Recent developments in modelling using supply functions

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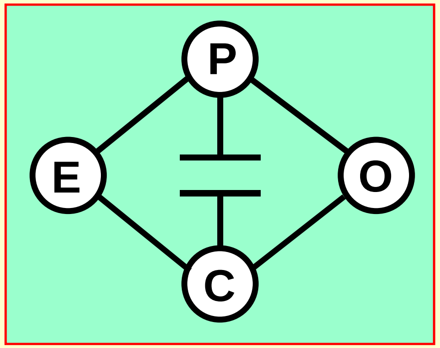


Summary

- Supply functions
- Price-taking optimization models
- Market distribution functions
- Unit commitment
- Nash equilibria
- Dynamic and repeated games

Supply functions

- Offer stacks = {(q,p) pairs}
- Expresses p(q)
- Sometimes easier to work analytically with smooth functions rather than step functions
- Supply function = q(p)

  (parametrically) = $\{(q(t),p(t) \mid t \,\varepsilon\, [0,T]\}$

SOL BidQuery

- Mentioned in last year’s workshop
- Available in Beta-test form
- Free (for the moment)

Price-taking optimization models

- Offer short-run marginal cost of dispatch
- What is
  - SRMC function for a reservoir?
  - SRMC function for a block dispatched river chain?
  - SRMC function for a thermal unit with startup costs, shut-down costs and ramp rates?

SRMC function for a reservoir

Opportunity cost of stored water

- Equals water value of SPECTRA, SDDP when all agents offer SRMC.
- If not then prices depend on other agents behaviour
- Can model price process (and inflow process) and do stochastic DP (or SDDP) using this.
- HERO model does this for a single reservoir (Pritchard, Philpott, Neame, *Math Programming,* 2004)

Dynamic programming

x	$=$	level of stored water in reservoir
W_t	$=$	inflow to reservoir (assumed independent of W_{t-1})
s	$=$	generation stack submitted by generator
$U_t(s)$	$=$	water release from submitting stack s in stage t
$R_t(s)$	$=$	profit earned from submitting stack s in stage t
α	$=$	discount factor
$V_t(x)$	$=$	optimal (random) future profit from t to ∞ given a reservoir level x at start of stage t
Y_t		market and inflow state

$$V_t(x) = \max_{s \in S} \{R_t(s, Y_t) + E[\alpha V_{t+1}(x - U_t(s) + W_t) \mid Y_t]\}$$

Split formulation

$$g_t(\mu, \sigma, y) = \max_s \quad R_t(s, y)$$

$$\text{subject to} \quad \mu_t(s) = \mu$$

$$\sigma_t(s)^2 \leq \sigma^2$$

$$V_t(x) = \max_{\mu,\sigma} \left\{ g_t(\mu, \sigma, Y_t) + \alpha E_{Z,W_t}[V_{t+1}(x - \mu - \sigma Z + W_t) \mid Y_t] \right\}$$

Computing $g(\mu, \sigma)$ with (random) price duration curves

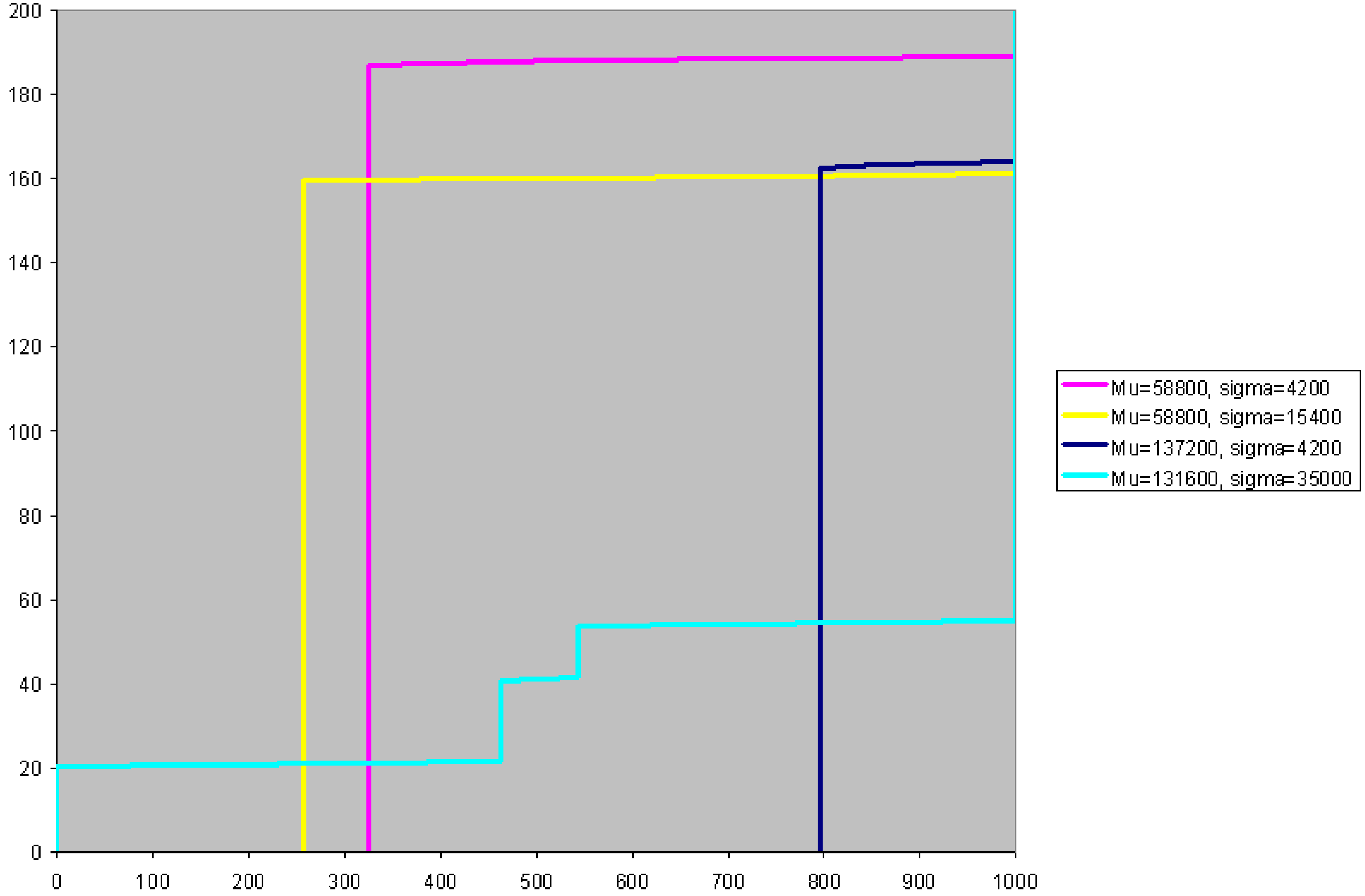
Let p_i be breakpoints of piecewise linear price-duration curve and let A_i be the random slope of the segment between p_{i-1} and p_i.

