EPOC Winter Workshop 2008

Cost-allocation Models in Electricity Systems

Presented by
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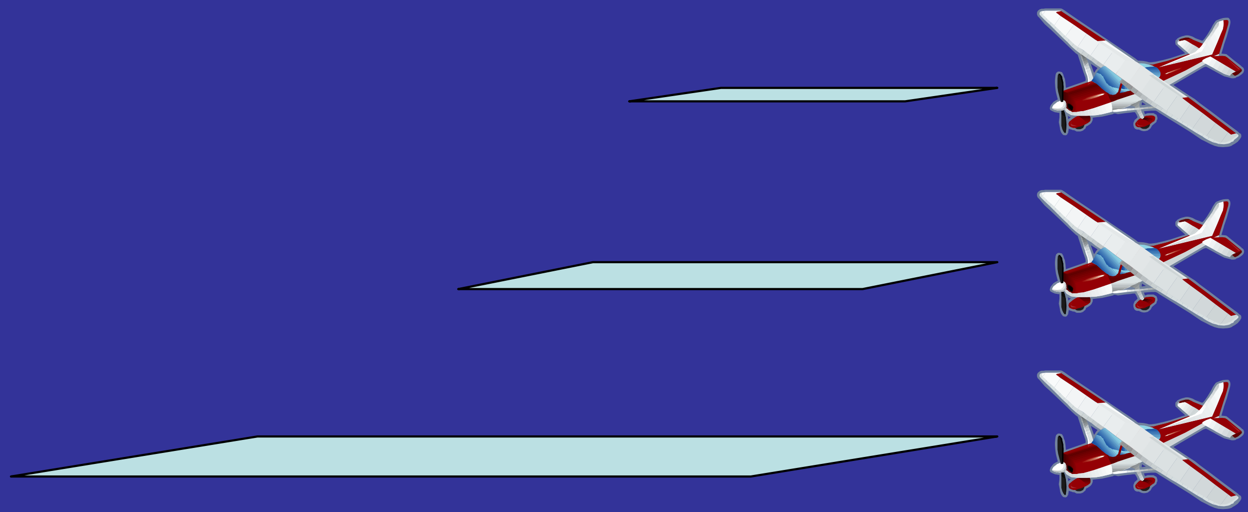
Cost Recovery Problem

- Extract payments for shared resource
  - Public utility cost
- Example: airlines building runway



Cost-allocation Objectives

- Complete cost recovery
- Incentives for efficiency
- Symmetry (non-discriminate)
- Easy to calculate / implement

Cost Recovery for Transmission Investment

- Open-access with competition
  - Share to compete
  - Cost recovery schemes can alter their behaviours
- Recovery through congestion revenues
  - Discrete investments
  - Economy of scale
  - Security constraints (creates redundancy)
  - Networks appear “under-used”
  - Only recover ~20%

Cost Recovery for Transmission Investment (New Zealand)

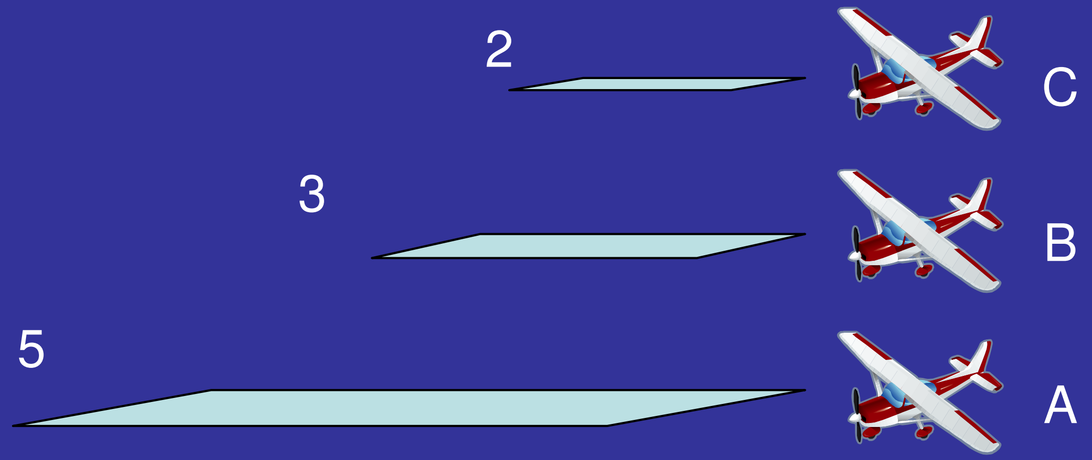
- New Zealand
  - North: population
  - South: resource
- Transmission Charge
  - Large proportion
  - Important in decision making (e.g. wind generation, HVDC)

A few definitions

- Sets of agents
  - N all agents, S subset of N
- Cost functions
  - c(N), c(S), opportunity cost
- Value functions
  - v(N), v(S), any valuation function
- Payments
  - P_i is the payment for i^{th} agent

Economics Concept – the Core

- The Core
  - An allocation *x* is in the *core* of v(N), iff *x* is feasible, and no coalition improve on *x*.
- Runway problem
  - N={A, B, C}
  - S, subset of N
  - c(S)



The Core – runway example

S	c(S)
empty set	0
{A}	5
{B}	3
{C}	2
{A, B}	5
{B, C}	3
{A, C}	5
{A, B, C}	5

Conditions to satisfy:

- $P_A <= c(\{A\}) = 5$,
- $P_B <= c(\{B\}) = 3$,
- $P_C <= c(\{C\}) = 2$,
- $P_B + P_C <= c(\{B,C\}) = 3$,
- $P_A + P_C <= c(\{A,C\}) = 5$,
- $P_A + P_B <= c(\{A,B\}) = 5$,
- $P_A + P_B + P_C = c(\{A,B,C\}) = 5$,

The Core
– runway example

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{A, C}	5
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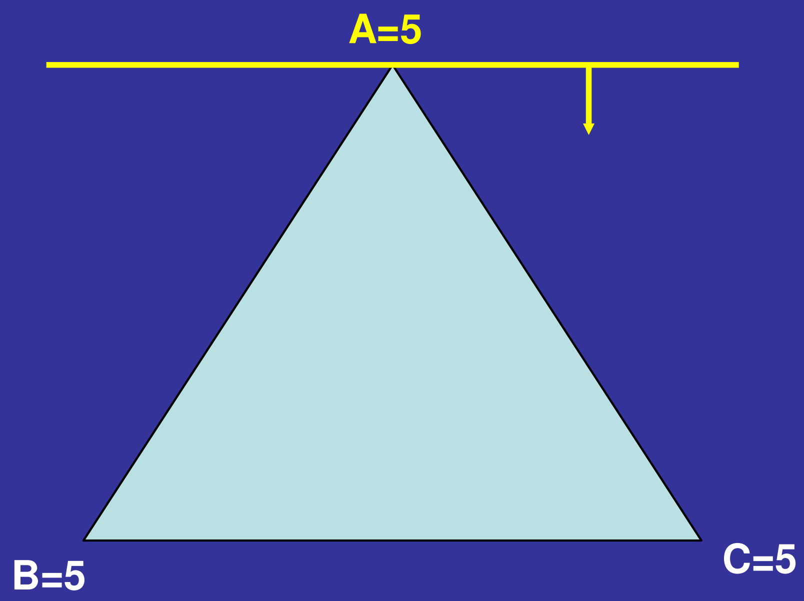
The Core
– runway example



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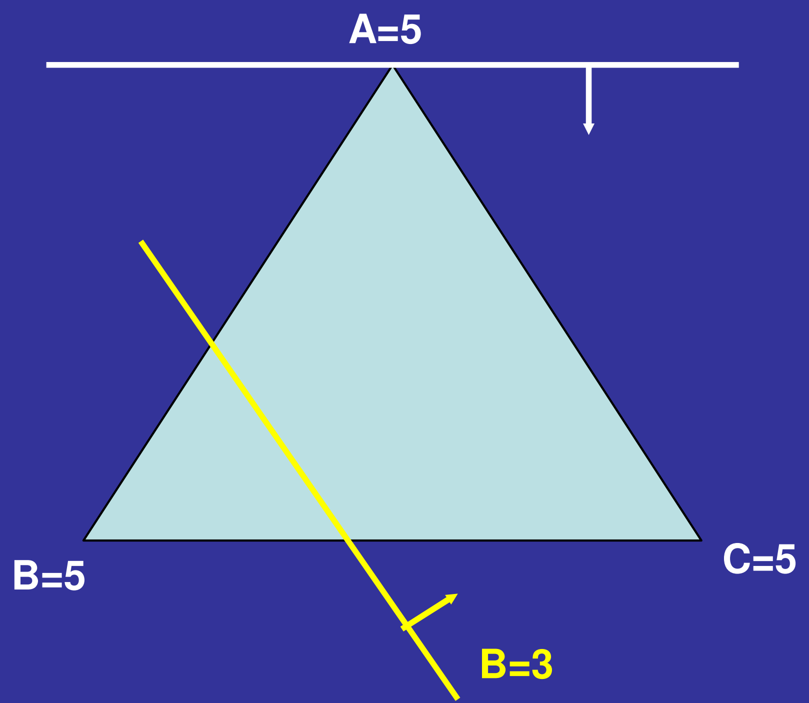
The Core
– runway example



