Forward contracting in markets with private information

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The setting for our talk

- We consider a market for a divisible good (e.g. electricity).
- Prices w in the future are uncertain.
- Suppliers and consumers have different views about these prices.
- They seek to arrange a sale of quantity Q at a forward price f.
- In electricity these are called hedge contracts or contracts for differences.
- Suppliers and consumers are risk averse with same utility function U.

Negotiating forward contracts

- How are contract quantities and prices determined?
- Contracting driven by information asymmetry as well as risk aversion.
- Contracts can be traded by
  - using a broker;
  - direct negotiation between buyer and seller;
  - buying on an exchange.
- We compare the first two mechanisms.

Summary

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Contracts for differences

- Recall the commodity price is denoted by the random variable w.
- The holder of Q forward contracts at forward price f receives a random payoff of $Q(w - f)$.
- The writer of the contracts (counterparty) receives a random payoff of $Q(f - w)$.

This is the same as if a purchaser buying Q at the random price w had to pay the fixed price f.
Similarly the seller receives an equivalent fixed price f for this quantity when he sells it.

Payoff notation

With random commodity price w, the expected utilities of the buyer (agent 1) and seller (agent 2) are:

Uncontracted profit	$R_1(w), R_2(w)$
Probability	$\mathbb{P}_1(w), \mathbb{P}_2(w)$
Utility	$U(\cdot)$
$\Pi_1(Q, f)$	$\int_{-\infty}^{\infty} U\left(R_1(w) + Q(w - f)\right) d\mathbb{P}_1(w)$
$\Pi_2(Q, f)$	$\int_{-\infty}^{\infty} U\left(R_2(w) + Q(f - w)\right) d\mathbb{P}_2(w)$

Example: two random outcomes

Suppose commodity price is w_L or w_H, and assume CARA utility function. For example:

$w_L = 1$ and $w_H = 2$
$R_1(w_L) = R_2(w_H) = 4$ and $R_1(w_H) = R_2(w_L) = 1$
$\rho_1 = \mathbb{P}_1(w_H)$ and $\rho_2 = \mathbb{P}_2(w_H)$.
$U(x) = 1 - e^{-\alpha x}$
$\Pi_1(Q, f) = (1 - \rho_1)(1 - e^{-\alpha(4+Q(w_L - f)}) + \rho_1(1 - e^{-\alpha(1+Q(w_H - f)})$
$\Pi_2(Q, f) = (1 - \rho_2)(1 - e^{-\alpha(1-Q(w_L - f)}) + \rho_2(1 - e^{-\alpha(4-Q(w_H - f)})$

Summary

Using a broker

Contract traders sometimes use a broker to maintain anonymity. Agent 1 (the buyer) provides a demand curve to a broker that is a schedule $Q_1(f)$ of contract quantities that she would buy at varying prices f, where $Q_1(f)$ solves

$$\frac{\partial}{\partial Q}\Pi_1(Q, f) = 0.$$

Agent 2 (the seller) provides a supply curve to the broker that is a schedule of contract quantities $Q_2(f)$ that he would sell at varying prices f, where $Q_2(f)$ solves

$$\frac{\partial}{\partial Q}\Pi_2(Q, f) = 0.$$

The broker arranges a transaction where these curves cross.

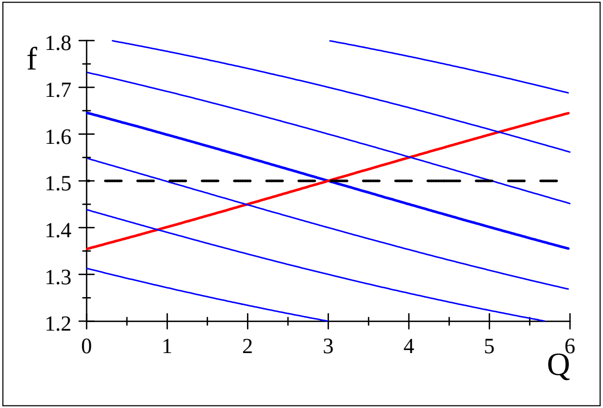
Recall two price outcomes

Price outcomes: $w_L = 1$ and $w_H = 2$,
Rewards $R_1(w_L) = R_2(w_H) = 4$ and $R_1(w_H) = R_2(w_L) = 1$,
Utility: $U(x) = 1 - e^{-\alpha x}$.
Probabilities: $\rho_1 = \mathbb{P}_1(w_H)$ and $\rho_2 = \mathbb{P}_2(w_H)$.