Suppose $\mathbf{A}$ has mean $\mathbf{a}$ and covariance matrix $\mathbf{V}$.

$$I_i = \int_{p_{i-1}}^{p_i} q(p)\,dp, \text{ for } i = 1, \ldots, m.$$

$$\begin{array}{rll} g(\mu,\sigma) = \max & \sum_{i=1}^{m} a_i \int_{p_{i-1}}^{p_i} pq(p)\,dp & \\ \text{subject to} & q : [p_0, p_m] \to [0, q_{\max}] \text{ is a non-decreasing function} & \\ & I_i = \int_{p_{i-1}}^{p_i} q(p)\,dp, & i = 1, \ldots, m, \\ & \mathbf{a}^\top \mathbf{I} = \mu, & \\ & \mathbf{I}^\top \mathbf{V} \mathbf{I} \le \sigma^2, & \end{array}$$

Solutions (from GAMS/MINOS)

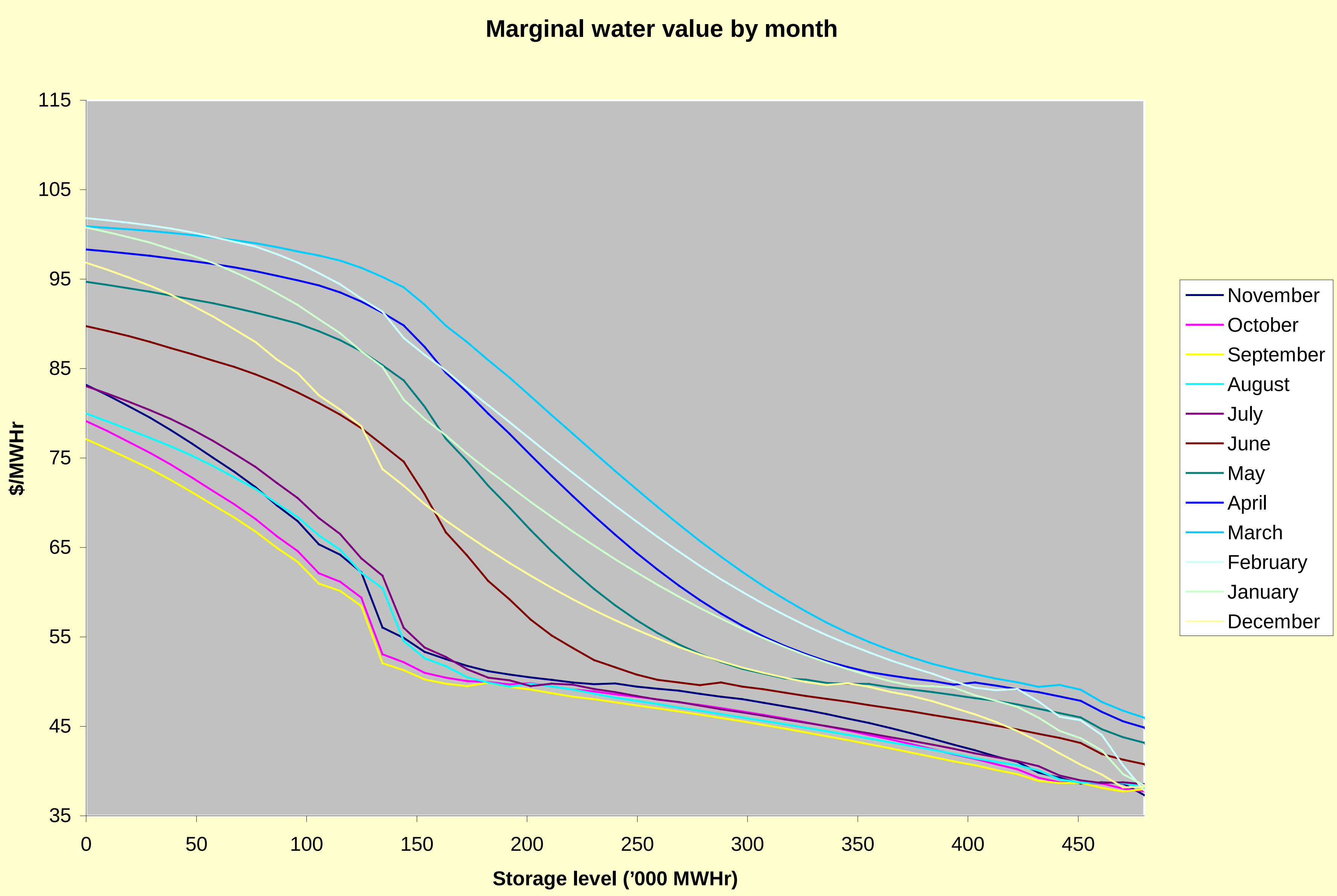


Solutions to intra-week problem

Compute $g_t(\mu, \sigma)$ offline for $M = \left\{\mu_j : j \in J\right\}$ and $S = \{\sigma_k : k \in K\}$

	8400	14000	19600	25200	30800	36400	42000	47600	53200	58800	64400	70000
4200	1.2	1.6	2.1	2.6	3.0	3.5	4.0	4.4	4.9	5.3	5.8	6.3
7000	1.5	2.0	2.4	2.9	3.3	3.8	4.3	4.7	5.2	5.6	6.1	6.5
9800	1.6	2.3	2.7	3.2	3.6	4.1	4.5	5.0	5.5	5.9	6.4	6.8
12600	1.6	2.5	3.0	3.4	3.9	4.4	4.8	5.3	5.7	6.2	6.6	7.1
15400	1.6	2.6	3.3	3.7	4.2	4.6	5.1	5.5	6.0	6.4	6.9	7.3
18200	1.6	2.6	3.5	4.0	4.4	4.9	5.3	5.7	6.2	6.6	7.1	7.5
21000	1.6	2.6	3.6	4.2	4.6	5.1	5.5	6.0	6.4	6.8	7.3	7.7
23800	1.6	2.6	3.6	4.4	4.9	5.3	5.7	6.2	6.6	7.0	7.5	7.9
26600	1.6	2.6	3.6	4.5	5.1	5.5	5.9	6.4	6.8	7.2	7.7	8.1
29400	1.6	2.6	3.6	4.5	5.3	5.7	6.1	6.6	7.0	7.4	7.8	8.2
32200	1.6	2.6	3.6	4.5	5.3	5.9	6.3	6.7	7.1	7.6	8.0	8.4
35000	1.6	2.6	3.6	4.5	5.3	6.0	6.5	6.9	7.3	7.7	8.1	8.5

Weekly revenue $g_t(\mu, \sigma)$ in \$M for a range of σ_k (rows) and μ_j (columns)


Marginal water value by month
$/MWHr
115
105
95
85
75
65
55
45
35
0
50
100
150
200
250
300
350
400
450
Storage level ('000 MWHr)
November
October
September
August
July
June
May
April
March
February
January
December

SRMC function for a river chain

- Constraints affect feasible dispatch
- Tailor offers to meet river constraints
- Discretize prices, tribflows into bands
  - use price/tribflow scenarios
  - solve a stochastic LP
  - different models for block/station dispatch
  - can include two-hour rule, environmental constraints
- Model coded in AMPL/CPLEX for generic river chain (Philpott and Paulus, 2004)

SRMC function for thermal unit

SRMC = fuel cost if unit commitment not counted

We can tailor offers to cover startup and shutdown costs

(Neame, Philpott, Pritchard, *Operations Research,* 2003)

Price-making models

- Estimate the impact of offer on clearing price and dispatch
  - can be done by simulation (e.g. BOOMER code)
- Represent this as a

  market distribution function (denoted ψ).

