Mixed Integer Programming Models for Wind Farm Design

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Outline

Wind Farm Design in NZ

Complex design problem that includes

- Wind farm layout
- External transmission and access
- Ecological and visual impacts
- Internal reticulation layout

Difficult economic conditions

- Inflated turbine prices due to excess international demand
- Depressed national electricity prices due to gas subsidies
- Lack of carbon pricing mechanisms

Developers driven to seek comparatively large-scale high-wind sites

Huge scope for the application of optimisation techniques

Existing Practice

Determine the optimal location of turbines that maximises the net power produced, subject to constraints on the number of turbines, turbine proximity, and turbine wake

Industry use “off-the-shelf” commercial software packages

- WindFarm, WindFarmer, and WindPRO
- Heuristic in nature
    - User specifies an initial feasible layout
    - Perform local neighbourhood search with random restarts
- Limited in the types of constraints that can be modelled

Previous work in the literature

- Genetic algorithms (Mosetti, 1996; Grady et al, 2004)

Heuristic vs Exact Methods

Heuristic methods

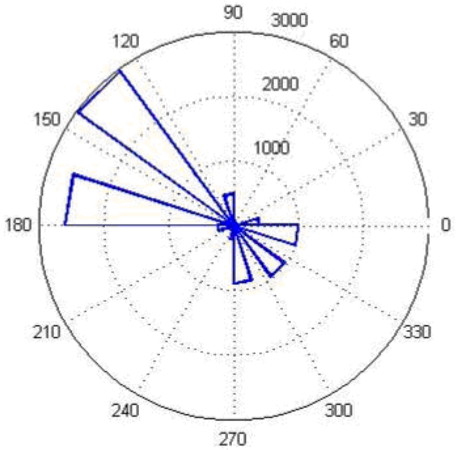
- Typically fast, perhaps too fast
- Cannot guarantee to find an optimal solution
- Cannot provide a measure of the quality of a solution

Exact methods

- Typically slower, but can be terminated early
- Guaranteed to find an optimal solution
- Provide a bound on how good the current solution is

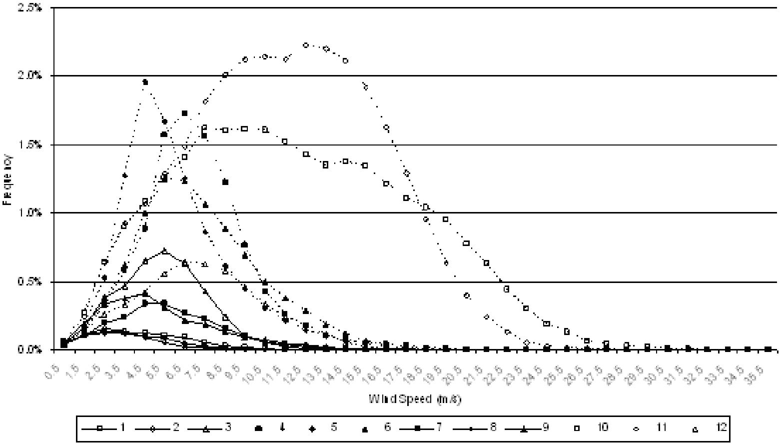
Mixed integer programming (MIP) using LP-based branch-and-bound

Wind Direction



Wind rose for six years of data from the lower North Island partitioned into 30° sectors

Wind Speed



Wind speed frequency distribution for six years of data from the lower North Island partitioned into 30° sectors and $1ms^{-1}$ ranges

Wind Flow Models

Industry standard models use a linear flow model

- WAsP and MS-Micro

Model inputs

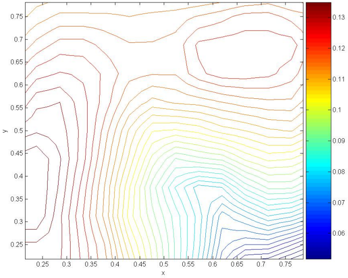
- Wind speed frequency distribution
- Digital terrain model

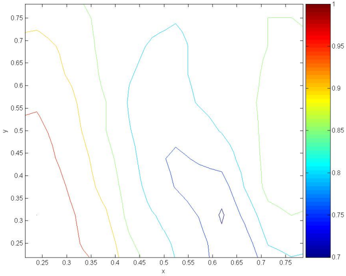
Two-parameter Weibull distributions are used to continuously approximate the empirical wind speed frequency distributions