$$\Pi_1(Q, f) = (1 - \rho_1)(1 - e^{-0.2(4+Q(1-f)}) + \rho_1(1 - e^{-0.2(1+Q(2-f)})$$

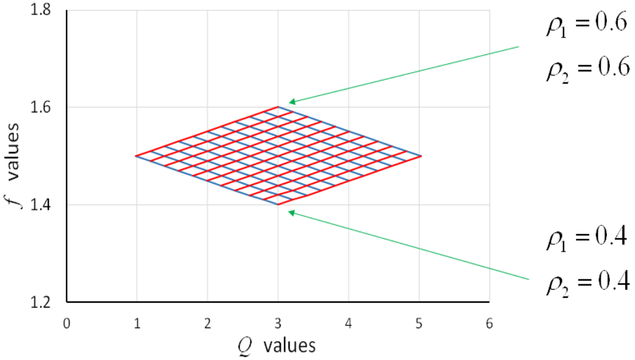
$$\frac{\partial}{\partial Q}\Pi_1(Q, f) = 0 \Longrightarrow Q = 3 + \frac{1}{\alpha}\log\left(\frac{\rho_1(2-f)}{(1-\rho_1)(f-1)}\right)$$

Example



Buyer demand curves $Q_1(f)$ for $\rho_1 = 0.2, 0.3, \ldots, 0.8$ (blue) and supply curve $Q_2(f)$ for $\rho_2 = 0.5$ (red).

Example



Buyer demand curves $Q_1(f)$ for $\rho_1 = 0.40, 0.42, ..., 0.6$ (blue) and supply curves $Q_2(f)$ for $\rho_2 = 0.40, 0.42, ..., 0.6$ (red).

Summary

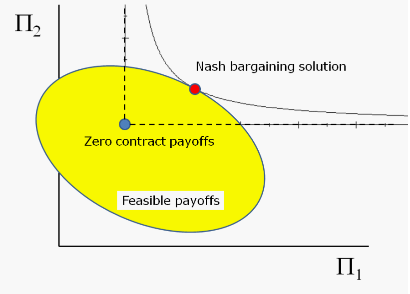
[Rubinstein and Wolinsky, 1986]

Each agent alternately offers the counterparty an ordered pair (Q, f). If the counterparty accepts this then this forms an agreement, and the agents receive $\Pi_1(Q, f)$ and $\Pi_2(Q, f)$ respectively. If the counterparty does not accept (Q, f) then they can make a counter offer (Q', f'), or walk away with some small probability α (negotiation breakdown), which terminates the negotiation. When this happens, resulting payoffs are $\Pi_1(0, f)$, and $\Pi_2(0, f)$.

Nash bargaining solution
[Nash, 1950]

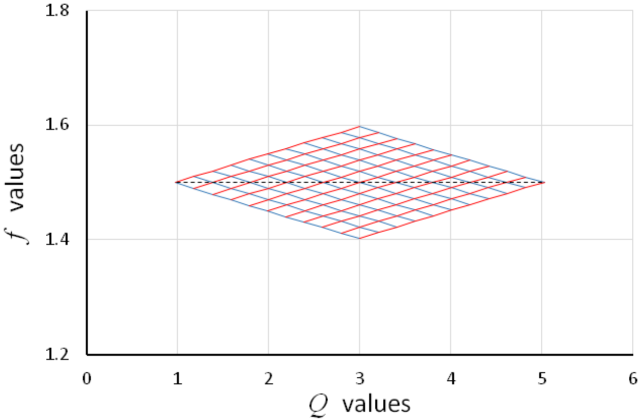
In the limit as the breakdown probability tends to 0, the subgame-perfect equilibrium of the negotiation repeated game gives Q and f that solves

$$\max_{Q,f} \left(\Pi_1(Q,f) - \Pi_1(0,f)\right)\left(\Pi_2(Q,f) - \Pi_2(0,f)\right).$$



Nash bargaining solution for contract negotiation

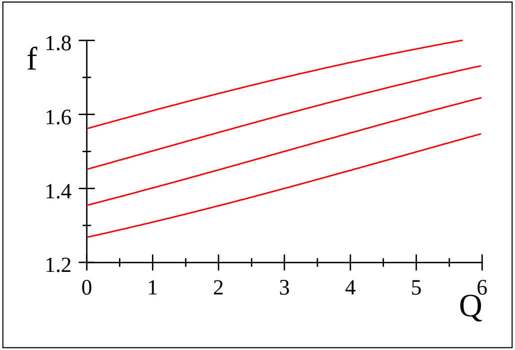
Example



Nash bargaining solutions for $\rho_1 = 0.40, 0.42, ..., 0.6$, and $\rho_2 = 0.40, 0.42, ..., 0.6$.