  $\psi(q,p)$ = Pr(Not being fully dispatched if offer(q,p))

Market distribution functions

$$R(q,p) = qp + q_c(f - p) - C(q)$$

$$\begin{aligned} Z(q,p) &= \frac{\partial R(q,p)}{\partial q}\frac{\partial \psi(q,p)}{\partial p} - \frac{\partial R(q,p)}{\partial p}\frac{\partial \psi(q,p)}{\partial q} \\ &= (p - C'(q))\frac{\partial \psi(q,p)}{\partial p} - (q - q_c)\frac{\partial \psi(q,p)}{\partial q} \end{aligned}$$

Optimal stack follows trajectory

$$Z(q,p) = 0$$

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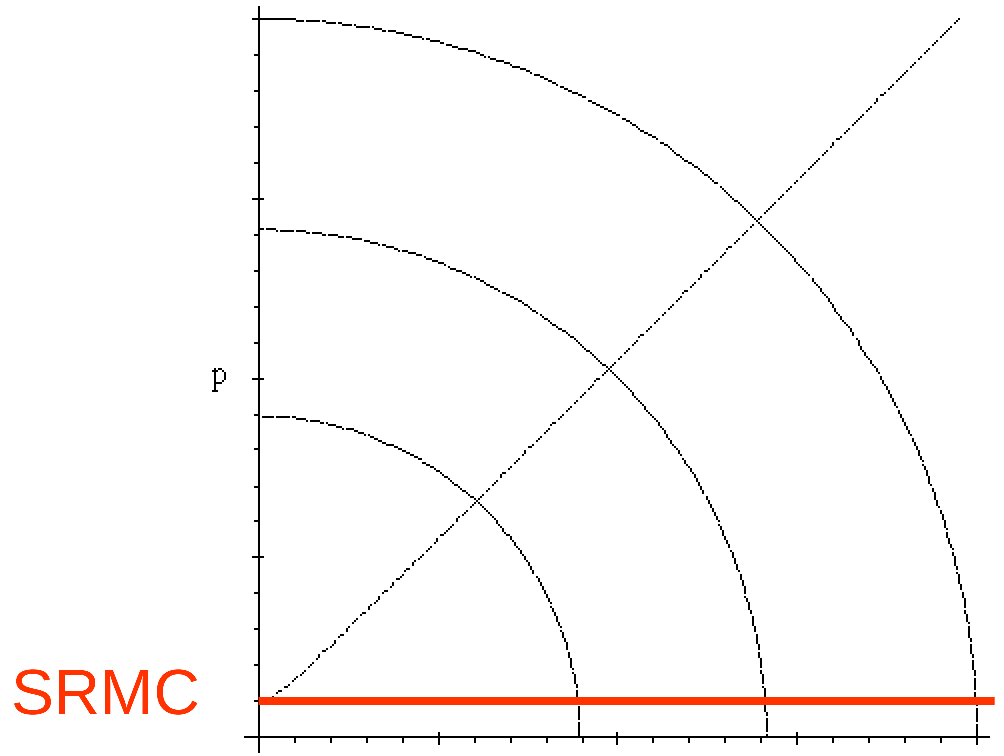
Market distribution functions



$Z(q,p) = 0$ and $\psi(q,p)$ contours.

Single unit with MDF

(Philpott and Schultz, 2004)

Stages $k = 1, 2, 3, \ldots, K$ with MDF $\psi_k(q, p)$.

