Risk averse equilibrium in electricity market

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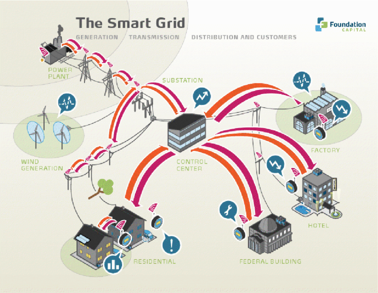
CERMICS - EPOC

Why do we look for equilibria in electricity market ?



- In electricity market, agents are more and more subject to
  - uncertainty (e.g. weather forecast)
  - risks (e.g. black out)
- Each agent has to take decision at each time steps (e.g. hour/day/month)
- Prices on the market should balance demand and supply
- We look for equilibrium in multistage risk averse problem

Our objectives

We want to

- understand the impact of risk on equilibrium
- manage large scale problem
- study the distribution of welfare

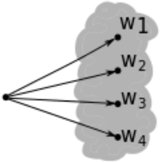
Outline

1. Numerical results on a toy problem
   - Statement of the two stage problem
   - Computing an equilibrium
   - Extension to a multistage framework
2. Recall in economy and first results
   - Statement of a equilibrium problem
   - Existence of an equilibrium
   - Pareto efficiency and link with multistage stochastic equilibrium
3. Ongoing work and open questions

Outline

Ingredients for the toy problem

This problem is largely inspired from Philpott, Ferris, and Wets (2013)



The problem has the following features

- two stage problem
- thermal producer
- hydro producer
- scenario tree structure
- uncertainty = inflows
- a deterministic demand D

Agent's objective function

At each step t, agent a has :

- an endowment $\boldsymbol{x}_t^a$ and a control $\boldsymbol{u}_t^a$
- an instantaneous cost function C_t^a and a production function g_t^a.
- an objective function L_t^a defined by

$$L_t^a(\boldsymbol{x}^a, \boldsymbol{u}^a) = \underbrace{C_t^a(\boldsymbol{x}^a, \boldsymbol{u}^a)}_{\text{costs}} - \underbrace{\pi_t g_t^a(\boldsymbol{x}^a, \boldsymbol{u}^a)}_{\text{incomes}}$$

Agent's risk measure

Each agent is endowed of a risk measure to measure uncertainty of second stage

$$\mathbb{F}_{\lambda_a,\beta_a}[\boldsymbol{x}] = (1-\lambda_a)\mathbb{E}_{\mathbb{P}}[\boldsymbol{x}] + \lambda_a \mathrm{CV@R}_{1-\alpha_a}[\boldsymbol{x}]$$

We recall

- $\alpha \to 0$, we converge to the Worst Case risk measure
- $\alpha = 1$ is equivalent to the Expectation

Hydro producer's dynamic

The state of the hydro producer is linked between stages by the constraint

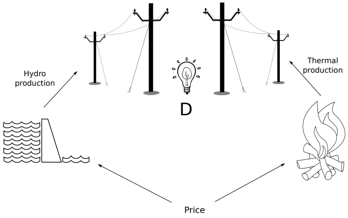
$$\boldsymbol{x}_2^a(\omega) = x_1^a - u_1^a + \omega$$

Equilibrium prices and complementarity constraint

The prices at equilibrium is determined through the complementarity constraints

$$0 \leq \underbrace{\sum_{a \in A} g_t^a(\boldsymbol{x}_t^a, \boldsymbol{u}_t^a)}_{\text{supply}} - \underbrace{\boldsymbol{D}}_{\text{demand}} \perp \boldsymbol{\pi}_t \geq 0$$

Summing up



We want to find prices $\pi_1^\sharp$ and $\pi_2(\omega)^\sharp$ so that

- Each agent solves a two stage risk averse optimization problem with prices $\pi_1^\sharp$ and $\pi_2(\omega)^\sharp$
- Each agent returns at each node a plan of production
- At each node we want Production = Demand