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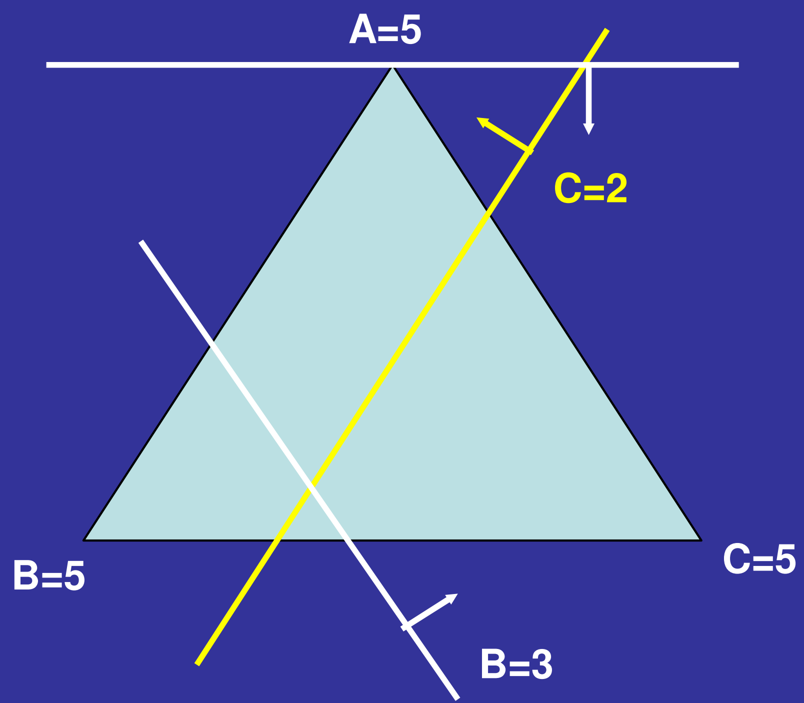
– runway example



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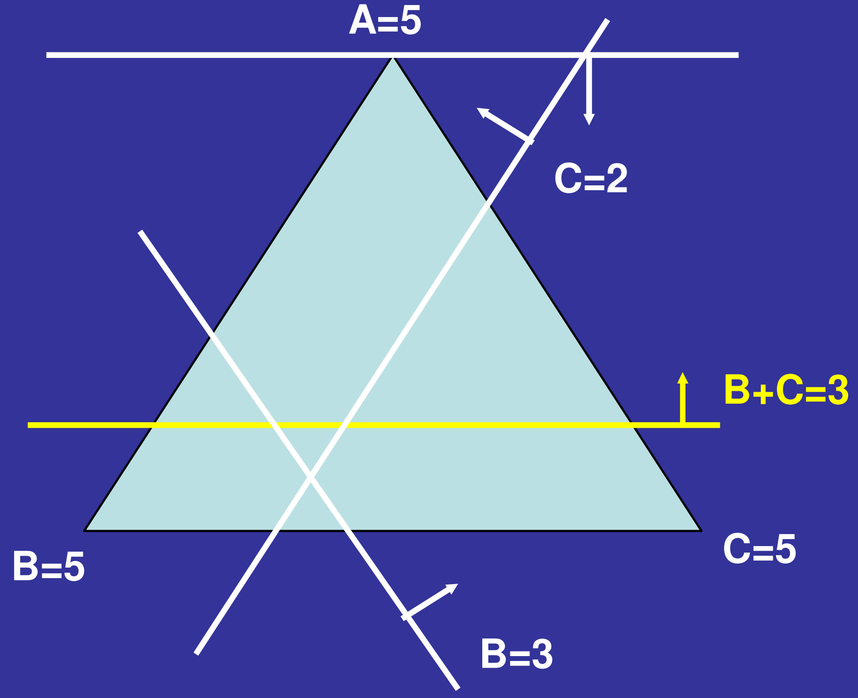
– runway example



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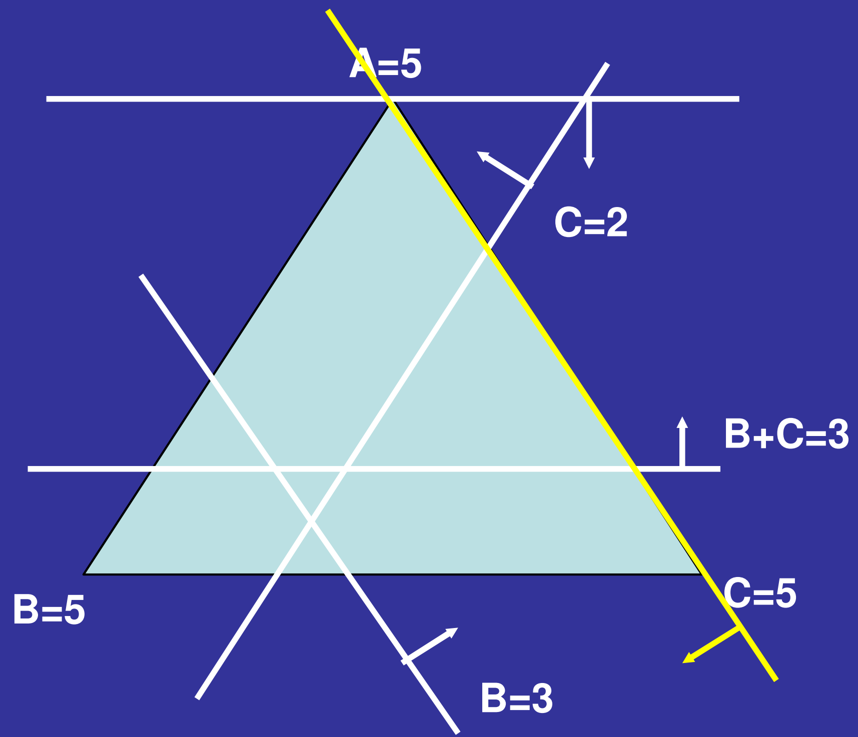

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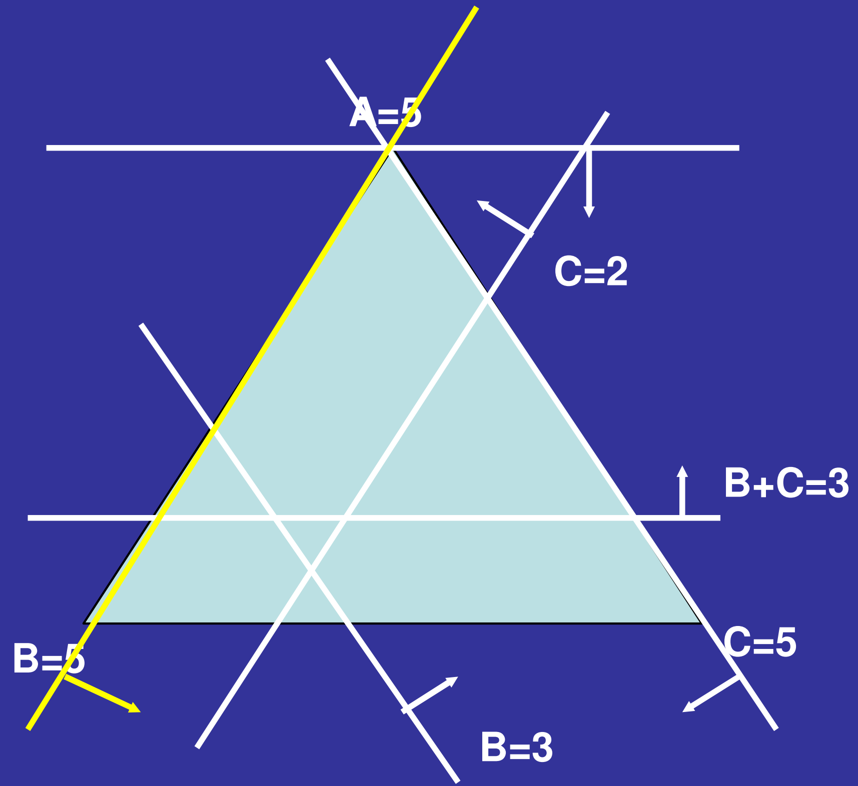
The Core – runway example