States 0=stopped, 1=running

$$V_K(0) = V_K(1) = 0.$$

$$R_k = \max_s \int_s (pq - C(q) + q_c(f - p))d\psi_k(q, p),$$

and

$$S_k = \int_0^\infty q_c(f - p)d\psi_k(0, p).$$

DP recursion

$$V_{k-1}(0) = \max\{S_k + V_k(0), R_k - U + V_k(1)\},$$

$$V_{k-1}(1) = \max\{S_k - D + V_k(0), R_k + V_k(1)\}.$$

N identical units

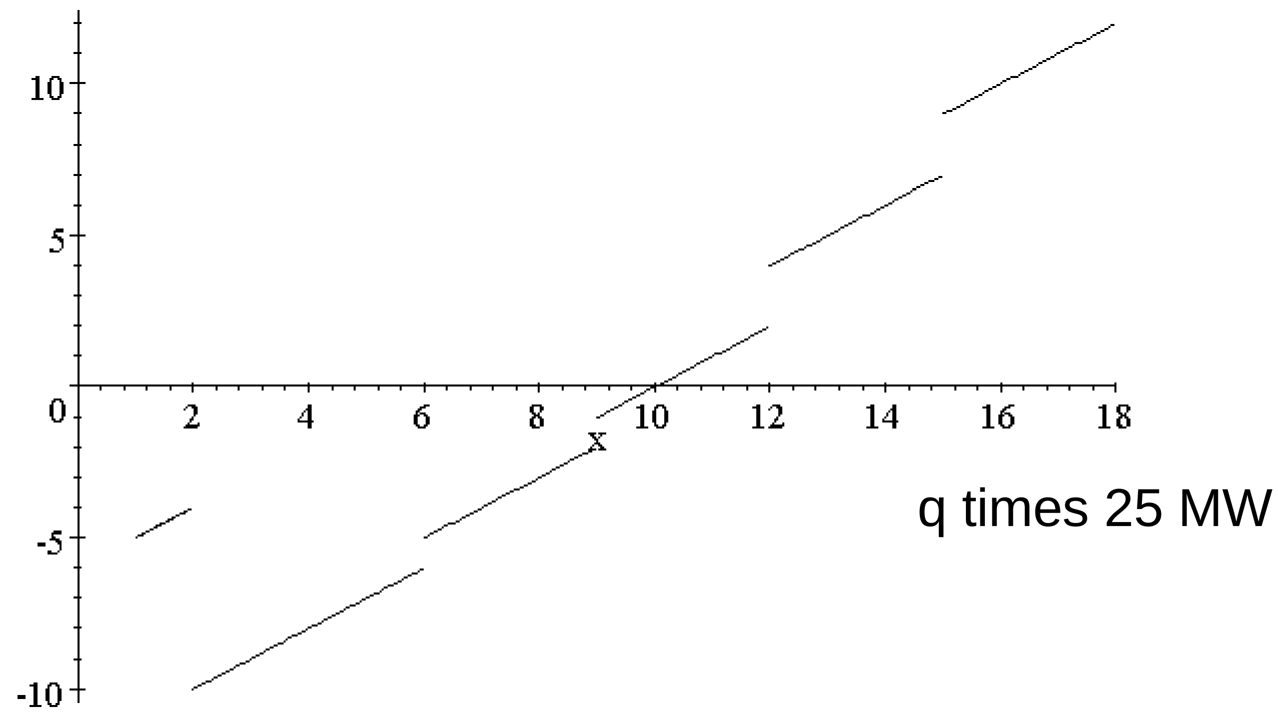
$$J(q) = \{ j \mid j \text{ units can generate the amount } q \}$$