Statement of the toy problem Producer/Producer

$$\min_{x_1^a, u_2^a, \boldsymbol{x}_2^a, \boldsymbol{u}_2^a} L_1^a(x_1^a, u_1^a) + \underbrace{\underbrace{(1-\lambda_a)}_{\text{Proportion of risk}} \mathbb{E}_{\mathbb{P}}\left[L_2^a(\boldsymbol{x}_2^a, \boldsymbol{u}_2^a)\right] + \lambda_a \underbrace{\mathrm{CV@R}_{1-\alpha}}_{\text{risk measure}} \left(L_2^a(\boldsymbol{x}_2^a, \boldsymbol{u}_2^a)\right)}_{\text{numerical evalutaion of second stage}}$$

$$\text{subject to} \begin{cases} \text{Production cost: } \overbrace{L_t^a}^{\text{Objective cost}} = \overbrace{C_t^a}^{\text{instantaneous cost}} - \overbrace{\boldsymbol{\pi}_t}^{\text{price}} \overbrace{g_t^a}^{\text{production}} \\ \text{Dynamics: } \boldsymbol{x}_2^a(\omega) = x_1^a - u_1^a + \omega \\ \text{Bounds: } x_t^a \in \mathcal{X}_t^a \;, \quad u_t^a \in \mathcal{U}_t^a \end{cases}$$

$$0 \leq \underbrace{\sum_{a \in A} g_t^a(\boldsymbol{x}_t^a, \boldsymbol{u}_t^a)}_{\text{supply}} - \underbrace{\boldsymbol{D}_t}_{\text{demand}} \perp \boldsymbol{\pi}_t \geq 0$$

1. Numerical results on a toy problem
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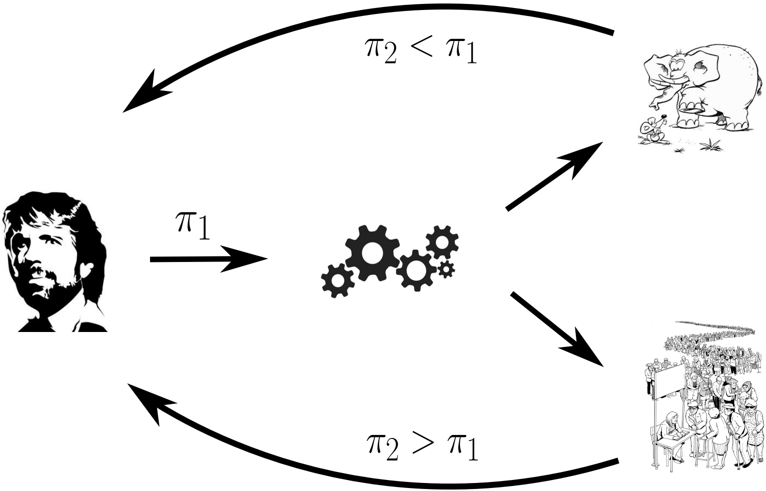
Computing an equilibrium

We have computed equilibria using two tools

- GAMS and EMP: generation of a system of KKT conditions
  Ferris et al. (2009)
- Julia and JuMP: implementation of an iterative algorithm (Uzawa algorithm, Walras tâtonnement, ...)
  Cohen (2004)

General idea of Walras' tâtonnement



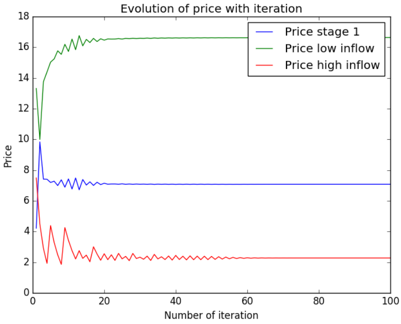
Walras's tâtonemment

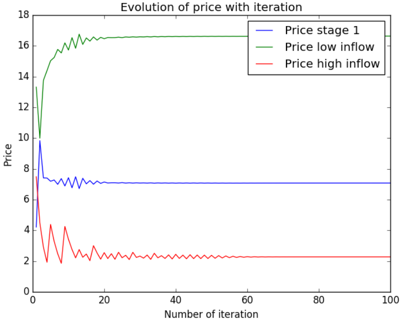
- Initialize π
- For i from 1 to *maximum_iteration* do
  - update the step size: $\tau = \frac{1}{\sqrt{i}}$
  - compute optimal decision:

$$(\boldsymbol{x}_t^{\sharp a}, \boldsymbol{u}_t^{\sharp a}) = \underset{\boldsymbol{x}^a \in \mathbb{X}_t^a, \boldsymbol{u}_t^a \in \mathbb{U}_t^a}{\arg\min} \; C_t^a(\boldsymbol{x}_t^{\sharp a}, \boldsymbol{u}_t^{\sharp a}) - \boldsymbol{\pi}_t g_t^a(\boldsymbol{x}_t^{\sharp a}, \boldsymbol{u}_t^{\sharp a})$$

