Wets, "Equilibrium, uncertainty and risk in hydro-thermal electricity systems", 2016.

Reference work for Section 2

Main contribution of the talk

Vincent Leclère and Andy Philpott, "Strategic behavior of risk-averse agents under stochastic market clearing", *Operations Research Letters*, 2025.

- start from the complete-market risked-equilibrium construction;
- make risk sets private objects declared to the system operator;
- characterize the induced non-cooperative game;
- show that truthful declaration of risk aversion can fail.

Related but different. Vespermann–Hamacher–Kazempour (2022) study ambiguity-averse local market equilibrium; Shilov–Le Cadre–Bušić (2021) study risk-averse prosumer equilibria with financial hedging. Here the issue is strategic declaration of risk sets to a centralized stochastic market-clearing operator.

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Why risk sets must be declared

To solve the complete-market risk-adjusted dispatch, the system operator needs

$$\bigcap_{a \in \mathcal{A}} \mathcal{P}_a .$$

But the true risk set $\mathcal{P}_a$ is a private preference object. In a market-clearing mechanism, the operator can only use declared information.

agents declare risk sets $\mathcal{Q}_a$ → ISO solves with $\mathcal{Q}^{\cap} = \cap_a \mathcal{Q}_a$ → agents evaluate with true $\mathcal{P}_a$

This transforms a risked equilibrium construction into a strategic declaration game.[4]

[4] Leclère–Philpott, "Strategic behavior of risk-averse agents under stochastic market clearing", 2025.

Risk-adjusted economic dispatch with declarations

Notation: x_a is the day-ahead setpoint, $U_a(\omega)$ the real-time deviation, $X_a(\omega)$ the real-time dispatch, and $F(\omega)$ the network flow.

The ISO receives declared risk sets $\mathcal{Q}_a$ and uses only their intersection $\mathcal{Q}^\cap = \cap_a \mathcal{Q}_a$:

$$\text{RADP:} \quad \min_{x,U,X,F} \quad \sup_{\mathbb{Q}\in\mathcal{Q}^\cap} \mathbb{E}_{\mathbb{Q}}\left[\sum_{a\in\mathcal{A}} c_a(x_a) + r_a(U_a(\omega))\right]$$

s.t. standard dispatch constraints: demand balance, recourse, network feasibility, capacity bounds.

The ISO's dispatch and prices depend on declarations only through $\mathcal{Q}^\cap$. Actual incentives then depend on true private risk sets.

Settlement properties of the declared-risk mechanism

Revenue adequacy

Here $\pi_n(\omega)$ is the nodal price and $\tau_n(F(\omega))$ is the net inflow from the network into node n. Revenue adequacy means

$$\sum_n \pi_n(\omega)\tau_n(F(\omega)) \geq 0, \qquad \omega \in \Omega.$$

Risk-adjusted cost recovery

Let $C_a(\omega)$ be production cost, $R_a(\omega)$ market settlement revenue, and $W_a(\omega)$ the return of AD trades. Declared cost recovery means

$$\rho_{\mathcal{Q}_a}(C_a - R_a - W_a) = \sup_{\mathbb{Q} \in \mathcal{Q}_a} \mathbb{E}_{\mathbb{Q}}[C_a - R_a - W_a] \leq 0.$$

Positive theorem [L.–Philpott, 2025]

If declared risk sets are treated as the agents' actual risk sets, RADP gives scenario-wise revenue adequacy and cost recovery in the corresponding risk-adjusted expectation.

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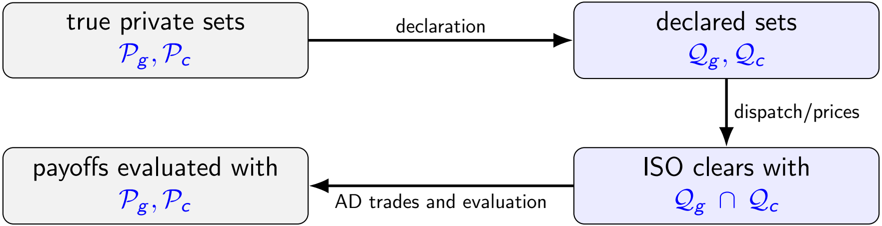
The induced strategic question



Question

What are the Nash equilibria of this declaration game, and can agents gain by misrepresenting risk aversion?

The game can be studied as a set-intersection game: agents choose declared sets, while the ISO reacts only to their intersection.

Admissible declarations and intersection games

Each declaration is a risk set. We denote by $\mathcal{R}$ the family of declarations the ISO accepts.