Model output

- Spatial wind speed distribution called a wind resource grid

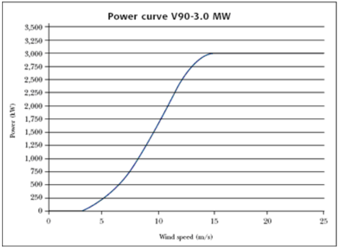
Digital Terrain Map and Wind Resource Data





Digital terrain map contours and wind resource grid velocity distribution at a height of 80m

Turbine Power Output



Power curve for a Vestas V90-3.0MW turbine

Ideal power

- Total expected power output by a turbine in isolation
- Ideal power $P = \int P(u) \cdot F(u)\, du$
  - $P(u)$ is the turbine power function
  - $F(u)$ is the wind speed frequency cumulative distribution function

Turbine Proximity and Wake Interference

Turbine proximity

- Minimum turbine separation requirements prescribe an exclusion zone around any turbine location

Turbine wake interference

- Turbine wakes create an energy deficit downstream
- The interference I between a pair of turbines is the net expected power loss that results from the superposition of the energy deficits at one turbine due to the wake of the other turbine
- Interference is calculated using the PARK turbine wake model (Jensen,1983; Katic, Hojstrup and Jensen, 1986)

Notation

Consider a graph $G = (V, E)$ where

- Set of nodes V denote potential turbine locations
- Set of edges E are partitioned into mutually exclusive sets
  - E_P denote proximity edges
  - E_I denote interference edges
- An edge $(u, v) \in E$ corresponds to a pair of turbine locations
  - If $(u, v) \in E_P$ then they violate the minimum separation distance
  - If $(u, v) \in E_I$ then the turbine wakes impact one another

Let

- P_v denote the ideal power of a turbine located at $v \in V$
- I_{uv} denote the power loss due to interference between turbines located at u and v where $(u, v) \in E_I$
- $\mathcal{Q}$ denote the set of all maximum complete subgraphs of the graph $G_P = (V, E_P)$

Mixed Integer Programming Model

$$
\begin{aligned}
\text{maximise} \quad & \sum_{v \in V} P_v x_v - \sum_{(u,v) \in E_I} I_{uv} y_{uv} & & \text{(Net power)} \\
\text{subject to} \quad & \sum_{v \in Q} x_v \leq 1, & Q \in \mathcal{Q} \quad & \text{(Proximity)} \\
& x_u + x_v \leq 1 + y_{uv}, & (u,v) \in E_I \quad & \text{(Interference)} \\
& \sum_{v \in V} x_v \leq k & & \text{(Turbine limit)} \\
& x_v \in \{0,1\},\ y_{uv} \geq 0, & v \in V,\ (u,v) \in E_I &
\end{aligned}
$$

Case study

- 1.7km x 2.0km domain overlaid by a 48m regularly spaced grid
- 1069 nodes, 2388 proximity cliques, 66903 interference edges
- 67972 variables, 69292 constraints, 226313 non-zeros

Results

- Priority branching
- 3600 second time limit

Turbines	Relaxations LP	Relaxations Root	Cuts	Nodes	Best Bound	Best Integer	(% Nodes)	Gap (%)
10	37985	36672	67 / 490	15764	37595	36831	(37.57)	2.07
11	41695	40199	84 / 435	14703	41276	40320	(10.88+)	2.37
12	45409	43355	96 / 365	10876	45011	43734	(36.72)	2.92
13	49109	47141	118 / 346	8404	48689	47257	(94.00+)	3.03
14	52809	50258	133 / 2	7385	52359	50636	(75.83+)	3.40
15	56492	53872	165 / 189	6272	56066	54074	(20.09+)	3.68
16	60171	57245	173 / 89	4794	59741	57410	(14.60+)	4.06
17	63846	60659	160 / 141	5296	63380	60738	(96.30+)	4.35
18	67499	63860	169 / 94	4501	67099	64068	(31.10+)	4.73
19	71152	67322	191 / 83	3859	70739	67521	(66.08+)	4.77
20	74791	70472	206 / 65	3357	74242	70811	(23.83+)	4.85

CPLEX 10.0 with default options running on a 2.2GHz Pentium IV with 2GB RAM

Cuts = Clique cuts / Implied bound cuts

Results

- Interference constraints placed in a cut pool
- Priority branching
- 1200 second time limit