  - update prices :

$$\boldsymbol{\pi}_t = \max\Big\{0, \boldsymbol{\pi}_t + \tau\big(D - \sum_{a \in A} g_t^a(\boldsymbol{x}_t^{\sharp a}, \boldsymbol{u}_t^{\sharp a})\big)\Big\}$$


Evolution of price with iteration
Price
Number of iteration
Price stage 1
Price low inflow
Price high inflow
0
2
4
6
8
10
12
14
16
18
0
20
40
60
80
100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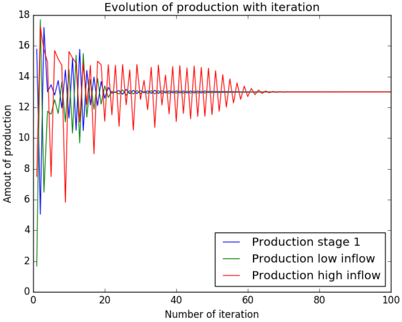
Evolution of production with iteration
Amout of production
Number of iteration
Production stage 1
Production low inflow
Production high inflow
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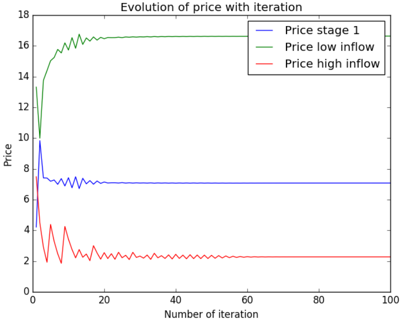
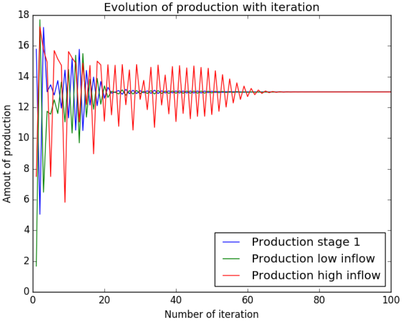


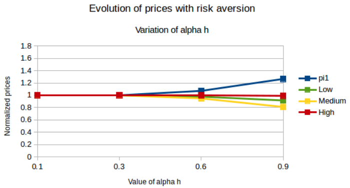


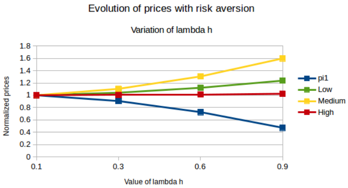
	Mean error	Max error	Standard deviation error
GAMS vs Julia	0.01%	0.14%	0.01%

How price evolves with risk aversion ?

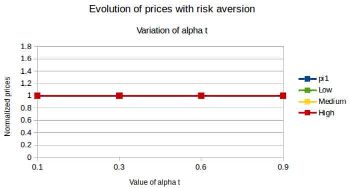
Variation of α_a | Variation of λ_a

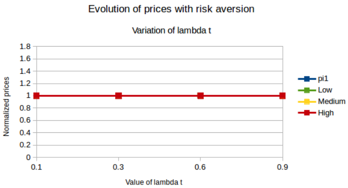
Hydro





Thermal





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From two stages to multistage

We introduce the notation to denote the risk measure of agent a

$$\mathbb{F}_{\lambda_a,\beta_a}[\boldsymbol{X}] = (1-\lambda_a)\mathbb{E}_{\mathbb{P}}[\boldsymbol{X}] + \lambda_a \mathrm{CV@R}_{\beta_a}[\boldsymbol{X}]$$

Given a $\sigma-$field $\mathcal{F}$ we can define a conditional risk measure by

$$\mathbb{F}_{\lambda_a,\beta_a}[\boldsymbol{X}|\mathcal{F}] = \lambda_a \mathbb{E}_{\mathbb{P}}[\boldsymbol{X}|\mathcal{F}] + (1-\lambda_a)\mathrm{CV@R}_{\beta_a}[\boldsymbol{X}|\mathcal{F}]$$