Admissible family

$\mathcal{R}$ is a family of subsets of $\Delta(\Omega)$ such that

$$\mathcal{Q}, \widetilde{\mathcal{Q}} \in \mathcal{R} \Rightarrow \mathcal{Q} \cap \widetilde{\mathcal{Q}} \in \mathcal{R}, \qquad \mathbb{P}^0 \in \mathcal{Q} \quad \forall \mathcal{Q} \in \mathcal{R}.$$

Typical choices: all polyhedral risk sets containing $\mathbb{P}^0$, AVaR-type risk sets, or nested one-parameter sets $\mathcal{Q}_t = (1-t)\{\mathbb{P}^0\} + t\mathcal{D}$.

For a profile $(\mathcal{Q}_a)_a$, write $\mathcal{Q}^\cap = \cap_a \mathcal{Q}_a$ and $\mathcal{Q}^{\cap -a} = \cap_{b \neq a} \mathcal{Q}_b$.

Nash characterization and reductions

The loss of agent a depends on a strategy profile only through the induced intersection:

$$L^a((\mathcal{Q}_b)_{b\in\mathcal{A}}) = \ell^a(\mathcal{Q}^\cap).$$

Nash characterization

A profile $(\mathcal{Q}_a)_{a\in\mathcal{A}}$ is a Nash equilibrium if and only if, for every agent a,

$$\mathcal{Q} \subseteq \mathcal{Q}^{\cap -a} \implies \ell^a(\mathcal{Q}^\cap) \leq \ell^a(\mathcal{Q}) \qquad \forall \mathcal{Q} \in \mathcal{R}.$$

Two useful consequences:

- any equilibrium has an equivalent symmetric equilibrium $(\mathcal{Q}^\cap, \dots, \mathcal{Q}^\cap)$;
- if $\mathcal{Q}_t$ is nested, then $\bigcap_a \mathcal{Q}_{t_a} = \mathcal{Q}_{\min_a t_a}$.

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Non-truthfulness: minimal counterexample

Two agents declare $t_g, t_c \in [0,1]$. Since the risk sets are nested, the ISO uses

$$t = \min\{t_g, t_c\}, \qquad x(t) = 8t + 30, \qquad U(g) = 80 - 4t, \qquad U(h) = 40 - 4t.$$

Agent	true risk aversion	preferred declaration if pivotal
Generator	$\lambda_g > 0$	under-declare: $t_g = 0$
Consumer	$\lambda_c > 0$	over-declare: $t_c = 1$

Mechanism of the example

Low t gives a less conservative dispatch, which the generator prefers. High t gives a more conservative dispatch, which the consumer prefers. But the consumer cannot offset a generator declaration $t_g = 0$, because the effective parameter is the minimum.

Conclusion: truthful declaration is not generally a Nash equilibrium.

Reference work for Section 3

Ongoing inverse problem

Lucien Le Gall, “A method for inferring information on risk sets from a market equilibrium”, internship report, 2025.

The strategic failure raises an inverse question:

- if declarations can be strategic, can observed trades audit them?
- equilibrium AD trades give normal-cone certificates;
- for MAVaR risk sets, one can infer the intersection more easily than each individual risk set.

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Can AD trades audit declarations?

Suppose we observe, at equilibrium,

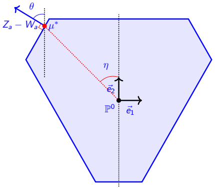
$$\mu^*, \qquad Z_a, \qquad W_a,$$

where μ^* is the AD price vector, normalized here as a state-price probability vector, Z_a is the nominal disutility vector, and W_a is the AD position.

For risk sets $\mathcal{P}_a$, the risk-trading equilibrium satisfies

$$\mu^* \in \operatorname*{argmax}_{\mu \in \cap_a \mathcal{P}_a} \mathbb{E}_\mu \left[\sum_{a \in \mathcal{A}} Z_a \right], \qquad Z_a - W_a \in \mathcal{N}_{\mu^*}(\mathcal{P}_a), \qquad \sum_{a \in \mathcal{A}} W_a(\omega) = 0.$$

- μ^* supports the intersection of risk sets.
- $Z_a - W_a$ is a normal vector to agent a's private risk set.
- Certificate of compatibility, not full identification in general.