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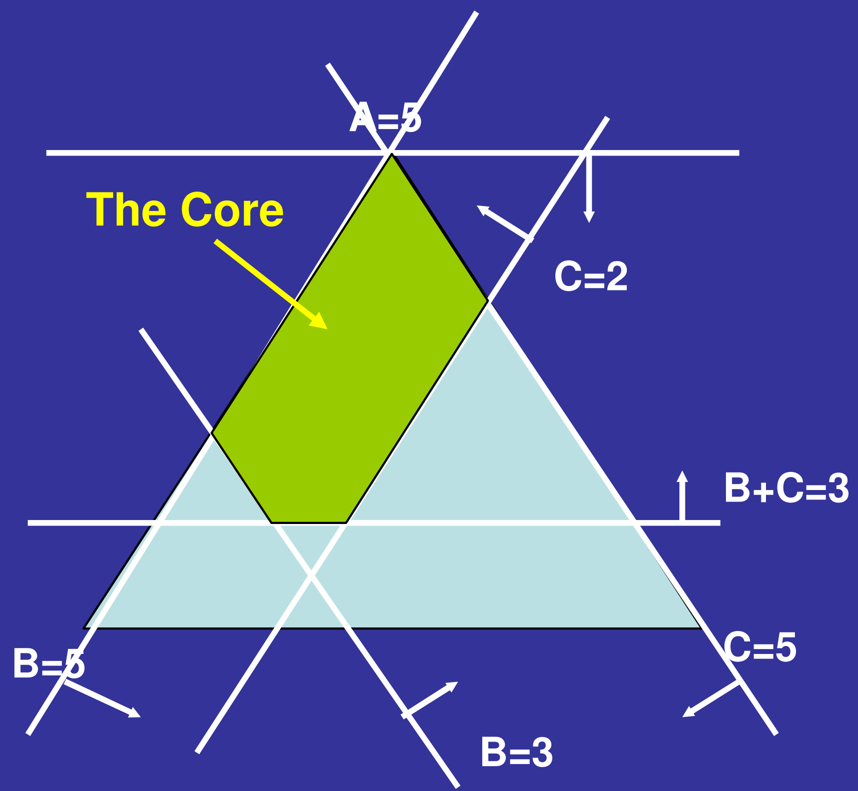

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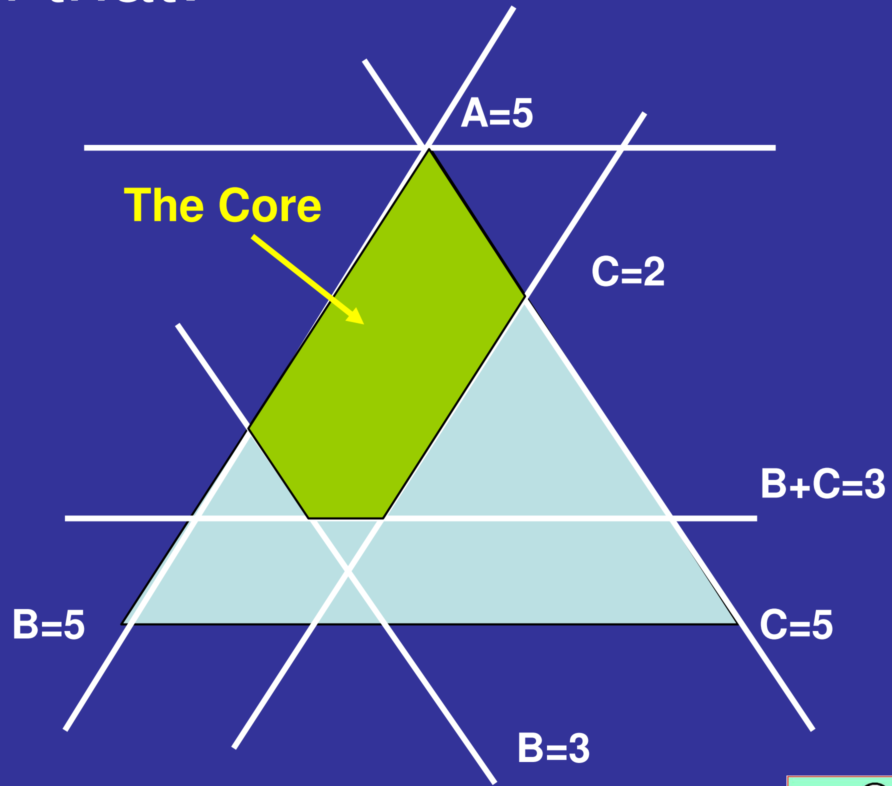
The Core
– runway example

- The Core is defined by a set of linear constraints
- But… how to find a solution that:
  - satisfies objectives
  - is unique



The Shapley Value

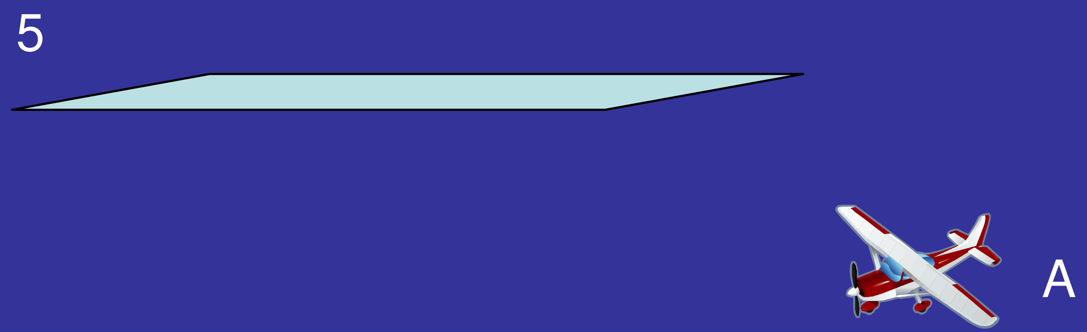
- Lloyd Shapley (1953)
  - Fair allocation of gains / costs
  - By cooperation among several agents
- The Intuitive Idea
  - Average incremental cost for each player
  - Computed over all possible orders in total set - N

The Shapley Value

- Properties
  - **Efficiency** – complete cost allocation
  - **Symmetry** – treat identical players the same
  - **Dummy** – no use no pay
  - **Additivity** – additive function -> additive rules
- Cost non-decreasing returns to scale
  - The Core of the function is nonempty and contains the Shapley Value (Shapley, 1971)

The Shapley Value – the runway problem

Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0



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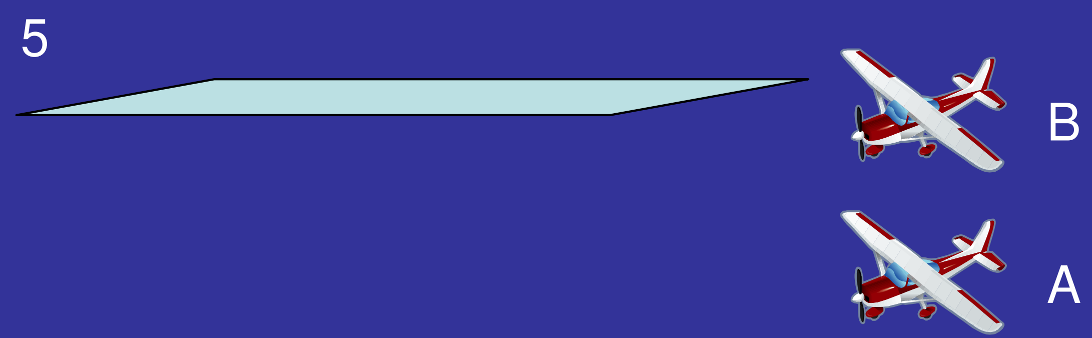
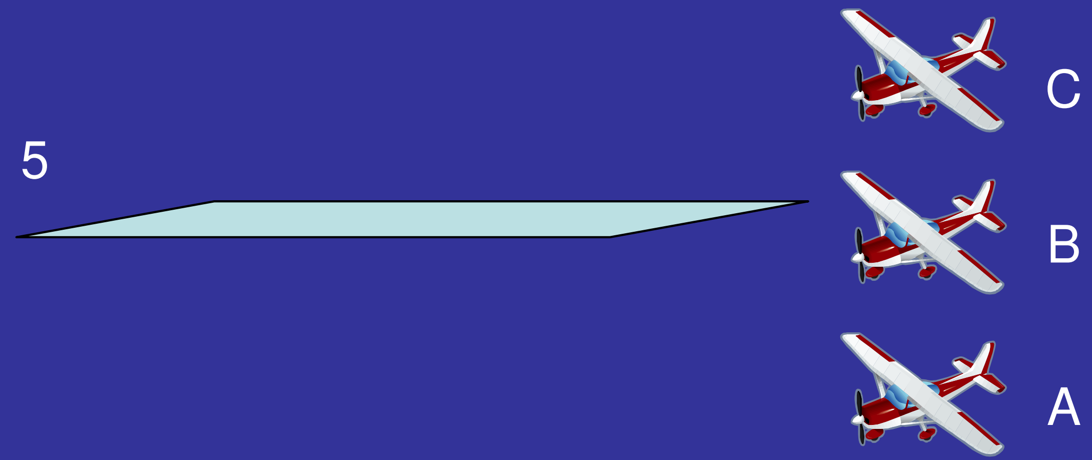