In this framework, risk is controlled at each stage

Multistage risk averse equilibrium

Given a filtration $(\mathcal{F}_t)$, the risk averse multistage equilibrium problem is written

$$\begin{aligned} \min_{\boldsymbol{x}_t^a, \boldsymbol{u}_t^a} \quad & \mathbb{F}_{\lambda_a,\beta_a}\Big[L_1^a(\boldsymbol{x}_1^a, \boldsymbol{u}_1^a) + \mathbb{F}_{\lambda_a,\beta_a}\Big[L_2^a(\boldsymbol{x}_2^a, \boldsymbol{u}_2^a) + \cdots + \mathbb{F}_{\lambda_a,\beta_a}\big[L_T^a(\boldsymbol{x}_T^a, \boldsymbol{u}_T^a)|\mathcal{F}_T\big] \\ s.t. \quad & L_t^a(\boldsymbol{x}_t^a, \boldsymbol{u}_t^a) = C_t^a(\boldsymbol{x}_t^a, \boldsymbol{u}_t^a) - \boldsymbol{\pi}_t g^a(\boldsymbol{x}_t^a, \boldsymbol{u}_t^a) \\ & \boldsymbol{x}_t^a = f_t^a(x_{t-1}^a, u_t^a, \omega_t) \\ & \boldsymbol{x}_t^a \in (\mathbb{X}_t^a) \ , \ \ \boldsymbol{u}_t^a \in (\mathbb{U}_t^a) \end{aligned}$$

Equilibrium prices are determined through $|\mathcal{N}|$ constraints which are called complementarity constraints

$$0 \leq \sum_{a \in A} g^a(\boldsymbol{x}_t^a, \boldsymbol{u}_t^a) - \boldsymbol{D}_t \perp \boldsymbol{\pi}_t \geq 0 \ , \ \ \forall t \in [\![1 : T]\!], \mathbb{P} - a.s.$$

Conclusion on the toy problem

So far we have

- defined a risk averse toy problem with two producers
- presented an iterative algorithm to compute equilibrium prices
- shown that risk aversion have an impact on equilibrium
- defined a multistage problem

We will now

- state the problem in a more general framework
- give a theorem of existence
- discuss on efficiency of the solution

Outline

2 Recall in economy and first results

- Statement of a equilibrium problem
- Existence of an equilibrium
- Pareto efficiency and link with multistage stochastic equilibrium

Ingredients for the market clearing model in a economy of exchange

We consider a market

- with $g \in \mathbb{G}$ goods traded at a price π_g
- $|A|$ agents with a utility function $F^a(x^a)$
- e_g^a is the initial endowment
- $x_g^a \in \mathbb{X}_g^a$ is the quantity traded
- $\mathbb{X}^a = \prod_{g \in \mathbb{G}} \mathbb{X}_g^a$
- $\mathbb{X}^{|A|} = \prod_{a \in A} \mathbb{X}^a$

Definition

If agents act as if they have no influence on the prices, they are called price takers

Extension to an economy of production

- A cost function C^p and a production function g^p
- $|\mathcal{P}|$ producer with a utility function

$$F^a(x^p, \pi) = \underbrace{C_p(x^p, \pi)}_{\text{Costs}} - \underbrace{\pi g_p(x^p, \pi)}_{\text{Incomes}}$$

- e_g^p is the initial endowment
- $x_g^p \in \mathbb{X}_g^p$ is the quantity produced
- $\mathbb{X}^p = \prod_{g \in \mathbb{G}} \mathbb{X}_g^p$
- $\mathbb{X}^{|\mathcal{P}|} = \prod_{p \in \mathcal{P}} \mathbb{X}^p$

An equilibrium $(x_\sharp, y_\sharp, \pi_\sharp)$ in an economy of production satisfies

- $x_\sharp^a \in \arg\min_{x^a \in \mathbb{X}^a} F^a(x^a)$
- $y_\sharp^p \in \arg\min_{x^p \in \mathbb{X}^p} F^p(x^p, \pi_\sharp)$
- $0 \leq \underbrace{\sum_{p \in \mathcal{P}} (x_\sharp^p - e^p)}_{\text{Production}} - \underbrace{\sum_{a \in A} (x_\sharp^a - e^a)}_{\text{Demande}} \perp \pi_\sharp \geq 0$