$$G_n(q) = \max_{j \in J(q)} \{ V_k(j) - U[j-n]_+ - D[n-j]_+ \}$$

$$R_{k-1}(n) = \max_s \int (pq + q_c(f-p) - C(q) + G_n(q)) d\psi_k(q,p)$$

$$V_{k-1}(n) = \max\{ S_k - nD + V_k(0), R_{k-1}(n) \}$$

Expected Future Cost with 2 units on



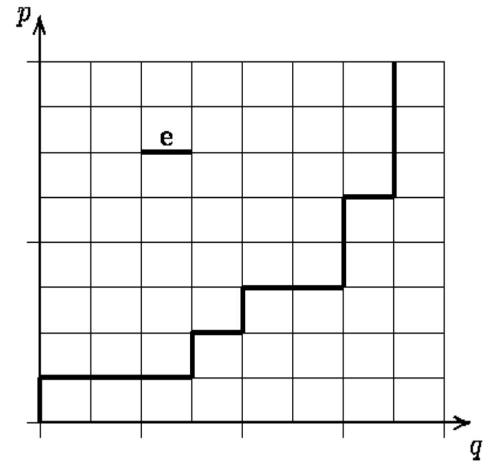
Plot of $C_2(q)$

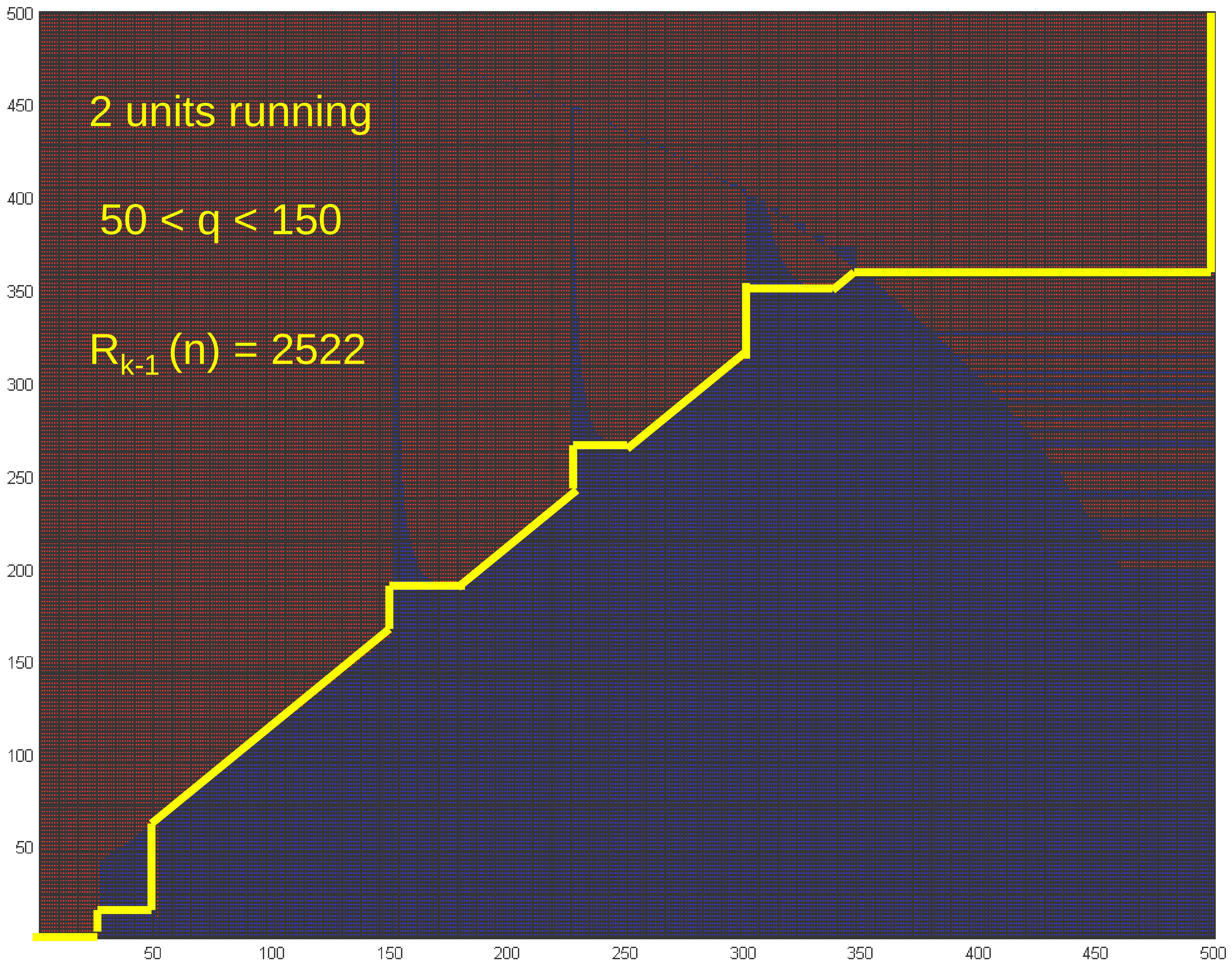
Optimization technique

Work with a pre-determined grid.

V(e) = expected payoff due to dispatches on edge e,
if e is included in the stack.
(Psi function)

Given V(e) for each edge, can find the stack that includes the most value.