The Shapley Value
– the runway problem

Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0



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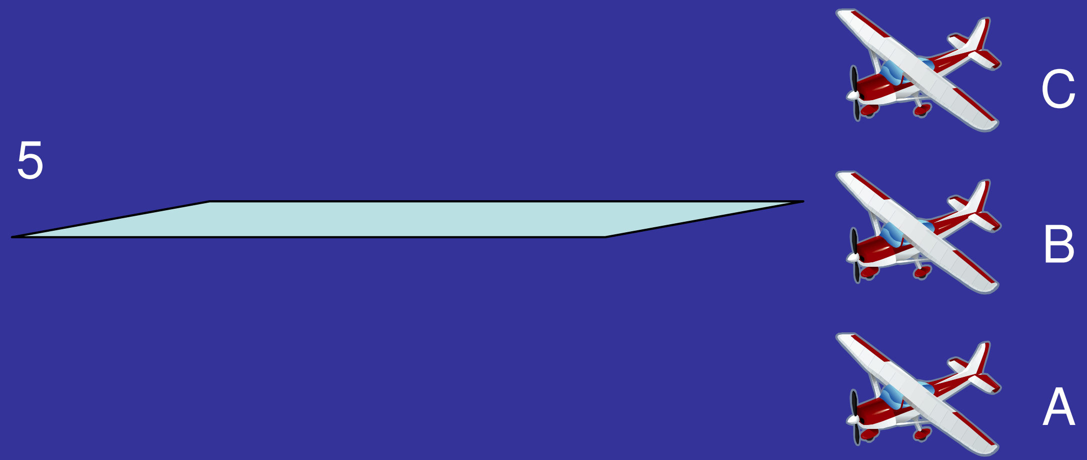
(A, B, C)	5	0	0	5	0	0

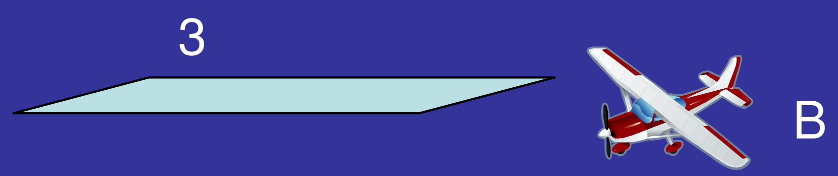


The Shapley Value – the runway problem

Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0
(A, C, B)	5	0	0	5	0	0

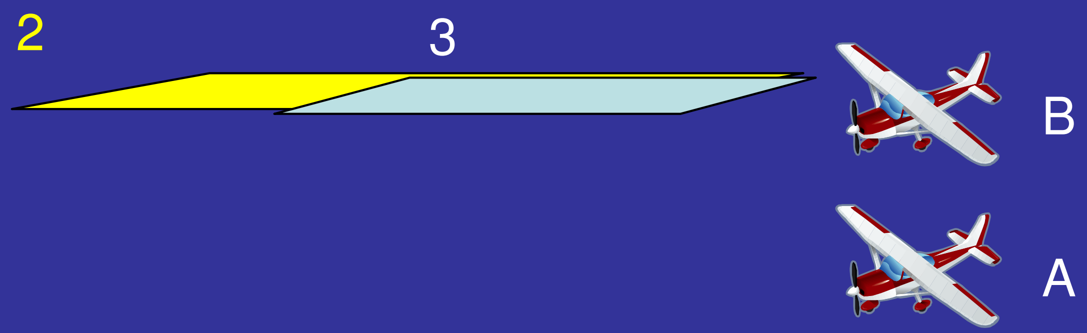
The Shapley Value – the runway problem

Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0
(A, C, B)	5	0	0	5	0	0
(B, A, C)	3	2	0	2	3	0



The Shapley Value – the runway problem

Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0
(A, C, B)	5	0	0	5	0	0
(B, A, C)	3	2	0	2	3	0



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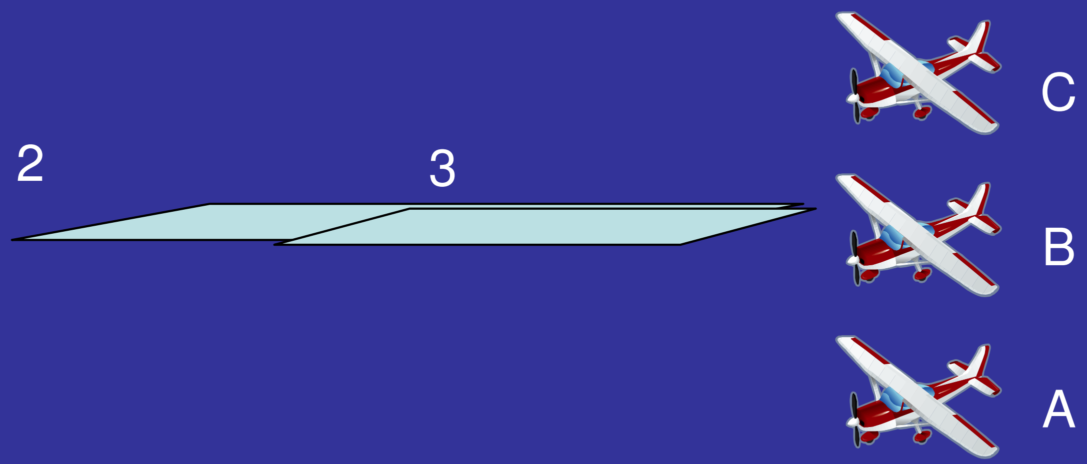
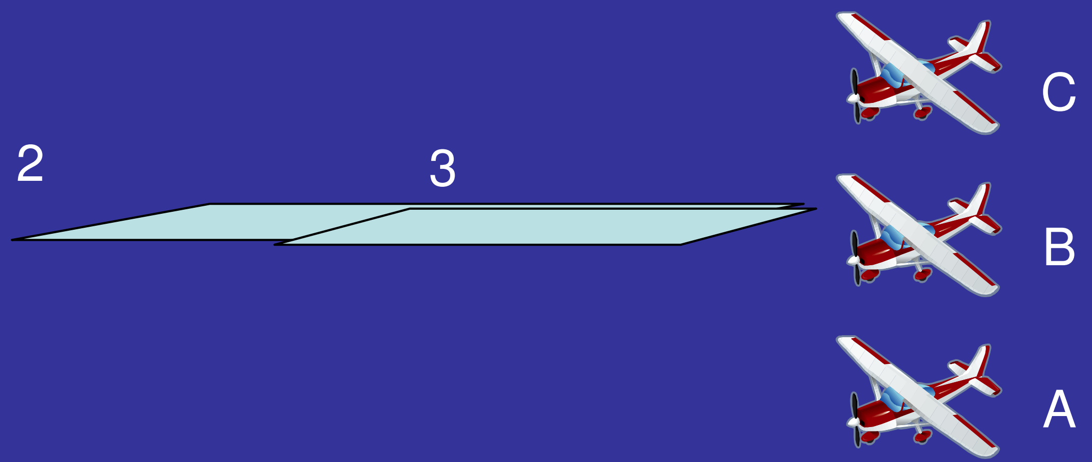
(A, C, B)	5	0	0	5	0	0
(B, A, C)	3	2	0	2	3	0



The Shapley Value
– the runway problem

Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0
(A, C, B)	5	0	0	5	0	0
(B, A, C)	3	2	0	2	3	0



The Shapley Value
– the runway problem

Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0
(A, C, B)	5	0	0	5	0	0
(B, A, C)	3	2	0	2	3	0
(B, C, A)	3	0	2	2	3	0