Summary

Using Information

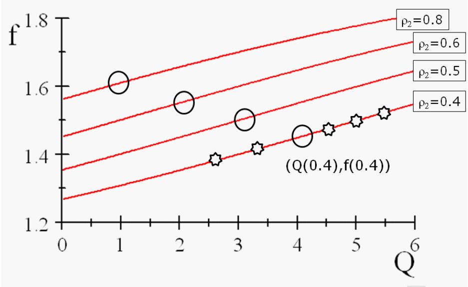


The seller offers red curves to the purchaser of the form $(Q(t,\rho_2), f(t,\rho_2))$ for varying ρ_2.

Using Information

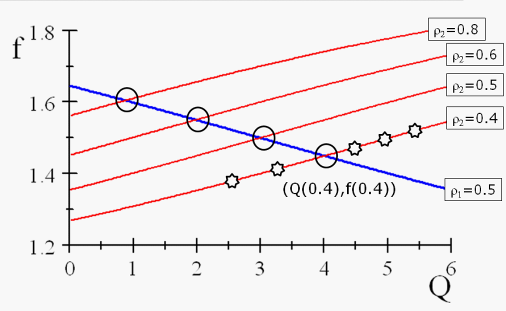
The buyer with belief ρ_1 wants the best outcome on each curve so for each ρ_2 she chooses t to maximize

$$\begin{aligned}\Pi^{(1)} &= \rho_1 U(R_1(w_H) - Q(t,\rho_2)(f(t,\rho_2) - w_H)) \\ &+ (1-\rho_1) U(R_1(w_L) - Q(t,\rho_2)(f(t,\rho_2) - w_L)).\end{aligned}$$



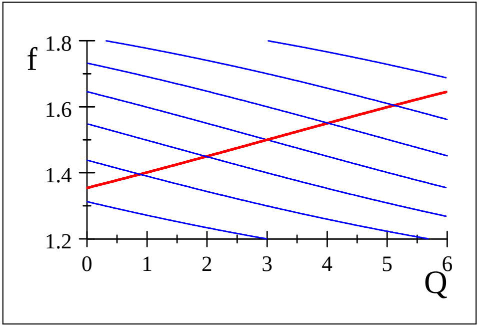
When $\rho_1 = 0.5$, the buyer chooses optimal (Q, f) for each seller curve.

Using Information



Joining the optimal points gives a buyer best response (blue) for $\rho_1 = 0.5$ to a set of seller curves (red).

Supply function equilibrium (limited information)



The seller with belief ρ_2 wants the best outcome on each curve so for each ρ_1 he chooses t to maximize

$$\begin{aligned}\Pi^{(2)} &= \rho_2 U(R_2(w_H) - Q(\rho_1, t)(f(\rho_1, t) - w_H)) \\ &\quad +(1-\rho_2)U(R_2(w_L) - Q(\rho_1, t)(f(\rho_1, t) - w_L))\end{aligned}$$