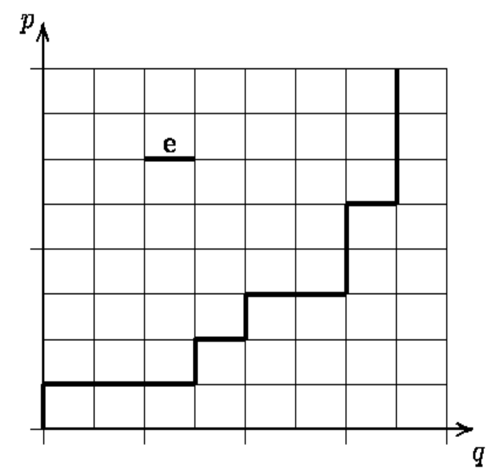


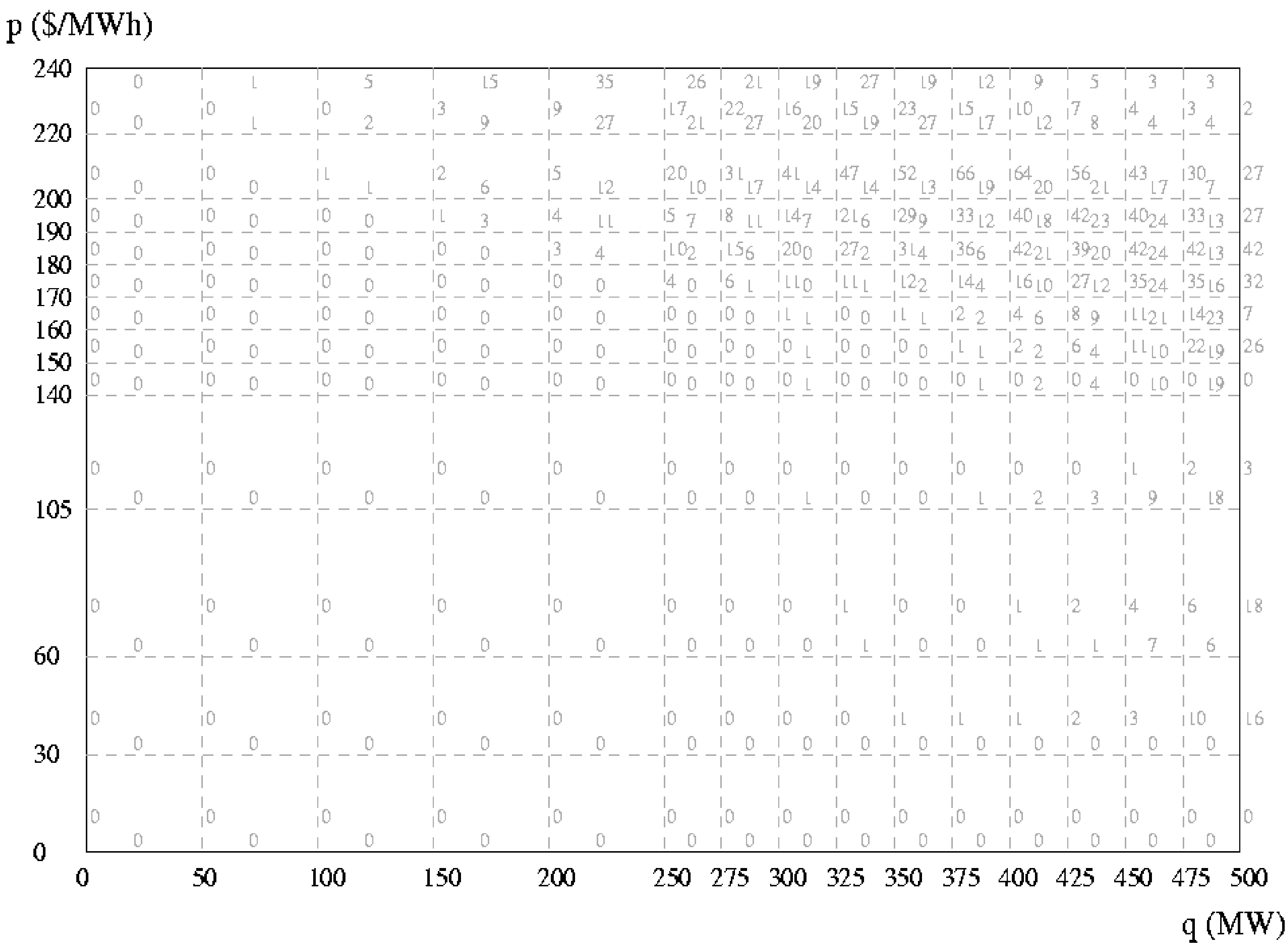

2 units running
50 < q < 150
R_{k-1} (n) = 2522
500
450
400
350
300
250
200
150
100
50
50
100
150
200
250
300
350
400
450
500

Edge values from BOOMER

Generate scenarios of load and other agents offers.
(antithetic sampling)

Estimate V(e) as the sample average payoff from dispatches on edge e.