The Shapley Value
– the runway problem

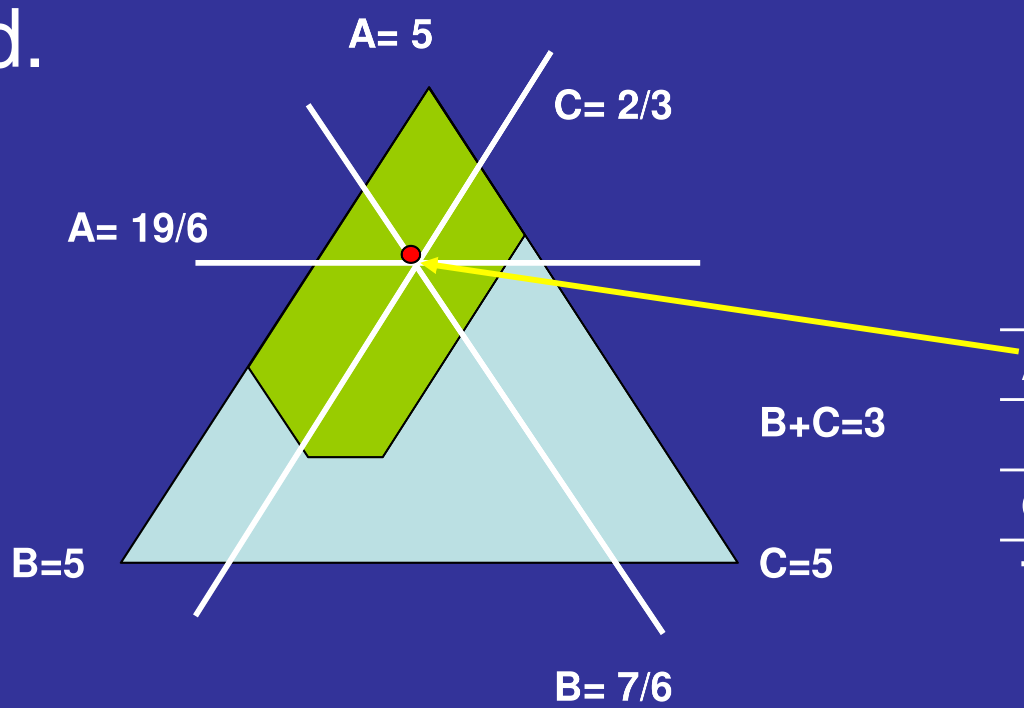
Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0
(A, C, B)	5	0	0	5	0	0
(B, A, C)	3	2	0	2	3	0
(B, C, A)	3	0	2	2	3	0
(C, A, B)	2	3	0	3	0	2

The Shapley Value
– the runway problem

Permutations	ΔC1	ΔC2	ΔC3	PA	PB	PC
(A, B, C)	5	0	0	5	0	0
(A, C, B)	5	0	0	5	0	0
(B, A, C)	3	2	0	2	3	0
(B, C, A)	3	0	2	2	3	0
(C, A, B)	2	3	0	3	0	2
(C, B, A)	2	1	2	2	1	2
			Average:	19/6	7/6	2/3

The Shapley Value – runway example

- The Shapley Value solution
  - is in the Core, and satisfies all the properties as promised.



	Cost Allocation
A =	3 1/6
B =	1 1/6
C =	2/3
Total	5

The Shapley Value

- General formulation

$$\phi_i(c) = \sum_{S \subseteq N-i} \frac{|S|!\,(|N-S|-1)!}{|N|!}\left[c(S+i) - c(S)\right]$$

- Properties
  - Efficiency
  - Symmetry
  - Dummy
  - Additivity

The Shapley Value – in electricity market (Chile [1])

- Assumptions
  - Inelastic demand
  - Power flow on link as measure of cost
- Minimise generation cost
  - subject to Generation = Demand
  - for all subset S of grand coalition N

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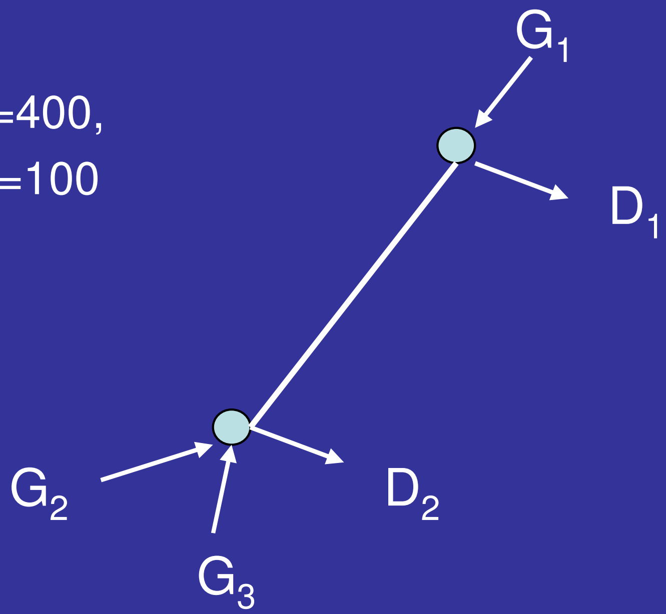
[1] J. M. Zolezzi and H. Rudnick, "Transmission cost allocation by cooperative games and coalition formation," IEEE Trans. Power Syst., vol. 17, no.4, pp. 1008-1015, Nov. 2002.

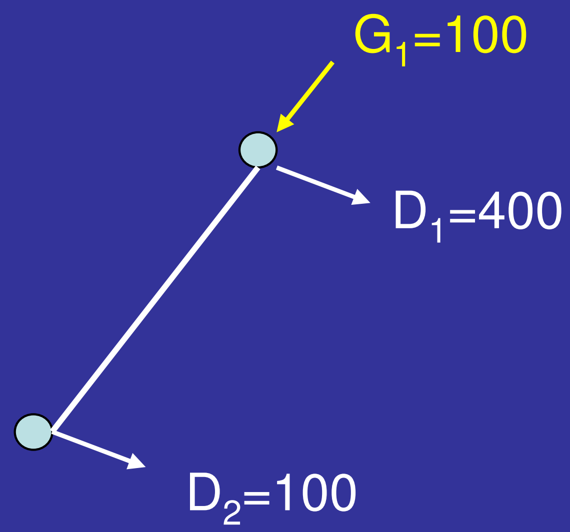
The Shapley Value
– in electricity market (NZ)

- The two-node NZ model
  - Generations
    - North: G_1=100 ,
    - South: G_2=100 , G_3=300
  - Demands
    - North: D_1=400,
    - South: D_2=100



The Shapley Value – in electricity market (NZ)

Permutations	ΔT1	ΔT2	ΔT3	P1	P2	P3
(1, 2, 3)	0	0	300	0	0	300



The Shapley Value
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Permutations	ΔT1	ΔT2	ΔT3	P1	P2	P3
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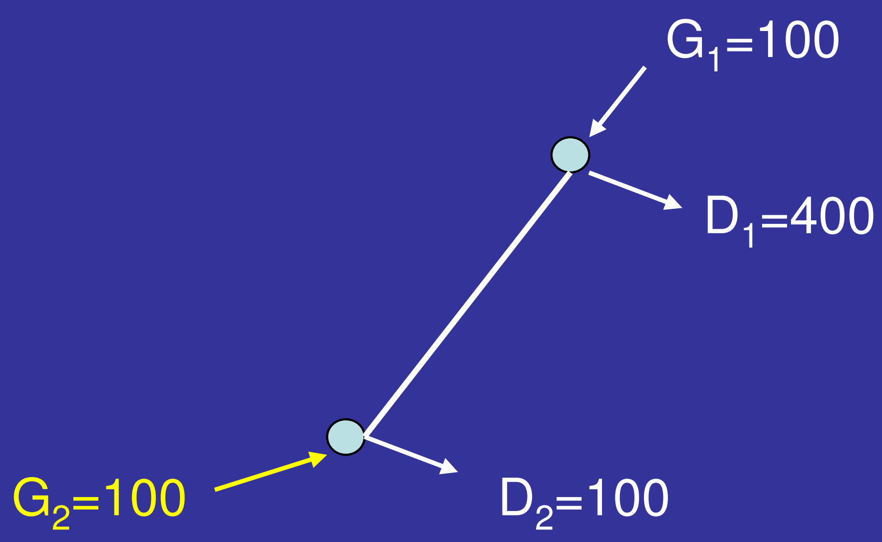