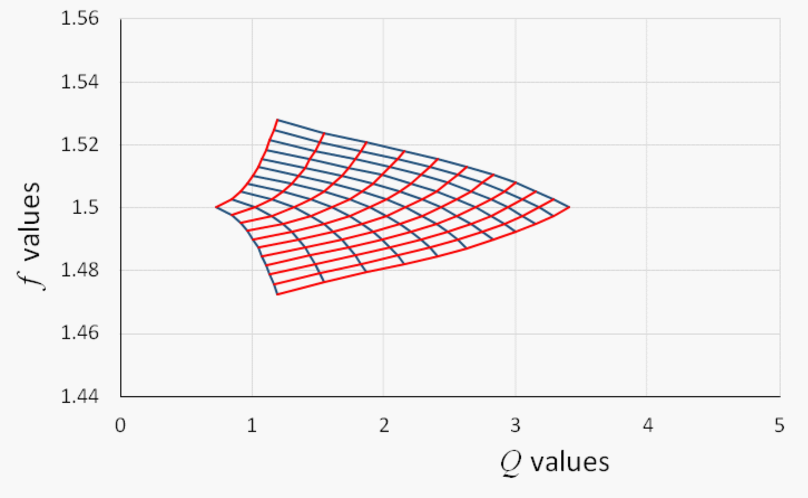
Supply function equilibrium

The conditions

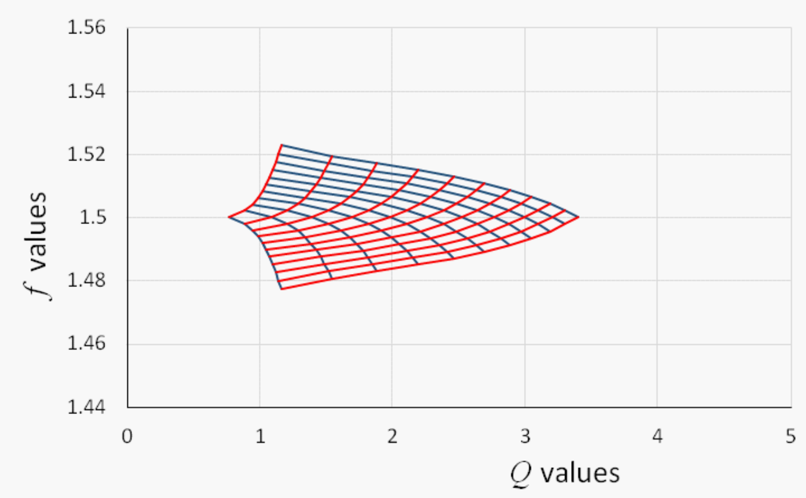
$$\left[\frac{\partial}{\partial t}\Pi^{(1)}\right]_{\rho_1} = 0$$
$$\left[\frac{\partial}{\partial t}\Pi^{(2)}\right]_{\rho_2} = 0$$

involve derivatives $\frac{\partial}{\partial t}Q(t,\rho_2)$, $\frac{\partial}{\partial t}f(t,\rho_2)$, $\frac{\partial}{\partial t}Q(\rho_1,t)$, $\frac{\partial}{\partial t}f(\rho_1,t)$ in nonlinear equations with $Q(\rho_1,\rho_2)$ and $f(\rho_1,\rho_2)$. We can solve these simultaneous differential equations numerically.

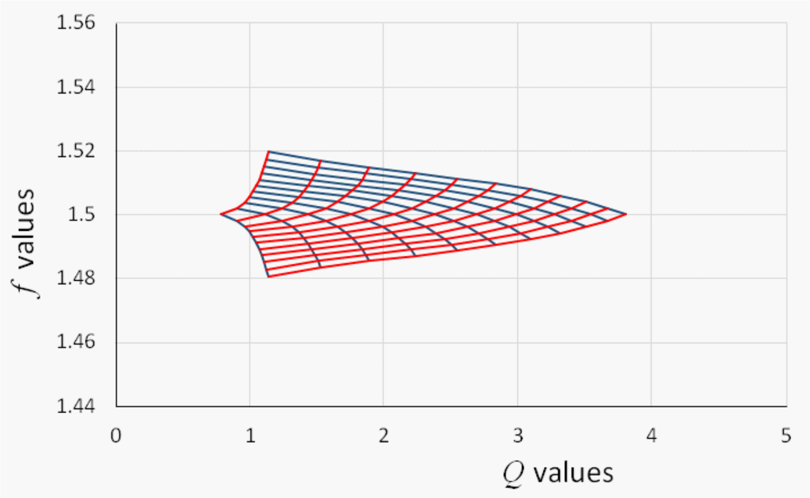
Examples



Examples



Examples



Supply function equilibrium (information deduction)