Turbines	Relaxations LP	Root	Cuts	Nodes	Best Bound	Best Integer	(% Nodes)	Gap (%)
10	37988	36339	64 / 10857	30916	37524	36830	(88.57)	1.88
11	41704	39346	66 / 13220	24501	41264	40323	(30.28)	2.33
12	45410	42833	86 / 15180	18323	44988	43767	(86.69)	2.79
13	49112	46366	110 / 17083	16002	48699	47145	(54.04)	3.30
14	52805	49661	108 / 18334	13144	52424	50578	(76.73)	3.65
15	56490	52762	140 / 19536	10756	56115	53993	(78.59)	3.93
16	60171	56208	145 / 20106	10398	59781	57339	(97.76)	4.26
17	63845	59506	146 / 21599	8748	63532	60592	(65.66)	**4.85**
18	67504	62497	150 / 22435	7999	67194	63966	(98.17)	**5.05**
19	71153	65768	164 / 22535	7901	70839	67312	(71.02)	**5.24**
20	74792	69517	169 / 22081	7179	74509	70591	(83.34)	**5.55**

CPLEX 10.0 with default options running on a 2.2GHz Pentium IV with 2GB RAM

Cuts = Clique cuts / Lazy constraints

Comparison with WindFarmer

GH WindFarmer (Garrad Hassan Ltd)

- Widely used in industry
- Proprietary improvements to ideal power calculations and the PARK turbine wake model give better estimates of net power

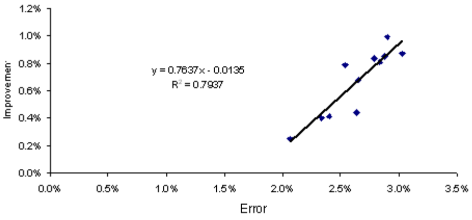
Error

- Relative difference between the net power calculated by WindFarmer and the MIP for the same turbine locations

Improvement

- Relative improvement of a WindFarmer solution found using the MIP solution as a starting solution

Results



Observations

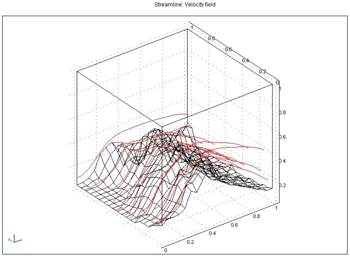
- Strong positive correlation between the error and improvement
- Suggests improved techniques are needed for calculating model data rather than there being a weakness with the MIP model

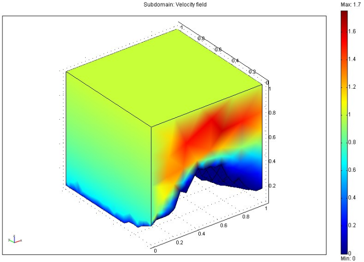
Linear vs Non-Linear Flow Models

Linear flow models

- Industry standard
- WAsP and MS-Micro
- Non-linear effects induced by complex terrain and high wind speeds

Non-linear CFD flow models





Qualitative Wind Statistics

Important at sites with complex terrain and high wind speeds

Background and induced turbulence

- Variation in wind velocity over short lengths of time
- Cyclical loading patterns on turbine structures

Maximum gusts

- Maximum wind speed at a particular location within time frame
- Difficult to predict the spatial variation over the domain

Inflow angles

- Large vertical velocity components increase the inflow angle
- Negative impact on turbine operation and maintenance

Eliminate potential turbine locations in MIP model

Additional Design Parameters

Capital budget constraints

- Generalisation of turbine limit constraint

Line of sight constraints

- Eliminate potential turbine locations in MIP model

NZS sound level restrictions

- Model in a similar way to interference

Reticulation layout

- Costs negligible next to turbine capital costs

Conclusions

Wind farm design

- Complex design problem under difficult economic conditions
- Huge scope for the application of optimisation techniques

Wind flow modelling

- CFD has many advantages compared to linear flow models
- Non-linear turbulent wind flow in NZ's complex terrain

Wind farm layout optimisation

- Mixed integer programming models are viable
- More amenable to modelling NZ's unique conditions
- Require state-of-the-art mixed integer programming techniques
- Adapt the extensive literature for similar MIP problems

Thank you for your attention

Questions?

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