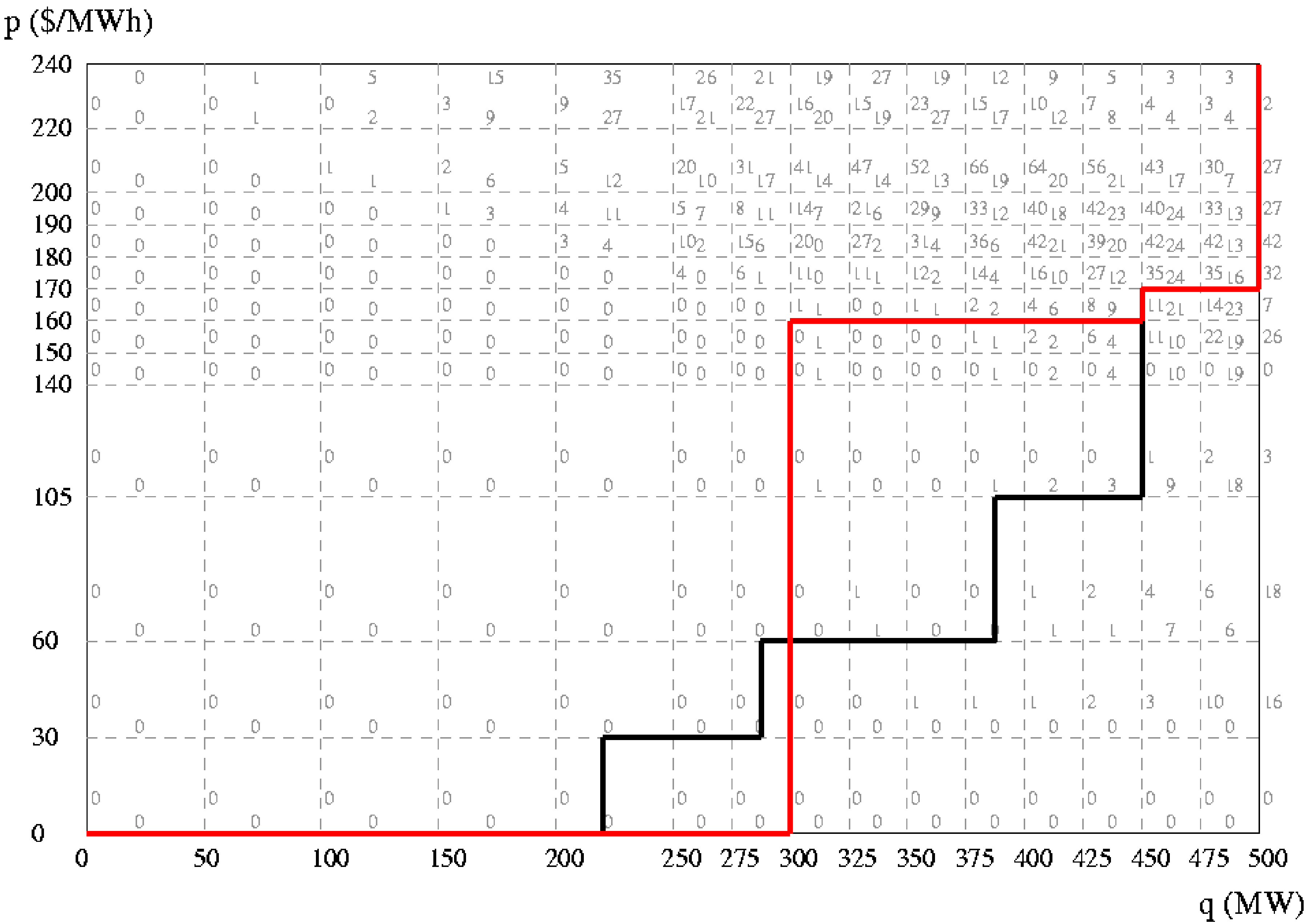
Offers for HLY1

Generator’s objective function

Owner of Huntly might have wanted to maximize

- revenue
  - HLY+TKU+RPO
- less generation cost
  - $35/MWh
- less cost of own retail loads
  - 10% market share at MDN, OTA, HND, WKM, HAY, ISL
- As modified by contracts for differences
  - 100MW each at OTA, HAY, ISL, HWB; all at $50/MWh.

(Numbers are illustrative only!)



Offers for HLY1

200 scenarios
Value of stack 38682

Nash equilibrium models

Classical model (Cournot)

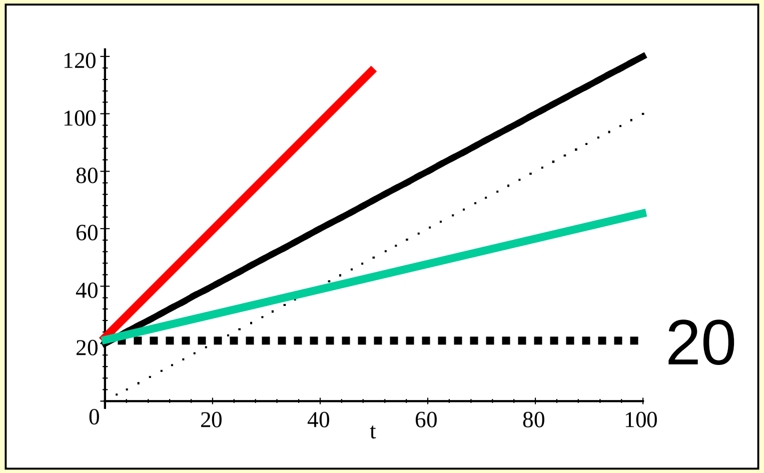
- Generator *i* offers quantity *Q(i)* at price 0
- Deterministic demand curve determines clearing price
- Transmission network can be modelled
- Non identical players can be modelled