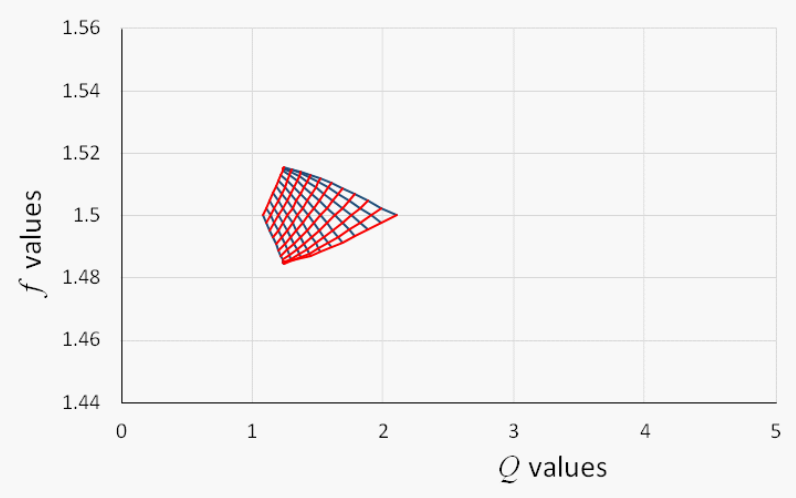
Repeat the above analysis with

$$\rho = \frac{\rho_1 + \rho_1}{2}$$

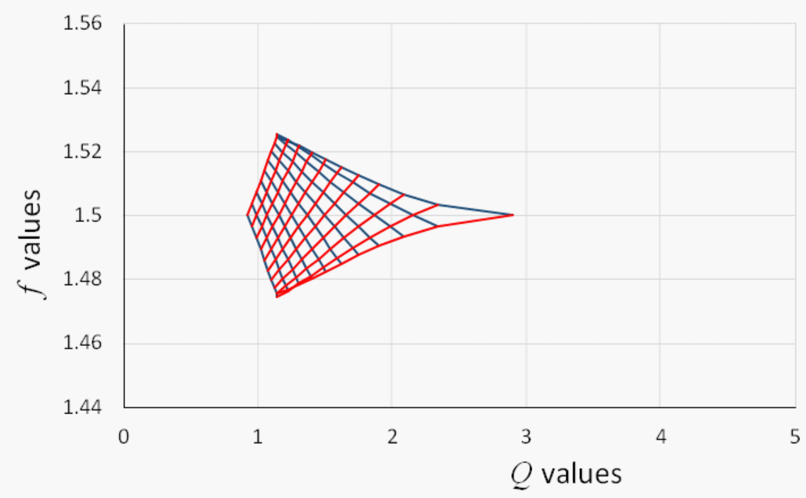
$$\begin{aligned}\Pi^{(1)} &= \rho U(R_1(w_H) - Q(\rho_1, \rho_2)(f(\rho_1, \rho_2) - w_H)) \\ &\quad +(1-\rho)U(R_1(w_L) - Q(\rho_1, \rho_2)(f(\rho_1, \rho_2) - w_L))\end{aligned}$$

$$\begin{aligned}\Pi^{(2)} &= \rho U(R_2(w_H) + Q(\rho_1, \rho_2)(f(\rho_1, \rho_2) - w_H)) \\ &\quad +(1-\rho)U(R_2(w_L) + Q(\rho_1, \rho_2)(f(\rho_1, \rho_2) - w_L))\end{aligned}$$

Examples



Examples



Evaluating a supply function equilibrium

- Construct the equilibrium
- Assume a prior distribution of ρ values (e.g.uniform on (0.4,0.6)).
- Nature selects a value of ρ according to the prior.
- Both players take a sample of size N (N =10 here). On the basis of the number of w_L and w_H values in the sample they make a maximum likelihood estimation of ρ and submit their offers.
- The expected utility is obtained by taking expectations over ρ according to the prior. There is correlation between ρ_1 and ρ_2 (a high ρ is likely to give a large number of w_H values).

Supply function equilibrium results

alpha	$R_H^{(1)}$	$R_H^{(2)}$	Expected utility with no contract	% improvement using simple broker	% improvement using supply function equilibrium	% improvement using supply function with deduction
0.25	1	3	0.3744	**3.848**	3.172	3.137
0.3	1	3	0.4263	**4.821**	4.209	4.140
0.25	1	4	0.4267	**7.881**	7.280	7.099
0.3	1	4	0.4790	**9.282**	8.845	8.658
0.25	1	6	0.4990	**15.979**	15.722	15.297
0.3	1	6	0.5469	**18.090**	17.955	17.552

Payoffs in symmetric equilibrium with increasing risk aversion.

Supply function equilibrium results

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0.3	1	4	0.4790	**9.282**	8.845	8.658
0.25	1	6	0.4990	**15.979**	15.722	15.297
0.3	1	6	0.5469	**18.090**	17.955	17.552
0.5	1	6	0.6718	22.448	**22.607**	22.534

Payoffs in symmetric equilibrium with increasing risk aversion.

This is the end

THE END

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