Walras' law in an economy of exchange

Definition

We define the aggregate demand at a price π by

$$z(\pi) = \sum_{a \in A} \left(x_\sharp^a(\pi) - e^a \right)$$

Proposition

In an economy of exchange and under technical assumptions, we have

$$\pi z(\pi) = 0$$

2. Recall in economy and first results
   - Statement of a equilibrium problem
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Existence of an equilibrium in a game

Definition

Given a decision space R, we endowed each agent with a utility function $F^a(x^a, x^{-a})$
An equilibrium $(x_\sharp^a)_{a \in A}$ of the game $\mathcal{G} = ((F^a)_{a \in A}, R)$ satisfies

$$x_\sharp^a \in \arg\min_{x^a} \left\{ F^a(x_\sharp^1, \cdots, x^a, \cdots, x_\sharp^{|A|}) | (x_\sharp^1, \cdots, x^a, \cdots, x_\sharp^{|A|}) \in R \right\}$$

Theorem (Existence of equilibrium Rosen (1965))

Under technical assumptions (convexity, conpacity), an equilibrium exists for every convex $|A|$-person game.

Assumption on demand D

To be able to use Rosen's theorem, we have to make the following assumption

Assumption (Price bounded)

Price is bounded by $\pi_{\max}$ and we use the rule

$$D(\pi_{\max}) = \min\left\{D, \sum_{a \in A} g_a\left(x_\sharp^a(\pi_{\max})\right)\right\}$$

Existence of an equilibrium

Proposition (Rosen (1965))

If

- *the decision spaces $\mathbb{X}_a$ are closed, convex and bounded*
- *the cost functions C_a are continuous and convex in x_a*
- *the production functions g_a are continuous and concave in x_a*

then there exist a competitive equilibrium in the electricity market

Some work remains to be done on uniqueness of the equilibrium (strict convexity)

Sketch of proof in an economy of exchange

To simplify notation, we study an economy of exchange

- The players are the $|A|$ agents and the Price Player (PP)
- Each a's payoff function is $F^a(x^a)$
- Agent a's best response to a price π is given $x^a_\sharp(\pi)$
- The Price Player's payoff by the value of the aggregate excess demand

$$F^{PP}(\pi, x) = \pi \sum_{a \in A} \left(x^a_\sharp(\pi) - e_a \right) = \pi z(\pi)$$

- There exist $\pi_\sharp$ and a $x^a_\sharp(\pi_\sharp)$ (Rosen's theorem)
- We have $0 = \pi z(\pi)$ and $\pi_\sharp z(\pi_\sharp) = 0$
- By minimization, $\pi_\sharp z(\pi_\sharp) \leq \pi z(\pi_\sharp)$
- These imply $z(\pi_\sharp) \geq 0$

2 Recall in economy and first results

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Pareto efficiency of an equilibrium

Definition

A feasible production is said to be Pareto efficient if we cannot improve the welfare of some agent without deteriorating the welfare of an other

Proposition (Levin (2006))

Assume that agent are price takers
A competitve equilibrium is Pareto efficient

Just the idea on how to apply the previous result

- We study equilibrium in a multistage risk averse problem
- We use a structure of scenario tree of which Ω is finite
- We make a correspondence between state of the world and goods

Recall on the theory

- In this section we have
  - given a theorem of existence
  - shown that equilibria are Pareto efficient
- We still have to discuss about uniqueness
- We present now some on going work

Outline

On large scale problems, exact implementation is slow

EMP is unefficient for multistage problem

- On a big scenario tree, calculating the optimal price at each node is inefficient (GAMS is limited to three stages problem)
- We want to approximate the price using iterative algorithm

What is the impact of risk aversion on the distribution of Welfare

- If we had contracts to trade risk, we know there exists a social planning problem which gives the same solution than the market clearing problem
- We want to study how evolves the distribution of welfare with risk aversion on incomplete market

Conclusion

In this talk, we have

- studied impact on risk on equilibirum with a toy problem
- given some abstract results
- given clues to scale the problem

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