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Permutations	ΔT1	ΔT2	ΔT3	P1	P2	P3
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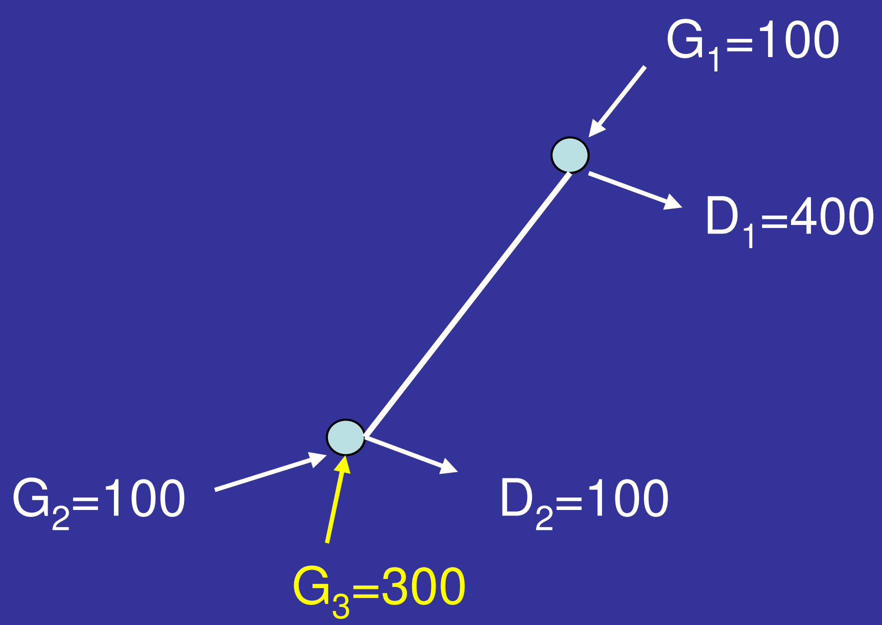

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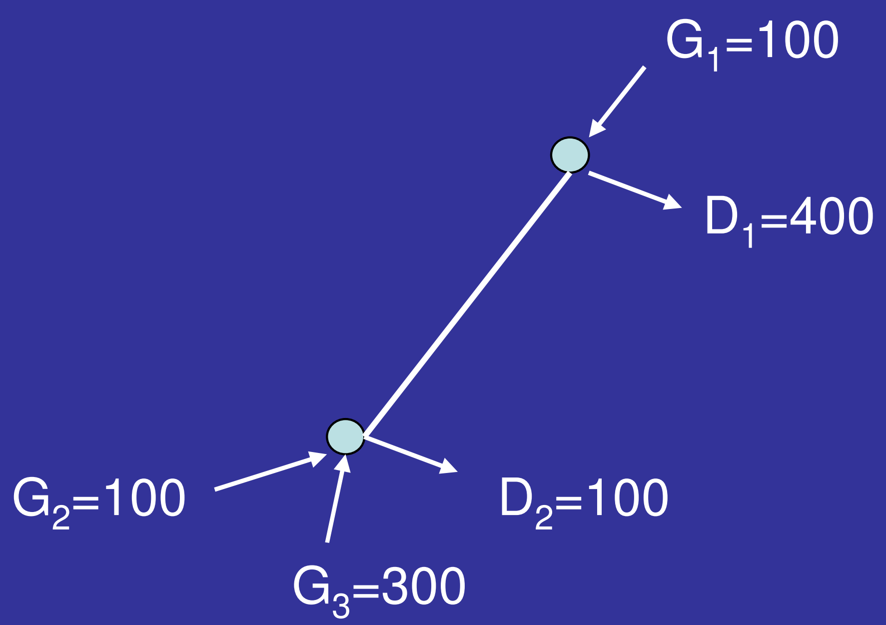


The Shapley Value
– in electricity market (NZ)

Permutations	ΔT1	ΔT2	ΔT3	P1	P2	P3
(1, 2, 3)	0	0	300	0	0	300
(1, 3, 2)	0	200	100	0	100	200

The Shapley Value
– in electricity market (NZ)

Permutations	ΔT1	ΔT2	ΔT3	P1	P2	P3
(1, 2, 3)	0	0	300	0	0	300
(1, 3, 2)	0	200	100	0	100	200
(2, 1, 3)	0	0	300	0	0	300

The Shapley Value
– in electricity market (NZ)

Permutations	ΔT1	ΔT2	ΔT3	P1	P2	P3
(1, 2, 3)	0	0	300	0	0	300
(1, 3, 2)	0	200	100	0	100	200
(2, 1, 3)	0	0	300	0	0	300
(2, 3, 1)	0	300	0	0	0	300

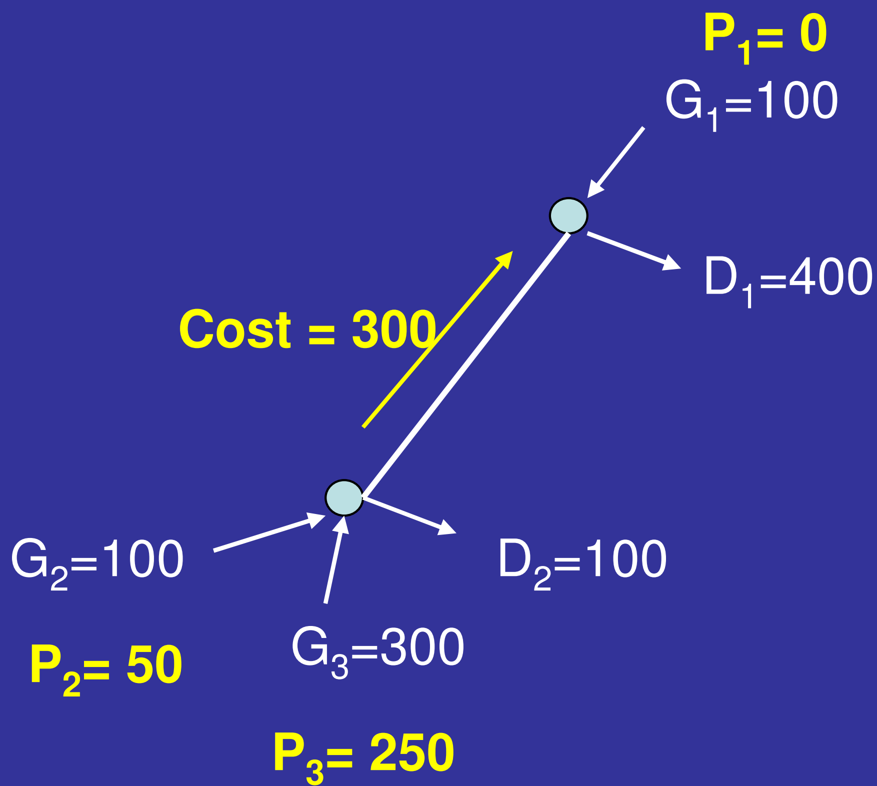
The Shapley Value
– in electricity market (NZ)

Permutations	$\Delta T1$	$\Delta T2$	$\Delta T3$	P1	P2	P3
(1, 2, 3)	0	0	300	0	0	300
(1, 3, 2)	0	200	100	0	100	200
(2, 1, 3)	0	0	300	0	0	300
(2, 3, 1)	0	300	0	0	0	300
(3, 1, 2)	200	0	100	0	100	200

The Shapley Value – in electricity market (NZ)

Permutations	ΔT1	ΔT2	ΔT3	P1	P2	P3
(1, 2, 3)	0	0	300	0	0	300
(1, 3, 2)	0	200	100	0	100	200
(2, 1, 3)	0	0	300	0	0	300
(2, 3, 1)	0	300	0	0	0	300
(3, 1, 2)	200	0	100	0	100	200
(3, 2, 1)	200	100	0	0	100	200
			Average:	**0**	**50**	**250**

The Shapley Value
– in electricity market (NZ)



From Shapley Value to Aumann-Shapley prices

- Aumann-Shapley prices
  - Marginal cost averaged over all vectors [2]

$$\forall i, \quad p_i = f_i(C, q^*) = \int_0^1 (\partial C(tq^*)/\partial q_i)\,\mathrm{d}t.$$

$$tq^* : 0 \leqslant t \leqslant 1$$

  - Consider the progression of cost function rather than just two states

[2] H. P. Young, "Cost allocation," in *Handbook of Game Theory,* R. J. Aumann and S. Hart, Eds. New York: Elsevier, 1994, vol. 2, ch. 34.