- Solve using calculus or numerical method (PATH or MILES)

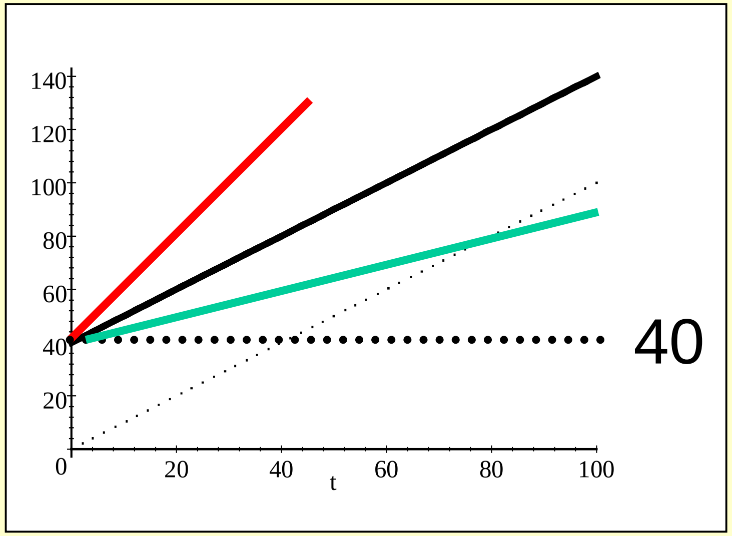
Supply-function model (Klemperer & Meyer, 1989)

- Players offer supply functions ("marginal cost" curves)
- Demand can be elastic or inelastic (Anderson &Philpott, 2002)
- Uncertainty in demand (D(p) + h, h a random shock)
- Mixed strategy s.f. equilibria exist in network models (Jofre, 2004)
- Difficult to model and compute with

Inelastic SFE

Suppose two generators have fuel costs of $20/MWh and $40/MWh, and demand is random on [0,200]





Equilibrium with a price cap

Suppose the Electricity Commission sets price cap of $120/MWh

Two competing generators each with marginal cost $20 and capacity 100 MW offer supply functions.

Demand randomly distributed on [0,200]