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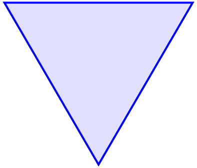
MAVaR risk sets and identifiability

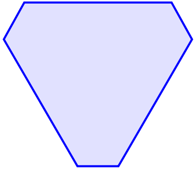
The report studies MAVaR-type risk sets in the three-scenario, uniform-reference setting.[5]

$$\rho_{\lambda,\beta}(Z) = (1-\lambda)\mathbb{E}_{\mathbb{P}^0}[Z] + \lambda\,\mathrm{AVaR}_\beta(Z), \qquad \mathcal{P}_\lambda(\beta) = (1-\lambda)\{\mathbb{P}^0\} + \lambda\big(\Delta \cap C(\beta)\big).$$

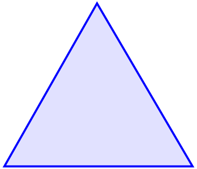
Here β changes the shape and λ is a homothety around $\mathbb{P}^0$.



$0 \leq \beta \leq \frac{1}{3}$



$\frac{1}{3} < \beta < \frac{2}{3}$



$\beta \geq \frac{2}{3}$

Intersection closure [Le Gall, 2025]

For a uniform reference probability and three scenarios, the intersection of MAVaR risk sets is again a MAVaR risk set: $\bigcap_i \mathcal{P}_{\lambda_i}(\beta_i) = \mathcal{P}_\lambda(\beta)$.

[5] Rockafellar–Uryasev, "Optimization of Conditional Value-at-Risk", 2000.

What can be inferred from one supporting price?

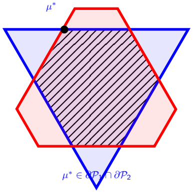
Consider the specific setting of the report: three scenarios, uniform $\mathbb{P}^0$, and MAVaR risk sets $\mathcal{P}_\lambda(\beta)$.

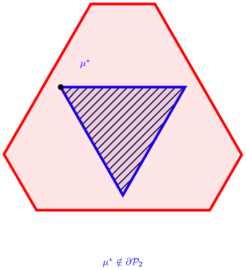
Intersection result [Le Gall, 2025]

If an observed equilibrium price μ^* is an extreme point of the intersection, then one extreme supporting price determines the intersection $\cap_i \mathcal{P}_i = \mathcal{P}_\lambda(\beta)$ in this family.

Identifiability limit [Le Gall, 2025]

For several agents, different individual risk sets can have the same intersection. Individual parameters are then only partially identifiable, through boundary cases, bounds, or functions.





Takeaways

1. In the finite/polyhedral setting, complete Arrow–Debreu markets turn coherent-risk equilibrium into a social planner problem using $\cap_a \mathcal{P}_a$.
2. If risk sets must be declared to the ISO, the mechanism induces a set-intersection game.
3. The game has a clean Nash characterization, but truthful risk-set revelation is not guaranteed.
4. AD trades give normal-cone certificates; in the MAVaR three-scenario setting one extreme supporting price identifies the intersection, but individual risk sets may remain only partially identifiable.

Final message

Risk sets are not only modeling primitives; once used in a market-clearing mechanism, they become strategic objects.

References: stochastic market clearing and risked equilibrium

G. Pritchard, G. Zakeri and A. Philpott.
A single-settlement, energy-only electric power market for unpredictable and intermittent participants.
Operations Research, 2010.

G. Zakeri, G. Pritchard, M. Bjørndal and E. Bjørndal.
Pricing wind: a revenue adequate, cost recovering uniform price auction for electricity markets with intermittent generation.
INFORMS Journal on Optimization, 2019.

P. Artzner, F. Delbaen, J.-M. Eber and D. Heath.
Coherent measures of risk.
Mathematical Finance, 1999.

M. C. Ferris, A. Philpott and R. J.-B. Wets.
Equilibrium, uncertainty and risk in hydro-thermal electricity systems.
Mathematical Programming, 2016.

H. Gérard, V. Leclère and A. Philpott.
On risk averse competitive equilibrium.
Operations Research Letters, 46:19–26, 2018.

References: strategic declarations and related work

V. Leclère and A. Philpott.
Strategic behavior of risk-averse agents under stochastic market clearing.
Operations Research Letters, 63:107365, 2025.

I. Shilov, H. Le Cadre and A. Bušić.
Risk-averse equilibrium analysis and computation.
Operations Research Letters, 2021.

N. Vespermann, T. Hamacher and J. Kazempour.
On ambiguity-averse market equilibrium.
Optimization Letters, published online 2022.

References: inverse problem and MAVaR geometry

L. Le Gall.
A method for inferring information on risk sets from a market equilibrium.
Internship report, CERMICS / ENPC, 2025.

R. T. Rockafellar and S. Uryasev.
Optimization of conditional value-at-risk.
Journal of Risk, 2000.