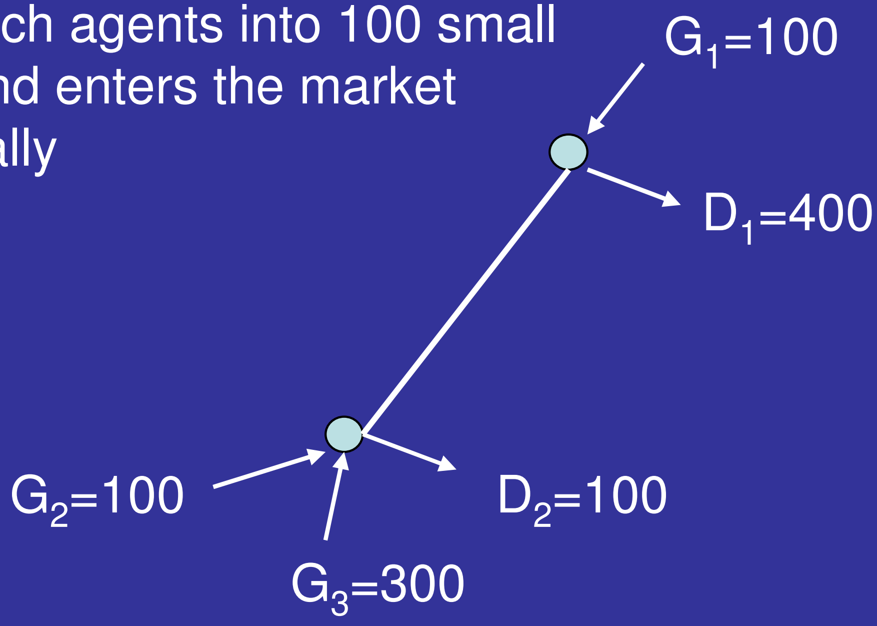
Aumann-Shapley continued – the Brazilian idea

- Model from PSR, Brazil [3]
  - Divide agents into very small agents
  - Entrance order does not matter
  - Number of calculation step = number of small agents
  - Calculate using same model as in Shapley Value
  - Price to pay is the sum of all small agents

[3] M. Junqueira , L. C. Costa, Jr. , L. A. Barroso , G. C. Oliveira , L. M. Thomé and M. V. Pereira “An Aumann–Shapley approach to allocate transmission service cost among network users in electricity markets,” IEEE Trans. Power Syst., vol. 22, pp. 1532-1546, Nov. 2007.

Aumann-Shapley continued – New Zealand example

- Again, the two-node NZ model
  – Divide each agents into 100 small agents and enters the market sequentially



Aumann-Shapley continued – New Zealand example

Step	G1	G2	G3	P1	P2	P3	Demand Supplied	D1	D2
1	1	1	3	0	0	0	5	1	4

Aumann-Shapley continued – New Zealand example

Step	G1	G2	G3	P1	P2	P3	Demand Supplied	D1	D2
1	1	1	3	0	0	0	5	1	4
2	1	1	3	0	0	0	10	2	8

Aumann-Shapley continued – New Zealand example

Step	G1	G2	G3	P1	P2	P3	Demand Supplied	D1	D2
1	1	1	3	0	0	0	5	1	4
2	1	1	3	0	0	0	10	2	8
3	1	1	3	0	0	0	15	3	12
4	1	1	3	0	0	0	20	4	16
5	1	1	3	0	0	0	25	5	20

Aumann-Shapley continued – New Zealand example

Step	G1	G2	G3	P1	P2	P3	Demand Supplied	D1	D2
1	1	1	3	0	0	0	5	1	4
2	1	1	3	0	0	0	10	2	8
3	1	1	3	0	0	0	15	3	12
4	1	1	3	0	0	0	20	4	16
5	1	1	3	0	0	0	25	5	20
25	1	1	3	0	0	0	125	25	100

Aumann-Shapley continued – New Zealand example

Step	G1	G2	G3	P1	P2	P3	Demand Supplied	D1	D2
1	1	1	3	0	0	0	5	1	4
2	1	1	3	0	0	0	10	2	8
3	1	1	3	0	0	0	15	3	12
4	1	1	3	0	0	0	20	4	16
5	1	1	3	0	0	0	25	5	20
25	1	1	3	0	0	0	125	25	100
26	1	1	3	0	1	3	130	30	100
27	1	1	3	0	1	3	135	35	100

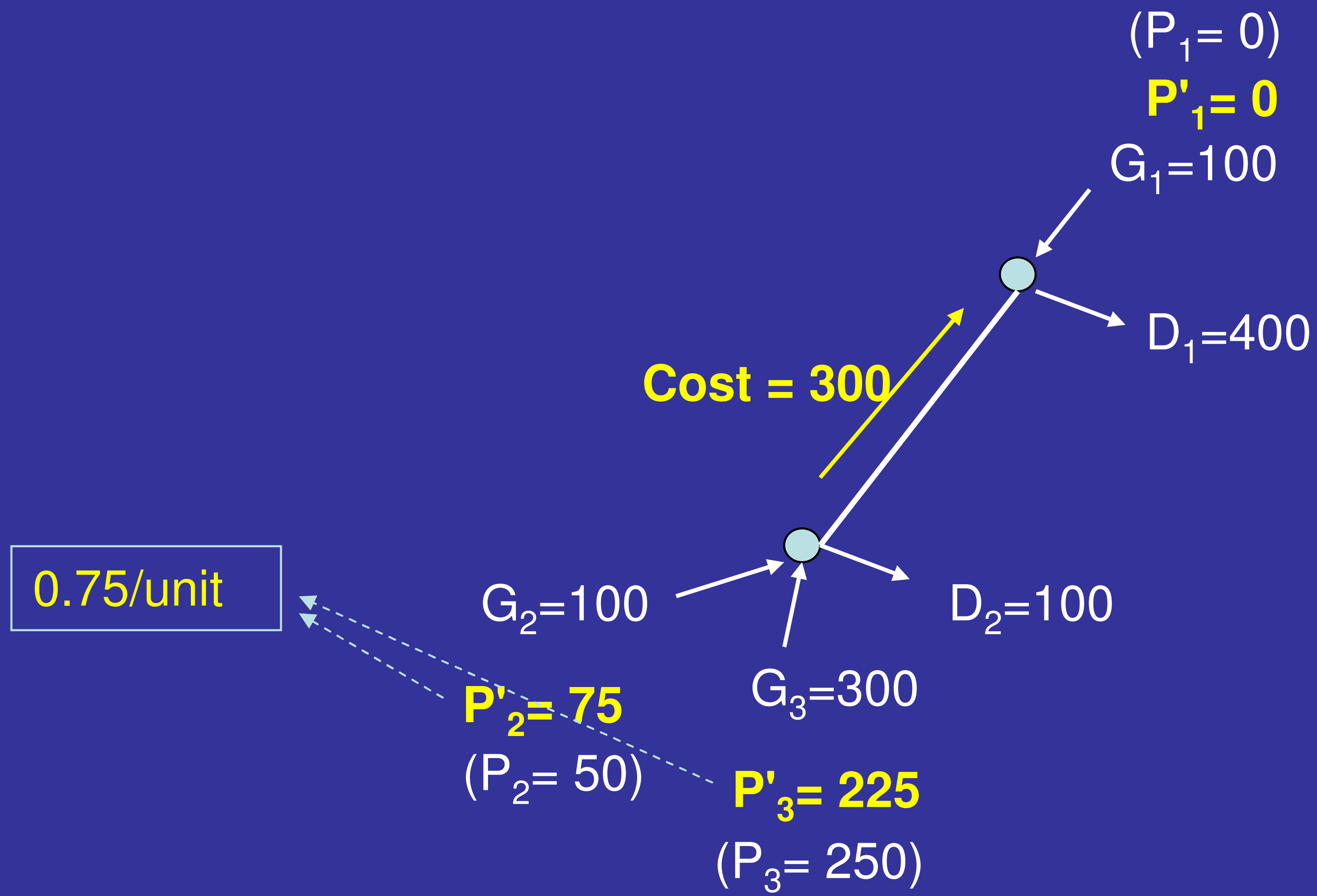
Aumann-Shapley continued – New Zealand example

Step	G1	G2	G3	P1	P2	P3	Demand Supplied	D1	D2
1	1	1	3	0	0	0	5	1	4
2	1	1	3	0	0	0	10	2	8
3	1	1	3	0	0	0	15	3	12
4	1	1	3	0	0	0	20	4	16
5	1	1	3	0	0	0	25	5	20
25	1	1	3	0	0	0	125	25	100
26	1	1	3	0	1	3	130	30	100
27	1	1	3	0	1	3	135	35	100
99	1	1	3	0	1	3	495	395	100

Aumann-Shapley continued – New Zealand example

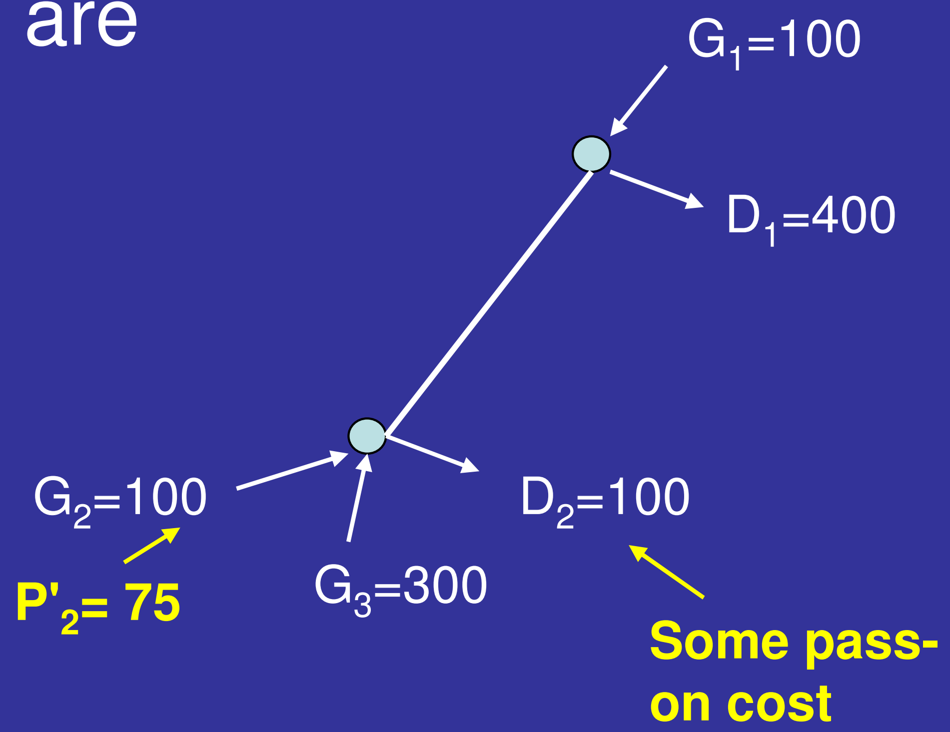
Step	G1	G2	G3	P1	P2	P3	Demand Supplied	D1	D2
1	1	1	3	0	0	0	5	1	4
2	1	1	3	0	0	0	10	2	8
3	1	1	3	0	0	0	15	3	12
4	1	1	3	0	0	0	20	4	16
5	1	1	3	0	0	0	25	5	20
25	1	1	3	0	0	0	125	25	100
26	1	1	3	0	1	3	130	30	100
27	1	1	3	0	1	3	135	35	100
99	1	1	3	0	1	3	495	395	100
100	1	1	3	0	1	3	500	400	100
Total:	100	100	300	0	75	225			

Aumann-Shapley continued – New Zealand example



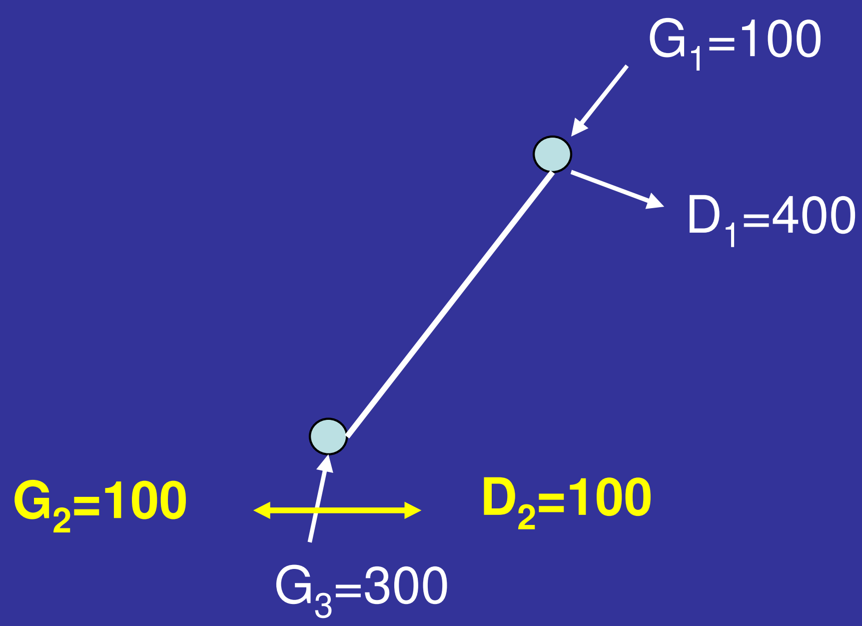
Where in the Core?

- Only if define by cost function
- Chile & Brazil markets are central planned
- New Zealand
  - Competitive
  - Oligopoly



In the Core?

- Depend on demand / supply functions
  - D_2 & G_2 can form coalition
  - Not in the core…
- Perverse incentives



The Benefit Game

- Cost Games vs Benefit Games [4]
  - Taking gain as well as cost into consideration
- Defining the characteristic function

$$v(S) = \underset{R \subseteq S}{Max}\{\sum_{i \in R} B_i - C(R)\} \qquad \text{for all } S$$

  - B_i is the max value consumer i is willing to pay

$$\sum_{i \in N} r_i = C(N)$$

  - r_i is the cost component

[4] W.W.Sharkey, "Suggestions for a game theoretic approach for public utility pricing and cost allocation," The Bell Journal of Economics 13 1, pp. 57–68, 1982

The Benefit Game

- The net surplus for each agent

$$y_i = B_i - r_i$$

- The allocation (outcome) is in the core if

$$\sum_{i \in S} y_i \geq v(S) \quad \text{for } S \subseteq N$$

$$\text{and} \quad \sum_{i \in N} y_i = v(N)$$

[4] W.W.Sharkey, "Suggestions for a game theoretic approach for public utility pricing and cost allocation," The Bell Journal of Economics 13 1, pp. 57–68, 1982

Benefit Games and beyond

- Cannot re-allocate welfare gains in competitive market
- Allocate cost in proportion to benefit game outcomes
  - doesn't necessarily lie in the Core
  - gives a good echo of pricing signal

Benefit Games and beyond

- All these charging schemes assumed no change to agents behaviour, however…
- HVDC link charge
  - Charge on historical maximum injection
  - In fact a capacity constraint in the long term
- Cournot Equilibrium Models
  - Non-cooperative game theory
  - Explored together with the cost-allocation scheme (cooperative)

Thank You!

What’s Happening Elsewhere?

	Sharing of network operator costs among customers (Producer / consumer)	Price signal: Seasonal time-of-day (1)	Price signal: Distance, location	Do the losses fall within the TSO cost basis?	Are system services included in the transmission tariffs?
Spain	0 % / 100 %	XXX	_	No	No
England & Wales	TNUoS: 27 % / 73 % BSUoS: 50 % / 50 %	XX	Location	No recovered in the energy market	Yes
Germany	0 % / 100 %	–	–	Yes	Yes
Sweden	25 % / 75 %	X (via losses)	Location	Yes	Yes, to a partial extent
Norway	36 % / 64 %	XXX (via losses)	Location	Yes	Yes, partially (excl. congestion)
France	2 % / 98%		–	Yes	Yes
Nether-lands	25 % / 75 %	–	–	Yes	There is a specific system services tariff

From European Transmission System Operators (ETSO) Report – *Benchmarking on transmission pricing in Europe: Synthesis 2003*

What’s Happening Elsewhere?

Portugal	0 % 100 %	XX	–	No	Recovered by a special charge (global use of system charge)
Finland	<10 % > 90 %	X	–	Yes	Yes
Italy	1 à 5 % 95%	XX	–	No	Through a specific fee to generators and consumers
Austria	16,5% 83,5 %	XX	–	Yes	Through a specific component to generators
Denmark East / West	3 à 6 % 97 à 94 %	XXX	–	Yes	Yes, partially + PSO tariff
Poland	< 1 % 99%	–	–	Yes	Yes
Slovenia	100%	XXX	–	Yes	Yes
Belgium	100%	XXX	–	No	Tariff for ancillary services

(1) : The number of signs X is in accordance with the application of differentiated periods and the application of the differentiation to all or some of the tariff components

From European Transmission System Operators (ETSO) Report – *Benchmarking on transmission pricing in Europe: Synthesis 2003*